

Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.2.1-a+b-sec-^m-c+d-sec-^n

Nasser M. Abbasi

July 17, 2021

Compiled on July 17, 2021 at 5:50pm

Contents

1	Introduction	3
1.1	Listing of CAS systems tested	3
1.2	Results	3
1.3	Performance	7
1.4	list of integrals that has no closed form antiderivative	8
1.5	list of integrals solved by CAS but has no known antiderivative	8
1.6	list of integrals solved by CAS but failed verification	8
1.7	Timing	9
1.8	Verification	9
1.9	Important notes about some of the results	9
1.9.1	Important note about Maxima results	9
1.9.2	Important note about FriCAS and Giac/XCAS results	10
1.9.3	Important note about finding leaf size of antiderivative	10
1.9.4	Important note about Mupad results	11
1.10	Design of the test system	11
2	detailed summary tables of results	13
2.1	List of integrals sorted by grade for each CAS	13
2.1.1	Rubi	13
2.1.2	Mathematica	13
2.1.3	Maple	13
2.1.4	Maxima	14
2.1.5	FriCAS	14
2.1.6	Sympy	14
2.1.7	Giac	15
2.1.8	Mupad	15
2.2	Detailed conclusion table per each integral for all CAS systems	16
2.3	Detailed conclusion table specific for Rubi results	56
3	Listing of integrals	65
3.1	$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^5 dx$	65
3.2	$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^4 dx$	69
3.3	$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^3 dx$	73
3.4	$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^2 dx$	76
3.5	$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx)) dx$	79
3.6	$\int \frac{(a+a \sec(e+fx))^2}{c-c \sec(e+fx)} dx$	82

3.7	$\int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^2} dx$	85
3.8	$\int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^3} dx$	88
3.9	$\int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^4} dx$	92
3.10	$\int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^5} dx$	96
3.11	$\int (a+a \sec(e+fx))^3 (c-c \sec(e+fx))^5 dx$	100
3.12	$\int (a+a \sec(e+fx))^3 (c-c \sec(e+fx))^4 dx$	104
3.13	$\int (a+a \sec(e+fx))^3 (c-c \sec(e+fx))^3 dx$	108
3.14	$\int (a+a \sec(e+fx))^3 (c-c \sec(e+fx))^2 dx$	111
3.15	$\int (a+a \sec(e+fx))^3 (c-c \sec(e+fx)) dx$	114
3.16	$\int \frac{(a+a \sec(e+fx))^3}{c-c \sec(e+fx)} dx$	117
3.17	$\int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^2} dx$	121
3.18	$\int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^3} dx$	125
3.19	$\int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^4} dx$	129
3.20	$\int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^5} dx$	134
3.21	$\int \frac{(c-c \sec(e+fx))^5}{(a+a \sec(e+fx))^2} dx$	139
3.22	$\int \frac{(c-c \sec(e+fx))^4}{(a+a \sec(e+fx))^2} dx$	144
3.23	$\int \frac{(c-c \sec(e+fx))^3}{(a+a \sec(e+fx))^2} dx$	149
3.24	$\int \frac{(c-c \sec(e+fx))^2}{(a+a \sec(e+fx))^2} dx$	153
3.25	$\int \frac{c-c \sec(e+fx)}{(a+a \sec(e+fx))^2} dx$	156
3.26	$\int \frac{1}{(a+a \sec(e+fx))^2 (c-c \sec(e+fx))} dx$	159
3.27	$\int \frac{1}{(a+a \sec(e+fx))^2 (c-c \sec(e+fx))^2} dx$	162
3.28	$\int \frac{1}{(a+a \sec(e+fx))^2 (c-c \sec(e+fx))^3} dx$	165
3.29	$\int \frac{1}{(a+a \sec(e+fx))^2 (c-c \sec(e+fx))^4} dx$	168
3.30	$\int \frac{1}{(a+a \sec(e+fx))^2 (c-c \sec(e+fx))^5} dx$	172
3.31	$\int \frac{(c-c \sec(e+fx))^5}{(a+a \sec(e+fx))^3} dx$	176
3.32	$\int \frac{(c-c \sec(e+fx))^4}{(a+a \sec(e+fx))^3} dx$	181
3.33	$\int \frac{(c-c \sec(e+fx))^3}{(a+a \sec(e+fx))^3} dx$	186
3.34	$\int \frac{(c-c \sec(e+fx))^2}{(a+a \sec(e+fx))^3} dx$	190
3.35	$\int \frac{c-c \sec(e+fx)}{(a+a \sec(e+fx))^3} dx$	194
3.36	$\int \frac{1}{(a+a \sec(e+fx))^3 (c-c \sec(e+fx))} dx$	198
3.37	$\int \frac{1}{(a+a \sec(e+fx))^3 (c-c \sec(e+fx))^2} dx$	202
3.38	$\int \frac{1}{(a+a \sec(e+fx))^3 (c-c \sec(e+fx))^3} dx$	205
3.39	$\int \frac{1}{(a+a \sec(e+fx))^3 (c-c \sec(e+fx))^4} dx$	208
3.40	$\int \frac{1}{(a+a \sec(e+fx))^3 (c-c \sec(e+fx))^5} dx$	212
3.41	$\int \frac{1}{(a+a \sec(e+fx))^3 (c-c \sec(e+fx))^6} dx$	216
3.42	$\int \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^4 dx$	221
3.43	$\int \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^3 dx$	225
3.44	$\int \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^2 dx$	229

3.45	$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx)) dx$	233
3.46	$\int \frac{\sqrt{a + a \sec(e + fx)}}{c - c \sec(e + fx)} dx$	237
3.47	$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^2} dx$	241
3.48	$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^3} dx$	245
3.49	$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^4} dx$	249
3.50	$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^3 dx$	253
3.51	$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^2 dx$	257
3.52	$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx)) dx$	261
3.53	$\int \frac{(a + a \sec(e + fx))^{3/2}}{c - c \sec(e + fx)} dx$	265
3.54	$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^2} dx$	269
3.55	$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^3} dx$	273
3.56	$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^4} dx$	277
3.57	$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^3 dx$	282
3.58	$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^2 dx$	286
3.59	$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx)) dx$	290
3.60	$\int \frac{(a + a \sec(e + fx))^{5/2}}{c - c \sec(e + fx)} dx$	294
3.61	$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^2} dx$	298
3.62	$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^3} dx$	302
3.63	$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^4} dx$	306
3.64	$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^5} dx$	310
3.65	$\int \frac{(c - c \sec(e + fx))^4}{\sqrt{a + a \sec(e + fx)}} dx$	315
3.66	$\int \frac{(c - c \sec(e + fx))^3}{\sqrt{a + a \sec(e + fx)}} dx$	322
3.67	$\int \frac{(c - c \sec(e + fx))^2}{\sqrt{a + a \sec(e + fx)}} dx$	329
3.68	$\int \frac{c - c \sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx$	335
3.69	$\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))} dx$	340
3.70	$\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^2} dx$	346
3.71	$\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^3} dx$	352
3.72	$\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^{3/2}} dx$	359
3.73	$\int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^{3/2}} dx$	366
3.74	$\int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^{3/2}} dx$	373
3.75	$\int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^{3/2}} dx$	379
3.76	$\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))} dx$	385
3.77	$\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^2} dx$	391
3.78	$\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^3} dx$	397
3.79	$\int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^{5/2}} dx$	404
3.80	$\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^{5/2}} dx$	412
3.81	$\int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^{5/2}} dx$	420

3.82	$\int \frac{(c-c \sec(e+fx))^2}{(a+a \sec(e+fx))^{5/2}} dx$	427
3.83	$\int \frac{c-c \sec(e+fx)}{(a+a \sec(e+fx))^{5/2}} dx$	434
3.84	$\int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))} dx$	441
3.85	$\int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^2} dx$	448
3.86	$\int \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{7/2} dx$	455
3.87	$\int \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2} dx$	471
3.88	$\int \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{3/2} dx$	487
3.89	$\int \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)} dx$	502
3.90	$\int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c-c \sec(e+fx)}} dx$	516
3.91	$\int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{3/2}} dx$	527
3.92	$\int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{5/2}} dx$	530
3.93	$\int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{7/2}} dx$	548
3.94	$\int (a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^{5/2} dx$	552
3.95	$\int (a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^{3/2} dx$	568
3.96	$\int (a+a \sec(e+fx))^{3/2} \sqrt{c-c \sec(e+fx)} dx$	583
3.97	$\int \frac{(a+a \sec(e+fx))^{3/2}}{\sqrt{c-c \sec(e+fx)}} dx$	598
3.98	$\int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{3/2}} dx$	609
3.99	$\int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{5/2}} dx$	612
3.100	$\int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{7/2}} dx$	616
3.101	$\int (a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^{5/2} dx$	621
3.102	$\int (a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^{3/2} dx$	637
3.103	$\int (a+a \sec(e+fx))^{5/2} \sqrt{c-c \sec(e+fx)} dx$	653
3.104	$\int \frac{(a+a \sec(e+fx))^{5/2}}{\sqrt{c-c \sec(e+fx)}} dx$	669
3.105	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{3/2}} dx$	681
3.106	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{5/2}} dx$	684
3.107	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{7/2}} dx$	687
3.108	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{9/2}} dx$	692
3.109	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{11/2}} dx$	698
3.110	$\int \frac{(c-c \sec(e+fx))^{7/2}}{\sqrt{a+a \sec(e+fx)}} dx$	706
3.111	$\int \frac{(c-c \sec(e+fx))^{5/2}}{\sqrt{a+a \sec(e+fx)}} dx$	722
3.112	$\int \frac{(c-c \sec(e+fx))^{3/2}}{\sqrt{a+a \sec(e+fx)}} dx$	738
3.113	$\int \frac{\sqrt{c-c \sec(e+fx)}}{\sqrt{a+a \sec(e+fx)}} dx$	754
3.114	$\int \frac{1}{\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} dx$	757
3.115	$\int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{3/2}} dx$	768
3.116	$\int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2}} dx$	780
3.117	$\int \frac{(c-c \sec(e+fx))^{7/2}}{(a+a \sec(e+fx))^{3/2}} dx$	793
3.118	$\int \frac{(c-c \sec(e+fx))^{5/2}}{(a+a \sec(e+fx))^{3/2}} dx$	797

3.119	$\int \frac{(c-c \sec(e+fx))^{3/2}}{(a+a \sec(e+fx))^{3/2}} dx$	800
3.120	$\int \frac{\sqrt{c-c \sec(e+fx)}}{(a+a \sec(e+fx))^{3/2}} dx$	803
3.121	$\int \frac{1}{(a+a \sec(e+fx))^{3/2} \sqrt{c-c \sec(e+fx)}} dx$	806
3.122	$\int \frac{1}{(a+a \sec(e+fx))^{3/2} (c-c \sec(e+fx))^{3/2}} dx$	818
3.123	$\int \frac{1}{(a+a \sec(e+fx))^{3/2} (c-c \sec(e+fx))^{5/2}} dx$	830
3.124	$\int \frac{(c-c \sec(e+fx))^{7/2}}{(a+a \sec(e+fx))^{5/2}} dx$	844
3.125	$\int \frac{(c-c \sec(e+fx))^{5/2}}{(a+a \sec(e+fx))^{5/2}} dx$	847
3.126	$\int \frac{(c-c \sec(e+fx))^{3/2}}{(a+a \sec(e+fx))^{5/2}} dx$	850
3.127	$\int \frac{\sqrt{c-c \sec(e+fx)}}{(a+a \sec(e+fx))^{5/2}} dx$	854
3.128	$\int \frac{1}{(a+a \sec(e+fx))^{5/2} \sqrt{c-c \sec(e+fx)}} dx$	858
3.129	$\int \frac{1}{(a+a \sec(e+fx))^{5/2} (c-c \sec(e+fx))^{3/2}} dx$	871
3.130	$\int \frac{1}{(a+a \sec(e+fx))^{5/2} (c-c \sec(e+fx))^{5/2}} dx$	885
3.131	$\int (1 + \sec(e + fx))^m (c - c \sec(e + fx))^n dx$	898
3.132	$\int (a + a \sec(e + fx))^m (c - c \sec(e + fx))^n dx$	900
3.133	$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^n dx$	903
3.134	$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^n dx$	906
3.135	$\int (a + a \sec(e + fx))(c - c \sec(e + fx))^n dx$	909
3.136	$\int \frac{(c-c \sec(e+fx))^n}{a+a \sec(e+fx)} dx$	912
3.137	$\int \frac{(c-c \sec(e+fx))^n}{(a+a \sec(e+fx))^2} dx$	915
3.138	$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^n dx$	918
3.139	$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^n dx$	921
3.140	$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^n dx$	924
3.141	$\int \frac{(c-c \sec(e+fx))^n}{\sqrt{a+a \sec(e+fx)}} dx$	926
3.142	$\int \frac{(c-c \sec(e+fx))^n}{(a+a \sec(e+fx))^{3/2}} dx$	929
3.143	$\int \frac{\sqrt{a+a \sec(e+fx)}}{c+c \sec(e+fx)} dx$	932
3.144	$\int \frac{(c+d \sec(e+fx))^{3/2}}{a+a \sec(e+fx)} dx$	936
3.145	$\int \frac{\sqrt{c+d \sec(e+fx)}}{a+a \sec(e+fx)} dx$	940
3.146	$\int \frac{1}{(a+a \sec(e+fx)) \sqrt{c+d \sec(e+fx)}} dx$	943
3.147	$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^4 dx$	947
3.148	$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^3 dx$	952
3.149	$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^2 dx$	956
3.150	$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx)) dx$	960
3.151	$\int \frac{\sqrt{a+a \sec(e+fx)}}{c+d \sec(e+fx)} dx$	964
3.152	$\int \frac{\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^2} dx$	969
3.153	$\int \frac{\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^3} dx$	974
3.154	$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^3 dx$	980
3.155	$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^2 dx$	985
3.156	$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx)) dx$	989
3.157	$\int \frac{(a+a \sec(e+fx))^{3/2}}{c+d \sec(e+fx)} dx$	993

3.158	$\int \frac{(a+a \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^2} dx$	997
3.159	$\int \frac{(a+a \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^3} dx$	1003
3.160	$\int (a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))^3 dx$	1010
3.161	$\int (a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))^2 dx$	1015
3.162	$\int (a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx)) dx$	1019
3.163	$\int \frac{(a+a \sec(e+fx))^{5/2}}{c+d \sec(e+fx)} dx$	1024
3.164	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^2} dx$	1029
3.165	$\int \frac{(a+a \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^3} dx$	1034
3.166	$\int \frac{(c+d \sec(e+fx))^3}{\sqrt{a+a \sec(e+fx)}} dx$	1042
3.167	$\int \frac{(c+d \sec(e+fx))^2}{\sqrt{a+a \sec(e+fx)}} dx$	1049
3.168	$\int \frac{c+d \sec(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx$	1055
3.169	$\int \frac{1}{\sqrt{a+a \sec(e+fx)}(c+d \sec(e+fx))} dx$	1060
3.170	$\int \frac{1}{\sqrt{a+a \sec(e+fx)}(c+d \sec(e+fx))^2} dx$	1066
3.171	$\int \frac{1}{\sqrt{a+a \sec(e+fx)}(c+d \sec(e+fx))^3} dx$	1072
3.172	$\int \frac{(c+d \sec(e+fx))^3}{(a+a \sec(e+fx))^{3/2}} dx$	1078
3.173	$\int \frac{(c+d \sec(e+fx))^2}{(a+a \sec(e+fx))^{3/2}} dx$	1085
3.174	$\int \frac{c+d \sec(e+fx)}{(a+a \sec(e+fx))^{3/2}} dx$	1091
3.175	$\int \frac{1}{(a+a \sec(e+fx))^{3/2}(c+d \sec(e+fx))} dx$	1097
3.176	$\int \frac{1}{(a+a \sec(e+fx))^{3/2}(c+d \sec(e+fx))^2} dx$	1103
3.177	$\int \frac{1}{(a+a \sec(e+fx))^{3/2}(c+d \sec(e+fx))^3} dx$	1109
3.178	$\int \frac{(c+d \sec(e+fx))^3}{(a+a \sec(e+fx))^{5/2}} dx$	1116
3.179	$\int \frac{(c+d \sec(e+fx))^2}{(a+a \sec(e+fx))^{5/2}} dx$	1123
3.180	$\int \frac{c+d \sec(e+fx)}{(a+a \sec(e+fx))^{5/2}} dx$	1130
3.181	$\int \frac{1}{(a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))} dx$	1136
3.182	$\int \frac{1}{(a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))^2} dx$	1144
3.183	$\int \frac{1}{(a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))^3} dx$	1151
3.184	$\int \sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)} dx$	1158
3.185	$\int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx$	1162
3.186	$\int \frac{\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^{3/2}} dx$	1165
3.187	$\int \frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{a+a \sec(e+fx)}} dx$	1169
3.188	$\int \frac{1}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}} dx$	1173
3.189	$\int \frac{a+b \sec(e+fx)}{c+d \sec(e+fx)} dx$	1177
3.190	$\int \frac{a+b \sec(e+fx)}{(c+d \sec(e+fx))^2} dx$	1181
3.191	$\int \frac{a+b \sec(e+fx)}{(c+d \sec(e+fx))^3} dx$	1186
3.192	$\int \frac{(a+b \sec(e+fx))^2}{(c+d \sec(e+fx))^2} dx$	1193
3.193	$\int \frac{(a+b \sec(e+fx))^2}{(c+d \sec(e+fx))^3} dx$	1198

3.194	$\int \frac{(a+b \sec(e+fx))^2}{(c+d \sec(e+fx))^4} dx$	1206
3.195	$\int \frac{(a+b \sec(e+fx))^3}{(c+d \sec(e+fx))^3} dx$	1217
3.196	$\int \frac{(a+b \sec(e+fx))^3}{(c+d \sec(e+fx))^4} dx$	1226
3.197	$\int \frac{(a+b \sec(e+fx))^3}{(c+d \sec(e+fx))^5} dx$	1239
3.198	$\int \sqrt{a+b \sec(e+fx)} (c+d \sec(e+fx)) dx$	1255
3.199	$\int \frac{\sqrt{a+b \sec(e+fx)}}{c+d \sec(e+fx)} dx$	1259
3.200	$\int (a+b \sec(e+fx))^{3/2} (c+d \sec(e+fx)) dx$	1262
3.201	$\int \frac{(a+b \sec(e+fx))^{3/2}}{c+d \sec(e+fx)} dx$	1266
3.202	$\int (a+b \sec(e+fx))^{5/2} (c+d \sec(e+fx)) dx$	1270
3.203	$\int \frac{c+d \sec(e+fx)}{\sqrt{a+b \sec(e+fx)}} dx$	1275
3.204	$\int \frac{1}{\sqrt{a+b \sec(e+fx)} (c+d \sec(e+fx))} dx$	1278
3.205	$\int \frac{c+d \sec(e+fx)}{(a+b \sec(e+fx))^{3/2}} dx$	1281
3.206	$\int \frac{c+d \sec(e+fx)}{(a+b \sec(e+fx))^{5/2}} dx$	1286
3.207	$\int \sqrt{a+b \sec(e+fx)} \sqrt{c+d \sec(e+fx)} dx$	1291
3.208	$\int \frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx$	1294
3.209	$\int \frac{\sqrt{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{3/2}} dx$	1297
3.210	$\int \frac{\sqrt{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{5/2}} dx$	1303
3.211	$\int \frac{(a+b \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^{3/2}} dx$	1308
3.212	$\int \frac{(a+b \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^{5/2}} dx$	1314
3.213	$\int \frac{(a+b \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^{7/2}} dx$	1319
3.214	$\int \frac{(a+b \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^{5/2}} dx$	1325
3.215	$\int \frac{(a+b \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^{7/2}} dx$	1330
3.216	$\int \frac{(a+b \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^{9/2}} dx$	1336
3.217	$\int \frac{(c+d \sec(e+fx))^{3/2}}{\sqrt{a+b \sec(e+fx)}} dx$	1342
3.218	$\int \frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{a+b \sec(e+fx)}} dx$	1345
3.219	$\int \frac{1}{\sqrt{a+b \sec(e+fx)} \sqrt{c+d \sec(e+fx)}} dx$	1348
3.220	$\int \frac{1}{\sqrt{a+b \sec(e+fx)} (c+d \sec(e+fx))^{3/2}} dx$	1351
3.221	$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{\sqrt[3]{c+d \sec(e+fx)}} dx$	1357
3.222	$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}} dx$	1359
3.223	$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}} dx$	1361
3.224	$\int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{2/3}} dx$	1363
3.225	$\int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{5/3}} dx$	1365
3.226	$\int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{8/3}} dx$	1367
3.227	$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{4/3}} dx$	1369
3.228	$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}} dx$	1371

3.229	$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{10/3}} dx$	1373
3.230	$\int (c(d \sec(e+fx))^p)^n (a+a \sec(e+fx))^m dx$	1375
3.231	$\int (c(d \sec(e+fx))^p)^n (a+a \sec(e+fx))^3 dx$	1379
3.232	$\int (c(d \sec(e+fx))^p)^n (a+a \sec(e+fx))^2 dx$	1383
3.233	$\int (c(d \sec(e+fx))^p)^n (a+a \sec(e+fx)) dx$	1386
3.234	$\int \frac{(c(d \sec(e+fx))^p)^n}{a+a \sec(e+fx)} dx$	1389
3.235	$\int \frac{(c(d \sec(e+fx))^p)^n}{(a+a \sec(e+fx))^2} dx$	1392
3.236	$\int (c(d \sec(e+fx))^p)^n (a+b \sec(e+fx))^m dx$	1396
3.237	$\int (c(d \sec(e+fx))^p)^n (a+b \sec(e+fx))^3 dx$	1398
3.238	$\int (c(d \sec(e+fx))^p)^n (a+b \sec(e+fx))^2 dx$	1402
3.239	$\int (c(d \sec(e+fx))^p)^n (a+b \sec(e+fx)) dx$	1405
3.240	$\int \frac{(c(d \sec(e+fx))^p)^n}{a+b \sec(e+fx)} dx$	1408
3.241	$\int \frac{(c(d \sec(e+fx))^p)^n}{(a+b \sec(e+fx))^2} dx$	1411
4	Listing of Grading functions	1415
4.0.1	Mathematica and Rubi grading function	1415
4.0.2	Maple grading function	1417
4.0.3	Sympy grading function	1420
4.0.4	SageMath grading function	1422

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [241]. This is test number [121].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.59 (240)	% 0.41 (1)
Mathematica	% 94.19 (227)	% 5.81 (14)
Maple	% 89.63 (216)	% 10.37 (25)
Maxima	% 39.83 (96)	% 60.17 (145)
Fricas	% 60.17 (145)	% 39.83 (96)
Sympy	% 2.07 (5)	% 97.93 (236)
Giac	% 18.26 (44)	% 81.74 (197)
Mupad	% 23.24 (56)	% 76.76 (185)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

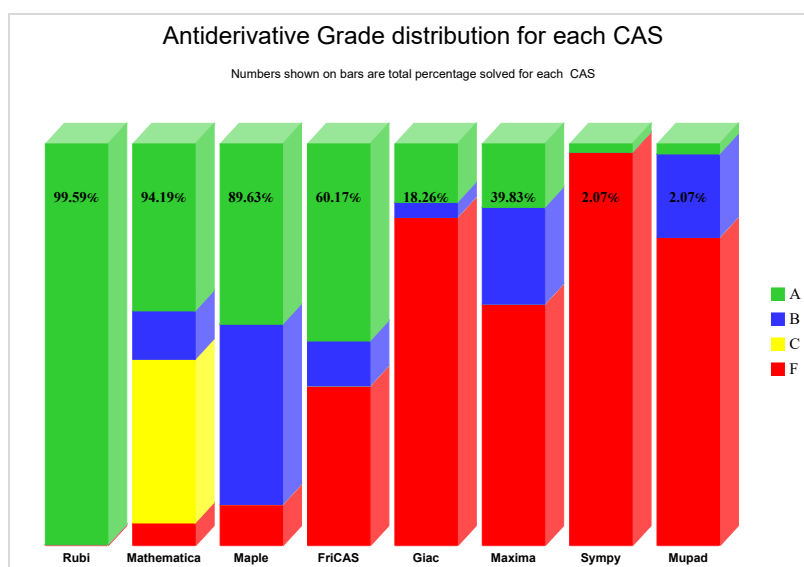
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

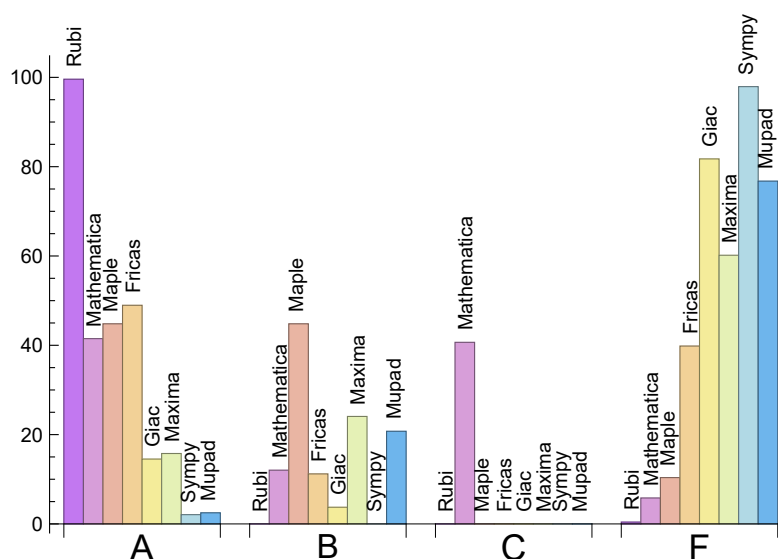
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.59	0.00	0.00	0.41
Mathematica	41.49	12.03	40.66	5.81
Maple	44.81	44.81	0.00	10.37
Maxima	15.77	24.07	0.00	60.17
Fricas	48.96	11.20	0.00	39.83
Sympy	2.07	0.00	0.00	97.93
Giac	14.52	3.73	0.00	81.74
Mupad	2.49	20.75	0.00	76.76

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	14	100.00 %	0.00 %	0.00 %
Maple	25	92.00 %	0.00 %	8.00 %
Maxima	145	52.41 %	30.34 %	17.24 %
Fricas	96	67.71 %	32.29 %	0.00 %
Sympy	236	83.05 %	16.95 %	0.00 %
Giac	197	27.41 %	6.09 %	66.50 %
Mupad	185	92.97 %	7.03 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

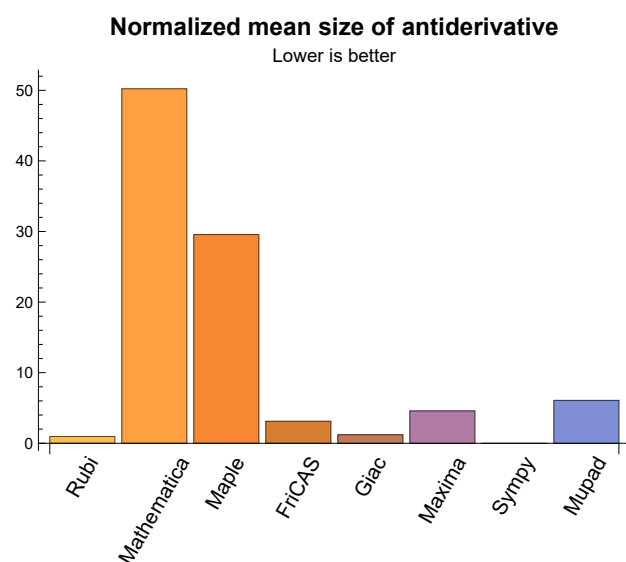
1.3 Performance

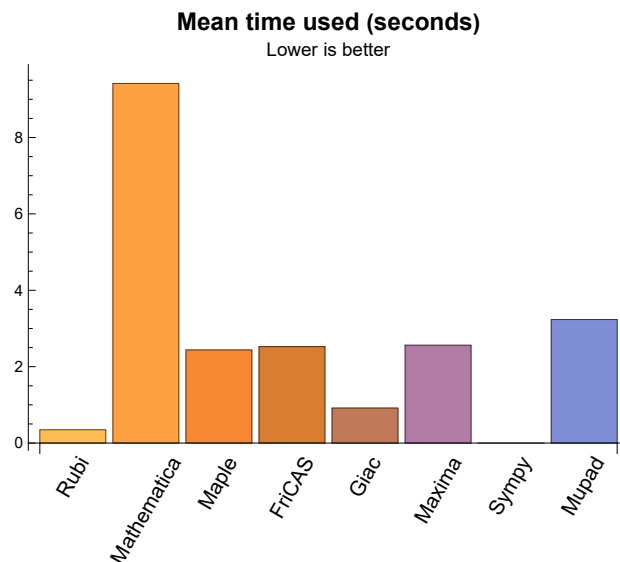
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	211.50	0.96	153.00	1.00
Mathematica	9.42	24977.16	50.22	171.00	1.22
Maple	2.44	14803.56	29.57	286.50	1.80
Maxima	2.56	769.52	4.58	243.00	2.34
Fricas	2.53	544.61	3.11	405.00	2.85
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.92	260.95	1.21	93.00	0.85
Mupad	3.23	1611.38	6.08	122.00	1.08

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{221, 222, 223, 224, 225, 226, 227, 228, 229, 236}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {62, 63, 64, 70, 71, 77, 78, 85, 147, 148, 149, 151, 152, 153, 158, 159, 163, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 200, 202, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 230, 231, 232, 240, 241}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```



```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

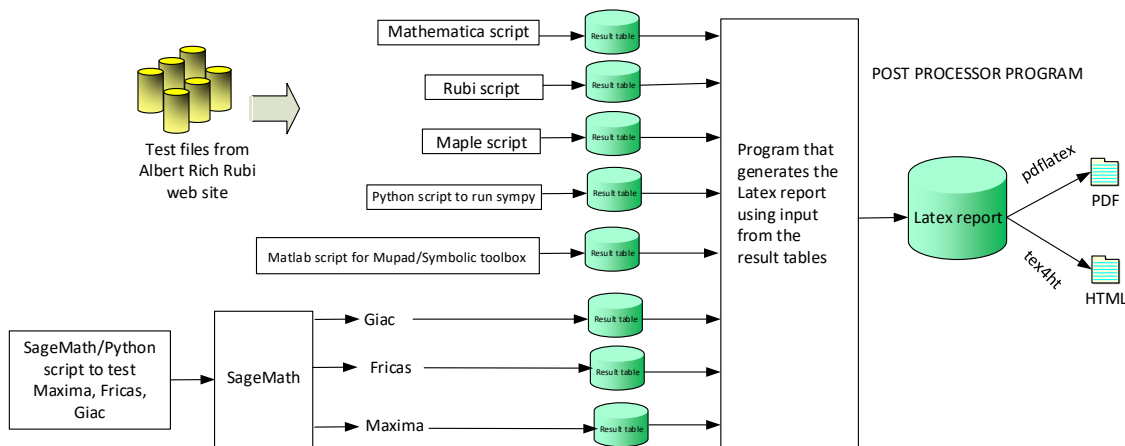
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241 }

B grade: { }

C grade: { }

F grade: { 217 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 24, 25, 26, 29, 30, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 145, 146, 150, 154, 155, 156, 157, 160, 161, 162, 164, 168, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 199, 201, 203, 204, 208, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 233, 236, 237, 238, 239 }

B grade: { 6, 16, 17, 21, 22, 23, 28, 31, 37, 39, 40, 144, 193, 195, 197, 200, 202, 209, 210, 211, 212, 213, 214, 215, 216, 220, 230, 240, 241 }

C grade: { 7, 18, 27, 38, 47, 48, 49, 55, 56, 62, 63, 64, 70, 71, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 143, 147, 148, 149, 151, 152, 153, 158, 159, 163, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 198, 205, 206, 207, 217, 219, 231, 232 }

F grade: { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 234, 235 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 44, 45, 46, 51, 60, 68, 69, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 122,

123, 124, 125, 126, 127, 128, 129, 130, 143, 144, 145, 146, 149, 189, 201, 203, 204, 207, 208, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 42, 43, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 105, 106, 107, 114, 118, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 220 }

C grade: { }

F grade: { 27, 38, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.4 Maxima

A grade: { 1, 2, 4, 5, 13, 15, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 89, 90, 97, 98, 106, 112, 113, 114, 119, 125, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 3, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 45, 52, 59, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 109, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 150, 156, 162 }

C grade: { }

F grade: { 42, 43, 44, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 104, 105, 110, 111, 118, 124, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 94, 95, 96, 101, 102, 103, 130, 143, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 178, 179, 184, 185, 187, 188, 189, 236 }

B grade: { 5, 22, 23, 61, 62, 63, 74, 75, 83, 89, 105, 114, 118, 122, 153, 159, 174, 180, 186, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 90, 91, 92, 93, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 170, 171, 175, 176, 177, 181, 182, 183, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.6 Sympy

A grade: { 221, 222, 224, 225, 236 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108,

109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.7 Giac

A grade: { 4, 7, 8, 9, 10, 13, 18, 19, 20, 24, 25, 26, 28, 29, 30, 33, 34, 35, 36, 37, 39, 40, 41, 190, 192, 221, 222, 223, 224, 225, 226, 227, 228, 229, 236 }

B grade: { 27, 38, 189, 191, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 11, 12, 14, 15, 16, 17, 21, 22, 23, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.1.8 Mupad

A grade: { 221, 222, 224, 225, 227, 236 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade: { }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	165	186	334	179	0	0	228
normalized size	1	1.00	0.84	0.95	1.70	0.91	0.00	0.00	1.16
time (sec)	N/A	0.302	2.130	1.578	0.345	0.463	0.000	0.000	2.534
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	146	161	240	163	0	0	195
normalized size	1	1.00	1.04	1.15	1.71	1.16	0.00	0.00	1.39
time (sec)	N/A	0.199	1.166	1.406	0.341	0.518	0.000	0.000	2.300
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	122	136	203	147	0	0	163
normalized size	1	1.00	1.26	1.40	2.09	1.52	0.00	0.00	1.68
time (sec)	N/A	0.118	0.735	1.219	0.333	0.488	0.000	0.000	2.189
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	58	57	65	0	51	84
normalized size	1	1.00	0.96	1.23	1.21	1.38	0.00	1.09	1.79
time (sec)	N/A	0.065	0.031	0.820	0.321	0.433	0.000	0.303	3.709
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	72	76	95	103	0	0	91
normalized size	1	1.00	1.31	1.38	1.73	1.87	0.00	0.00	1.65
time (sec)	N/A	0.063	0.290	0.856	0.326	0.476	0.000	0.000	1.513

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	169	90	153	87	0	0	46
normalized size	1	1.00	3.02	1.61	2.73	1.55	0.00	0.00	0.82
time (sec)	N/A	0.164	0.293	0.777	0.431	0.484	0.000	0.000	1.484
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	67	174	88	0	60	40
normalized size	1	1.00	0.75	0.94	2.45	1.24	0.00	0.85	0.56
time (sec)	N/A	0.236	0.058	0.862	0.443	0.458	0.000	0.291	1.389
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	171	89	215	128	0	76	96
normalized size	1	1.00	1.68	0.87	2.11	1.25	0.00	0.75	0.94
time (sec)	N/A	0.329	0.629	1.046	0.445	0.425	0.000	0.462	1.450
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	227	111	294	172	0	93	124
normalized size	1	1.00	1.71	0.83	2.21	1.29	0.00	0.70	0.93
time (sec)	N/A	0.427	0.635	0.904	0.435	0.435	0.000	0.329	1.496
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	283	133	335	212	0	110	146
normalized size	1	1.00	1.73	0.81	2.04	1.29	0.00	0.67	0.89
time (sec)	N/A	0.543	0.995	0.845	0.446	0.463	0.000	0.439	1.536
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	189	211	356	195	0	0	259
normalized size	1	1.00	1.01	1.12	1.89	1.04	0.00	0.00	1.38
time (sec)	N/A	0.237	2.236	1.621	0.338	0.485	0.000	0.000	2.619

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	165	186	334	179	0	0	227
normalized size	1	1.00	1.25	1.41	2.53	1.36	0.00	0.00	1.72
time (sec)	N/A	0.151	1.871	1.646	0.338	0.526	0.000	0.000	2.593
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	93	94	81	0	69	122
normalized size	1	1.00	0.90	1.37	1.38	1.19	0.00	1.01	1.79
time (sec)	N/A	0.074	0.039	1.056	0.323	0.465	0.000	0.553	4.959
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	122	136	203	147	0	0	163
normalized size	1	1.00	1.26	1.40	2.09	1.52	0.00	0.00	1.68
time (sec)	N/A	0.114	0.798	1.254	0.337	0.496	0.000	0.000	2.133
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	101	98	107	118	0	0	104
normalized size	1	1.00	1.31	1.27	1.39	1.53	0.00	0.00	1.35
time (sec)	N/A	0.147	0.453	0.863	0.312	0.448	0.000	0.000	1.552
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	240	137	274	125	0	0	85
normalized size	1	1.00	3.08	1.76	3.51	1.60	0.00	0.00	1.09
time (sec)	N/A	0.209	2.580	0.701	0.434	0.515	0.000	0.000	1.479
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	177	90	274	156	0	0	45
normalized size	1	1.00	2.01	1.02	3.11	1.77	0.00	0.00	0.51
time (sec)	N/A	0.361	1.182	0.833	0.439	0.455	0.000	0.000	1.437

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	53	89	282	128	0	77	96
normalized size	1	1.00	0.52	0.87	2.76	1.25	0.00	0.75	0.94
time (sec)	N/A	0.452	0.099	0.819	0.443	0.429	0.000	0.396	1.378
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	227	111	383	172	0	93	122
normalized size	1	1.00	1.71	0.83	2.88	1.29	0.00	0.70	0.92
time (sec)	N/A	0.579	0.599	0.881	0.447	0.453	0.000	0.588	1.438
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	283	133	403	212	0	110	146
normalized size	1	1.00	1.73	0.81	2.46	1.29	0.00	0.67	0.89
time (sec)	N/A	0.733	0.902	0.893	0.458	0.438	0.000	0.555	1.461
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	153	384	207	603	242	0	0	145
normalized size	1	1.12	2.82	1.52	4.43	1.78	0.00	0.00	1.07
time (sec)	N/A	0.402	3.151	0.708	0.438	0.459	0.000	0.000	1.495
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	753	159	413	220	0	0	112
normalized size	1	1.00	7.38	1.56	4.05	2.16	0.00	0.00	1.10
time (sec)	N/A	0.309	6.262	0.679	0.442	0.488	0.000	0.000	1.471
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	216	90	268	173	0	0	46
normalized size	1	1.00	2.54	1.06	3.15	2.04	0.00	0.00	0.54
time (sec)	N/A	0.331	1.096	0.818	0.431	0.463	0.000	0.000	1.410

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	65	170	94	0	63	38
normalized size	1	1.00	1.00	0.97	2.54	1.40	0.00	0.94	0.57
time (sec)	N/A	0.228	0.051	0.859	0.428	0.450	0.000	0.392	1.375
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	113	59	119	86	0	56	41
normalized size	1	1.00	1.85	0.97	1.95	1.41	0.00	0.92	0.67
time (sec)	N/A	0.145	0.312	0.900	0.422	0.470	0.000	0.277	1.336
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	135	87	102	70	0	85	69
normalized size	1	1.00	1.96	1.26	1.48	1.01	0.00	1.23	1.00
time (sec)	N/A	0.114	0.548	0.840	0.428	0.425	0.000	0.398	1.422
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	0	46	81	0	100	58
normalized size	1	1.00	0.85	0.00	1.00	1.76	0.00	2.17	1.26
time (sec)	N/A	0.071	0.049	180.000	0.422	0.414	0.000	0.288	1.468
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	257	130	147	154	0	116	161
normalized size	1	1.00	2.62	1.33	1.50	1.57	0.00	1.18	1.64
time (sec)	N/A	0.145	1.411	1.100	0.426	0.424	0.000	0.434	1.544
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	315	153	167	166	0	129	185
normalized size	1	1.00	1.90	0.92	1.01	1.00	0.00	0.78	1.11
time (sec)	N/A	0.211	1.271	0.950	0.428	0.490	0.000	0.948	1.635

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	383	175	186	232	0	143	209
normalized size	1	1.00	1.82	0.83	0.89	1.10	0.00	0.68	1.00
time (sec)	N/A	0.286	1.199	0.945	0.429	0.424	0.000	0.476	1.779
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	557	179	562	289	0	0	134
normalized size	1	1.00	3.44	1.10	3.47	1.78	0.00	0.00	0.83
time (sec)	N/A	0.443	5.880	0.641	0.435	0.452	0.000	0.000	1.490
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	231	110	396	242	0	0	50
normalized size	1	1.00	1.56	0.74	2.68	1.64	0.00	0.00	0.34
time (sec)	N/A	0.606	1.202	0.772	0.435	0.480	0.000	0.000	1.421
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	90	87	277	138	0	84	93
normalized size	1	1.00	0.94	0.91	2.89	1.44	0.00	0.88	0.97
time (sec)	N/A	0.421	0.078	0.794	0.432	0.439	0.000	0.374	1.404
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	171	87	211	138	0	84	93
normalized size	1	1.00	1.78	0.91	2.20	1.44	0.00	0.88	0.97
time (sec)	N/A	0.304	0.466	0.768	0.435	0.442	0.000	0.333	1.396
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	169	79	159	124	0	75	85
normalized size	1	1.00	1.92	0.90	1.81	1.41	0.00	0.85	0.97
time (sec)	N/A	0.202	0.460	0.752	0.430	0.427	0.000	1.193	1.377

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	197	109	122	109	0	107	82
normalized size	1	1.00	1.56	0.87	0.97	0.87	0.00	0.85	0.65
time (sec)	N/A	0.193	0.988	0.985	0.425	0.428	0.000	0.331	1.425
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	257	131	146	154	0	122	161
normalized size	1	1.00	2.57	1.31	1.46	1.54	0.00	1.22	1.61
time (sec)	N/A	0.143	1.008	1.083	0.425	0.426	0.000	0.350	1.504
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	0	56	118	0	136	94
normalized size	1	1.00	0.58	0.00	0.84	1.76	0.00	2.03	1.40
time (sec)	N/A	0.081	0.070	180.000	0.427	0.448	0.000	0.459	1.569
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	362	174	187	232	0	150	209
normalized size	1	1.00	2.81	1.35	1.45	1.80	0.00	1.16	1.62
time (sec)	N/A	0.173	1.405	0.981	0.440	0.447	0.000	0.438	1.852
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	441	197	205	271	0	163	233
normalized size	1	1.00	2.10	0.94	0.98	1.29	0.00	0.78	1.11
time (sec)	N/A	0.236	1.742	0.982	0.444	0.462	0.000	0.507	2.041
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	499	219	227	310	0	179	257
normalized size	1	1.00	1.98	0.87	0.90	1.23	0.00	0.71	1.02
time (sec)	N/A	0.300	2.362	0.999	0.449	0.455	0.000	0.981	2.328

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	121	391	0	373	0	0	-1
normalized size	1	1.00	0.69	2.23	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.176	1.048	1.896	0.000	0.507	0.000	0.000	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	111	302	0	347	0	0	-1
normalized size	1	1.00	0.79	2.16	0.00	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.168	1.195	1.747	0.000	0.481	0.000	0.000	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	142	0	313	0	0	-1
normalized size	1	1.00	0.92	1.35	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.885	1.714	0.000	0.494	0.000	0.000	0.000
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	115	147	234	0	0	-1
normalized size	1	1.00	1.06	1.74	2.23	3.55	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.321	1.420	0.835	0.444	0.000	0.000	0.000
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	90	116	0	266	0	0	-1
normalized size	1	1.00	1.30	1.68	0.00	3.86	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.588	1.585	0.000	0.589	0.000	0.000	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	78	214	0	339	0	0	-1
normalized size	1	1.00	0.75	2.06	0.00	3.26	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.242	1.682	0.000	0.545	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	78	311	0	405	0	0	-1
normalized size	1	1.00	0.56	2.24	0.00	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.265	1.899	0.000	0.580	0.000	0.000	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	78	402	0	475	0	0	-1
normalized size	1	1.00	0.45	2.31	0.00	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.244	2.200	0.000	0.639	0.000	0.000	0.000
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	392	0	385	0	0	-1
normalized size	1	1.00	0.69	2.21	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.949	2.332	0.000	0.483	0.000	0.000	0.000
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	112	232	0	355	0	0	-1
normalized size	1	1.00	0.79	1.63	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.908	1.840	0.000	0.462	0.000	0.000	0.000
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	96	212	998	303	0	0	-1
normalized size	1	1.00	0.95	2.10	9.88	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.677	1.431	0.818	0.459	0.000	0.000	0.000
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	194	0	269	0	0	-1
normalized size	1	1.00	1.33	2.77	0.00	3.84	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.584	1.374	0.000	0.540	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	113	215	0	351	0	0	-1
normalized size	1	1.00	1.11	2.11	0.00	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.643	1.550	0.000	0.549	0.000	0.000	0.000
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	102	304	0	417	0	0	-1
normalized size	1	1.00	0.74	2.22	0.00	3.04	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.804	1.762	0.000	0.560	0.000	0.000	0.000
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	102	401	0	495	0	0	-1
normalized size	1	1.00	0.59	2.33	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.291	1.768	0.000	0.603	0.000	0.000	0.000
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	134	483	0	441	0	0	-1
normalized size	1	1.00	0.63	2.28	0.00	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.191	1.305	1.837	0.000	0.525	0.000	0.000	0.000
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	124	323	0	409	0	0	-1
normalized size	1	1.00	0.70	1.82	0.00	2.31	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.924	1.572	0.000	0.490	0.000	0.000	0.000
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	303	1396	353	0	0	-1
normalized size	1	1.00	0.83	2.30	10.58	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.813	1.368	1.146	0.500	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	96	120	0	291	0	0	-1
normalized size	1	1.00	0.93	1.17	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.719	1.356	0.000	0.577	0.000	0.000	0.000
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	351	0	339	0	0	-1
normalized size	1	1.00	1.38	4.74	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.168	4.224	1.419	0.000	0.567	0.000	0.000	0.000
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	196	306	0	441	0	0	-1
normalized size	1	1.00	1.88	2.94	0.00	4.24	0.00	0.00	-0.01
time (sec)	N/A	0.177	5.331	1.619	0.000	0.570	0.000	0.000	0.000
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	361	395	0	527	0	0	-1
normalized size	1	1.00	2.58	2.82	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.184	7.977	1.627	0.000	0.567	0.000	0.000	0.000
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	205	492	0	601	0	0	-1
normalized size	1	1.00	1.19	2.86	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.198	3.561	1.899	0.000	0.576	0.000	0.000	0.000
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	153	544	0	552	0	0	-1
normalized size	1	1.00	0.83	2.94	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.276	1.452	1.974	0.000	1.257	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	166	372	0	518	0	0	-1
normalized size	1	1.00	1.09	2.45	0.00	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.228	1.331	1.921	0.000	1.029	0.000	0.000	0.000
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	124	329	0	438	0	0	-1
normalized size	1	1.00	1.04	2.76	0.00	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.506	1.897	0.000	0.725	0.000	0.000	0.000
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	144	0	298	0	0	-1
normalized size	1	1.00	0.94	1.66	0.00	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.297	1.523	0.000	0.597	0.000	0.000	0.000
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	101	194	0	436	0	0	-1
normalized size	1	1.00	0.83	1.60	0.00	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.575	1.719	0.000	0.618	0.000	0.000	0.000
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	5576	377	0	520	0	0	-1
normalized size	1	1.00	34.63	2.34	0.00	3.23	0.00	0.00	-0.01
time (sec)	N/A	0.235	24.117	1.952	0.000	0.604	0.000	0.000	0.000
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	5592	545	0	608	0	0	-1
normalized size	1	1.00	28.53	2.78	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.284	23.742	2.566	0.000	0.610	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	196	552	0	634	0	0	-1
normalized size	1	1.00	0.97	2.72	0.00	3.12	0.00	0.00	-0.00
time (sec)	N/A	0.286	1.579	1.970	0.000	1.869	0.000	0.000	0.000
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	132	377	0	550	0	0	-1
normalized size	1	1.00	0.78	2.23	0.00	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.240	1.772	1.992	0.000	1.018	0.000	0.000	0.000
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	134	128	369	0	542	0	0	-1
normalized size	1	1.13	1.08	3.10	0.00	4.55	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.114	1.762	0.000	0.934	0.000	0.000	0.000
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	130	130	371	0	505	0	0	-1
normalized size	1	1.15	1.15	3.28	0.00	4.47	0.00	0.00	-0.01
time (sec)	N/A	0.166	1.003	1.870	0.000	0.666	0.000	0.000	0.000
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	154	377	0	514	0	0	-1
normalized size	1	1.00	0.87	2.13	0.00	2.90	0.00	0.00	-0.01
time (sec)	N/A	0.244	1.185	1.904	0.000	0.631	0.000	0.000	0.000
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	5612	387	0	560	0	0	-1
normalized size	1	1.00	26.22	1.81	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.278	24.032	2.498	0.000	0.652	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	5629	725	0	714	0	0	-1
normalized size	1	1.00	22.61	2.91	0.00	2.87	0.00	0.00	-0.00
time (sec)	N/A	0.340	23.977	2.508	0.000	0.693	0.000	0.000	0.000
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	180	726	0	742	0	0	-1
normalized size	1	1.00	0.69	2.79	0.00	2.85	0.00	0.00	-0.00
time (sec)	N/A	0.341	3.642	2.549	0.000	2.963	0.000	0.000	0.000
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	164	550	0	655	0	0	-1
normalized size	1	1.00	0.72	2.40	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.296	2.680	2.058	0.000	2.476	0.000	0.000	0.000
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	136	553	0	645	0	0	-1
normalized size	1	1.00	0.71	2.90	0.00	3.38	0.00	0.00	-0.01
time (sec)	N/A	0.252	1.590	2.112	0.000	1.982	0.000	0.000	0.000
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	136	545	0	645	0	0	-1
normalized size	1	1.00	0.72	2.88	0.00	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.230	1.561	1.879	0.000	1.529	0.000	0.000	0.000
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	181	134	543	0	605	0	0	-1
normalized size	1	1.22	0.91	3.67	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.490	1.618	0.000	1.035	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	158	545	0	608	0	0	-1
normalized size	1	1.00	0.69	2.37	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.306	1.519	2.158	0.000	0.658	0.000	0.000	0.000
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	5650	725	0	706	0	0	-1
normalized size	1	1.00	21.00	2.70	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.335	24.173	2.328	0.000	0.675	0.000	0.000	0.000
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	149	195	1289	459	0	0	-1
normalized size	1	1.00	0.81	1.05	6.97	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.372	6.136	2.575	0.887	0.592	0.000	0.000	0.000
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	162	185	710	425	0	0	-1
normalized size	1	1.00	1.17	1.33	5.11	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.273	2.311	2.354	1.041	0.566	0.000	0.000	0.000
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	99	165	243	350	0	0	-1
normalized size	1	1.00	1.06	1.77	2.61	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.178	1.244	2.306	0.774	0.569	0.000	0.000	0.000
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	102	128	39	200	0	0	-1
normalized size	1	1.00	2.12	2.67	0.81	4.17	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.582	2.398	0.813	0.571	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	86	100	65	0	0	0	-1
normalized size	1	1.00	1.69	1.96	1.27	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.978	2.120	0.593	0.567	0.000	0.000	0.000
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	107	164	399	0	0	0	-1
normalized size	1	1.00	1.11	1.71	4.16	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.042	2.548	1.028	0.545	0.000	0.000	0.000
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	152	226	1173	0	0	0	-1
normalized size	1	1.00	1.07	1.59	8.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.271	1.254	2.678	1.097	0.552	0.000	0.000	0.000
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	198	288	2444	0	0	0	-1
normalized size	1	1.00	1.05	1.53	13.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.366	1.942	2.179	4.634	0.564	0.000	0.000	0.000
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	157	190	1356	467	0	0	-1
normalized size	1	1.00	0.83	1.00	7.14	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.364	1.278	2.002	1.052	0.571	0.000	0.000	0.000
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	159	172	477	346	0	0	-1
normalized size	1	1.00	1.54	1.67	4.63	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.110	1.391	2.084	0.887	0.587	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	128	152	243	347	0	0	-1
normalized size	1	1.00	1.38	1.63	2.61	3.73	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.784	2.031	0.991	0.566	0.000	0.000	0.000
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	105	157	60	0	0	0	-1
normalized size	1	1.00	1.01	1.51	0.58	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	1.201	1.948	0.556	0.632	0.000	0.000	0.000
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	115	161	95	0	0	0	-1
normalized size	1	1.00	1.15	1.61	0.95	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.685	1.973	0.723	0.567	0.000	0.000	0.000
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	153	227	1786	0	0	0	-1
normalized size	1	1.00	1.05	1.55	12.23	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	1.272	2.095	1.213	0.569	0.000	0.000	0.000
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	199	289	3480	0	0	0	-1
normalized size	1	1.00	1.02	1.47	17.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	2.202	2.295	3.656	0.614	0.000	0.000	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	164	192	1619	405	0	0	-1
normalized size	1	1.00	1.07	1.25	10.58	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.536	2.104	1.181	0.599	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	149	200	1356	467	0	0	-1
normalized size	1	1.00	0.78	1.05	7.14	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.363	1.196	2.047	1.114	0.579	0.000	0.000	0.000
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	164	180	710	420	0	0	-1
normalized size	1	1.00	1.18	1.29	5.11	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.263	1.404	2.324	1.051	0.560	0.000	0.000	0.000
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	292	184	0	0	0	0	-1
normalized size	1	1.00	1.92	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	6.764	2.061	0.000	0.616	0.000	0.000	0.000
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	111	239	0	442	0	0	-1
normalized size	1	1.00	1.16	2.49	0.00	4.60	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.294	2.256	0.000	0.585	0.000	0.000	0.000
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	155	229	139	0	0	0	-1
normalized size	1	1.00	1.55	2.29	1.39	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	1.386	2.251	0.567	0.558	0.000	0.000	0.000
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	202	281	3738	0	0	0	-1
normalized size	1	1.00	1.36	1.90	25.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	2.670	2.369	4.396	0.571	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	285	353	6134	0	0	0	-1
normalized size	1	1.00	1.47	1.82	31.62	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.376	5.498	2.352	24.801	0.616	0.000	0.000	0.000
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	299	415	9150	0	0	0	-1
normalized size	1	1.00	1.23	1.70	37.50	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.474	5.986	2.192	146.306	0.613	0.000	0.000	0.000
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	153	190	0	0	0	0	-1
normalized size	1	1.00	0.75	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	16.348	2.216	0.000	0.633	0.000	0.000	0.000
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	181	170	0	0	0	0	-1
normalized size	1	1.00	1.20	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	4.017	2.238	0.000	0.615	0.000	0.000	0.000
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	103	93	60	0	0	0	-1
normalized size	1	1.00	1.01	0.91	0.59	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	9.715	2.396	0.835	0.618	0.000	0.000	0.000
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	127	75	34	0	0	0	-1
normalized size	1	1.00	2.59	1.53	0.69	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.981	2.411	0.729	0.523	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	104	101	39	272	0	0	-1
normalized size	1	1.00	2.26	2.20	0.85	5.91	0.00	0.00	-0.02
time (sec)	N/A	0.090	1.085	2.376	0.838	0.725	0.000	0.000	0.000
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	217	143	167	818	0	0	0	-1
normalized size	1	1.29	0.85	0.99	4.87	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	8.673	2.591	0.950	0.650	0.000	0.000	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	194	229	2206	0	0	0	-1
normalized size	1	1.00	0.71	0.84	8.05	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	1.930	2.587	1.279	1.018	0.000	0.000	0.000
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	204	278	2393	0	0	0	-1
normalized size	1	1.00	0.95	1.29	11.13	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	2.275	2.271	1.484	0.736	0.000	0.000	0.000
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	116	238	0	453	0	0	-1
normalized size	1	1.00	1.21	2.48	0.00	4.72	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.744	2.163	0.000	0.576	0.000	0.000	0.000
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	114	106	70	0	0	0	-1
normalized size	1	1.00	1.16	1.08	0.71	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	1.140	2.191	0.726	0.526	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	119	395	0	0	0	-1
normalized size	1	1.00	1.13	1.27	4.20	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.633	2.302	0.957	0.551	0.000	0.000	0.000
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	141	159	818	0	0	0	-1
normalized size	1	1.00	0.66	0.74	3.80	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	1.418	2.433	0.958	0.806	0.000	0.000	0.000
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	121	173	486	492	0	0	-1
normalized size	1	1.00	1.20	1.71	4.81	4.87	0.00	0.00	-0.01
time (sec)	N/A	0.121	1.528	2.391	0.880	0.863	0.000	0.000	0.000
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	275	293	4272	0	0	0	-1
normalized size	1	1.00	0.79	0.84	12.31	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	2.619	2.948	4.183	0.801	0.000	0.000	0.000
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	157	338	0	0	0	0	-1
normalized size	1	1.00	0.71	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	2.540	2.602	0.000	0.692	0.000	0.000	0.000
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	154	144	102	0	0	0	-1
normalized size	1	1.00	1.57	1.47	1.04	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.572	2.747	0.714	0.559	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	152	152	1786	0	0	0	-1
normalized size	1	1.00	1.06	1.06	12.40	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.866	2.653	1.146	0.557	0.000	0.000	0.000
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	151	152	1165	0	0	0	-1
normalized size	1	1.00	1.08	1.09	8.32	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	0.675	2.787	1.397	0.574	0.000	0.000	0.000
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	195	223	2206	0	0	0	-1
normalized size	1	1.00	0.72	0.83	8.17	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	1.659	2.707	1.144	0.771	0.000	0.000	0.000
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	275	284	4272	0	0	0	-1
normalized size	1	1.00	0.80	0.82	12.38	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	2.400	2.604	4.632	0.795	0.000	0.000	0.000
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	237	1386	564	0	0	-1
normalized size	1	1.00	0.99	1.57	9.18	3.74	0.00	0.00	-0.01
time (sec)	N/A	0.131	2.192	2.480	1.338	1.022	0.000	0.000	0.000
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.103	2.135	0.000	0.478	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.445	2.389	0.000	0.507	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	3.242	2.857	0.000	0.452	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.526	2.293	0.000	0.460	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	1.598	2.099	0.000	0.431	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.999	2.274	0.000	0.433	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	1.726	1.752	0.000	0.442	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	8.540	1.739	0.000	0.453	0.000	0.000	0.000
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	11.826	1.727	0.000	0.466	0.000	0.000	0.000
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.143	1.883	0.000	0.455	0.000	0.000	0.000
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.363	1.928	0.000	0.447	0.000	0.000	0.000
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	1.765	2.005	0.000	0.461	0.000	0.000	0.000
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	133	141	0	293	0	0	-1
normalized size	1	1.00	1.46	1.55	0.00	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.652	1.781	0.000	0.556	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	810	295	0	0	0	0	-1
normalized size	1	1.00	3.51	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	18.228	1.923	0.000	0.826	0.000	0.000	0.000
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	178	285	0	0	0	0	-1
normalized size	1	1.00	0.79	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	8.746	1.985	0.000	22.527	0.000	0.000	0.000
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	187	327	0	0	0	0	-1
normalized size	1	1.00	0.59	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	12.708	2.100	0.000	0.818	0.000	0.000	0.000
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	587	546	0	472	0	0	-1
normalized size	1	1.00	2.17	2.01	0.00	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.173	14.296	2.134	0.000	0.512	0.000	0.000	0.000
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	517	389	0	392	0	0	-1
normalized size	1	1.00	2.52	1.90	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.141	14.186	1.955	0.000	0.483	0.000	0.000	0.000
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	444	248	0	320	0	0	-1
normalized size	1	1.00	3.08	1.72	0.00	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.116	6.589	1.735	0.000	0.478	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	147	235	0	0	-1
normalized size	1	1.00	1.15	1.79	2.23	3.56	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.315	1.546	0.592	0.501	0.000	0.000	0.000
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	2650	502	0	669	0	0	-1
normalized size	1	1.00	25.24	4.78	0.00	6.37	0.00	0.00	-0.01
time (sec)	N/A	0.230	25.068	1.750	0.000	0.868	0.000	0.000	0.000
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	2907	97265	0	1413	0	0	-1
normalized size	1	1.00	13.27	444.13	0.00	6.45	0.00	0.00	-0.00
time (sec)	N/A	0.222	28.578	2.998	0.000	2.865	0.000	0.000	0.000
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	3070	330749	0	2368	0	0	-1
normalized size	1	1.00	10.70	1152.44	0.00	8.25	0.00	0.00	-0.00
time (sec)	N/A	0.307	24.493	19.097	0.000	7.738	0.000	0.000	0.000
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	219	539	0	482	0	0	-1
normalized size	1	1.00	0.91	2.24	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	3.892	1.853	0.000	0.502	0.000	0.000	0.000
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	145	382	0	398	0	0	-1
normalized size	1	1.00	0.82	2.17	0.00	2.26	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.335	1.674	0.000	0.504	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	998	316	0	0	-1
normalized size	1	1.00	0.97	2.26	9.50	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.590	1.536	1.400	0.511	0.000	0.000	0.000
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	135	864	0	731	0	0	-1
normalized size	1	1.00	1.23	7.85	0.00	6.65	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.497	1.493	0.000	1.625	0.000	0.000	0.000
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	2862	62283	0	1640	0	0	-1
normalized size	1	1.00	12.50	271.98	0.00	7.16	0.00	0.00	-0.00
time (sec)	N/A	0.249	24.922	3.116	0.000	7.403	0.000	0.000	0.000
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	3166	234091	0	2729	0	0	-1
normalized size	1	1.00	10.21	755.13	0.00	8.80	0.00	0.00	-0.00
time (sec)	N/A	0.342	24.704	11.531	0.000	20.444	0.000	0.000	0.000
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	286	677	0	620	0	0	-1
normalized size	1	1.00	0.85	2.01	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.209	6.496	1.908	0.000	0.522	0.000	0.000	0.000
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	191	504	0	500	0	0	-1
normalized size	1	1.00	0.74	1.95	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.177	2.669	1.797	0.000	0.539	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1396	390	0	0	-1
normalized size	1	1.00	0.90	2.40	9.83	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.232	1.026	1.481	0.799	0.461	0.000	0.000	0.000
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	343	1487	0	1140	0	0	-1
normalized size	1	1.00	1.69	7.33	0.00	5.62	0.00	0.00	-0.00
time (sec)	N/A	0.232	6.547	1.498	0.000	5.105	0.000	0.000	0.000
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	280	46082	0	2031	0	0	-1
normalized size	1	1.00	0.85	140.07	0.00	6.17	0.00	0.00	-0.00
time (sec)	N/A	0.337	3.645	2.123	0.000	23.656	0.000	0.000	0.000
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	3344	209489	0	3351	0	0	-1
normalized size	1	1.00	6.24	390.84	0.00	6.25	0.00	0.00	-0.00
time (sec)	N/A	0.492	26.115	8.149	0.000	40.755	0.000	0.000	0.000
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	787	907	0	619	0	0	-1
normalized size	1	1.00	3.05	3.52	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.207	7.240	1.913	0.000	13.682	0.000	0.000	0.000
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	295	358	0	481	0	0	-1
normalized size	1	1.00	1.61	1.96	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.158	2.496	1.692	0.000	3.346	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	314	0	0	-1
normalized size	1	1.00	1.01	2.13	0.00	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.295	1.500	0.000	1.226	0.000	0.000	0.000
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	431980	662	0	1050	0	0	-1
normalized size	1	1.00	2602.29	3.99	0.00	6.33	0.00	0.00	-0.01
time (sec)	N/A	0.372	39.128	2.051	0.000	18.623	0.000	0.000	0.000
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	473385	117715	0	0	0	0	-1
normalized size	1	1.00	1137.94	282.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	36.128	5.174	0.000	0.000	0.000	0.000	0.000
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	654358	402966	0	0	0	0	-1
normalized size	1	1.00	1002.08	617.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.646	38.992	24.788	0.000	0.000	0.000	0.000	0.000
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	856	957	0	701	0	0	-1
normalized size	1	1.00	2.64	2.95	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.237	6.575	1.882	0.000	34.523	0.000	0.000	0.000
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	16153	756	0	620	0	0	-1
normalized size	1	1.00	55.70	2.61	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.217	27.749	1.548	0.000	12.896	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	10105	552	0	548	0	0	-1
normalized size	1	1.00	79.57	4.35	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.183	26.696	1.380	0.000	3.182	0.000	0.000	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	378865	2076	0	0	0	0	-1
normalized size	1	1.00	961.59	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	35.207	1.783	0.000	0.000	0.000	0.000	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	582620	164796	0	0	0	0	-1
normalized size	1	1.00	1040.39	294.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	38.103	5.454	0.000	0.000	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	802	802	776222	480553	0	0	0	0	-1
normalized size	1	1.00	967.86	599.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.797	41.230	31.938	0.000	0.000	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	21194	1444	0	880	0	0	-1
normalized size	1	1.00	44.15	3.01	0.00	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.316	29.390	1.898	0.000	56.607	0.000	0.000	0.000
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	16249	1133	0	782	0	0	-1
normalized size	1	1.00	34.72	2.42	0.00	1.67	0.00	0.00	-0.00
time (sec)	N/A	0.295	27.953	1.636	0.000	23.944	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	11243	824	0	670	0	0	-1
normalized size	1	1.00	68.55	5.02	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.265	26.947	1.727	0.000	6.363	0.000	0.000	0.000
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	486155	3860	0	0	0	0	-1
normalized size	1	1.00	821.21	6.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	36.996	2.046	0.000	0.000	0.000	0.000	0.000
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	756	756	688080	197500	0	0	0	0	-1
normalized size	1	1.00	910.16	261.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	40.037	7.791	0.000	0.000	0.000	0.000	0.000
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	999	999	893714	556423	0	0	0	0	-1
normalized size	1	1.00	894.61	556.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.914	43.442	44.265	0.000	0.000	0.000	0.000	0.000
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	240	1562	0	806	0	0	-1
normalized size	1	1.00	1.95	12.70	0.00	6.55	0.00	0.00	-0.01
time (sec)	N/A	0.339	18.422	2.243	0.000	1.469	0.000	0.000	0.000
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	102	189	0	206	0	0	-1
normalized size	1	1.00	1.67	3.10	0.00	3.38	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.217	1.957	0.000	0.591	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	135	377	0	517	0	0	-1
normalized size	1	1.00	1.22	3.40	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.960	2.046	0.000	0.601	0.000	0.000	0.000
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	184	494	0	883	0	0	-1
normalized size	1	1.00	1.30	3.50	0.00	6.26	0.00	0.00	-0.01
time (sec)	N/A	0.369	14.615	1.885	0.000	1.004	0.000	0.000	0.000
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	171	424	0	913	0	0	-1
normalized size	1	1.00	1.21	3.01	0.00	6.48	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.350	2.133	0.000	1.688	0.000	0.000	0.000
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	250	0	278	573
normalized size	1	1.00	1.01	1.69	0.00	3.73	0.00	4.15	8.55
time (sec)	N/A	0.126	0.165	0.736	0.000	0.476	0.000	0.284	2.676
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	155	328	0	561	0	209	3763
normalized size	1	1.00	1.26	2.67	0.00	4.56	0.00	1.70	30.59
time (sec)	N/A	0.247	0.661	0.769	0.000	0.495	0.000	0.581	9.327
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	267	1063	0	1152	0	477	6909
normalized size	1	1.00	1.31	5.21	0.00	5.65	0.00	2.34	33.87
time (sec)	N/A	0.510	1.418	0.773	0.000	0.589	0.000	8.542	11.353

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	136	462	0	671	0	246	4934
normalized size	1	1.00	1.02	3.47	0.00	5.05	0.00	1.85	37.10
time (sec)	N/A	0.285	0.794	0.782	0.000	0.531	0.000	0.298	10.205
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	493	1593	0	1409	0	686	8682
normalized size	1	1.00	2.08	6.72	0.00	5.95	0.00	2.89	36.63
time (sec)	N/A	0.796	2.044	0.738	0.000	0.582	0.000	0.456	12.136
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	438	3293	0	2362	0	1253	12818
normalized size	1	1.00	1.16	8.73	0.00	6.27	0.00	3.32	34.00
time (sec)	N/A	1.999	3.451	0.802	0.000	0.731	0.000	0.465	15.074
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	517	2031	0	1629	0	852	10759
normalized size	1	1.00	2.04	8.00	0.00	6.41	0.00	3.35	42.36
time (sec)	N/A	1.132	2.303	0.723	0.000	0.680	0.000	1.609	14.496
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	459	4330	0	2776	0	1639	15647
normalized size	1	1.00	1.11	10.51	0.00	6.74	0.00	3.98	37.98
time (sec)	N/A	1.062	3.777	0.764	0.000	0.809	0.000	0.931	16.091
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	1285	8573	0	4346	0	3311	21021
normalized size	1	1.00	2.07	13.78	0.00	6.99	0.00	5.32	33.80
time (sec)	N/A	1.769	6.926	0.795	0.000	1.129	0.000	15.070	16.348

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	913	1372	0	0	0	0	-1
normalized size	1	1.00	2.85	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	17.935	1.944	0.000	0.794	0.000	0.000	0.000
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	225	443	0	0	0	0	-1
normalized size	1	1.00	1.02	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	4.591	1.693	0.000	0.000	0.000	0.000	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	6063	2337	0	0	0	0	-1
normalized size	1	1.00	15.96	6.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	24.765	1.884	0.000	1.280	0.000	0.000	0.000
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	230	581	0	0	0	0	-1
normalized size	1	1.00	0.71	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	6.308	1.704	0.000	0.000	0.000	0.000	0.000
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	7138	3285	0	0	0	0	-1
normalized size	1	1.00	16.15	7.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.629	25.995	2.199	0.000	1.466	0.000	0.000	0.000
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	145	215	0	0	0	0	-1
normalized size	1	1.00	0.70	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.117	2.687	1.720	0.000	1.381	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	251	318	0	0	0	0	-1
normalized size	1	1.00	1.16	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	8.624	1.804	0.000	0.000	0.000	0.000	0.000
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	1491	2010	0	0	0	0	-1
normalized size	1	1.00	3.97	5.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	14.819	1.804	0.000	0.000	0.000	0.000	0.000
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	2083	5710	0	0	0	0	-1
normalized size	1	1.00	4.21	11.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	17.431	1.806	0.000	1.592	0.000	0.000	0.000
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	39925	543	0	0	0	0	-1
normalized size	1	1.00	102.63	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	32.947	2.293	0.000	0.000	0.000	0.000	0.000
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	336	352	0	0	0	0	-1
normalized size	1	1.00	1.70	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	5.873	2.146	0.000	0.000	0.000	0.000	0.000
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	1708	2847	0	0	0	0	-1
normalized size	1	1.00	2.86	4.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.902	9.283	2.686	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	899	899	1990	15724	0	0	0	0	-1
normalized size	1	1.00	2.21	17.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.258	6.904	3.079	0.000	0.000	0.000	0.000	0.000
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	744	744	1750	4298	0	0	0	0	-1
normalized size	1	1.00	2.35	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.106	9.722	2.098	0.000	0.000	0.000	0.000	0.000
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	919	919	1960	13060	0	0	0	0	-1
normalized size	1	1.00	2.13	14.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.132	6.736	2.234	0.000	9.291	0.000	0.000	0.000
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2385	39420	0	0	0	0	-1
normalized size	1	1.00	2.13	35.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.163	7.414	3.402	0.000	8.699	0.000	0.000	0.000
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	891	891	2026	15922	0	0	0	0	-1
normalized size	1	1.00	2.27	17.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.009	6.726	2.296	0.000	0.000	0.000	0.000	0.000
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1150	2344	32283	0	0	0	0	-1
normalized size	1	1.00	2.04	28.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.435	7.316	2.977	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1428	1428	2979	75468	0	0	0	0	-1
normalized size	1	1.00	2.09	52.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.438	8.285	4.944	0.000	0.000	0.000	0.000	0.000
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F	F(-1)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	652	0	49385	491	0	0	0	0	-1
normalized size	1	0.00	75.74	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.093	32.785	2.103	0.000	0.000	0.000	0.000	0.000
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	325	352	0	0	0	0	-1
normalized size	1	1.00	1.64	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	5.536	2.129	0.000	4.521	0.000	0.000	0.000
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	249	292	0	0	0	0	-1
normalized size	1	1.00	0.63	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	2.372	2.152	0.000	5.836	0.000	0.000	0.000
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	763	1761	3451	0	0	0	0	-1
normalized size	1	1.23	2.83	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.316	9.642	2.361	0.000	0.000	0.000	0.000	0.000
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	2.367	1.601	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.092	52.113	1.327	0.000	0.000	0.000	0.000	0.000
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	89.285	1.263	0.000	0.000	0.000	0.000	0.000
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	2.613	1.448	0.000	0.000	0.000	0.000	0.000
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	53.308	1.270	0.000	0.000	0.000	0.000	0.000
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.096	92.805	1.366	0.000	0.000	0.000	0.000	0.000
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	62.593	1.344	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	102.048	1.270	0.000	0.000	0.000	0.000	0.000
Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.097	148.174	1.317	0.000	0.000	0.000	0.000	0.000
Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	2425	0	0	0	0	0	-1
normalized size	1	1.00	22.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	14.612	2.509	0.000	1.010	0.000	0.000	0.000
Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	343	0	0	0	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	2.305	1.273	0.000	0.784	0.000	0.000	0.000
Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	299	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	2.819	1.178	0.000	0.917	0.000	0.000	0.000
Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.209	1.224	0.000	0.956	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	1.178	1.139	0.000	0.677	0.000	0.000	0.000
Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.451	10.827	1.172	0.000	0.471	0.000	0.000	0.000
Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.116	2.691	1.450	0.000	0.521	0.000	0.000	0.000
Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	278	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.515	1.306	0.921	0.000	0.438	0.000	0.000	0.000
Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	200	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.542	1.050	0.000	0.467	0.000	0.000	0.000
Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	125	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.240	1.049	0.000	0.452	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	5411	0	0	0	0	0	-1
normalized size	1	1.00	26.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.402	25.625	0.845	0.000	0.441	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	14108	0	0	0	0	0	-1
normalized size	1	1.00	43.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	45.375	1.000	0.000	0.488	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [31] had the largest ratio of [.5769]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	9	1.00	26	0.346
2	A	11	8	1.00	26	0.308
3	A	5	3	1.00	26	0.115
4	A	4	3	1.00	26	0.115
5	A	4	3	1.00	24	0.125
6	A	8	6	1.00	26	0.231
7	A	9	6	1.00	26	0.231
8	A	12	7	1.00	26	0.269
9	A	15	7	1.00	26	0.269
10	A	18	7	1.00	26	0.269
11	A	13	8	1.00	26	0.308
12	A	6	3	1.00	26	0.115
13	A	5	3	1.00	26	0.115

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	5	3	1.00	26	0.115
15	A	9	8	1.00	24	0.333
16	A	15	11	1.00	26	0.423
17	A	13	9	1.00	26	0.346
18	A	15	9	1.00	26	0.346
19	A	19	9	1.00	26	0.346
20	A	23	9	1.00	26	0.346
21	A	26	14	1.12	26	0.538
22	A	21	13	1.00	26	0.500
23	A	13	9	1.00	26	0.346
24	A	9	6	1.00	26	0.231
25	A	7	5	1.00	24	0.208
26	A	4	3	1.00	26	0.115
27	A	4	3	1.00	26	0.115
28	A	5	3	1.00	26	0.115
29	A	13	8	1.00	26	0.308
30	A	17	9	1.00	26	0.346
31	A	29	15	1.00	26	0.577
32	A	20	13	1.00	26	0.500
33	A	15	9	1.00	26	0.346
34	A	12	7	1.00	26	0.269
35	A	9	6	1.00	24	0.250
36	A	12	8	1.00	26	0.308
37	A	5	3	1.00	26	0.115
38	A	5	3	1.00	26	0.115
39	A	6	3	1.00	26	0.115
40	A	14	8	1.00	26	0.308
41	A	18	9	1.00	26	0.346
42	A	5	4	1.00	28	0.143
43	A	5	4	1.00	28	0.143
44	A	5	4	1.00	28	0.143
45	A	4	4	1.00	26	0.154
46	A	4	4	1.00	28	0.143
47	A	5	4	1.00	28	0.143
48	A	6	4	1.00	28	0.143
49	A	7	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	6	5	1.00	28	0.179
51	A	6	5	1.00	28	0.179
52	A	5	5	1.00	26	0.192
53	A	4	4	1.00	28	0.143
54	A	5	5	1.00	28	0.179
55	A	6	5	1.00	28	0.179
56	A	7	5	1.00	28	0.179
57	A	5	4	1.00	28	0.143
58	A	5	4	1.00	28	0.143
59	A	5	4	1.00	26	0.154
60	A	5	4	1.00	28	0.143
61	A	5	4	1.00	28	0.143
62	A	5	4	1.00	28	0.143
63	A	5	4	1.00	28	0.143
64	A	5	4	1.00	28	0.143
65	A	8	6	1.00	28	0.214
66	A	7	6	1.00	28	0.214
67	A	6	5	1.00	28	0.179
68	A	5	4	1.00	26	0.154
69	A	6	5	1.00	28	0.179
70	A	7	6	1.00	28	0.214
71	A	8	6	1.00	28	0.214
72	A	8	6	1.00	28	0.214
73	A	7	6	1.00	28	0.214
74	A	7	6	1.13	28	0.214
75	A	6	5	1.15	26	0.192
76	A	7	6	1.00	28	0.214
77	A	8	6	1.00	28	0.214
78	A	9	6	1.00	28	0.214
79	A	9	7	1.00	28	0.250
80	A	8	7	1.00	28	0.250
81	A	7	6	1.00	28	0.214
82	A	7	6	1.00	28	0.214
83	A	7	6	1.22	26	0.231
84	A	8	7	1.00	28	0.250
85	A	9	7	1.00	28	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	3	1.00	30	0.100
87	A	4	3	1.00	30	0.100
88	A	3	3	1.00	30	0.100
89	A	2	2	1.00	30	0.067
90	A	2	2	1.00	30	0.067
91	A	3	3	1.00	30	0.100
92	A	4	3	1.00	30	0.100
93	A	5	3	1.00	30	0.100
94	A	5	4	1.00	30	0.133
95	A	3	3	1.00	30	0.100
96	A	3	3	1.00	30	0.100
97	A	3	2	1.00	30	0.067
98	A	3	3	1.00	30	0.100
99	A	4	4	1.00	30	0.133
100	A	5	4	1.00	30	0.133
101	A	4	3	1.00	30	0.100
102	A	5	4	1.00	30	0.133
103	A	4	3	1.00	30	0.100
104	A	3	2	1.00	30	0.067
105	A	3	3	1.00	30	0.100
106	A	3	3	1.00	30	0.100
107	A	4	4	1.00	30	0.133
108	A	5	4	1.00	30	0.133
109	A	6	4	1.00	30	0.133
110	A	3	2	1.00	30	0.067
111	A	3	2	1.00	30	0.067
112	A	3	2	1.00	30	0.067
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	30	0.067
115	A	3	2	1.29	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	30	0.067
118	A	3	3	1.00	30	0.100
119	A	3	3	1.00	30	0.100
120	A	3	3	1.00	30	0.100
121	A	3	2	1.00	30	0.067

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	3	3	1.00	30	0.100
123	A	3	2	1.00	30	0.067
124	A	3	2	1.00	30	0.067
125	A	3	3	1.00	30	0.100
126	A	4	4	1.00	30	0.133
127	A	4	3	1.00	30	0.100
128	A	3	2	1.00	30	0.067
129	A	3	2	1.00	30	0.067
130	A	4	3	1.00	30	0.100
131	A	2	2	1.00	24	0.083
132	A	3	3	1.00	26	0.115
133	A	3	3	1.00	26	0.115
134	A	3	3	1.00	26	0.115
135	A	3	3	1.00	24	0.125
136	A	3	3	1.00	26	0.115
137	A	3	3	1.00	26	0.115
138	A	4	3	1.00	28	0.107
139	A	3	3	1.00	28	0.107
140	A	2	2	1.00	28	0.071
141	A	4	4	1.00	28	0.143
142	A	5	5	1.00	28	0.179
143	A	6	5	1.00	27	0.185
144	A	3	3	1.00	27	0.111
145	A	3	3	1.00	27	0.111
146	A	5	5	1.00	27	0.185
147	A	5	4	1.00	27	0.148
148	A	5	4	1.00	27	0.148
149	A	5	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	5	5	1.00	27	0.185
152	A	7	6	1.00	27	0.222
153	A	8	7	1.00	27	0.259
154	A	6	5	1.00	27	0.185
155	A	5	5	1.00	27	0.185
156	A	5	5	1.00	25	0.200
157	A	5	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	7	6	1.00	27	0.222
159	A	8	6	1.00	27	0.222
160	A	5	4	1.00	27	0.148
161	A	5	4	1.00	27	0.148
162	A	6	5	1.00	25	0.200
163	A	7	5	1.00	27	0.185
164	A	10	6	1.00	27	0.222
165	A	14	6	1.00	27	0.222
166	A	9	5	1.00	27	0.185
167	A	7	4	1.00	27	0.148
168	A	5	4	1.00	25	0.160
169	A	8	7	1.00	27	0.259
170	A	12	6	1.00	27	0.222
171	A	16	6	1.00	27	0.222
172	A	10	5	1.00	27	0.185
173	A	10	5	1.00	27	0.185
174	A	6	5	1.00	25	0.200
175	A	12	6	1.00	27	0.222
176	A	15	6	1.00	27	0.222
177	A	19	6	1.00	27	0.222
178	A	14	5	1.00	27	0.185
179	A	14	5	1.00	27	0.185
180	A	7	5	1.00	25	0.200
181	A	16	6	1.00	27	0.222
182	A	19	6	1.00	27	0.222
183	A	23	6	1.00	27	0.222
184	A	5	5	1.00	29	0.172
185	A	2	2	1.00	29	0.069
186	A	5	5	1.00	29	0.172
187	A	5	4	1.00	29	0.138
188	A	5	4	1.00	29	0.138
189	A	4	4	1.00	23	0.174
190	A	5	5	1.00	23	0.217
191	A	6	6	1.00	23	0.261
192	A	5	5	1.00	25	0.200
193	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	7	7	1.00	25	0.280
195	A	6	6	1.00	25	0.240
196	A	7	7	1.00	25	0.280
197	A	8	8	1.00	25	0.320
198	A	5	5	1.00	25	0.200
199	A	3	3	1.00	27	0.111
200	A	6	6	1.00	25	0.240
201	A	5	5	1.00	27	0.185
202	A	7	7	1.00	25	0.280
203	A	3	3	1.00	25	0.120
204	A	3	3	1.00	27	0.111
205	A	6	6	1.00	25	0.240
206	A	7	7	1.00	25	0.280
207	A	3	3	1.00	29	0.103
208	A	1	1	1.00	29	0.034
209	A	5	5	1.00	29	0.172
210	A	7	7	1.00	29	0.241
211	A	6	6	1.00	29	0.207
212	A	7	7	1.00	29	0.241
213	A	8	8	1.00	29	0.276
214	A	7	7	1.00	29	0.241
215	A	8	8	1.00	29	0.276
216	A	9	8	1.00	29	0.276
217	F	0	0	N/A	0	N/A
218	A	1	1	1.00	29	0.034
219	A	3	3	1.00	29	0.103
220	A	6	6	1.23	29	0.207
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000
224	A	0	0	0.00	0	0.000
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	0	0	0.00	0	0.000
229	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
230	A	4	4	1.00	27	0.148
231	A	8	6	1.00	27	0.222
232	A	7	5	1.00	27	0.185
233	A	6	4	1.00	25	0.160
234	A	7	5	1.00	27	0.185
235	A	8	6	1.00	27	0.222
236	A	0	0	0.00	0	0.000
237	A	8	6	1.00	27	0.222
238	A	7	5	1.00	27	0.185
239	A	6	4	1.00	25	0.160
240	A	7	5	1.00	27	0.185
241	A	10	5	1.00	27	0.185

Chapter 3

Listing of integrals

3.1 $\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^5 dx$

Optimal. Leaf size=196

$$\frac{3a^2c^5 \tan^5(e + fx)}{5f} + \frac{a^2c^5 \tan^3(e + fx)}{3f} - \frac{a^2c^5 \tan(e + fx)}{f} - \frac{19a^2c^5 \tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^2c^5 \tan^3(e + fx) \sec^3(e + fx)}{6f}$$

[Out] $a^2c^5x - 19/16a^2c^5 \arctanh(\sin(fx+e))/f - a^2c^5 \tan(fx+e)/f + 17/16a^2c^5 \sec(fx+e) \tan(fx+e)/f + 1/8a^2c^5 \sec^3(fx+e) \tan(fx+e)/f + 1/3a^2c^5 \tan^3(fx+e)/f - 3/4a^2c^5 \sec(fx+e) \tan^3(fx+e)/f - 1/6a^2c^5 \sec^3(fx+e) \tan^3(fx+e)/f + 3/5a^2c^5 \tan^5(fx+e)/f$

Rubi [A] time = 0.30, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30, 3768}

$$\frac{3a^2c^5 \tan^5(e + fx)}{5f} + \frac{a^2c^5 \tan^3(e + fx)}{3f} - \frac{a^2c^5 \tan(e + fx)}{f} - \frac{19a^2c^5 \tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^2c^5 \tan^3(e + fx) \sec^3(e + fx)}{6f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \text{Sec}[e + fx])^2 (c - c \text{Sec}[e + fx])^5, x]$

[Out] $a^2c^5x - (19a^2c^5 \text{ArcTanh}[\text{Sin}[e + fx]])/(16f) - (a^2c^5 \text{Tan}[e + fx])/f + (17a^2c^5 \text{Sec}[e + fx] \text{Tan}[e + fx])/(16f) + (a^2c^5 \text{Sec}[e + fx]^3 \text{Tan}[e + fx])/(8f) + (a^2c^5 \text{Tan}[e + fx]^3)/(3f) - (3a^2c^5 \text{Sec}[e + fx] \text{Tan}[e + fx]^3)/(4f) - (a^2c^5 \text{Sec}[e + fx]^3 \text{Tan}[e + fx]^3)/(6f) + (3a^2c^5 \text{Tan}[e + fx]^5)/(5f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 2607

$\text{Int}[\sec[(e_.) + (f_.)(x_)]^{(m_.)} ((b_.) \tan[(e_.) + (f_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n (1+x^2)^{(m/2-1)}, x], x, \text{Tan}[e + fx]], x] /; \text{FreeQ}\{b, e, f, n\}, x \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ !(\text{IntegerQ}[(n-1)/2]) \ \&\& \ \text{LtQ}[0, n, m-1]$

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m, 2*n]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^5 dx &= (a^2 c^2) \int (c - c \sec(e + fx))^3 \tan^4(e + fx) dx \\
&= (a^2 c^2) \int (c^3 \tan^4(e + fx) - 3c^3 \sec(e + fx) \tan^4(e + fx) + 3c^3 \sec^3(e + fx) \tan^4(e + fx)) dx \\
&= (a^2 c^5) \int \tan^4(e + fx) dx - (a^2 c^5) \int \sec^3(e + fx) \tan^4(e + fx) dx \\
&= \frac{a^2 c^5 \tan^3(e + fx)}{3f} - \frac{3a^2 c^5 \sec(e + fx) \tan^3(e + fx)}{4f} - \frac{a^2 c^5 \sec^3(e + fx) \tan^3(e + fx)}{4f} \\
&= -\frac{a^2 c^5 \tan(e + fx)}{f} + \frac{9a^2 c^5 \sec(e + fx) \tan(e + fx)}{8f} + \frac{a^2 c^5 \sec^3(e + fx) \tan^3(e + fx)}{4f} \\
&= a^2 c^5 x - \frac{9a^2 c^5 \tanh^{-1}(\sin(e + fx))}{8f} - \frac{a^2 c^5 \tan(e + fx)}{f} + \frac{17a^2 c^5 \sec^3(e + fx) \tan^3(e + fx)}{4f} \\
&= a^2 c^5 x - \frac{19a^2 c^5 \tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^2 c^5 \tan(e + fx)}{f} + \frac{17a^2 c^5 \sec^3(e + fx) \tan^3(e + fx)}{4f}
\end{aligned}$$

Mathematica [A] time = 2.13, size = 165, normalized size = 0.84

$$a^2 c^5 \sec^6(e + fx) \left(-210 \sin(e + fx) - 120 \sin(2(e + fx)) + 865 \sin(3(e + fx)) - 768 \sin(4(e + fx)) + 435 \sin(5(e + fx)) - 88 \sin(6(e + fx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]

[Out] (a^2*c^5*Sec[e + f*x]^6*(1200*e + 1200*f*x - 4560*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 720*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)] - 210*Sin[e + f*x] - 120*Sin[2*(e + f*x)] + 865*Sin[3*(e + f*x)] - 768*Sin[4*(e + f*x)] + 435*Sin[5*(e + f*x)] - 88*Sin[6*(e + f*x)])/(3840*f)

fricas [A] time = 0.46, size = 179, normalized size = 0.91

$$480 a^2 c^5 f x \cos(fx + e)^6 - 285 a^2 c^5 \cos(fx + e)^6 \log(\sin(fx + e) + 1) + 285 a^2 c^5 \cos(fx + e)^6 \log(-\sin(fx + e) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/480*(480*a^2*c^5*f*x*cos(f*x + e)^6 - 285*a^2*c^5*cos(f*x + e)^6*log(sin(f*x + e) + 1) + 285*a^2*c^5*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(176*a^2*c^5*cos(f*x + e)^5 - 435*a^2*c^5*cos(f*x + e)^4 + 208*a^2*c^5*cos(f*x + e)^3 + 110*a^2*c^5*cos(f*x + e)^2 - 144*a^2*c^5*cos(f*x + e) + 40*a^2*c^5)*sin(f*x + e))/(f*cos(f*x + e)^6)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-2*a^2*c^5/2*(f*x+exp(1))/2-19*a^2*c^5/32*ln(abs(tan((f*x+exp(1))/2)-1))+19*a^2*c^5/32*ln(abs(tan((f*x+exp(1))/2)+1))-(525*tan((f*x+exp(1))/2)^11*a^2*c^5-3135*tan((f*x+exp(1))/2)^9*a^2*c^5+1746*tan((f*x+exp(1))/2)^7*a^2*c^5-366*tan((f*x+exp(1))/2)^5*a^2*c^5-95*tan((f*x+exp(1))/2)^3*a^2*c^5+45*tan((f*x+exp(1))/2)*a^2*c^5)*1/240/(tan((f*x+exp(1))/2)^2-1)^6)

maple [A] time = 1.58, size = 186, normalized size = 0.95

$$\frac{11a^2c^5(\sec^3(fx + e))\tan(fx + e)}{24f} + \frac{29a^2c^5\sec(fx + e)\tan(fx + e)}{16f} - \frac{19c^5a^2\ln(\sec(fx + e) + \tan(fx + e))}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x)

[Out] -11/24*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+29/16*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f-19/16/f*c^5*a^2*ln(sec(f*x+e)+tan(f*x+e))-11/15*a^2*c^5*tan(f*x+e)/f-13/15/f*c^5*a^2*tan(f*x+e)*sec(f*x+e)^2+a^2*c^5*x+1/f*a^2*c^5*e+3/5/f*c^5*a^2*tan(f*x+e)*sec(f*x+e)^4-1/6/f*c^5*a^2*tan(f*x+e)*sec(f*x+e)^5

maxima [A] time = 0.35, size = 334, normalized size = 1.70

$$96 \left(3 \tan(fx + e)^5 + 10 \tan(fx + e)^3 + 15 \tan(fx + e) \right) a^2 c^5 - 800 \left(\tan(fx + e)^3 + 3 \tan(fx + e) \right) a^2 c^5 + 480$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] $\frac{1}{480} \cdot (96 \cdot (3 \cdot \tan(fx + e)^5 + 10 \cdot \tan(fx + e)^3 + 15 \cdot \tan(fx + e)) \cdot a^2 \cdot c^5 - 800 \cdot (\tan(fx + e)^3 + 3 \cdot \tan(fx + e)) \cdot a^2 \cdot c^5 + 480 \cdot (fx + e) \cdot a^2 \cdot c^5 + 5 \cdot a^2 \cdot c^5 \cdot (2 \cdot (15 \cdot \sin(fx + e)^5 - 40 \cdot \sin(fx + e)^3 + 33 \cdot \sin(fx + e)) / (\sin(fx + e)^6 - 3 \cdot \sin(fx + e)^4 + 3 \cdot \sin(fx + e)^2 - 1) - 15 \cdot \log(\sin(fx + e) + 1) + 15 \cdot \log(\sin(fx + e) - 1)) + 30 \cdot a^2 \cdot c^5 \cdot (2 \cdot (3 \cdot \sin(fx + e)^3 - 5 \cdot \sin(fx + e)) / (\sin(fx + e)^4 - 2 \cdot \sin(fx + e)^2 + 1) - 3 \cdot \log(\sin(fx + e) + 1) + 3 \cdot \log(\sin(fx + e) - 1)) - 600 \cdot a^2 \cdot c^5 \cdot (2 \cdot \sin(fx + e) / (\sin(fx + e)^2 - 1) - \log(\sin(fx + e) + 1) + \log(\sin(fx + e) - 1)) - 1440 \cdot a^2 \cdot c^5 \cdot \log(\sec(fx + e) + \tan(fx + e)) + 480 \cdot a^2 \cdot c^5 \cdot \tan(fx + e)) / f$

mupad [B] time = 2.53, size = 228, normalized size = 1.16

$$a^2 c^5 x - \frac{-\frac{35 a^2 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{11}}{8} + \frac{209 a^2 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9}{8} - \frac{291 a^2 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{20} + \frac{61 a^2 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{20} + \frac{19 a^2 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{24} - \frac{3 a^2 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{24}}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} - 6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} + 15 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 20 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 15 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5,x)

[Out] $a^2 \cdot c^5 \cdot x - \left(\frac{19 \cdot a^2 \cdot c^5 \cdot \tan(e/2 + (fx)/2)^3}{24} + \frac{61 \cdot a^2 \cdot c^5 \cdot \tan(e/2 + (fx)/2)^5}{20} - \frac{291 \cdot a^2 \cdot c^5 \cdot \tan(e/2 + (fx)/2)^7}{20} + \frac{209 \cdot a^2 \cdot c^5 \cdot \tan(e/2 + (fx)/2)^9}{8} - \frac{35 \cdot a^2 \cdot c^5 \cdot \tan(e/2 + (fx)/2)^{11}}{8} - \frac{3 \cdot a^2 \cdot c^5 \cdot \tan(e/2 + (fx)/2)^{13}}{24} \right) / (f \cdot (15 \cdot \tan(e/2 + (fx)/2)^4 - 6 \cdot \tan(e/2 + (fx)/2)^2 - 20 \cdot \tan(e/2 + (fx)/2)^6 + 15 \cdot \tan(e/2 + (fx)/2)^8 - 6 \cdot \tan(e/2 + (fx)/2)^{10} + \tan(e/2 + (fx)/2)^{12} + 1) - (19 \cdot a^2 \cdot c^5 \cdot \operatorname{atanh}(\tan(e/2 + (fx)/2))) / (8 \cdot f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2 c^5 \left(\int (-1) dx + \int 3 \sec(e + fx) dx + \int (-\sec^2(e + fx)) dx + \int (-5 \sec^3(e + fx)) dx + \int 5 \sec^4(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)

[Out] $-a^{**2} \cdot c^{**5} \cdot (\operatorname{Integral}(-1, x) + \operatorname{Integral}(3 \cdot \sec(e + fx), x) + \operatorname{Integral}(-\sec(e + fx)^{**2}, x) + \operatorname{Integral}(-5 \cdot \sec(e + fx)^{**3}, x) + \operatorname{Integral}(5 \cdot \sec(e + fx)^{**4}, x) + \operatorname{Integral}(\sec(e + fx)^{**5}, x) + \operatorname{Integral}(-3 \cdot \sec(e + fx)^{**6}, x) + \operatorname{Integral}(\sec(e + fx)^{**7}, x))$

3.2 $\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^4 dx$

Optimal. Leaf size=140

$$\frac{a^2 c^4 \tan^5(e + fx)}{5f} + \frac{a^2 c^4 \tan^3(e + fx)}{3f} - \frac{a^2 c^4 \tan(e + fx)}{f} - \frac{3a^2 c^4 \tanh^{-1}(\sin(e + fx))}{4f} - \frac{a^2 c^4 \tan^3(e + fx) \sec(e + fx)}{2f}$$

[Out] a^2*c^4*x-3/4*a^2*c^4*arctanh(sin(f*x+e))/f-a^2*c^4*tan(f*x+e)/f+3/4*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f+1/3*a^2*c^4*tan(f*x+e)^3/f-1/2*a^2*c^4*sec(f*x+e)*tan(f*x+e)^3/f+1/5*a^2*c^4*tan(f*x+e)^5/f

Rubi [A] time = 0.20, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^2 c^4 \tan^5(e + fx)}{5f} + \frac{a^2 c^4 \tan^3(e + fx)}{3f} - \frac{a^2 c^4 \tan(e + fx)}{f} - \frac{3a^2 c^4 \tanh^{-1}(\sin(e + fx))}{4f} - \frac{a^2 c^4 \tan^3(e + fx) \sec(e + fx)}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]

[Out] a^2*c^4*x - (3*a^2*c^4*ArcTanh[Sin[e + f*x]])/(4*f) - (a^2*c^4*Tan[e + f*x])/f + (3*a^2*c^4*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (a^2*c^4*Tan[e + f*x]^3)/(3*f) - (a^2*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (a^2*c^4*Tan[e + f*x]^5)/(5*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2607

Int[sec[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^(n*(1 + x^2)^(m/2 - 1)), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 2611

Int[((a_)*sec[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3473

Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_.))^(n_), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^4 dx &= (a^2 c^2) \int (c - c \sec(e + fx))^2 \tan^4(e + fx) dx \\
&= (a^2 c^2) \int (c^2 \tan^4(e + fx) - 2c^2 \sec(e + fx) \tan^4(e + fx) + c^2 \sec^2(e + fx) \tan^4(e + fx)) dx \\
&= (a^2 c^4) \int \tan^4(e + fx) dx + (a^2 c^4) \int \sec^2(e + fx) \tan^4(e + fx) dx \\
&= \frac{a^2 c^4 \tan^3(e + fx)}{3f} - \frac{a^2 c^4 \sec(e + fx) \tan^3(e + fx)}{2f} - (a^2 c^4) \int \tan^2(e + fx) dx \\
&= -\frac{a^2 c^4 \tan(e + fx)}{f} + \frac{3a^2 c^4 \sec(e + fx) \tan(e + fx)}{4f} + \frac{a^2 c^4 \tan^3(e + fx)}{3f} \\
&= a^2 c^4 x - \frac{3a^2 c^4 \tanh^{-1}(\sin(e + fx))}{4f} - \frac{a^2 c^4 \tan(e + fx)}{f} + \frac{3a^2 c^4 \sec^3(e + fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 1.17, size = 146, normalized size = 1.04

$$\frac{a^2 c^4 \sec^5(e + fx) (40 \sin(e + fx) + 60 \sin(2(e + fx)) - 220 \sin(3(e + fx)) + 150 \sin(4(e + fx)) - 68 \sin(5(e + fx)))}{(960 * f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*Sec[e + f*x]^5*(600*(e + f*x)*Cos[e + f*x] - 720*ArcTanh[Sin[e + f
*x]]*Cos[e + f*x]^5 + 300*e*Cos[3*(e + f*x)] + 300*f*x*Cos[3*(e + f*x)] + 6
0*e*Cos[5*(e + f*x)] + 60*f*x*Cos[5*(e + f*x)] + 40*Sin[e + f*x] + 60*Sin[2
*(e + f*x)] - 220*Sin[3*(e + f*x)] + 150*Sin[4*(e + f*x)] - 68*Sin[5*(e + f
*x)]))/(960*f)
```

fricas [A] time = 0.52, size = 163, normalized size = 1.16

$$120 a^2 c^4 f x \cos(fx + e)^5 - 45 a^2 c^4 \cos(fx + e)^5 \log(\sin(fx + e) + 1) + 45 a^2 c^4 \cos(fx + e)^5 \log(-\sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(120*a^2*c^4*f*x*cos(f*x + e)^5 - 45*a^2*c^4*cos(f*x + e)^5*log(sin(f*x + e) + 1) + 45*a^2*c^4*cos(f*x + e)^5*log(-sin(f*x + e) + 1) - 2*(68*a^2*c^4*cos(f*x + e)^4 - 75*a^2*c^4*cos(f*x + e)^3 + 4*a^2*c^4*cos(f*x + e)^2 + 30*a^2*c^4*cos(f*x + e) - 12*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*a^2*c^4/2*(f*x+exp(1))/2+3*a^2*c^4/8*ln(abs(tan((f*x+exp(1))/2)-1))-3*a^2*c^4/8*ln(abs(tan((f*x+exp(1))/2)+1))-(-105*tan((f*x+exp(1))/2)^9*a^2*c^4+530*tan((f*x+exp(1))/2)^7*a^2*c^4-328*tan((f*x+exp(1))/2)^5*a^2*c^4+110*tan((f*x+exp(1))/2)^3*a^2*c^4-15*tan((f*x+exp(1))/2)*a^2*c^4)*1/60/(tan((f*x+exp(1))/2)^2-1)^5)
```

maple [A] time = 1.41, size = 161, normalized size = 1.15

$$\frac{17a^2c^4 \tan(fx + e)}{15f} - \frac{a^2c^4 \tan(fx + e) (\sec^2(fx + e))}{15f} + \frac{5a^2c^4 \sec(fx + e) \tan(fx + e)}{4f} - \frac{3a^2c^4 \ln(\sec(fx + e))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -17/15*a^2*c^4*tan(f*x+e)/f-1/15/f*a^2*c^4*tan(f*x+e)*sec(f*x+e)^2+5/4*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f-3/4/f*a^2*c^4*ln(sec(f*x+e)+tan(f*x+e))+a^2*c^4*x+1/f*a^2*c^4*e-1/2/f*a^2*c^4*tan(f*x+e)*sec(f*x+e)^3+1/5/f*a^2*c^4*tan(f*x+e)*sec(f*x+e)^4
```

maxima [A] time = 0.34, size = 240, normalized size = 1.71

$$8 \left(3 \tan(fx + e)^5 + 10 \tan(fx + e)^3 + 15 \tan(fx + e) \right) a^2 c^4 - 40 \left(\tan(fx + e)^3 + 3 \tan(fx + e) \right) a^2 c^4 + 120$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4 - 40*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 120*(f*x + e)*a^2*c^4 + 15*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 240*a^2*c^4*log(sec(f*x + e) + tan(f*x + e)) - 120*a^2*c^4*tan(f*x + e))/f
```

mupad [B] time = 2.30, size = 195, normalized size = 1.39

$$a^2 c^4 x + \frac{7 a^2 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9}{2} - \frac{53 a^2 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{3} + \frac{164 a^2 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{15} - \frac{11 a^2 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3} + \frac{a^2 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{2} - \frac{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} - 5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 + 10 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 - 10 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + 5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4,x)
```

```
[Out] a^2*c^4*x + ((164*a^2*c^4*tan(e/2 + (f*x)/2)^5)/15 - (11*a^2*c^4*tan(e/2 +
(f*x)/2)^3)/3 - (53*a^2*c^4*tan(e/2 + (f*x)/2)^7)/3 + (7*a^2*c^4*tan(e/2 +
(f*x)/2)^9)/2 + (a^2*c^4*tan(e/2 + (f*x)/2))/2)/(f*(5*tan(e/2 + (f*x)/2)^2
- 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 - 1)) - (3*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(2*
f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^4 \left(\int 1 dx + \int (-2 \sec(e + fx)) dx + \int (-\sec^2(e + fx)) dx + \int 4 \sec^3(e + fx) dx + \int (-\sec^4(e + fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(1, x) + Integral(-2*sec(e + f*x), x) + Integral(-sec(e
+ f*x)**2, x) + Integral(4*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4,
x) + Integral(-2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

3.3 $\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^3 dx$

Optimal. Leaf size=97

$$-\frac{3a^2c^3 \tanh^{-1}(\sin(e + fx))}{8f} + \frac{a^2 \tan^3(e + fx) (4c^3 - 3c^3 \sec(e + fx))}{12f} - \frac{a^2 \tan(e + fx) (8c^3 - 3c^3 \sec(e + fx))}{8f} +$$

[Out] a^2*c^3*x-3/8*a^2*c^3*arctanh(sin(f*x+e))/f-1/8*a^2*(8*c^3-3*c^3*sec(f*x+e))*tan(f*x+e)/f+1/12*a^2*(4*c^3-3*c^3*sec(f*x+e))*tan(f*x+e)^3/f

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3881, 3770}

$$-\frac{3a^2c^3 \tanh^{-1}(\sin(e + fx))}{8f} + \frac{a^2 \tan^3(e + fx) (4c^3 - 3c^3 \sec(e + fx))}{12f} - \frac{a^2 \tan(e + fx) (8c^3 - 3c^3 \sec(e + fx))}{8f} +$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]

[Out] a^2*c^3*x - (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (a^2*(8*c^3 - 3*c^3*Sec[e + f*x])*Tan[e + f*x])/(8*f) + (a^2*(4*c^3 - 3*c^3*Sec[e + f*x])*Tan[e + f*x]^3)/(12*f)

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3881

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^m*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^n, x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^3 dx &= (a^2 c^2) \int (c - c \sec(e + fx)) \tan^4(e + fx) dx \\
&= \frac{a^2 (4c^3 - 3c^3 \sec(e + fx)) \tan^3(e + fx)}{12f} - \frac{1}{4} (a^2 c^2) \int (4c - 3c \sec(e + fx)) \tan^2(e + fx) dx \\
&= -\frac{a^2 (8c^3 - 3c^3 \sec(e + fx)) \tan(e + fx)}{8f} + \frac{a^2 (4c^3 - 3c^3 \sec(e + fx)) \tan^3(e + fx)}{12f} \\
&= a^2 c^3 x - \frac{a^2 (8c^3 - 3c^3 \sec(e + fx)) \tan(e + fx)}{8f} + \frac{a^2 (4c^3 - 3c^3 \sec(e + fx)) \tan^3(e + fx)}{12f} \\
&= a^2 c^3 x - \frac{3a^2 c^3 \tanh^{-1}(\sin(e + fx))}{8f} - \frac{a^2 (8c^3 - 3c^3 \sec(e + fx)) \tan^3(e + fx)}{8f}
\end{aligned}$$

Mathematica [A] time = 0.73, size = 122, normalized size = 1.26

$$\frac{a^2 c^3 \sec^4(e + fx) (-18 \sin(e + fx) - 32 \sin(2(e + fx)) + 30 \sin(3(e + fx)) - 32 \sin(4(e + fx)) + 96(e + fx) \cos(2(e + fx)) - 32 \sin(4(e + fx)))}{192f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]

[Out] (a^2*c^3*Sec[e + f*x]^4*(72*e + 72*f*x - 72*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[4*(e + f*x)] - 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] + 30*Sin[3*(e + f*x)] - 32*Sin[4*(e + f*x)]))/(192*f)

fricas [A] time = 0.49, size = 147, normalized size = 1.52

$$\frac{48 a^2 c^3 f x \cos(fx + e)^4 - 9 a^2 c^3 \cos(fx + e)^4 \log(\sin(fx + e) + 1) + 9 a^2 c^3 \cos(fx + e)^4 \log(-\sin(fx + e) + 1)}{48 f \cos(fx + e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/48*(48*a^2*c^3*f*x*cos(f*x + e)^4 - 9*a^2*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) + 9*a^2*c^3*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^2*c^3*cos(f*x + e)^3 - 15*a^2*c^3*cos(f*x + e)^2 - 8*a^2*c^3*cos(f*x + e) + 6*a^2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-2*a^2*c^3/2*(f*x+exp(1))/2-3*a^2*c^3/16*ln(abs(tan((f*x+exp(1))/2)-1))+3*a^2*c^3/16*ln(abs(tan((f*x+exp(1))/2)+1))+(-33*tan((f*x+exp(1))/2)^7*a^2*c^3+137*tan((f*x+exp(1))/2)^5*a^2*c^3-71*tan((f*x+exp(1))/2)^3*a^2*c^3+15*tan((f*x+exp(1))/2)*a^2*c^3)*1/24/(tan((f*x+exp(1))/2)^2-1)^4)

maple [A] time = 1.22, size = 136, normalized size = 1.40

$$\frac{5c^3a^2 \sec(fx + e) \tan(fx + e)}{8f} - \frac{3c^3a^2 \ln(\sec(fx + e) + \tan(fx + e))}{8f} - \frac{4a^2c^3 \tan(fx + e)}{3f} + a^2c^3x + \frac{a^2c^3e}{f} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x)

[Out] 5/8/f*c^3*a^2*sec(f*x+e)*tan(f*x+e)-3/8/f*c^3*a^2*ln(sec(f*x+e)+tan(f*x+e))
-4/3*a^2*c^3*tan(f*x+e)/f+a^2*c^3*x+1/f*a^2*c^3*e+1/3/f*c^3*a^2*tan(f*x+e)*
sec(f*x+e)^2-1/4/f*c^3*a^2*tan(f*x+e)*sec(f*x+e)^3

maxima [B] time = 0.33, size = 203, normalized size = 2.09

$$16 \left(\tan(fx + e)^3 + 3 \tan(fx + e) \right) a^2 c^3 + 48 (fx + e) a^2 c^3 + 3 a^2 c^3 \left(\frac{2 \left(3 \sin(fx + e)^3 - 5 \sin(fx + e) \right)}{\sin(fx + e)^4 - 2 \sin(fx + e)^2 + 1} - 3 \log(\sin(fx + e) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 48*(f*x + e)*a^2*c^3 +
3*a^2*c^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 24*a^
2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(si
n(f*x + e) - 1)) - 48*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) - 96*a^2*c^3
*tan(f*x + e))/f

mupad [B] time = 2.19, size = 163, normalized size = 1.68

$$\frac{\frac{11a^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{4} - \frac{137a^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{12} + \frac{71a^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{12} - \frac{5a^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{4}}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1 \right)} + a^2c^3x - \frac{3a^2c^3 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3,x)

[Out] ((71*a^2*c^3*tan(e/2 + (f*x)/2)^3)/12 - (137*a^2*c^3*tan(e/2 + (f*x)/2)^5)/
12 + (11*a^2*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^2*c^3*tan(e/2 + (f*x)/2))/4
)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^2*c^3*x - (3*a^2*c^3*atanh(tan(e/2 + (f
*x)/2)))/(4*f)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^3 \left(\int (-1) dx + \int \sec(e + fx) dx + \int 2 \sec^2(e + fx) dx + \int (-2 \sec^3(e + fx)) dx + \int (-\sec^4(e + fx)) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)

[Out] -a**2*c**3*(Integral(-1, x) + Integral(sec(e + f*x), x) + Integral(2*sec(e
+ f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**4
, x) + Integral(sec(e + f*x)**5, x))

3.4 $\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^2 dx$

Optimal. Leaf size=47

$$\frac{a^2 c^2 \tan^3(e + fx)}{3f} - \frac{a^2 c^2 \tan(e + fx)}{f} + a^2 c^2 x$$

[Out] $a^2 c^2 x - a^2 c^2 \tan(fx + e)/f + 1/3 a^2 c^2 \tan(fx + e)^3/f$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3473, 8}

$$\frac{a^2 c^2 \tan^3(e + fx)}{3f} - \frac{a^2 c^2 \tan(e + fx)}{f} + a^2 c^2 x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \text{Sec}[e + f*x])^2 (c - c \text{Sec}[e + f*x])^2, x]$

[Out] $a^2 c^2 x - (a^2 c^2 \text{Tan}[e + f*x])/f + (a^2 c^2 \text{Tan}[e + f*x]^3)/(3*f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3473

$\text{Int}[(b \text{Tan}[c + d*x])^n, x_Symbol] \rightarrow \text{Simp}[(b \text{Tan}[c + d*x])^{n-1}]/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \text{Tan}[c + d*x])^{n-2}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1]$

Rule 3904

$\text{Int}[(\text{Csc}[e + f*x] + (f*x)) * (b + a) * (\text{Csc}[e + f*x] + (f*x)) * (d + c)]^n, x_Symbol] \rightarrow \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{2*m} * (c + d * \text{Csc}[e + f*x])^{n-m}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^2 dx &= (a^2 c^2) \int \tan^4(e + fx) dx \\ &= \frac{a^2 c^2 \tan^3(e + fx)}{3f} - (a^2 c^2) \int \tan^2(e + fx) dx \\ &= -\frac{a^2 c^2 \tan(e + fx)}{f} + \frac{a^2 c^2 \tan^3(e + fx)}{3f} + (a^2 c^2) \int 1 dx \\ &= a^2 c^2 x - \frac{a^2 c^2 \tan(e + fx)}{f} + \frac{a^2 c^2 \tan^3(e + fx)}{3f} \end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.96

$$a^2 c^2 \left(\frac{\tan^{-1}(\tan(e + fx))}{f} + \frac{\tan^3(e + fx)}{3f} - \frac{\tan(e + fx)}{f} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]

[Out] $a^2c^2(\text{ArcTan}[\text{Tan}[e + fx]]/f - \text{Tan}[e + fx]/f + \text{Tan}[e + fx]^3/(3f))$

fricas [A] time = 0.43, size = 65, normalized size = 1.38

$$\frac{3a^2c^2fx \cos(fx + e)^3 - \left(4a^2c^2 \cos(fx + e)^2 - a^2c^2\right) \sin(fx + e)}{3f \cos(fx + e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] $1/3*(3a^2c^2fx \cos(fx + e)^3 - (4a^2c^2 \cos(fx + e)^2 - a^2c^2) \sin(fx + e))/(f \cos(fx + e)^3)$

giac [A] time = 0.30, size = 51, normalized size = 1.09

$$\frac{a^2c^2 \tan(fx + e)^3 + 3(fx + e)a^2c^2 - 3a^2c^2 \tan(fx + e)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] $1/3*(a^2c^2 \tan(fx + e)^3 + 3*(fx + e)a^2c^2 - 3a^2c^2 \tan(fx + e))/f$

maple [A] time = 0.82, size = 58, normalized size = 1.23

$$\frac{-2a^2c^2 \tan(fx + e) + (fx + e)a^2c^2 - a^2c^2 \left(-\frac{2}{3} - \frac{(\sec^2(fx+e))}{3}\right) \tan(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x)

[Out] $1/f*(-2a^2c^2 \tan(fx+e) + (fx+e)a^2c^2 - a^2c^2*(-2/3 - 1/3 \sec^2(fx+e)) \tan(fx+e))$

maxima [A] time = 0.32, size = 57, normalized size = 1.21

$$\frac{(\tan(fx + e)^3 + 3 \tan(fx + e))a^2c^2 + 3(fx + e)a^2c^2 - 6a^2c^2 \tan(fx + e)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] $1/3*((\tan(fx + e)^3 + 3 \tan(fx + e))a^2c^2 + 3*(fx + e)a^2c^2 - 6a^2c^2 \tan(fx + e))/f$

mupad [B] time = 3.71, size = 84, normalized size = 1.79

$$a^2c^2x + \frac{2a^2c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 - \frac{20a^2c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3} + 2a^2c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2,x)
```

```
[Out] a^2*c^2*x + (2*a^2*c^2*tan(e/2 + (f*x)/2)^5 - (20*a^2*c^2*tan(e/2 + (f*x)/2)^3)/3 + 2*a^2*c^2*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2c^2 \left(\int 1 dx + \int (-2 \sec^2(e + fx)) dx + \int \sec^4(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*c**2*(Integral(1, x) + Integral(-2*sec(e + f*x)**2, x) + Integral(sec(e + f*x)**4, x))
```

3.5 $\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx)) dx$

Optimal. Leaf size=55

$$\frac{a^2 c \tanh^{-1}(\sin(e + fx))}{2f} - \frac{c \tan(e + fx) (a^2 \sec(e + fx) + 2a^2)}{2f} + a^2 cx$$

[Out] $a^2 c x + 1/2 a^2 c \operatorname{arctanh}(\sin(f x + e)) / f - 1/2 c (2 a^2 + a^2 \sec(f x + e)) \tan(f x + e) / f$

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3904, 3881, 3770}

$$\frac{a^2 c \tanh^{-1}(\sin(e + fx))}{2f} - \frac{c \tan(e + fx) (a^2 \sec(e + fx) + 2a^2)}{2f} + a^2 cx$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x]), x]$

[Out] $a^2 c x + (a^2 c \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]) / (2 f) - (c (2 a^2 + a^2 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]) / (2 f)$

Rule 3770

$\text{Int}[\operatorname{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d x]] / d, x] / ; \text{FreeQ}\{c, d\}, x]$

Rule 3881

$\text{Int}[(\operatorname{cot}[(c_.) + (d_.)(x_.)] * (e_.))^{(m_.)} * (\operatorname{csc}[(c_.) + (d_.)(x_.)] * (b_.) + (a_.)), x_Symbol] \rightarrow -\text{Simp}[(e * (e * \operatorname{Cot}[c + d x])^{(m - 1)} * (a * m + b * (m - 1) * \operatorname{Csc}[c + d x])) / (d * m * (m - 1)), x] - \text{Dist}[e^{2/m}, \text{Int}[(e * \operatorname{Cot}[c + d x])^{(m - 2)} * (a * m + b * (m - 1) * \operatorname{Csc}[c + d x]), x], x] / ; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{GtQ}[m, 1]$

Rule 3904

$\text{Int}[(\operatorname{csc}[(e_.) + (f_.)(x_.)] * (b_.) + (a_.))^{(m_.)} * (\operatorname{csc}[(e_.) + (f_.)(x_.)] * (d_.) + (c_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(-a * c)^m, \text{Int}[(\operatorname{Cot}[e + f x])^{(2 * m)} * (c + d * \operatorname{Csc}[e + f x])^{(n - m)}, x], x] / ; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{EqQ}[b * c + a * d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{RationalQ}[n] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{GtQ}[m - n, 0])]$

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^2 (c - c \sec(e + fx)) dx &= - \left((ac) \int (a + a \sec(e + fx)) \tan^2(e + fx) dx \right) \\ &= - \frac{c (2a^2 + a^2 \sec(e + fx)) \tan(e + fx)}{2f} + \frac{1}{2} (ac) \int (2a + a \sec(e + fx)) \tan(e + fx) dx \\ &= a^2 cx - \frac{c (2a^2 + a^2 \sec(e + fx)) \tan(e + fx)}{2f} + \frac{1}{2} (a^2 c) \int \sec(e + fx) dx \\ &= a^2 cx + \frac{a^2 c \tanh^{-1}(\sin(e + fx))}{2f} - \frac{c (2a^2 + a^2 \sec(e + fx)) \tan(e + fx)}{2f} \end{aligned}$$

Mathematica [A] time = 0.29, size = 72, normalized size = 1.31

$$\frac{a^2c \sec^2(e + fx) \left(-\sin(e + fx) - \sin(2(e + fx)) + (e + fx) \cos(2(e + fx)) + \cos^2(e + fx) \tanh^{-1}(\sin(e + fx)) + e \right)}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]

[Out] (a^2*c*Sec[e + f*x]^2*(e + f*x + ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^2 + (e + f*x)*Cos[2*(e + f*x)] - Sin[e + f*x] - Sin[2*(e + f*x)]))/(2*f)

fricas [B] time = 0.48, size = 103, normalized size = 1.87

$$\frac{4a^2cfx \cos(fx + e)^2 + a^2c \cos(fx + e)^2 \log(\sin(fx + e) + 1) - a^2c \cos(fx + e)^2 \log(-\sin(fx + e) + 1) - 2}{4f \cos(fx + e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] 1/4*(4*a^2*c*f*x*cos(f*x + e)^2 + a^2*c*cos(f*x + e)^2*log(sin(f*x + e) + 1) - a^2*c*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*(2*a^2*c*cos(f*x + e) + a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^2)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-2*a^2*c/2*(f*x+exp(1))/2+a^2*c/4*ln(abs(tan((f*x+exp(1))/2)-1))-a^2*c/4*ln(abs(tan((f*x+exp(1))/2)+1))-(tan((f*x+exp(1))/2)^3*a^2*c-3*tan((f*x+exp(1))/2)*a^2*c)*1/2/(tan((f*x+exp(1))/2)^2-1)^2)

maple [A] time = 0.86, size = 76, normalized size = 1.38

$$\frac{a^2c \ln(\sec(fx + e) + \tan(fx + e))}{2f} + a^2cx + \frac{a^2ce}{f} - \frac{a^2c \tan(fx + e)}{f} - \frac{a^2c \sec(fx + e) \tan(fx + e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)

[Out] 1/2/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))+a^2*c*x+1/f*a^2*c*e-a^2*c*tan(f*x+e)/f-1/2/f*a^2*c*sec(f*x+e)*tan(f*x+e)

maxima [A] time = 0.33, size = 95, normalized size = 1.73

$$\frac{4(fx + e)a^2c + a^2c \left(\frac{2 \sin(fx + e)}{\sin(fx + e)^2 - 1} - \log(\sin(fx + e) + 1) + \log(\sin(fx + e) - 1) \right) + 4a^2c \log(\sec(fx + e) + \tan(fx + e))}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] $\frac{1}{4}*(4*(f*x + e)*a^2*c + a^2*c*(2*\sin(f*x + e)/(\sin(f*x + e)^2 - 1) - \log(\sin(f*x + e) + 1) + \log(\sin(f*x + e) - 1)) + 4*a^2*c*\log(\sec(f*x + e) + \tan(f*x + e)) - 4*a^2*c*\tan(f*x + e))/f$

mupad [B] time = 1.51, size = 91, normalized size = 1.65

$$a^2 c x - \frac{3 a^2 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right) - a^2 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3}{f \left(\tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 - 2 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 + 1\right)} + \frac{a^2 c \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{f x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)),x)`

[Out] $a^2*c*x - (3*a^2*c*\tan(e/2 + (f*x)/2) - a^2*c*\tan(e/2 + (f*x)/2)^3)/(f*(\tan(e/2 + (f*x)/2)^4 - 2*\tan(e/2 + (f*x)/2)^2 + 1)) + (a^2*c*\operatorname{atanh}(\tan(e/2 + (f*x)/2)))/f$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2c \left(\int (-1) dx + \int (-\sec(e + fx)) dx + \int \sec^2(e + fx) dx + \int \sec^3(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)`

[Out] $-a**2*c*(\operatorname{Integral}(-1, x) + \operatorname{Integral}(-\sec(e + f*x), x) + \operatorname{Integral}(\sec(e + f*x)**2, x) + \operatorname{Integral}(\sec(e + f*x)**3, x))$

$$3.6 \quad \int \frac{(a+a \sec(e+fx))^2}{c-c \sec(e+fx)} dx$$

Optimal. Leaf size=56

$$\frac{a^2 \tanh^{-1}(\sin(e+fx))}{cf} - \frac{4a^2 \tan(e+fx)}{cf(1-\sec(e+fx))} + \frac{a^2 x}{c}$$

[Out] a^2*x/c-a^2*arctanh(sin(f*x+e))/c/f-4*a^2*tan(f*x+e)/c/f/(1-sec(f*x+e))

Rubi [A] time = 0.16, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3903, 3777, 8, 3794, 3789, 3770}

$$\frac{a^2 \tanh^{-1}(\sin(e+fx))}{cf} - \frac{4a^2 \tan(e+fx)}{cf(1-\sec(e+fx))} + \frac{a^2 x}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]

[Out] (a^2*x)/c - (a^2*ArcTanh[Sin[e + f*x]]/(c*f) - (4*a^2*Tan[e + f*x])/(c*f*(1 - Sec[e + f*x])))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3789

Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^2}{c - c \sec(e + fx)} dx &= \frac{\int \left(\frac{a^2}{1 - \sec(e + fx)} + \frac{2a^2 \sec(e + fx)}{1 - \sec(e + fx)} + \frac{a^2 \sec^2(e + fx)}{1 - \sec(e + fx)} \right) dx}{c} \\
&= \frac{a^2 \int \frac{1}{1 - \sec(e + fx)} dx}{c} + \frac{a^2 \int \frac{\sec^2(e + fx)}{1 - \sec(e + fx)} dx}{c} + \frac{(2a^2) \int \frac{\sec(e + fx)}{1 - \sec(e + fx)} dx}{c} \\
&= \frac{3a^2 \tan(e + fx)}{cf(1 - \sec(e + fx))} - \frac{a^2 \int -1 dx}{c} - \frac{a^2 \int \sec(e + fx) dx}{c} + \frac{a^2 \int \frac{\sec(e + fx)}{1 - \sec(e + fx)} dx}{c} \\
&= \frac{a^2 x}{c} - \frac{a^2 \tanh^{-1}(\sin(e + fx))}{cf} - \frac{4a^2 \tan(e + fx)}{cf(1 - \sec(e + fx))}
\end{aligned}$$

Mathematica [B] time = 0.29, size = 169, normalized size = 3.02

$$\frac{a^2 \csc\left(\frac{e}{2}\right) \sin\left(\frac{1}{2}(e + fx)\right) \left(-\cos\left(\frac{fx}{2}\right) \left(\log\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{2cf \sin\left(\frac{1}{2}(e + fx)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]

[Out] (a^2*Csc[e/2]*(-(Cos[(f*x)/2]*(f*x + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + Cos[e + (f*x)/2]*(f*x + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + 8*Sin[(f*x)/2])*Sin[(e + f*x)/2])/(c*f*(-1 + Cos[e + f*x]))

fricas [A] time = 0.48, size = 87, normalized size = 1.55

$$\frac{2a^2 fx \sin(fx + e) - a^2 \log(\sin(fx + e) + 1) \sin(fx + e) + a^2 \log(-\sin(fx + e) + 1) \sin(fx + e) + 8a^2 \cos(fx + e)}{2cf \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] 1/2*(2*a^2*f*x*sin(f*x + e) - a^2*log(sin(f*x + e) + 1)*sin(f*x + e) + a^2*log(-sin(f*x + e) + 1)*sin(f*x + e) + 8*a^2*cos(f*x + e) + 8*a^2)/(c*f*sin(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-a^2*1/2*c*ln(abs(tan((f*x+exp(1))/2)-1))+a^2*1/2*c*ln(abs(tan((f*x+exp(1))/2)+1))-2*a^2/c*1/2*(f*x+exp(1))/2-2*a^2/c/tan((f*x+exp(1))/2))

maple [A] time = 0.78, size = 90, normalized size = 1.61

$$\frac{a^2 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1\right)}{fc} - \frac{a^2 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}{fc} + \frac{4a^2}{fc \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2a^2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)`

[Out] $1/f*a^2/c*\ln(\tan(1/2*e+1/2*f*x)-1)-1/f*a^2/c*\ln(\tan(1/2*e+1/2*f*x)+1)+4/f*a^2/c/\tan(1/2*e+1/2*f*x)+2/f*a^2/c*\arctan(\tan(1/2*e+1/2*f*x))$

maxima [B] time = 0.43, size = 153, normalized size = 2.73

$$\frac{a^2 \left(\frac{2 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} + \frac{\cos(fx+e)+1}{c \sin(fx+e)} \right) - a^2 \left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)+1}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)-1}{c} - \frac{\cos(fx+e)+1}{c \sin(fx+e)} \right) + \frac{2a^2(\cos(fx+e)+1)}{c \sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")`

[Out] $(a^2*(2*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/c + (\cos(f*x + e) + 1)/(c*\sin(f*x + e))) - a^2*(\log(\sin(f*x + e)/(\cos(f*x + e) + 1) + 1)/c - \log(\sin(f*x + e)/(\cos(f*x + e) + 1) - 1)/c - (\cos(f*x + e) + 1)/(c*\sin(f*x + e))) + 2*a^2*(\cos(f*x + e) + 1)/(c*\sin(f*x + e))/f$

mupad [B] time = 1.48, size = 46, normalized size = 0.82

$$\frac{a^2 x}{c} - \frac{a^2 \left(2 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right) - \frac{4}{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)} \right)}{c f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x)),x)`

[Out] $(a^2*x)/c - (a^2*(2*\operatorname{atanh}(\tan(e/2 + (f*x)/2)) - 4/\tan(e/2 + (f*x)/2)))/(c*f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \left(\int \frac{2 \sec(e+fx)}{\sec(e+fx)-1} dx + \int \frac{\sec^2(e+fx)}{\sec(e+fx)-1} dx + \int \frac{1}{\sec(e+fx)-1} dx \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))*2/(c-c*sec(f*x+e)),x)`

[Out] $-a**2*(\operatorname{Integral}(2*\sec(e + f*x)/(\sec(e + f*x) - 1), x) + \operatorname{Integral}(\sec(e + f*x)**2/(\sec(e + f*x) - 1), x) + \operatorname{Integral}(1/(\sec(e + f*x) - 1), x))/c$

$$3.7 \quad \int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=71

$$-\frac{4a^2 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))} - \frac{4a^2 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))^2} + \frac{a^2 x}{c^2}$$

[Out] $a^2 x/c^2 - 4/3 a^2 \tan(fx+e)/c^2/f/(1-\sec(fx+e))^{-2} - 4/3 a^2 \tan(fx+e)/c^2/f/(1-\sec(fx+e))$

Rubi [A] time = 0.24, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3903, 3777, 3919, 3794, 3796, 3797}

$$-\frac{4a^2 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))} - \frac{4a^2 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))^2} + \frac{a^2 x}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2, x]

[Out] $(a^2 x)/c^2 - (4 a^2 \tan[e + f x])/(3 c^2 f (1 - \sec[e + f x])^2) - (4 a^2 \tan[e + f x])/(3 c^2 f (1 - \sec[e + f x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]

&& LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^2}{(c - c \sec(e + fx))^2} dx &= \frac{\int \left(\frac{a^2}{(1 - \sec(e + fx))^2} + \frac{2a^2 \sec(e + fx)}{(1 - \sec(e + fx))^2} + \frac{a^2 \sec^2(e + fx)}{(1 - \sec(e + fx))^2} \right) dx}{c^2} \\ &= \frac{a^2 \int \frac{1}{(1 - \sec(e + fx))^2} dx}{c^2} + \frac{a^2 \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^2} dx}{c^2} + \frac{(2a^2) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{c^2} \\ &= \frac{4a^2 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))^2} - \frac{a^2 \int \frac{-3 - \sec(e + fx)}{1 - \sec(e + fx)} dx}{3c^2} \\ &= \frac{a^2 x}{c^2} - \frac{4a^2 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))^2} + \frac{(4a^2) \int \frac{\sec(e + fx)}{1 - \sec(e + fx)} dx}{3c^2} \\ &= \frac{a^2 x}{c^2} - \frac{4a^2 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))^2} - \frac{4a^2 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))} \end{aligned}$$

Mathematica [C] time = 0.06, size = 53, normalized size = 0.75

$$\frac{2a^2 \cot^3\left(\frac{e}{2} + \frac{fx}{2}\right) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{3c^2 f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]

[Out] (-2*a^2*Cot[e/2 + (f*x)/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e/2 + (f*x)/2]^2])/(3*c^2*f)

fricas [A] time = 0.46, size = 88, normalized size = 1.24

$$\frac{8a^2 \cos(fx + e)^2 + 4a^2 \cos(fx + e) - 4a^2 + 3(a^2 fx \cos(fx + e) - a^2 fx) \sin(fx + e)}{3(c^2 f \cos(fx + e) - c^2 f) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*(8*a^2*cos(f*x + e)^2 + 4*a^2*cos(f*x + e) - 4*a^2 + 3*(a^2*f*x*cos(f*x + e) - a^2*f*x)*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))

giac [A] time = 0.29, size = 60, normalized size = 0.85

$$\frac{\frac{3(fx+e)a^2}{c^2} + \frac{2\left(3a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a^2\right)}{c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{3} \cdot (3 \cdot (f \cdot x + e) \cdot a^2 / c^2 + 2 \cdot (3 \cdot a^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - a^2) / (c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3)) / f$

maple [A] time = 0.86, size = 67, normalized size = 0.94

$$-\frac{2a^2}{3f c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{2a^2}{f c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2a^2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)

[Out] $-2/3/f \cdot a^2/c^2/\tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^3 + 2/f \cdot a^2/c^2/\tan(1/2 \cdot e + 1/2 \cdot f \cdot x) + 2/f \cdot a^2/c^2 \cdot \arctan(\tan(1/2 \cdot e + 1/2 \cdot f \cdot x))$

maxima [B] time = 0.44, size = 174, normalized size = 2.45

$$\frac{a^2 \left(\frac{12 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} - 1\right) (\cos(fx+e)+1)^3}{c^2 \sin^3(fx+e)} \right) - \frac{a^2 \left(\frac{3 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1\right) (\cos(fx+e)+1)^3}{c^2 \sin^3(fx+e)} + \frac{2a^2 \left(\frac{3 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} - 1\right) (\cos(fx+e)+1)^3}{c^2 \sin^3(fx+e)}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot (a^2 \cdot (12 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1))) / c^2 + (9 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 - 1) \cdot (\cos(f \cdot x + e) + 1)^3 / (c^2 \cdot \sin(f \cdot x + e)^3)) - a^2 \cdot (3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 1) \cdot (\cos(f \cdot x + e) + 1)^3 / (c^2 \cdot \sin(f \cdot x + e)^3) + 2 \cdot a^2 \cdot (3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 - 1) \cdot (\cos(f \cdot x + e) + 1)^3 / (c^2 \cdot \sin(f \cdot x + e)^3)) / f$

mupad [B] time = 1.39, size = 40, normalized size = 0.56

$$\frac{a^2 \left(-2 \cot\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 6 \cot\left(\frac{e}{2} + \frac{fx}{2}\right) + 3fx \right)}{3c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^2,x)

[Out] $(a^2 \cdot (6 \cdot \cot(e/2 + (f \cdot x)/2) - 2 \cdot \cot(e/2 + (f \cdot x)/2)^3 + 3 \cdot f \cdot x)) / (3 \cdot c^2 \cdot f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \left(\int \frac{2 \sec(e+fx)}{\sec^2(e+fx) - 2 \sec(e+fx) + 1} dx + \int \frac{\sec^2(e+fx)}{\sec^2(e+fx) - 2 \sec(e+fx) + 1} dx + \int \frac{1}{\sec^2(e+fx) - 2 \sec(e+fx) + 1} dx \right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)

[Out] $a^2 \cdot (\text{Integral}(2 \cdot \sec(e + f \cdot x) / (\sec(e + f \cdot x)^2 - 2 \cdot \sec(e + f \cdot x) + 1), x) + \text{Integral}(\sec(e + f \cdot x)^2 / (\sec(e + f \cdot x)^2 - 2 \cdot \sec(e + f \cdot x) + 1), x) + \text{Integral}(1 / (\sec(e + f \cdot x)^2 - 2 \cdot \sec(e + f \cdot x) + 1), x)) / c^2$

$$3.8 \quad \int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=102

$$-\frac{23a^2 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))} - \frac{8a^2 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))^2} - \frac{4a^2 \tan(e+fx)}{5c^3 f(1-\sec(e+fx))^3} + \frac{a^2 x}{c^3}$$

[Out] $a^2 x / c^3 - 4/5 a^2 \tan(fx+e) / c^3 / f / (1-\sec(fx+e))^{3-8} / 15 a^2 \tan(fx+e) / c^3 / f / (1-\sec(fx+e))^{2-23} / 15 a^2 \tan(fx+e) / c^3 / f / (1-\sec(fx+e))$

Rubi [A] time = 0.33, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$-\frac{23a^2 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))} - \frac{8a^2 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))^2} - \frac{4a^2 \tan(e+fx)}{5c^3 f(1-\sec(e+fx))^3} + \frac{a^2 x}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3, x]

[Out] $(a^2 x) / c^3 - (4 a^2 \tan[e + f x]) / (5 c^3 f (1 - \sec[e + f x])^3) - (8 a^2 \tan[e + f x]) / (15 c^3 f (1 - \sec[e + f x])^2) - (23 a^2 \tan[e + f x]) / (15 c^3 f (1 - \sec[e + f x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]

&& LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^2}{(c - c \sec(e + fx))^3} dx &= \frac{\int \left(\frac{a^2}{(1 - \sec(e + fx))^3} + \frac{2a^2 \sec(e + fx)}{(1 - \sec(e + fx))^3} + \frac{a^2 \sec^2(e + fx)}{(1 - \sec(e + fx))^3} \right) dx}{c^3} \\ &= \frac{a^2 \int \frac{1}{(1 - \sec(e + fx))^3} dx}{c^3} + \frac{a^2 \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^3} dx}{c^3} + \frac{(2a^2) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{c^3} \\ &= -\frac{4a^2 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} - \frac{a^2 \int \frac{-5 - 2 \sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{5c^3} - \frac{(3a^2) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{5c^3} + \frac{(4a^2) \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^2} dx}{5c^3} \\ &= -\frac{4a^2 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} - \frac{8a^2 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))^2} + \frac{a^2 \int \frac{15 + 7 \sec(e + fx)}{1 - \sec(e + fx)} dx}{15c^3} - \frac{a^2 \int \frac{\sec^2(e + fx)}{1 - \sec(e + fx)} dx}{15c^3} \\ &= \frac{a^2 x}{c^3} - \frac{4a^2 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} - \frac{8a^2 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))^2} - \frac{a^2 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))} \\ &= \frac{a^2 x}{c^3} - \frac{4a^2 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} - \frac{8a^2 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))^2} - \frac{23a^2 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.63, size = 171, normalized size = 1.68

$$\frac{a^2 \csc\left(\frac{e}{2}\right) \csc^5\left(\frac{1}{2}(e + fx)\right) \left(-360 \sin\left(e + \frac{fx}{2}\right) + 280 \sin\left(e + \frac{3fx}{2}\right) + 150 \sin\left(2e + \frac{3fx}{2}\right) - 86 \sin\left(2e + \frac{5fx}{2}\right) - 15 \sin\left(2e + \frac{7fx}{2}\right)\right)}{(480c^3 f)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3, x]

[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^5*(150*f*x*Cos[(f*x)/2] - 150*f*x*Cos[e + (f*x)/2] - 75*f*x*Cos[e + (3*f*x)/2] + 75*f*x*Cos[2*e + (3*f*x)/2] + 15*f*x*Cos[2*e + (5*f*x)/2] - 15*f*x*Cos[3*e + (5*f*x)/2] - 500*Sin[(f*x)/2] - 360*Sin[e + (f*x)/2] + 280*Sin[e + (3*f*x)/2] + 150*Sin[2*e + (3*f*x)/2] - 86*Sin[2*e + (5*f*x)/2]))/(480*c^3*f)

fricas [A] time = 0.43, size = 128, normalized size = 1.25

$$\frac{43 a^2 \cos(fx + e)^3 - 11 a^2 \cos(fx + e)^2 - 31 a^2 \cos(fx + e) + 23 a^2 + 15 \left(a^2 fx \cos(fx + e)^2 - 2 a^2 fx \cos(fx + e) \right)}{15 \left(c^3 f \cos(fx + e)^2 - 2 c^3 f \cos(fx + e) + c^3 f \right) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{15} \cdot (43a^2 \cos(fx + e)^3 - 11a^2 \cos(fx + e)^2 - 31a^2 \cos(fx + e) + 23a^2 + 15(a^2 f x \cos(fx + e)^2 - 2a^2 f x \cos(fx + e) + a^2 f x) \sin(fx + e)) / ((c^3 f \cos(fx + e)^2 - 2c^3 f \cos(fx + e) + c^3 f) \sin(fx + e))$

giac [A] time = 0.46, size = 76, normalized size = 0.75

$$\frac{\frac{15(fx+e)a^2}{c^3} + \frac{30a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 10a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3a^2}{c^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] $\frac{1}{15} \cdot (15(fx + e)a^2/c^3 + (30a^2 \tan(1/2fx + 1/2e)^4 - 10a^2 \tan(1/2fx + 1/2e)^2 + 3a^2) / (c^3 \tan(1/2fx + 1/2e)^5)) / f$

maple [A] time = 1.05, size = 89, normalized size = 0.87

$$\frac{a^2}{5f c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{2a^2}{3f c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{2a^2}{f c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2a^2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)

[Out] $\frac{1}{5} \cdot f a^2 / c^3 / \tan(1/2e + 1/2fx)^5 - 2/3 \cdot f a^2 / c^3 / \tan(1/2e + 1/2fx)^3 + 2/5 \cdot f a^2 / c^3 / \tan(1/2e + 1/2fx) + 2/5 \cdot f a^2 / c^3 \cdot \arctan(\tan(1/2e + 1/2fx))$

maxima [B] time = 0.45, size = 215, normalized size = 2.11

$$\frac{a^2 \left(\frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3} - \frac{\left(\frac{20 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{105 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right) (\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5} \right)}{60f} - \frac{2a^2 \left(\frac{10 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{15 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3 \right) (\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{1}{60} \cdot (a^2 \cdot (120 \arctan(\sin(fx + e) / (\cos(fx + e) + 1)) / c^3 - (20 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 - 105 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 3) \cdot (\cos(fx + e) + 1)^5 / (c^3 \sin(fx + e)^5)) - 2a^2 \cdot (10 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 - 15 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 3) \cdot (\cos(fx + e) + 1)^5 / (c^3 \sin(fx + e)^5) - 3a^2 \cdot (5 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 1) \cdot (\cos(fx + e) + 1)^5 / (c^3 \sin(fx + e)^5)) / f$

mupad [B] time = 1.45, size = 96, normalized size = 0.94

$$\frac{a^2 x}{c^3} + \frac{\frac{a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{5} - \frac{2a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{3} + 2a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{c^3 f \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^3,x)`

[Out] $(a^2*x)/c^3 + ((a^2*\cos(e/2 + (f*x)/2)^5)/5 + 2*a^2*\cos(e/2 + (f*x)/2)*\sin(e/2 + (f*x)/2)^4 - (2*a^2*\cos(e/2 + (f*x)/2)^3*\sin(e/2 + (f*x)/2)^2)/3)/(c^3*f*\sin(e/2 + (f*x)/2)^5)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \left(\int \frac{2 \sec(e+fx)}{\sec^3(e+fx) - 3 \sec^2(e+fx) + 3 \sec(e+fx) - 1} dx + \int \frac{\sec^2(e+fx)}{\sec^3(e+fx) - 3 \sec^2(e+fx) + 3 \sec(e+fx) - 1} dx + \int \frac{1}{\sec^3(e+fx) - 3 \sec^2(e+fx) + 3 \sec(e+fx) - 1} dx \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)`

[Out] $-a**2*(Integral(2*\sec(e + f*x)/(\sec(e + f*x)**3 - 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) - 1), x) + Integral(\sec(e + f*x)**2/(\sec(e + f*x)**3 - 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) - 1), x) + Integral(1/(\sec(e + f*x)**3 - 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) - 1), x))/c**3$

$$3.9 \quad \int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=133

$$\frac{164a^2 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))} - \frac{59a^2 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))^2} - \frac{12a^2 \tan(e+fx)}{35c^4 f(1-\sec(e+fx))^3} - \frac{4a^2 \tan(e+fx)}{7c^4 f(1-\sec(e+fx))^4} + \frac{a^2 x}{c^4}$$

[Out] $a^2 x/c^4 - 4/7 a^2 \tan(fx+e)/c^4/f/(1-\sec(fx+e))^4 - 12/35 a^2 \tan(fx+e)/c^4/f/(1-\sec(fx+e))^3 - 59/105 a^2 \tan(fx+e)/c^4/f/(1-\sec(fx+e))^2 - 164/105 a^2 \tan(fx+e)/c^4/f/(1-\sec(fx+e))$

Rubi [A] time = 0.43, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$\frac{164a^2 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))} - \frac{59a^2 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))^2} - \frac{12a^2 \tan(e+fx)}{35c^4 f(1-\sec(e+fx))^3} - \frac{4a^2 \tan(e+fx)}{7c^4 f(1-\sec(e+fx))^4} + \frac{a^2 x}{c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4, x]

[Out] $(a^2 x)/c^4 - (4 a^2 \tan[e + f x])/(7 c^4 f (1 - \sec[e + f x])^4) - (12 a^2 \tan[e + f x])/(35 c^4 f (1 - \sec[e + f x])^3) - (59 a^2 \tan[e + f x])/(105 c^4 f (1 - \sec[e + f x])^2) - (164 a^2 \tan[e + f x])/(105 c^4 f (1 - \sec[e + f x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f

$\text{in}[e + (f*x)/2] + 8568*\text{Sin}[e + (3*f*x)/2] + 4830*\text{Sin}[2*e + (3*f*x)/2] - 3206*\text{Sin}[2*e + (5*f*x)/2] - 1260*\text{Sin}[3*e + (5*f*x)/2] + 638*\text{Sin}[3*e + (7*f*x)/2]))/(13440*c^4*f)$

fricas [A] time = 0.43, size = 172, normalized size = 1.29

$$\frac{319 a^2 \cos(fx + e)^4 - 327 a^2 \cos(fx + e)^3 - 95 a^2 \cos(fx + e)^2 + 387 a^2 \cos(fx + e) - 164 a^2 + 105 \left(a^2 f x \cos(fx + e)^3 - 3 a^2 f x \cos(fx + e)^2 + 3 a^2 f x \cos(fx + e) - a^2 f x \right)}{105 \left(c^4 f \cos(fx + e)^3 - 3 c^4 f \cos(fx + e)^2 + 3 c^4 f \cos(fx + e) - c^4 f \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] $\frac{1}{105} * (319 * a^2 * \cos(f * x + e)^4 - 327 * a^2 * \cos(f * x + e)^3 - 95 * a^2 * \cos(f * x + e)^2 + 387 * a^2 * \cos(f * x + e) - 164 * a^2 + 105 * (a^2 * f * x * \cos(f * x + e)^3 - 3 * a^2 * f * x * \cos(f * x + e)^2 + 3 * a^2 * f * x * \cos(f * x + e) - a^2 * f * x * \sin(f * x + e))) / ((c^4 * f * \cos(f * x + e)^3 - 3 * c^4 * f * \cos(f * x + e)^2 + 3 * c^4 * f * \cos(f * x + e) - c^4 * f * \sin(f * x + e)))$

giac [A] time = 0.33, size = 93, normalized size = 0.70

$$\frac{\frac{210 (fx+e)a^2}{c^4} + \frac{420 a^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 - 140 a^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 63 a^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 15 a^2}{c^4 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^7}}{210 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")

[Out] $\frac{1}{210} * (210 * (f * x + e) * a^2 / c^4 + (420 * a^2 * \tan(1/2 * f * x + 1/2 * e)^6 - 140 * a^2 * \tan(1/2 * f * x + 1/2 * e)^4 + 63 * a^2 * \tan(1/2 * f * x + 1/2 * e)^2 - 15 * a^2) / (c^4 * \tan(1/2 * f * x + 1/2 * e)^7)) / f$

maple [A] time = 0.90, size = 111, normalized size = 0.83

$$-\frac{a^2}{14 f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{3a^2}{10 f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{2a^2}{3 f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{2a^2}{f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2a^2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)

[Out] $-1/14/f*a^2/c^4/\tan(1/2*e+1/2*f*x)^7+3/10/f*a^2/c^4/\tan(1/2*e+1/2*f*x)^5-2/3/f*a^2/c^4/\tan(1/2*e+1/2*f*x)^3+2/f*a^2/c^4/\tan(1/2*e+1/2*f*x)+2/f*a^2/c^4*\arctan(\tan(1/2*e+1/2*f*x))$

maxima [B] time = 0.44, size = 294, normalized size = 2.21

$$5 a^2 \left(\frac{336 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right) (\cos(fx+e)+1)^7} {c^4 \sin(fx+e)^7} \right) + \frac{a^2 \left(\frac{21 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{105 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} \right)}{c^4 \sin(fx+e)}$$

840 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

```
[Out] 1/840*(5*a^2*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7)) + a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 6*a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

mupad [B] time = 1.50, size = 124, normalized size = 0.93

$$\frac{a^2 x}{c^4} \frac{\frac{a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{14} - \frac{3a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{10} + \frac{2a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{3} - 2a^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6}{c^4 f \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^4, x)
```

```
[Out] (a^2*x)/c^4 - ((a^2*cos(e/2 + (f*x)/2)^7)/14 - 2*a^2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 + (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4)/3 - (3*a^2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/10)/(c^4*f*sin(e/2 + (f*x)/2)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \left(\int \frac{2 \sec(e+fx)}{\sec^4(e+fx) - 4 \sec^3(e+fx) + 6 \sec^2(e+fx) - 4 \sec(e+fx) + 1} dx + \int \frac{\sec^2(e+fx)}{\sec^4(e+fx) - 4 \sec^3(e+fx) + 6 \sec^2(e+fx) - 4 \sec(e+fx) + 1} dx + \int \frac{\sec^4(e+fx)}{\sec^4(e+fx) - 4 \sec^3(e+fx) + 6 \sec^2(e+fx) - 4 \sec(e+fx) + 1} dx \right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4, x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

$$3.10 \quad \int \frac{(a+a \sec(e+fx))^2}{(c-c \sec(e+fx))^5} dx$$

Optimal. Leaf size=164

$$\frac{494a^2 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))} - \frac{179a^2 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))^2} - \frac{37a^2 \tan(e+fx)}{105c^5 f(1-\sec(e+fx))^3} - \frac{16a^2 \tan(e+fx)}{63c^5 f(1-\sec(e+fx))^4} - \frac{4a^2 \tan(e+fx)}{9c^5 f(1-\sec(e+fx))^5}$$

[Out] a^2*x/c^5-4/9*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5-16/63*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^4-37/105*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-179/315*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-494/315*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^1

Rubi [A] time = 0.54, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$\frac{494a^2 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))} - \frac{179a^2 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))^2} - \frac{37a^2 \tan(e+fx)}{105c^5 f(1-\sec(e+fx))^3} - \frac{16a^2 \tan(e+fx)}{63c^5 f(1-\sec(e+fx))^4} - \frac{4a^2 \tan(e+fx)}{9c^5 f(1-\sec(e+fx))^5}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]

[Out] (a^2*x)/c^5 - (4*a^2*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) - (16*a^2*Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105*c^5*f*(1 - Sec[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x])^2) - (494*a^2*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^2}{(c - c \sec(e + fx))^5} dx &= \frac{\int \left(\frac{a^2}{(1 - \sec(e + fx))^5} + \frac{2a^2 \sec(e + fx)}{(1 - \sec(e + fx))^5} + \frac{a^2 \sec^2(e + fx)}{(1 - \sec(e + fx))^5} \right) dx}{c^5} \\ &= \frac{a^2 \int \frac{1}{(1 - \sec(e + fx))^5} dx}{c^5} + \frac{a^2 \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^5} dx}{c^5} + \frac{(2a^2) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^5} dx}{c^5} \\ &= \frac{4a^2 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{a^2 \int \frac{-9 - 4 \sec(e + fx)}{(1 - \sec(e + fx))^4} dx}{9c^5} - \frac{(5a^2) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^4} dx}{9c^5} + \frac{(8a^2) \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^4} dx}{9c^5} \\ &= \frac{4a^2 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{16a^2 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} + \frac{a^2 \int \frac{63 + 39 \sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{63c^5} - \frac{(5a^2) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{63c^5} \\ &= \frac{4a^2 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{16a^2 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{37a^2 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} \\ &= \frac{4a^2 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{16a^2 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{37a^2 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} \\ &= \frac{a^2 x}{c^5} - \frac{4a^2 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{16a^2 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{37a^2 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} \\ &= \frac{a^2 x}{c^5} - \frac{4a^2 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{16a^2 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{37a^2 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} \end{aligned}$$

Mathematica [A] time = 0.99, size = 283, normalized size = 1.73

$$\frac{a^2 \csc\left(\frac{e}{2}\right) \csc^9\left(\frac{1}{2}(e + fx)\right) \left(-117810 \sin\left(e + \frac{fx}{2}\right) + 100002 \sin\left(e + \frac{3fx}{2}\right) + 68670 \sin\left(2e + \frac{3fx}{2}\right) - 48978 \sin\left(3e + \frac{3fx}{2}\right)\right)}{c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]

[Out] (a^2*Csc[e/2]*Csc[(e + f*x)/2]^9*(39690*f*x*Cos[(f*x)/2] - 39690*f*x*Cos[e + (f*x)/2] - 26460*f*x*Cos[e + (3*f*x)/2] + 26460*f*x*Cos[2*e + (3*f*x)/2] + 11340*f*x*Cos[2*e + (5*f*x)/2] - 11340*f*x*Cos[3*e + (5*f*x)/2] - 2835*f*x*Cos[3*e + (7*f*x)/2] + 2835*f*x*Cos[4*e + (7*f*x)/2] + 315*f*x*Cos[4*e + (9*f*x)/2] - 315*f*x*Cos[5*e + (9*f*x)/2] - 135198*Sin[(f*x)/2] - 117810*Sin[e + (f*x)/2] + 100002*Sin[e + (3*f*x)/2] + 68670*Sin[2*e + (3*f*x)/2] - 48978*Sin[2*e + (5*f*x)/2] - 23310*Sin[3*e + (5*f*x)/2] + 13662*Sin[3*e + (7*f*x)/2] + 4410*Sin[4*e + (7*f*x)/2] - 2008*Sin[4*e + (9*f*x)/2]))/(161280*c^5*f)

fricas [A] time = 0.46, size = 212, normalized size = 1.29

$$\frac{1004 a^2 \cos (f x+e)^5-1811 a^2 \cos (f x+e)^4+797 a^2 \cos (f x+e)^3+1457 a^2 \cos (f x+e)^2-1661 a^2 \cos (f x+e)+494 a^2+315\left(c^5 f \cos (f x+e)^4-4 c^5 f \cos (f x+e)^3\right)}{315\left(c^5 f \cos (f x+e)^4-4 c^5 f \cos (f x+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/315*(1004*a^2*cos(f*x + e)^5 - 1811*a^2*cos(f*x + e)^4 + 797*a^2*cos(f*x + e)^3 + 1457*a^2*cos(f*x + e)^2 - 1661*a^2*cos(f*x + e) + 494*a^2 + 315*(a^2*f*x*cos(f*x + e)^4 - 4*a^2*f*x*cos(f*x + e)^3 + 6*a^2*f*x*cos(f*x + e)^2 - 4*a^2*f*x*cos(f*x + e) + a^2*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))

giac [A] time = 0.44, size = 110, normalized size = 0.67

$$\frac{\frac{1260(f x+e) a^2}{c^5}+\frac{2520 a^2 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^8-840 a^2 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^6+441 a^2 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^4-180 a^2 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^2+35 a^2}{c^5 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^9}}{1260 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")

[Out] 1/1260*(1260*(f*x + e)*a^2/c^5 + (2520*a^2*tan(1/2*f*x + 1/2*e)^8 - 840*a^2*tan(1/2*f*x + 1/2*e)^6 + 441*a^2*tan(1/2*f*x + 1/2*e)^4 - 180*a^2*tan(1/2*f*x + 1/2*e)^2 + 35*a^2)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f

maple [A] time = 0.84, size = 133, normalized size = 0.81

$$\frac{a^2}{36 f c^5 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)^9}-\frac{a^2}{7 f c^5 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)^7}+\frac{7 a^2}{20 f c^5 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)^5}-\frac{2 a^2}{3 f c^5 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)^3}+\frac{2 a^2}{f c^5 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)}+\frac{2 a^2}{3 f c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)

[Out] 1/36/f*a^2/c^5/tan(1/2*e+1/2*f*x)^9-1/7/f*a^2/c^5/tan(1/2*e+1/2*f*x)^7+7/20/f*a^2/c^5/tan(1/2*e+1/2*f*x)^5-2/3/f*a^2/c^5/tan(1/2*e+1/2*f*x)^3+2/f*a^2/c^5/tan(1/2*e+1/2*f*x)+2/f*a^2/c^5*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.45, size = 335, normalized size = 2.04

$$a^2\left(\frac{10080 \arctan\left(\frac{\sin (f x+e)}{\cos (f x+e)+1}\right)}{c^5}-\frac{\left(\frac{270 \sin (f x+e)^2}{(\cos (f x+e)+1)^2}-\frac{1008 \sin (f x+e)^4}{(\cos (f x+e)+1)^4}+\frac{2730 \sin (f x+e)^6}{(\cos (f x+e)+1)^6}-\frac{9765 \sin (f x+e)^8}{(\cos (f x+e)+1)^8}-35\right)(\cos (f x+e)+1)^9}{c^5 \sin (f x+e)^9}\right)-\frac{2 a^2\left(\frac{180 \sin (f x+e)^2}{(\cos (f x+e)+1)^2}-\frac{315 \sin (f x+e)^4}{(\cos (f x+e)+1)^4}+\frac{1575 \sin (f x+e)^6}{(\cos (f x+e)+1)^6}-\frac{3969 \sin (f x+e)^8}{(\cos (f x+e)+1)^8}-45\right)(\cos (f x+e)+1)^9}{c^5 \sin (f x+e)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] 1/5040*(a^2*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 2*a^2*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^2*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f

mupad [B] time = 1.54, size = 146, normalized size = 0.89

$$a^2 \left(\frac{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)^9}{36} - \frac{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)^7 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{7} + \frac{7 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{20} - \frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6}{3} + 2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{fx}{2}\right) \right) \frac{1}{c^5 f \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^5,x)

[Out] (a^2*(cos(e/2 + (f*x)/2)^9/36 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8 + sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^6)/3 + (7*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/20 - (cos(e/2 + (f*x)/2)^7*sin(e/2 + (f*x)/2)^2)/7))/(c^5*f*sin(e/2 + (f*x)/2)^9)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{2 \sec(e+fx)}{\sec^5(e+fx) - 5 \sec^4(e+fx) + 10 \sec^3(e+fx) - 10 \sec^2(e+fx) + 5 \sec(e+fx) - 1} dx + \int \frac{\sec^2(e+fx)}{\sec^5(e+fx) - 5 \sec^4(e+fx) + 10 \sec^3(e+fx) - 10 \sec^2(e+fx) + 5 \sec(e+fx) - 1} dx \right) \frac{1}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)

[Out] -a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5

3.11 $\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^5 dx$

Optimal. Leaf size=188

$$\frac{a^3 c^5 \tan^7(e + fx)}{7f} - \frac{a^3 c^5 \tan^5(e + fx)}{5f} + \frac{a^3 c^5 \tan^3(e + fx)}{3f} - \frac{a^3 c^5 \tan(e + fx)}{f} - \frac{5a^3 c^5 \tanh^{-1}(\sin(e + fx))}{8f} + \frac{a^3 c^5 \tan(e + fx)}{f}$$

[Out] $a^3 c^5 x - 5/8 a^3 c^5 \operatorname{arctanh}(\sin(fx+e))/f - a^3 c^5 \tan(fx+e)/f + 5/8 a^3 c^5 \sec(fx+e) \tan(fx+e)/f + 1/3 a^3 c^5 \tan(fx+e)^3/f - 5/12 a^3 c^5 \sec(fx+e) \tan(fx+e)^3/f - 1/5 a^3 c^5 \tan(fx+e)^5/f + 1/3 a^3 c^5 \sec(fx+e) \tan(fx+e)^5/f - 1/7 a^3 c^5 \tan(fx+e)^7/f$

Rubi [A] time = 0.24, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^3 c^5 \tan^7(e + fx)}{7f} - \frac{a^3 c^5 \tan^5(e + fx)}{5f} + \frac{a^3 c^5 \tan^3(e + fx)}{3f} - \frac{a^3 c^5 \tan(e + fx)}{f} - \frac{5a^3 c^5 \tanh^{-1}(\sin(e + fx))}{8f} + \frac{a^3 c^5 \tan(e + fx)}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \operatorname{Sec}[e + fx])^3 (c - c \operatorname{Sec}[e + fx])^5, x]$

[Out] $a^3 c^5 x - (5 a^3 c^5 \operatorname{ArcTanh}[\sin[e + fx]])/(8f) - (a^3 c^5 \tan[e + fx])/f + (5 a^3 c^5 \operatorname{Sec}[e + fx] \tan[e + fx])/(8f) + (a^3 c^5 \tan[e + fx]^3)/(3f) - (5 a^3 c^5 \operatorname{Sec}[e + fx] \tan[e + fx]^3)/(12f) - (a^3 c^5 \tan[e + fx]^5)/(5f) + (a^3 c^5 \operatorname{Sec}[e + fx] \tan[e + fx]^5)/(3f) - (a^3 c^5 \tan[e + fx]^7)/(7f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 2607

$\text{Int}[\sec[(e_.) + (f_.)(x_)]^{(m_.)} ((b_.) \tan[(e_.) + (f_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n (1+x^2)^{(m/2-1)}, x], x, \tan[e + fx]], x] /; \text{FreeQ}\{b, e, f, n\}, x] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ !(\text{IntegerQ}[(n-1)/2]) \ \&\& \ \text{LtQ}[0, n, m-1]$

Rule 2611

$\text{Int}[(a_.) \sec[(e_.) + (f_.)(x_)]^{(m_.)} ((b_.) \tan[(e_.) + (f_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(a \operatorname{Sec}[e + fx])^m (b \operatorname{Tan}[e + fx])^{(n-1)})/(f*(m+n-1)), x] - \text{Dist}[(b^2*(n-1))/(m+n-1), \text{Int}[(a \operatorname{Sec}[e + fx])^m (b \operatorname{Tan}[e + fx])^{(n-2)}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{NeQ}[m+n-1, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rule 3473

$\text{Int}[(b_.) \tan[(c_.) + (d_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(b \operatorname{Tan}[c + dx])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \operatorname{Tan}[c + dx])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^5 dx &= - \left((a^3 c^3) \int (c - c \sec(e + fx))^2 \tan^6(e + fx) dx \right) \\
&= - \left((a^3 c^3) \int (c^2 \tan^6(e + fx) - 2c^2 \sec(e + fx) \tan^6(e + fx) + \dots \right) \\
&= - \left((a^3 c^5) \int \tan^6(e + fx) dx \right) - (a^3 c^5) \int \sec^2(e + fx) \tan^6(e + fx) dx \\
&= - \frac{a^3 c^5 \tan^5(e + fx)}{5f} + \frac{a^3 c^5 \sec(e + fx) \tan^5(e + fx)}{3f} + (a^3 c^5) \int \sec^2(e + fx) \tan^5(e + fx) dx \\
&= \frac{a^3 c^5 \tan^3(e + fx)}{3f} - \frac{5a^3 c^5 \sec(e + fx) \tan^3(e + fx)}{12f} - \frac{a^3 c^5 \tan^3(e + fx)}{5f} \\
&= - \frac{a^3 c^5 \tan(e + fx)}{f} + \frac{5a^3 c^5 \sec(e + fx) \tan(e + fx)}{8f} + \frac{a^3 c^5 \tan^3(e + fx)}{3f} \\
&= a^3 c^5 x - \frac{5a^3 c^5 \tanh^{-1}(\sin(e + fx))}{8f} - \frac{a^3 c^5 \tan(e + fx)}{f} + \frac{5a^3 c^5 \tan^3(e + fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 2.24, size = 189, normalized size = 1.01

$$a^3 c^5 \sec^7(e + fx) (-4200 \sin(e + fx) + 2975 \sin(2(e + fx)) - 2184 \sin(3(e + fx)) + 980 \sin(4(e + fx)) - 2408 \sin(5(e + fx)) + 1155 \sin(6(e + fx)) - 584 \sin(7(e + fx))) / (26880 f)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*c^5*Sec[e + f*x]^7*(14700*(e + f*x)*Cos[e + f*x] - 16800*ArcTanh[Sin[e
+ f*x]]*Cos[e + f*x]^7 + 8820*e*Cos[3*(e + f*x)] + 8820*f*x*Cos[3*(e + f*x
)] + 2940*e*Cos[5*(e + f*x)] + 2940*f*x*Cos[5*(e + f*x)] + 420*e*Cos[7*(e +
f*x)] + 420*f*x*Cos[7*(e + f*x)] - 4200*Sin[e + f*x] + 2975*Sin[2*(e + f*x
)] - 2184*Sin[3*(e + f*x)] + 980*Sin[4*(e + f*x)] - 2408*Sin[5*(e + f*x)] +
1155*Sin[6*(e + f*x)] - 584*Sin[7*(e + f*x)])/(26880*f)
```

fricas [A] time = 0.48, size = 195, normalized size = 1.04

$$1680 a^3 c^5 f x \cos(fx + e)^7 - 525 a^3 c^5 \cos(fx + e)^7 \log(\sin(fx + e) + 1) + 525 a^3 c^5 \cos(fx + e)^7 \log(-\sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] $\frac{1}{1680}*(1680*a^3*c^5*f*x*\cos(f*x + e)^7 - 525*a^3*c^5*\cos(f*x + e)^7*\log(\sin(f*x + e) + 1) + 525*a^3*c^5*\cos(f*x + e)^7*\log(-\sin(f*x + e) + 1) - 2*(1168*a^3*c^5*\cos(f*x + e)^6 - 1155*a^3*c^5*\cos(f*x + e)^5 - 256*a^3*c^5*\cos(f*x + e)^4 + 910*a^3*c^5*\cos(f*x + e)^3 - 192*a^3*c^5*\cos(f*x + e)^2 - 280*a^3*c^5*\cos(f*x + e) + 120*a^3*c^5)*\sin(f*x + e))/(f*\cos(f*x + e)^7)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: $(4*\pi/x/2)>(-4*\pi/x/2)$ Unable to check sign: $(4*\pi/x/2)>(-4*\pi/x/2)$ $-2/f*(-2*a^3*c^5/2*(f*x+\exp(1))/2-5*a^3*c^5/16*\ln(\abs(\tan((f*x+\exp(1))/2)-1))+5*a^3*c^5/16*\ln(\abs(\tan((f*x+\exp(1))/2)+1))+(-1365*\tan((f*x+\exp(1))/2)^{13}*a^3*c^5+9660*\tan((f*x+\exp(1))/2)^{11}*a^3*c^5-29673*\tan((f*x+\exp(1))/2)^9*a^3*c^5+21216*\tan((f*x+\exp(1))/2)^7*a^3*c^5-9863*\tan((f*x+\exp(1))/2)^5*a^3*c^5+2660*\tan((f*x+\exp(1))/2)^3*a^3*c^5-315*\tan((f*x+\exp(1))/2)*a^3*c^5)*1/840/(\tan((f*x+\exp(1))/2)^2-1)^7)$

maple [A] time = 1.62, size = 211, normalized size = 1.12

$$-\frac{13c^5a^3 \tan(fx + e) (\sec^3(fx + e))}{12f} + \frac{11a^3c^5 \sec(fx + e) \tan(fx + e)}{8f} - \frac{5c^5a^3 \ln(\sec(fx + e) + \tan(fx + e))}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x)

[Out] $-13/12/f*c^5*a^3*\tan(f*x+e)*\sec(f*x+e)^3+11/8*a^3*c^5*\sec(f*x+e)*\tan(f*x+e)/f-5/8/f*c^5*a^3*\ln(\sec(f*x+e)+\tan(f*x+e))-146/105*a^3*c^5*\tan(f*x+e)/f+a^3*c^5*x+1/f*a^3*c^5*e+8/35/f*c^5*a^3*\tan(f*x+e)*\sec(f*x+e)^4+32/105/f*c^5*a^3*\tan(f*x+e)*\sec(f*x+e)^2+1/3/f*c^5*a^3*\tan(f*x+e)*\sec(f*x+e)^5-1/7/f*c^5*a^3*\tan(f*x+e)*\sec(f*x+e)^6$

maxima [B] time = 0.34, size = 356, normalized size = 1.89

$$48 \left(5 \tan(fx + e)^7 + 21 \tan(fx + e)^5 + 35 \tan(fx + e)^3 + 35 \tan(fx + e) \right) a^3 c^5 - 224 \left(3 \tan(fx + e)^5 + 10 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] $-1/1680*(48*(5*\tan(f*x + e)^7 + 21*\tan(f*x + e)^5 + 35*\tan(f*x + e)^3 + 35*\tan(f*x + e))*a^3*c^5 - 224*(3*\tan(f*x + e)^5 + 10*\tan(f*x + e)^3 + 15*\tan(f*x + e))*a^3*c^5 - 1680*(f*x + e)*a^3*c^5 + 35*a^3*c^5*(2*(15*\sin(f*x + e)^5 - 40*\sin(f*x + e)^3 + 33*\sin(f*x + e)))/(\sin(f*x + e)^6 - 3*\sin(f*x + e)^4 + 3*\sin(f*x + e)^2 - 1) - 15*\log(\sin(f*x + e) + 1) + 15*\log(\sin(f*x + e) - 1)) - 630*a^3*c^5*(2*(3*\sin(f*x + e)^3 - 5*\sin(f*x + e)))/(\sin(f*x + e)^4 - 2*\sin(f*x + e)^2 + 1) - 3*\log(\sin(f*x + e) + 1) + 3*\log(\sin(f*x + e) - 1)) + 2520*a^3*c^5*(2*\sin(f*x + e)/(\sin(f*x + e)^2 - 1) - \log(\sin(f*x + e) + 1) + \log(\sin(f*x + e) - 1)) + 3360*a^3*c^5*\log(\sec(f*x + e) + \tan(f*x + e)) + 3360*a^3*c^5*\tan(f*x + e))/f$

mupad [B] time = 2.62, size = 259, normalized size = 1.38

$$\frac{13a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{13}}{4} - 23a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{11} + \frac{1413a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9}{20} - \frac{1768a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{35} + \frac{1409a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{60} - \frac{19a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{105} + \frac{a^3c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{158}$$

$$f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} - 7 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} + 21 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} - 35 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 + 35 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 - 21 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right) + a^3c^5x - (5a^3c^5 \operatorname{atanh}(\tan(\frac{e}{2} + \frac{fx}{2}))) / (4f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5,x)

[Out] ((1409*a^3*c^5*tan(e/2 + (f*x)/2)^5)/60 - (19*a^3*c^5*tan(e/2 + (f*x)/2)^3)/3 - (1768*a^3*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1413*a^3*c^5*tan(e/2 + (f*x)/2)^9)/20 - 23*a^3*c^5*tan(e/2 + (f*x)/2)^11 + (13*a^3*c^5*tan(e/2 + (f*x)/2)^13)/4 + (3*a^3*c^5*tan(e/2 + (f*x)/2))/4)/(f*(7*tan(e/2 + (f*x)/2)^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 - 1)) + a^3*c^5*x - (5*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(4*f)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^3c^5 \left(\int (-1) dx + \int 2 \sec(e + fx) dx + \int 2 \sec^2(e + fx) dx + \int (-6 \sec^3(e + fx)) dx + \int 6 \sec^5(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)

[Out] -a**3*c**5*(Integral(-1, x) + Integral(2*sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Integral(-6*sec(e + f*x)**3, x) + Integral(6*sec(e + f*x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x) + Integral(sec(e + f*x)**8, x))

3.12 $\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^4 dx$

Optimal. Leaf size=132

$$\frac{5a^3c^4 \tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^3 \tan^5(e + fx) (6c^4 - 5c^4 \sec(e + fx))}{30f} + \frac{a^3 \tan^3(e + fx) (8c^4 - 5c^4 \sec(e + fx))}{24f} - \frac{a^3 \tan(e + fx) (10c^4 - 5c^4 \sec(e + fx))}{24f}$$

[Out] $a^3c^4x - 5/16a^3c^4\operatorname{arctanh}(\sin(fx+e))/f - 1/16a^3(16c^4 - 5c^4\sec(fx+e))\tan(fx+e)/f + 1/24a^3(8c^4 - 5c^4\sec(fx+e))\tan(fx+e)^3/f - 1/30a^3(6c^4 - 5c^4\sec(fx+e))\tan(fx+e)^5/f$

Rubi [A] time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3881, 3770}

$$\frac{5a^3c^4 \tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^3 \tan^5(e + fx) (6c^4 - 5c^4 \sec(e + fx))}{30f} + \frac{a^3 \tan^3(e + fx) (8c^4 - 5c^4 \sec(e + fx))}{24f} - \frac{a^3 \tan(e + fx) (10c^4 - 5c^4 \sec(e + fx))}{24f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a\operatorname{Sec}[e + fx])^3(c - c\operatorname{Sec}[e + fx])^4, x]$

[Out] $a^3c^4x - (5a^3c^4\operatorname{ArcTanh}[\operatorname{Sin}[e + fx]])/(16f) - (a^3(16c^4 - 5c^4\operatorname{Sec}[e + fx])\operatorname{Tan}[e + fx])/(16f) + (a^3(8c^4 - 5c^4\operatorname{Sec}[e + fx])\operatorname{Tan}[e + fx]^3)/(24f) - (a^3(6c^4 - 5c^4\operatorname{Sec}[e + fx])\operatorname{Tan}[e + fx]^5)/(30f)$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)x], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3881

$\operatorname{Int}[(\operatorname{cot}[(c_.) + (d_.)x])^m(e_.)^n(\operatorname{csc}[(c_.) + (d_.)x])^p(a_.) + (a_.)], x_Symbol] \rightarrow -\operatorname{Simp}[(e_.)^n(\operatorname{cot}[(c_.) + (d_.)x])^{m-1}(a_.) + (a_.)\operatorname{Csc}[(c_.) + (d_.)x])^p/(d_.)m(m-1), x] - \operatorname{Dist}[e_./m, \operatorname{Int}[(e_.)^n(\operatorname{cot}[(c_.) + (d_.)x])^{m-2}(a_.) + (a_.)\operatorname{Csc}[(c_.) + (d_.)x])^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{GtQ}[m, 1]$

Rule 3904

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)x])^m(a_.)^n(\operatorname{csc}[(e_.) + (f_.)x])^p(d_.) + (c_.)^q], x_Symbol] \rightarrow \operatorname{Dist}[(-a_.)^m, \operatorname{Int}[(\operatorname{cot}[(e_.) + (f_.)x])^{2m}(c_.) + d_.\operatorname{Csc}[(e_.) + (f_.)x])^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \operatorname{EqQ}[b_.*c_. + a_.*d_., 0] \ \&\& \operatorname{EqQ}[a_.^2 - b_.^2, 0] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{RationalQ}[n] \ \&\& !(\operatorname{IntegerQ}[n] \ \&\& \operatorname{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^4 dx &= -\left((a^3 c^3) \int (c - c \sec(e + fx)) \tan^6(e + fx) dx \right) \\
&= -\frac{a^3 (6c^4 - 5c^4 \sec(e + fx)) \tan^5(e + fx)}{30f} + \frac{1}{6} (a^3 c^3) \int (6c - 5c \sec(e + fx)) \tan^5(e + fx) dx \\
&= \frac{a^3 (8c^4 - 5c^4 \sec(e + fx)) \tan^3(e + fx)}{24f} - \frac{a^3 (6c^4 - 5c^4 \sec(e + fx)) \tan^2(e + fx)}{30f} \\
&= -\frac{a^3 (16c^4 - 5c^4 \sec(e + fx)) \tan(e + fx)}{16f} + \frac{a^3 (8c^4 - 5c^4 \sec(e + fx)) \tan^2(e + fx)}{24f} \\
&= a^3 c^4 x - \frac{a^3 (16c^4 - 5c^4 \sec(e + fx)) \tan(e + fx)}{16f} + \frac{a^3 (8c^4 - 5c^4 \sec(e + fx)) \tan^2(e + fx)}{24f} \\
&= a^3 c^4 x - \frac{5a^3 c^4 \tanh^{-1}(\sin(e + fx))}{16f} - \frac{a^3 (16c^4 - 5c^4 \sec(e + fx)) \tan(e + fx)}{16f}
\end{aligned}$$

Mathematica [A] time = 1.87, size = 165, normalized size = 1.25

$$a^3 c^4 \sec^6(e + fx) (450 \sin(e + fx) - 600 \sin(2(e + fx)) - 25 \sin(3(e + fx)) - 384 \sin(4(e + fx)) + 165 \sin(5(e + fx)))$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]

[Out] (a^3*c^4*Sec[e + f*x]^6*(1200*e + 1200*f*x - 1200*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] + 720*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x)] + 450*Sin[e + f*x] - 600*Sin[2*(e + f*x)] - 25*Sin[3*(e + f*x)] - 384*Sin[4*(e + f*x)] + 165*Sin[5*(e + f*x)] - 184*Sin[6*(e + f*x)])/(3840*f)

fricas [A] time = 0.53, size = 179, normalized size = 1.36

$$480 a^3 c^4 f x \cos(fx + e)^6 - 75 a^3 c^4 \cos(fx + e)^6 \log(\sin(fx + e) + 1) + 75 a^3 c^4 \cos(fx + e)^6 \log(-\sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] 1/480*(480*a^3*c^4*f*x*cos(f*x + e)^6 - 75*a^3*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) + 75*a^3*c^4*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(368*a^3*c^4*cos(f*x + e)^5 - 165*a^3*c^4*cos(f*x + e)^4 - 176*a^3*c^4*cos(f*x + e)^3 + 130*a^3*c^4*cos(f*x + e)^2 + 48*a^3*c^4*cos(f*x + e) - 40*a^3*c^4)*sin(f*x + e))/(f*cos(f*x + e)^6)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*a^3*c^4/2*(f*x+exp(1))/2+5*a^3*c^4/32*ln(abs(tan((f*x+exp(1))))

$\left. \right)/2-1)) - 5a^3c^4/32 \ln(\tan((fx+\exp(1))/2)+1) - (-315 \tan((fx+\exp(1))/2)^{11} a^3c^4 + 1945 \tan((fx+\exp(1))/2)^9 a^3c^4 - 5118 \tan((fx+\exp(1))/2)^7 a^3c^4 + 3138 \tan((fx+\exp(1))/2)^5 a^3c^4 - 1095 \tan((fx+\exp(1))/2)^3 a^3c^4 + 165 \tan((fx+\exp(1))/2) a^3c^4) * 1/240 / (\tan((fx+\exp(1))/2)^2 - 1)^6$

maple [A] time = 1.65, size = 186, normalized size = 1.41

$$\frac{23a^3c^4 \tan(fx + e)}{15f} + \frac{11c^4a^3 \tan(fx + e) (\sec^2(fx + e))}{15f} + \frac{11c^4a^3 \sec(fx + e) \tan(fx + e)}{16f} - \frac{5c^4a^3 \ln(\sec(fx + e))}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x)`

[Out] $-23/15a^3c^4 \tan(fx+e)/f + 11/15/fc^4a^3 \tan(fx+e) \sec(fx+e)^2 + 11/16/fc^4a^3 \sec(fx+e) \tan(fx+e) - 5/16/fc^4a^3 \ln(\sec(fx+e) + \tan(fx+e)) + a^3c^4 * x + 1/f * a^3c^4 * e - 13/24/fc^4a^3 \tan(fx+e) \sec(fx+e)^3 - 1/5/fc^4a^3 \tan(fx+e) \sec(fx+e)^4 + 1/6/fc^4a^3 \tan(fx+e) \sec(fx+e)^5$

maxima [B] time = 0.34, size = 334, normalized size = 2.53

$$32 \left(3 \tan(fx + e)^5 + 10 \tan(fx + e)^3 + 15 \tan(fx + e) \right) a^3 c^4 - 480 \left(\tan(fx + e)^3 + 3 \tan(fx + e) \right) a^3 c^4 - 480$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="maxima")`

[Out] $-1/480 * (32 * (3 * \tan(fx + e)^5 + 10 * \tan(fx + e)^3 + 15 * \tan(fx + e)) * a^3 c^4 - 480 * (\tan(fx + e)^3 + 3 * \tan(fx + e)) * a^3 c^4 - 480 * (fx + e) * a^3 c^4 + 5 * a^3 c^4 * (2 * (15 * \sin(fx + e)^5 - 40 * \sin(fx + e)^3 + 33 * \sin(fx + e)) / (\sin(fx + e)^6 - 3 * \sin(fx + e)^4 + 3 * \sin(fx + e)^2 - 1) - 15 * \log(\sin(fx + e) + 1) + 15 * \log(\sin(fx + e) - 1)) - 90 * a^3 c^4 * (2 * (3 * \sin(fx + e)^3 - 5 * \sin(fx + e)) / (\sin(fx + e)^4 - 2 * \sin(fx + e)^2 + 1) - 3 * \log(\sin(fx + e) + 1) + 3 * \log(\sin(fx + e) - 1)) + 360 * a^3 c^4 * (2 * \sin(fx + e) / (\sin(fx + e)^2 - 1) - \log(\sin(fx + e) + 1) + \log(\sin(fx + e) - 1)) + 480 * a^3 c^4 * \log(\sec(fx + e) + \tan(fx + e)) + 1440 * a^3 c^4 * \tan(fx + e)) / f$

mupad [B] time = 2.59, size = 227, normalized size = 1.72

$$a^3 c^4 x + \frac{21 a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{11}}{8} - \frac{389 a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9}{24} + \frac{853 a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{20} - \frac{523 a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{20} + \frac{73 a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{8} - \frac{11 a^3 c^4}{8} \\ f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} - 6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} + 15 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 20 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 15 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4,x)`

[Out] $a^3c^4 * x + ((73 * a^3c^4 * \tan(e/2 + (fx)/2)^3)/8 - (523 * a^3c^4 * \tan(e/2 + (fx)/2)^5)/20 + (853 * a^3c^4 * \tan(e/2 + (fx)/2)^7)/20 - (389 * a^3c^4 * \tan(e/2 + (fx)/2)^9)/24 + (21 * a^3c^4 * \tan(e/2 + (fx)/2)^11)/8 - (11 * a^3c^4 * \tan(e/2 + (fx)/2))/8) / (f * (15 * \tan(e/2 + (fx)/2)^4 - 6 * \tan(e/2 + (fx)/2)^2 - 20 * \tan(e/2 + (fx)/2)^6 + 15 * \tan(e/2 + (fx)/2)^8 - 6 * \tan(e/2 + (fx)/2)^10 + \tan(e/2 + (fx)/2)^12 + 1)) - (5 * a^3c^4 * \operatorname{atanh}(\tan(e/2 + (fx)/2))) / (8 * f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3c^4 \left(\int 1 dx + \int (-\sec(e + fx)) dx + \int (-3 \sec^2(e + fx)) dx + \int 3 \sec^3(e + fx) dx + \int 3 \sec^4(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(1, x) + Integral(-sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + Integral(3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4, x) + Integral(-3*sec(e + f*x)**5, x) + Integral(-sec(e + f*x)**6, x) + Integral(sec(e + f*x)**7, x))
```

3.13 $\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^3 dx$

Optimal. Leaf size=68

$$-\frac{a^3 c^3 \tan^5(e + fx)}{5f} + \frac{a^3 c^3 \tan^3(e + fx)}{3f} - \frac{a^3 c^3 \tan(e + fx)}{f} + a^3 c^3 x$$

[Out] $a^3 c^3 x - a^3 c^3 \tan(fx + e)/f + 1/3 a^3 c^3 \tan(fx + e)^3/f - 1/5 a^3 c^3 \tan(fx + e)^5/f$

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3473, 8}

$$-\frac{a^3 c^3 \tan^5(e + fx)}{5f} + \frac{a^3 c^3 \tan^3(e + fx)}{3f} - \frac{a^3 c^3 \tan(e + fx)}{f} + a^3 c^3 x$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]

[Out] $a^3 c^3 x - (a^3 c^3 \tan[e + f*x])/f + (a^3 c^3 \tan[e + f*x]^3)/(3*f) - (a^3 c^3 \tan[e + f*x]^5)/(5*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.)^(n_.), x_Symbol] := Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^3 dx &= -\left((a^3 c^3) \int \tan^6(e + fx) dx \right) \\ &= -\frac{a^3 c^3 \tan^5(e + fx)}{5f} + (a^3 c^3) \int \tan^4(e + fx) dx \\ &= \frac{a^3 c^3 \tan^3(e + fx)}{3f} - \frac{a^3 c^3 \tan^5(e + fx)}{5f} - (a^3 c^3) \int \tan^2(e + fx) dx \\ &= -\frac{a^3 c^3 \tan(e + fx)}{f} + \frac{a^3 c^3 \tan^3(e + fx)}{3f} - \frac{a^3 c^3 \tan^5(e + fx)}{5f} + (a^3 c^3) \int \tan^0(e + fx) dx \\ &= a^3 c^3 x - \frac{a^3 c^3 \tan(e + fx)}{f} + \frac{a^3 c^3 \tan^3(e + fx)}{3f} - \frac{a^3 c^3 \tan^5(e + fx)}{5f} \end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.90

$$-a^3c^3 \left(-\frac{\tan^{-1}(\tan(e+fx))}{f} + \frac{\tan^5(e+fx)}{5f} - \frac{\tan^3(e+fx)}{3f} + \frac{\tan(e+fx)}{f} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]

[Out] -(a^3*c^3*(-(ArcTan[Tan[e + f*x]]/f) + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*f) + Tan[e + f*x]^5/(5*f)))

fricas [A] time = 0.46, size = 81, normalized size = 1.19

$$\frac{15a^3c^3fx \cos(fx+e)^5 - (23a^3c^3 \cos(fx+e)^4 - 11a^3c^3 \cos(fx+e)^2 + 3a^3c^3) \sin(fx+e)}{15f \cos(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(15*a^3*c^3*f*x*cos(f*x + e)^5 - (23*a^3*c^3*cos(f*x + e)^4 - 11*a^3*c^3*cos(f*x + e)^2 + 3*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)

giac [A] time = 0.55, size = 69, normalized size = 1.01

$$\frac{3a^3c^3 \tan(fx+e)^5 - 5a^3c^3 \tan(fx+e)^3 - 15(fx+e)a^3c^3 + 15a^3c^3 \tan(fx+e)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] -1/15*(3*a^3*c^3*tan(f*x + e)^5 - 5*a^3*c^3*tan(f*x + e)^3 - 15*(f*x + e)*a^3*c^3 + 15*a^3*c^3*tan(f*x + e))/f

maple [A] time = 1.06, size = 93, normalized size = 1.37

$$\frac{-3a^3c^3 \tan(fx+e) + (fx+e)a^3c^3 - 3a^3c^3 \left(-\frac{2}{3} - \frac{(\sec^2(fx+e))}{3} \right) \tan(fx+e) + a^3c^3 \left(-\frac{8}{15} - \frac{(\sec^4(fx+e))}{5} - \frac{4(\sec^2(fx+e))}{15} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x)

[Out] 1/f*(-3*a^3*c^3*tan(f*x+e)+(f*x+e)*a^3*c^3-3*a^3*c^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^3*c^3*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))

maxima [A] time = 0.32, size = 94, normalized size = 1.38

$$\frac{(3 \tan(fx+e)^5 + 10 \tan(fx+e)^3 + 15 \tan(fx+e))a^3c^3 - 15(\tan(fx+e)^3 + 3 \tan(fx+e))a^3c^3 - 15(fx+e)a^3c^3}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^3 - 15*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 - 15*(f*x + e)*a^3*c^3 + 45*a^3*c^3*tan(f*x + e))/f

mupad [B] time = 4.96, size = 122, normalized size = 1.79

$$a^3 c^3 x + \frac{2 a^3 c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9 - \frac{32 a^3 c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{3} + \frac{356 a^3 c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{15} - \frac{32 a^3 c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3} + 2 a^3 c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3,x)

[Out] a^3*c^3*x + ((356*a^3*c^3*tan(e/2 + (f*x)/2)^5)/15 - (32*a^3*c^3*tan(e/2 + (f*x)/2)^3)/3 - (32*a^3*c^3*tan(e/2 + (f*x)/2)^7)/3 + 2*a^3*c^3*tan(e/2 + (f*x)/2)^9 + 2*a^3*c^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^3 c^3 \left(\int (-1) dx + \int 3 \sec^2(e + fx) dx + \int (-3 \sec^4(e + fx)) dx + \int \sec^6(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)

[Out] -a**3*c**3*(Integral(-1, x) + Integral(3*sec(e + f*x)**2, x) + Integral(-3*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**6, x))

3.14 $\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^2 dx$

Optimal. Leaf size=97

$$\frac{3a^3c^2 \tanh^{-1}(\sin(e + fx))}{8f} + \frac{c^2 \tan^3(e + fx)(3a^3 \sec(e + fx) + 4a^3)}{12f} - \frac{c^2 \tan(e + fx)(3a^3 \sec(e + fx) + 8a^3)}{8f} + \dots$$

[Out] $a^3c^2x + 3/8a^3c^2 \arctanh(\sin(fx+e))/f - 1/8c^2(8a^3+3a^3\sec(fx+e)) \tan(fx+e)/f + 1/12c^2(4a^3+3a^3\sec(fx+e)) \tan(fx+e)^3/f$

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3881, 3770}

$$\frac{3a^3c^2 \tanh^{-1}(\sin(e + fx))}{8f} + \frac{c^2 \tan^3(e + fx)(3a^3 \sec(e + fx) + 4a^3)}{12f} - \frac{c^2 \tan(e + fx)(3a^3 \sec(e + fx) + 8a^3)}{8f} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \text{Sec}[e + f*x])^3 (c - c \text{Sec}[e + f*x])^2, x]$

[Out] $a^3c^2x + (3a^3c^2 \text{ArcTanh}[\text{Sin}[e + f*x]])/(8*f) - (c^2(8a^3 + 3a^3 \text{Sec}[e + f*x]) \text{Tan}[e + f*x])/(8*f) + (c^2(4a^3 + 3a^3 \text{Sec}[e + f*x]) \text{Tan}[e + f*x]^3)/(12*f)$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3881

$\text{Int}[(\text{cot}[(c_.) + (d_.)(x_.)]*(e_.))^{(m_.)} (\text{csc}[(c_.) + (d_.)(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow -\text{Simp}[(e*(e*\text{Cot}[c + d*x])^{(m-1)}*(a*m + b*(m-1)*\text{Csc}[c + d*x]))/(d*m*(m-1)), x] - \text{Dist}[e^{2/m}, \text{Int}[(e*\text{Cot}[c + d*x])^{(m-2)}*(a*m + b*(m-1)*\text{Csc}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{GtQ}[m, 1]$

Rule 3904

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)} (\text{csc}[(e_.) + (f_.)(x_.)]*(d_.) + (c_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n-m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^2 dx &= (a^2 c^2) \int (a + a \sec(e + fx)) \tan^4(e + fx) dx \\
&= \frac{c^2 (4a^3 + 3a^3 \sec(e + fx)) \tan^3(e + fx)}{12f} - \frac{1}{4} (a^2 c^2) \int (4a + 3a \sec(e + fx)) \tan^2(e + fx) dx \\
&= -\frac{c^2 (8a^3 + 3a^3 \sec(e + fx)) \tan(e + fx)}{8f} + \frac{c^2 (4a^3 + 3a^3 \sec(e + fx)) \tan^3(e + fx)}{12f} \\
&= a^3 c^2 x - \frac{c^2 (8a^3 + 3a^3 \sec(e + fx)) \tan(e + fx)}{8f} + \frac{c^2 (4a^3 + 3a^3 \sec(e + fx)) \tan^3(e + fx)}{12f} \\
&= a^3 c^2 x + \frac{3a^3 c^2 \tanh^{-1}(\sin(e + fx))}{8f} - \frac{c^2 (8a^3 + 3a^3 \sec(e + fx)) \tan(e + fx)}{8f}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 122, normalized size = 1.26

$$\frac{a^3 c^2 \sec^4(e + fx) (18 \sin(e + fx) - 32 \sin(2(e + fx)) - 30 \sin(3(e + fx)) - 32 \sin(4(e + fx)) + 96(e + fx) \cos(2(e + fx)) - 32 \sin(4(e + fx)))}{192f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]

[Out] (a^3*c^2*Sec[e + f*x]^4*(72*e + 72*f*x + 72*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Cos[4*(e + f*x)] + 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 30*Sin[3*(e + f*x)] - 32*Sin[4*(e + f*x)]))/(192*f)

fricas [A] time = 0.50, size = 147, normalized size = 1.52

$$\frac{48 a^3 c^2 f x \cos(fx + e)^4 + 9 a^3 c^2 \cos(fx + e)^4 \log(\sin(fx + e) + 1) - 9 a^3 c^2 \cos(fx + e)^4 \log(-\sin(fx + e) + 1)}{48 f \cos(fx + e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/48*(48*a^3*c^2*f*x*cos(f*x + e)^4 + 9*a^3*c^2*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 9*a^3*c^2*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^3*c^2*cos(f*x + e)^3 + 15*a^3*c^2*cos(f*x + e)^2 - 8*a^3*c^2*cos(f*x + e) - 6*a^3*c^2*sin(f*x + e))/(f*cos(f*x + e)^4)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*a^3*c^2/2*(f*x+exp(1))/2-3*a^3*c^2/16*ln(abs(tan((f*x+exp(1))/2)-1))+3*a^3*c^2/16*ln(abs(tan((f*x+exp(1))/2)+1)))+(15*tan((f*x+exp(1))/2)^7*a^3*c^2-71*tan((f*x+exp(1))/2)^5*a^3*c^2+137*tan((f*x+exp(1))/2)^3*a^3*c^2-33*tan((f*x+exp(1))/2)*a^3*c^2)*1/24/(tan((f*x+exp(1))/2)^2-1)^4

maple [A] time = 1.25, size = 136, normalized size = 1.40

$$-\frac{4a^3c^2 \tan(fx + e)}{3f} + \frac{3c^2a^3 \ln(\sec(fx + e) + \tan(fx + e))}{8f} + a^3c^2x + \frac{a^3c^2e}{f} - \frac{5c^2a^3 \sec(fx + e) \tan(fx + e)}{8f} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x)

[Out] $-4/3*a^3*c^2*\tan(f*x+e)/f+3/8/f*c^2*a^3*\ln(\sec(f*x+e)+\tan(f*x+e))+a^3*c^2*x+1/f*a^3*c^2*e-5/8/f*c^2*a^3*\sec(f*x+e)*\tan(f*x+e)+1/3/f*c^2*a^3*\tan(f*x+e)*\sec(f*x+e)^2+1/4/f*c^2*a^3*\tan(f*x+e)*\sec(f*x+e)^3$

maxima [B] time = 0.34, size = 203, normalized size = 2.09

$$16 \left(\tan(fx + e)^3 + 3 \tan(fx + e) \right) a^3 c^2 + 48 (fx + e) a^3 c^2 - 3 a^3 c^2 \left(\frac{2 \left(3 \sin(fx+e)^3 - 5 \sin(fx+e) \right)}{\sin(fx+e)^4 - 2 \sin(fx+e)^2 + 1} - 3 \log(\sin(fx + e) + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] $1/48*(16*(\tan(f*x + e)^3 + 3*\tan(f*x + e))*a^3*c^2 + 48*(f*x + e)*a^3*c^2 - 3*a^3*c^2*(2*(3*\sin(f*x + e)^3 - 5*\sin(f*x + e))/(\sin(f*x + e)^4 - 2*\sin(f*x + e)^2 + 1) - 3*\log(\sin(f*x + e) + 1) + 3*\log(\sin(f*x + e) - 1)) + 24*a^3*c^2*(2*\sin(f*x + e)/(\sin(f*x + e)^2 - 1) - \log(\sin(f*x + e) + 1) + \log(\sin(f*x + e) - 1)) + 48*a^3*c^2*\log(\sec(f*x + e) + \tan(f*x + e)) - 96*a^3*c^2*\tan(f*x + e))/f$

mupad [B] time = 2.13, size = 163, normalized size = 1.68

$$\frac{\frac{5a^3c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{4} - \frac{71a^3c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{12} + \frac{137a^3c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{12} - \frac{11a^3c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{4}}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1 \right)} + a^3c^2x + \frac{3a^3c^2 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2,x)

[Out] $((137*a^3*c^2*\tan(e/2 + (f*x)/2)^3)/12 - (71*a^3*c^2*\tan(e/2 + (f*x)/2)^5)/12 + (5*a^3*c^2*\tan(e/2 + (f*x)/2)^7)/4 - (11*a^3*c^2*\tan(e/2 + (f*x)/2))/4)/(f*(6*\tan(e/2 + (f*x)/2)^4 - 4*\tan(e/2 + (f*x)/2)^2 - 4*\tan(e/2 + (f*x)/2)^6 + \tan(e/2 + (f*x)/2)^8 + 1)) + a^3*c^2*x + (3*a^3*c^2*\operatorname{atanh}(\tan(e/2 + (f*x)/2)))/(4*f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3c^2 \left(\int 1 dx + \int \sec(e + fx) dx + \int (-2 \sec^2(e + fx)) dx + \int (-2 \sec^3(e + fx)) dx + \int \sec^4(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)

[Out] $a**3*c**2*(\operatorname{Integral}(1, x) + \operatorname{Integral}(\sec(e + f*x), x) + \operatorname{Integral}(-2*\sec(e + f*x)**2, x) + \operatorname{Integral}(-2*\sec(e + f*x)**3, x) + \operatorname{Integral}(\sec(e + f*x)**4, x) + \operatorname{Integral}(\sec(e + f*x)**5, x))$

3.15 $\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx)) dx$

Optimal. Leaf size=77

$$\frac{a^3 c \tan^3(e + fx)}{3f} - \frac{a^3 c \tan(e + fx)}{f} + \frac{a^3 c \tanh^{-1}(\sin(e + fx))}{f} - \frac{a^3 c \tan(e + fx) \sec(e + fx)}{f} + a^3 c x$$

[Out] $a^3 c x + a^3 c \operatorname{arctanh}(\sin(fx + e))/f - a^3 c \tan(fx + e)/f - a^3 c \sec(fx + e) \tan(fx + e)/f - 1/3 a^3 c \tan(fx + e)^3/f$

Rubi [A] time = 0.15, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3904, 3886, 3473, 8, 2611, 3770, 2607, 30}

$$\frac{a^3 c \tan^3(e + fx)}{3f} - \frac{a^3 c \tan(e + fx)}{f} + \frac{a^3 c \tanh^{-1}(\sin(e + fx))}{f} - \frac{a^3 c \tan(e + fx) \sec(e + fx)}{f} + a^3 c x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \operatorname{Sec}[e + f*x])^3 (c - c \operatorname{Sec}[e + f*x]), x]$

[Out] $a^3 c x + (a^3 c \operatorname{ArcTanh}[\operatorname{Sin}[e + f*x]])/f - (a^3 c \operatorname{Tan}[e + f*x])/f - (a^3 c \operatorname{Sec}[e + f*x] \operatorname{Tan}[e + f*x])/f - (a^3 c \operatorname{Tan}[e + f*x]^3)/(3*f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 2607

$\text{Int}[\sec[(e_.) + (f_.)(x_)]^{(m_.)} ((b_.) \tan[(e_.) + (f_.)(x_)]^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n (1 + x^2)^{(m/2 - 1)}], x], x, \operatorname{Tan}[e + f*x]] /; \text{FreeQ}\{b, e, f, n\}, x \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ !(\text{IntegerQ}[(n - 1)/2]) \ \&\& \ \text{LtQ}[0, n, m - 1]$

Rule 2611

$\text{Int}[(a_.) \sec[(e_.) + (f_.)(x_)]^{(m_.)} ((b_.) \tan[(e_.) + (f_.)(x_)]^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(b*(a \operatorname{Sec}[e + f*x])^m (b \operatorname{Tan}[e + f*x])^{(n-1)})/(f*(m+n-1)), x] - \text{Dist}[(b^2*(n-1))/(m+n-1), \text{Int}[(a \operatorname{Sec}[e + f*x])^m (b \operatorname{Tan}[e + f*x])^{(n-2)}], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{NeQ}[m+n-1, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rule 3473

$\text{Int}[(b_.) \tan[(c_.) + (d_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(b \operatorname{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \operatorname{Tan}[c + d*x])^{(n-2)}], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)(x_)], x_Symbol] \rightarrow -\text{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3886


```
Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^3 (c - c \sec(e + fx)) dx &= - \left((ac) \int (a + a \sec(e + fx))^2 \tan^2(e + fx) dx \right) \\ &= - \left((ac) \int (a^2 \tan^2(e + fx) + 2a^2 \sec(e + fx) \tan^2(e + fx) + a^2 \sec^2(e + fx)) dx \right) \\ &= - \left((a^3 c) \int \tan^2(e + fx) dx \right) - (a^3 c) \int \sec^2(e + fx) \tan^2(e + fx) dx \\ &= - \frac{a^3 c \tan(e + fx)}{f} - \frac{a^3 c \sec(e + fx) \tan(e + fx)}{f} + (a^3 c) \int 1 dx \\ &= a^3 cx + \frac{a^3 c \tanh^{-1}(\sin(e + fx))}{f} - \frac{a^3 c \tan(e + fx)}{f} - \frac{a^3 c \sec(e + fx)}{f} \end{aligned}$$

Mathematica [A] time = 0.45, size = 101, normalized size = 1.31

$$\frac{a^3 c \sec^3(e + fx) (-6 \sin(e + fx) - 6 \sin(2(e + fx)) - 2 \sin(3(e + fx)) + 9(e + fx) \cos(e + fx) + 3e \cos(3(e + fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*Sec[e + f*x]^3*(9*(e + f*x)*Cos[e + f*x] + 12*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^3 + 3*e*Cos[3*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] - 6*Sin[e + f*x] - 6*Sin[2*(e + f*x)] - 2*Sin[3*(e + f*x)]))/(12*f)
```

fricas [A] time = 0.45, size = 118, normalized size = 1.53

$$\frac{6 a^3 c f x \cos(fx + e)^3 + 3 a^3 c \cos(fx + e)^3 \log(\sin(fx + e) + 1) - 3 a^3 c \cos(fx + e)^3 \log(-\sin(fx + e) + 1)}{6 f \cos(fx + e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c*f*x*cos(f*x + e)^3 + 3*a^3*c*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a^3*c*cos(f*x + e)^3*log(-sin(f*x + e) + 1) - 2*(2*a^3*c*cos(f*x + e)^2 + 3*a^3*c*cos(f*x + e) + a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-2*a^3*c/2*(f*x+exp(1))/2+a^3*c/2*ln(abs(tan((f*x+exp(1))/2)-1))-a^3*c/2*ln(abs(tan((f*x+exp(1))/2)+1)))+(2*tan((f*x+exp(1))/2)^3*a^3*c-6*tan((f*x+exp(1))/2)*a^3*c)*1/3/(tan((f*x+exp(1))/2)^2-1)^3

maple [A] time = 0.86, size = 98, normalized size = 1.27

$$\frac{a^3 c \ln(\sec(fx + e) + \tan(fx + e))}{f} + a^3 c x + \frac{a^3 c e}{f} - \frac{a^3 c \sec(fx + e) \tan(fx + e)}{f} - \frac{2a^3 c \tan(fx + e)}{3f} - \frac{a^3 c \tan(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)

[Out] 1/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+a^3*c*x+1/f*a^3*c*e-a^3*c*sec(f*x+e)*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)/f-1/3/f*a^3*c*tan(f*x+e)*sec(f*x+e)^2

maxima [A] time = 0.31, size = 107, normalized size = 1.39

$$\frac{2 \left(\tan(fx + e)^3 + 3 \tan(fx + e) \right) a^3 c - 6 (fx + e) a^3 c - 3 a^3 c \left(\frac{2 \sin(fx + e)}{\sin(fx + e)^2 - 1} - \log(\sin(fx + e) + 1) + \log(\sin(fx + e) - 1) \right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] -1/6*(2*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 6*(f*x + e)*a^3*c - 3*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*a^3*c*log(sec(f*x + e) + tan(f*x + e)))/f

mupad [B] time = 1.55, size = 104, normalized size = 1.35

$$\frac{4a^3c \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - \frac{4a^3c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3}}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 - 3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + 3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right)} + a^3 c x + \frac{2a^3c \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)),x)

[Out] (4*a^3*c*tan(e/2 + (f*x)/2) - (4*a^3*c*tan(e/2 + (f*x)/2)^3)/3)/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + a^3*c*x + (2*a^3*c*atanh(tan(e/2 + (f*x)/2)))/f

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^3c \left(\int (-1) dx + \int (-2 \sec(e + fx)) dx + \int 2 \sec^3(e + fx) dx + \int \sec^4(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))*3*(c-c*sec(f*x+e)),x)

[Out] -a**3*c*(Integral(-1, x) + Integral(-2*sec(e + f*x), x) + Integral(2*sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))

$$3.16 \quad \int \frac{(a+a \sec(e+fx))^3}{c-c \sec(e+fx)} dx$$

Optimal. Leaf size=78

$$-\frac{a^3 \tan(e+fx)}{cf} + \frac{8a^3 \cot(e+fx)}{cf} + \frac{8a^3 \csc(e+fx)}{cf} - \frac{4a^3 \tanh^{-1}(\sin(e+fx))}{cf} + \frac{a^3 x}{c}$$

[Out] $a^3*x/c-4*a^3*\arctanh(\sin(f*x+e))/c/f+8*a^3*\cot(f*x+e)/c/f+8*a^3*\csc(f*x+e)/c/f-a^3*\tan(f*x+e)/c/f$

Rubi [A] time = 0.21, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3904, 3886, 3473, 8, 2606, 3767, 2621, 321, 207, 2620, 14}

$$-\frac{a^3 \tan(e+fx)}{cf} + \frac{8a^3 \cot(e+fx)}{cf} + \frac{8a^3 \csc(e+fx)}{cf} - \frac{4a^3 \tanh^{-1}(\sin(e+fx))}{cf} + \frac{a^3 x}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]

[Out] $(a^3*x)/c - (4*a^3*\text{ArcTanh}[\text{Sin}[e + f*x]])/(c*f) + (8*a^3*\text{Cot}[e + f*x])/(c*f) + (8*a^3*\text{Csc}[e + f*x])/(c*f) - (a^3*\text{Tan}[e + f*x])/(c*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2606

Int[((a_)*sec[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 2620

Int[csc[(e_) + (f_)*(x_)]^(m_)*sec[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Dist[1/f, Subst[Int[(1+x^2)^((m+n)/2-1)/x^m, x], x, Tan[e + f*x]],

x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rule 2621

Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3886

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sec(e + fx))^3}{c - c \sec(e + fx)} dx &= -\frac{\int \cot^2(e + fx)(a + a \sec(e + fx))^4 dx}{ac} \\
 &= -\frac{\int (a^4 \cot^2(e + fx) + 4a^4 \cot(e + fx) \csc(e + fx) + 6a^4 \csc^2(e + fx) + 4a^4 \csc^2(e + fx) \sec(e + fx) + a^4 \sec^2(e + fx)) dx}{ac} \\
 &= -\frac{a^3 \int \cot^2(e + fx) dx}{c} - \frac{a^3 \int \csc^2(e + fx) \sec^2(e + fx) dx}{c} - \frac{(4a^3) \int \cot(e + fx) \csc(e + fx) dx}{c} \\
 &= \frac{a^3 \cot(e + fx)}{cf} + \frac{a^3 \int 1 dx}{c} - \frac{a^3 \text{Subst}\left(\int \frac{1+x^2}{x^2} dx, x, \tan(e + fx)\right)}{cf} + \frac{(4a^3) \text{Subst}\left(\int \frac{1}{x} dx, x, \tan(e + fx)\right)}{cf} \\
 &= \frac{a^3 x}{c} + \frac{7a^3 \cot(e + fx)}{cf} + \frac{8a^3 \csc(e + fx)}{cf} - \frac{a^3 \text{Subst}\left(\int \left(1 + \frac{1}{x^2}\right) dx, x, \tan(e + fx)\right)}{cf} \\
 &= \frac{a^3 x}{c} - \frac{4a^3 \tanh^{-1}(\sin(e + fx))}{cf} + \frac{8a^3 \cot(e + fx)}{cf} + \frac{8a^3 \csc(e + fx)}{cf} - \frac{a^3 \tan(e + fx)}{cf}
 \end{aligned}$$

Mathematica [B] time = 2.58, size = 240, normalized size = 3.08

$$a^3 \cos^2(e + fx) \tan\left(\frac{1}{2}(e + fx)\right) \sec^4\left(\frac{1}{2}(e + fx)\right) (\sec(e + fx) + 1)^3 \left(8 \csc\left(\frac{e}{2}\right) \sin\left(\frac{fx}{2}\right) \sec\left(\frac{1}{2}(e + fx)\right) + \tan\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]

[Out] (a^3*Cos[e + f*x]^2*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^3*Tan[(e + f*x)/2] * (8*Csc[e/2]*Sec[(e + f*x)/2]*Sin[(f*x)/2] + (-f*x) - 4*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))) * Tan[(e + f*x)/2]) / (4*f*(c - c*Sec[e + f*x]))

fricas [A] time = 0.51, size = 125, normalized size = 1.60

$$\frac{a^3 fx \cos(fx + e) \sin(fx + e) - 2a^3 \cos(fx + e) \log(\sin(fx + e) + 1) \sin(fx + e) + 2a^3 \cos(fx + e) \log(-\sin(fx + e) + 1) \sin(fx + e) + 9a^3 \cos(fx + e)^2 + 8a^3 \cos(fx + e) - a^3}{cf \cos(fx + e) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] (a^3*f*x*cos(f*x + e)*sin(f*x + e) - 2*a^3*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) + 2*a^3*cos(f*x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) + 9*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) - a^3)/(c*f*cos(f*x + e)*sin(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-2*a^3/c*1/2*(f*x+exp(1))/2-2*a^3/c*ln(abs(tan((f*x+exp(1))/2)-1))+2*a^3/c*ln(abs(tan((f*x+exp(1))/2)+1))+(-5*tan((f*x+exp(1))/2)^2*a^3+4*a^3)/c/(tan((f*x+exp(1))/2)^3-tan((f*x+exp(1))/2)))

maple [A] time = 0.70, size = 137, normalized size = 1.76

$$\frac{8a^3}{fc \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{a^3}{fc \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1\right)} + \frac{4a^3 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1\right)}{fc} + \frac{a^3}{fc \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)} - \frac{4a^3 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)

[Out] 8/f*a^3/c/tan(1/2*e+1/2*f*x)+1/f*a^3/c/(tan(1/2*e+1/2*f*x)-1)+4/f*a^3/c*ln(tan(1/2*e+1/2*f*x)-1)+1/f*a^3/c/(tan(1/2*e+1/2*f*x)+1)-4/f*a^3/c*ln(tan(1/2*e+1/2*f*x)+1)+2/f*a^3/c*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.43, size = 274, normalized size = 3.51

$$\frac{a^3 \left(\frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2 - 1} + \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{c} \right) - a^3 \left(\frac{2 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} + \frac{\cos(fx+e)+1}{c \sin(fx+e)} \right) + 3 a^3 \left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{c} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] $-(a^3 * ((3 * \sin(f * x + e)^2 / (\cos(f * x + e) + 1)^2 - 1) / (c * \sin(f * x + e) / (\cos(f * x + e) + 1) - c * \sin(f * x + e)^3 / (\cos(f * x + e) + 1)^3) + \log(\sin(f * x + e) / (\cos(f * x + e) + 1) + 1) / c - \log(\sin(f * x + e) / (\cos(f * x + e) + 1) - 1) / c) - a^3 * (2 * \arctan(\sin(f * x + e) / (\cos(f * x + e) + 1)) / c + (\cos(f * x + e) + 1) / (c * \sin(f * x + e))) + 3 * a^3 * (\log(\sin(f * x + e) / (\cos(f * x + e) + 1) + 1) / c - \log(\sin(f * x + e) / (\cos(f * x + e) + 1) - 1) / c - (\cos(f * x + e) + 1) / (c * \sin(f * x + e))) - 3 * a^3 * (\cos(f * x + e) + 1) / (c * \sin(f * x + e))) / f$

mupad [B] time = 1.48, size = 85, normalized size = 1.09

$$\frac{a^3 x}{c} - \frac{10 a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 8 a^3}{f \left(c \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \right)} - \frac{8 a^3 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{c f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x)),x)

[Out] $(a^3 * x) / c - (10 * a^3 * \tan(e / 2 + (f * x) / 2)^2 - 8 * a^3) / (f * (c * \tan(e / 2 + (f * x) / 2) - c * \tan(e / 2 + (f * x) / 2)^3)) - (8 * a^3 * \operatorname{atanh}(\tan(e / 2 + (f * x) / 2))) / (c * f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \left(\int \frac{3 \sec(e+fx)}{\sec(e+fx)-1} dx + \int \frac{3 \sec^2(e+fx)}{\sec(e+fx)-1} dx + \int \frac{\sec^3(e+fx)}{\sec(e+fx)-1} dx + \int \frac{1}{\sec(e+fx)-1} dx \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)

[Out] $-a^{**3} * (\operatorname{Integral}(3 * \sec(e + f * x) / (\sec(e + f * x) - 1), x) + \operatorname{Integral}(3 * \sec(e + f * x)^{**2} / (\sec(e + f * x) - 1), x) + \operatorname{Integral}(\sec(e + f * x)^{**3} / (\sec(e + f * x) - 1), x) + \operatorname{Integral}(1 / (\sec(e + f * x) - 1), x)) / c$

$$3.17 \quad \int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=88

$$\frac{a^3 \tanh^{-1}(\sin(e+fx))}{c^2 f} + \frac{4a^3 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))} - \frac{8a^3 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))^2} + \frac{a^3 x}{c^2}$$

[Out] a^3*x/c^2+a^3*arctanh(sin(f*x+e))/c^2/f-8/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2+4/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x+e))

Rubi [A] time = 0.36, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3903, 3777, 3919, 3794, 3796, 3797, 3799, 3998, 3770}

$$\frac{a^3 \tanh^{-1}(\sin(e+fx))}{c^2 f} + \frac{4a^3 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))} - \frac{8a^3 \tan(e+fx)}{3c^2 f(1-\sec(e+fx))^2} + \frac{a^3 x}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]

[Out] (a^3*x)/c^2 + (a^3*ArcTanh[Sin[e + f*x]]/(c^2*f) - (8*a^3*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*c^2*f*(1 - Sec[e + f*x])))

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^n, x_Symbol] := -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_),
x_Symbol] := Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_.))^(n_), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^3}{(c - c \sec(e + fx))^2} dx &= \frac{\int \left(\frac{a^3}{(1 - \sec(e + fx))^2} + \frac{3a^3 \sec(e + fx)}{(1 - \sec(e + fx))^2} + \frac{3a^3 \sec^2(e + fx)}{(1 - \sec(e + fx))^2} + \frac{a^3 \sec^3(e + fx)}{(1 - \sec(e + fx))^2} \right) dx}{c^2} \\ &= \frac{a^3 \int \frac{1}{(1 - \sec(e + fx))^2} dx}{c^2} + \frac{a^3 \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{c^2} + \frac{(3a^3) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{c^2} + \frac{(3a^3) \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^2} dx}{c^2} \\ &= -\frac{8a^3 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))^2} - \frac{a^3 \int \frac{-3 - \sec(e + fx)}{1 - \sec(e + fx)} dx}{3c^2} + \frac{a^3 \int \frac{(-2 - 3 \sec(e + fx)) \sec(e + fx)}{1 - \sec(e + fx)} dx}{3c^2} + \frac{a^3 \int \frac{\sec^2(e + fx)}{1 - \sec(e + fx)} dx}{c^2} \\ &= \frac{a^3 x}{c^2} - \frac{8a^3 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))^2} + \frac{a^3 \tan(e + fx)}{c^2 f (1 - \sec(e + fx))} + \frac{a^3 \int \sec(e + fx) dx}{c^2} + \frac{(4a^3)}{c^2} \\ &= \frac{a^3 x}{c^2} + \frac{a^3 \tanh^{-1}(\sin(e + fx))}{c^2 f} - \frac{8a^3 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))^2} + \frac{4a^3 \tan(e + fx)}{3c^2 f (1 - \sec(e + fx))} \end{aligned}$$

Mathematica [B] time = 1.18, size = 177, normalized size = 2.01

$$\frac{a^3 (\cos(e + fx) + 1)^3 \tan\left(\frac{1}{2}(e + fx)\right) \sec^2\left(\frac{1}{2}(e + fx)\right) \left(-4 \cot\left(\frac{e}{2}\right) \tan\left(\frac{1}{2}(e + fx)\right) \sec^2\left(\frac{1}{2}(e + fx)\right) + 4 \csc\left(\frac{e}{2}\right) \sin\left(\frac{1}{2}(e + fx)\right)\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```


[Out] $(a^3(1 + \cos(e + fx))^3 \sec((e + fx)/2)^2 \tan((e + fx)/2) * (4 \csc(e/2) * \sec((e + fx)/2) \sin((fx)/2) - 4 \cot(e/2) * \sec((e + fx)/2)^2 \tan((e + fx)/2) + 3 * (fx - \log[\cos((e + fx)/2) - \sin((e + fx)/2)] + \log[\cos((e + fx)/2) + \sin((e + fx)/2)]) * \tan((e + fx)/2)^3) / (6 * c^2 * f * (-1 + \cos(e + fx))^2)$

fricas [A] time = 0.45, size = 156, normalized size = 1.77

$$\frac{8a^3 \cos(fx + e)^2 + 16a^3 \cos(fx + e) + 8a^3 + 3(a^3 \cos(fx + e) - a^3) \log(\sin(fx + e) + 1) \sin(fx + e) - 3(a^3 \cos(fx + e) - a^3) \log(-\sin(fx + e) + 1) \sin(fx + e) + 6(a^3 f \cos(fx + e) - a^3 f) \sin(fx + e)}{6(c^2 f \cos(fx + e) - c^2 f) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] $1/6 * (8 * a^3 * \cos(f * x + e)^2 + 16 * a^3 * \cos(f * x + e) + 8 * a^3 + 3 * (a^3 * \cos(f * x + e) - a^3) * \log(\sin(f * x + e) + 1) * \sin(f * x + e) - 3 * (a^3 * \cos(f * x + e) - a^3) * \log(-\sin(f * x + e) + 1) * \sin(f * x + e) + 6 * (a^3 * f * \cos(f * x + e) - a^3 * f) * \sin(f * x + e)) / ((c^2 * f * \cos(f * x + e) - c^2 * f) * \sin(f * x + e))$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: $(4 * \pi / x / 2) > (-4 * \pi / x / 2)$ Unable to check sign: $(4 * \pi / x / 2) > (-4 * \pi / x / 2)$ $2 / f * (-a^3 * 1 / 2 / c^2 * \ln(\tan((f * x + \exp(1)) / 2) - 1)) + a^3 * 1 / 2 / c^2 * \ln(\tan((f * x + \exp(1)) / 2) + 1)) + 2 * a^3 * 1 / 2 / c^2 * (f * x + \exp(1)) / 2 - 2 * a^3 * 1 / 3 / c^2 / \tan((f * x + \exp(1)) / 2)^3$

maple [A] time = 0.83, size = 90, normalized size = 1.02

$$\frac{4a^3}{3f c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} - \frac{a^3 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1\right)}{f c^2} + \frac{a^3 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}{f c^2} + \frac{2a^3 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)

[Out] $-4/3 * f * a^3 / c^2 / \tan(1/2 * e + 1/2 * f * x)^3 - 1/f * a^3 / c^2 * \ln(\tan(1/2 * e + 1/2 * f * x) - 1) + 1/f * a^3 / c^2 * \ln(\tan(1/2 * e + 1/2 * f * x) + 1) + 2/f * a^3 / c^2 * \arctan(\tan(1/2 * e + 1/2 * f * x))$

maxima [B] time = 0.44, size = 274, normalized size = 3.11

$$\frac{a^3 \left(\frac{12 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 1\right) (\cos(fx+e)+1)^3}{c^2 \sin(fx+e)^3} \right) + a^3 \left(\frac{6 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{c^2} - \frac{6 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{c^2} - \frac{\left(\frac{9 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 1\right)}{c^2 \sin(fx+e)} \right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] $1/6 * (a^3 * (12 * \arctan(\sin(f * x + e) / (\cos(f * x + e) + 1))) / c^2 + (9 * \sin(f * x + e)^2 / ((\cos(f * x + e) + 1)^2 - 1)) * (\cos(f * x + e) + 1)^3 / (c^2 * \sin(f * x + e)^3)) + a^3$

$$3*(6*\log(\sin(f*x + e)/(\cos(f*x + e) + 1) + 1)/c^2 - 6*\log(\sin(f*x + e)/(\cos(f*x + e) + 1) - 1)/c^2 - (9*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)*(\cos(f*x + e) + 1)^3/(c^2*\sin(f*x + e)^3)) - 3*a^3*(3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)*(\cos(f*x + e) + 1)^3/(c^2*\sin(f*x + e)^3) + 3*a^3*(3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 1)*(\cos(f*x + e) + 1)^3/(c^2*\sin(f*x + e)^3))/f$$

mupad [B] time = 1.44, size = 45, normalized size = 0.51

$$\frac{a^3 x}{c^2} + \frac{a^3 \left(2 \operatorname{atanh} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right) - \frac{4 \cot \left(\frac{e}{2} + \frac{fx}{2} \right)^3}{3} \right)}{c^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^2,x)

[Out] (a^3*x)/c^2 + (a^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*cot(e/2 + (f*x)/2)^3)/3))/(c^2*f)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \left(\int \frac{3 \sec(e+fx)}{\sec^2(e+fx)-2 \sec(e+fx)+1} dx + \int \frac{3 \sec^2(e+fx)}{\sec^2(e+fx)-2 \sec(e+fx)+1} dx + \int \frac{\sec^3(e+fx)}{\sec^2(e+fx)-2 \sec(e+fx)+1} dx + \int \frac{1}{\sec^2(e+fx)-2 \sec(e+fx)+1} dx \right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)

[Out] a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2

$$3.18 \quad \int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=102

$$-\frac{26a^3 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))} + \frac{4a^3 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))^2} - \frac{8a^3 \tan(e+fx)}{5c^3 f(1-\sec(e+fx))^3} + \frac{a^3 x}{c^3}$$

[Out] a^3*x/c^3-8/5*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3+4/15*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^2-26/15*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3

Rubi [A] time = 0.45, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{26a^3 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))} + \frac{4a^3 \tan(e+fx)}{15c^3 f(1-\sec(e+fx))^2} - \frac{8a^3 \tan(e+fx)}{5c^3 f(1-\sec(e+fx))^3} + \frac{a^3 x}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]

[Out] (a^3*x)/c^3 - (8*a^3*Tan[e + f*x])/(5*c^3*f*(1 - Sec[e + f*x])^3) + (4*a^3*Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*c^3*f*(1 - Sec[e + f*x]))

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^n, x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3799

Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a

$\wedge 2 - b^2, 0]$ && LtQ[m, $-2^{(-1)}$]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, $-2^{(-1)}$]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^3}{(c - c \sec(e + fx))^3} dx &= \frac{\int \left(\frac{a^3}{(1 - \sec(e + fx))^3} + \frac{3a^3 \sec(e + fx)}{(1 - \sec(e + fx))^3} + \frac{3a^3 \sec^2(e + fx)}{(1 - \sec(e + fx))^3} + \frac{a^3 \sec^3(e + fx)}{(1 - \sec(e + fx))^3} \right) dx}{c^3} \\ &= \frac{a^3 \int \frac{1}{(1 - \sec(e + fx))^3} dx}{c^3} + \frac{a^3 \int \frac{\sec^3(e + fx)}{(1 - \sec(e + fx))^3} dx}{c^3} + \frac{(3a^3) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{c^3} + \frac{(3a^3) \int \frac{\sec^3(e + fx)}{(1 - \sec(e + fx))^3} dx}{c^3} \\ &= -\frac{8a^3 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} - \frac{a^3 \int \frac{-5 - 2 \sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{5c^3} + \frac{a^3 \int \frac{(-3 - 5 \sec(e + fx)) \sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{5c^3} + \frac{a^3 \int \frac{15 + 7 \sec(e + fx)}{1 - \sec(e + fx)} dx}{15c^3} + \frac{(2a^3) \int \frac{\sec^3(e + fx)}{1 - \sec(e + fx)} dx}{15c^3} \\ &= \frac{a^3 x}{c^3} - \frac{8a^3 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} + \frac{4a^3 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))^2} - \frac{4a^3 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))} + \frac{a^3 x}{c^3} - \frac{8a^3 \tan(e + fx)}{5c^3 f(1 - \sec(e + fx))^3} + \frac{4a^3 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))^2} - \frac{26a^3 \tan(e + fx)}{15c^3 f(1 - \sec(e + fx))} \end{aligned}$$

Mathematica [C] time = 0.10, size = 53, normalized size = 0.52

$$\frac{2a^3 \cot^5\left(\frac{e}{2} + \frac{fx}{2}\right) {}_2F_1\left(-\frac{5}{2}, 1; -\frac{3}{2}; -\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{5c^3 f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]

[Out] (2*a^3*Cot[e/2 + (f*x)/2]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e/2 + (f*x)/2]^2])/(5*c^3*f)

fricas [A] time = 0.43, size = 128, normalized size = 1.25

$$\frac{46 a^3 \cos (f x+e)^3-2 a^3 \cos (f x+e)^2-22 a^3 \cos (f x+e)+26 a^3+15\left(a^3 f x \cos (f x+e)^2-2 a^3 f x \cos (f x+e)\right)}{15\left(c^3 f \cos (f x+e)^2-2 c^3 f \cos (f x+e)+c^3 f\right) \sin (f x+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(46*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)^2 - 22*a^3*cos(f*x + e) + 26*a^3 + 15*(a^3*f*x*cos(f*x + e)^2 - 2*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))

giac [A] time = 0.40, size = 77, normalized size = 0.75

$$\frac{\frac{15(f x+e) a^3}{c^3}+\frac{2\left(15 a^3 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^4-5 a^3 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^2+3 a^3\right)}{c^3 \tan \left(\frac{1}{2} f x+\frac{1}{2} e\right)^5}}{15 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] 1/15*(15*(f*x + e)*a^3/c^3 + 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 5*a^3*tan(1/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f

maple [A] time = 0.82, size = 89, normalized size = 0.87

$$-\frac{2 a^3}{3 f c^3 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)^3}+\frac{2 a^3}{5 f c^3 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)^5}+\frac{2 a^3}{f c^3 \tan \left(\frac{e}{2}+\frac{f x}{2}\right)}+\frac{2 a^3 \arctan \left(\tan \left(\frac{e}{2}+\frac{f x}{2}\right)\right)}{f c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)

[Out] -2/3/f*a^3/c^3/tan(1/2*e+1/2*f*x)^3+2/5/f*a^3/c^3/tan(1/2*e+1/2*f*x)^5+2/f*a^3/c^3/tan(1/2*e+1/2*f*x)+2/f*a^3/c^3*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.44, size = 282, normalized size = 2.76

$$a^3 \left(\frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3} - \frac{\left(\frac{20 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{105 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5} \right) + \frac{a^3 \left(\frac{10 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 3\right)(\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{1}{60} * (a^3 * (120 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1))) / c^3 - (20 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 - 105 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 - 3) * (\cos(f*x + e) + 1)^5 / (c^3 * \sin(f*x + e)^5)) + a^3 * (10 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 15 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 3) * (\cos(f*x + e) + 1)^5 / (c^3 * \sin(f*x + e)^5) - 3 * a^3 * (10 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 - 15 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 - 3) * (\cos(f*x + e) + 1)^5 / (c^3 * \sin(f*x + e)^5) - 9 * a^3 * (5 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 - 1) * (\cos(f*x + e) + 1)^5 / (c^3 * \sin(f*x + e)^5)) / f$

mupad [B] time = 1.38, size = 96, normalized size = 0.94

$$\frac{a^3 x}{c^3} + \frac{\frac{2a^3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{5} - \frac{2a^3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{3} + 2a^3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{c^3 f \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^3,x)

[Out] $(a^3 * x) / c^3 + ((2 * a^3 * \cos(e/2 + (f*x)/2)^5) / 5 + 2 * a^3 * \cos(e/2 + (f*x)/2) * \sin(e/2 + (f*x)/2)^4 - (2 * a^3 * \cos(e/2 + (f*x)/2)^3 * \sin(e/2 + (f*x)/2)^2) / 3) / (c^3 * f * \sin(e/2 + (f*x)/2)^5)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \left(\int \frac{3 \sec(e+fx)}{\sec^3(e+fx) - 3 \sec^2(e+fx) + 3 \sec(e+fx) - 1} dx + \int \frac{3 \sec^2(e+fx)}{\sec^3(e+fx) - 3 \sec^2(e+fx) + 3 \sec(e+fx) - 1} dx + \int \frac{\sec^3(e+fx)}{\sec^3(e+fx) - 3 \sec^2(e+fx) + 3 \sec(e+fx) - 1} dx \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)

[Out] $-a^{**3} * (\text{Integral}(3 * \sec(e + f*x) / (\sec(e + f*x)^{**3} - 3 * \sec(e + f*x)^{**2} + 3 * \sec(e + f*x) - 1), x) + \text{Integral}(3 * \sec(e + f*x)^{**2} / (\sec(e + f*x)^{**3} - 3 * \sec(e + f*x)^{**2} + 3 * \sec(e + f*x) - 1), x) + \text{Integral}(\sec(e + f*x)^{**3} / (\sec(e + f*x)^{**3} - 3 * \sec(e + f*x)^{**2} + 3 * \sec(e + f*x) - 1), x) + \text{Integral}(1 / (\sec(e + f*x)^{**3} - 3 * \sec(e + f*x)^{**2} + 3 * \sec(e + f*x) - 1), x)) / c^{**3}$

$$3.19 \quad \int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=133

$$-\frac{167a^3 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))} - \frac{62a^3 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))^2} + \frac{4a^3 \tan(e+fx)}{35c^4 f(1-\sec(e+fx))^3} - \frac{8a^3 \tan(e+fx)}{7c^4 f(1-\sec(e+fx))^4} + \frac{a^3 x}{c^4}$$

[Out] $a^3 x/c^4 - 8/7 * a^3 * \tan(f*x+e)/c^4/f/(1-\sec(f*x+e))^4 + 4/35 * a^3 * \tan(f*x+e)/c^4/f/(1-\sec(f*x+e))^3 - 62/105 * a^3 * \tan(f*x+e)/c^4/f/(1-\sec(f*x+e))^2 - 167/105 * a^3 * \tan(f*x+e)/c^4/f/(1-\sec(f*x+e))$

Rubi [A] time = 0.58, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{167a^3 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))} - \frac{62a^3 \tan(e+fx)}{105c^4 f(1-\sec(e+fx))^2} + \frac{4a^3 \tan(e+fx)}{35c^4 f(1-\sec(e+fx))^3} - \frac{8a^3 \tan(e+fx)}{7c^4 f(1-\sec(e+fx))^4} + \frac{a^3 x}{c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4, x]

[Out] $(a^3 x)/c^4 - (8 * a^3 * \tan[e + f*x])/(7 * c^4 * f * (1 - \sec[e + f*x])^4) + (4 * a^3 * \tan[e + f*x])/(35 * c^4 * f * (1 - \sec[e + f*x])^3) - (62 * a^3 * \tan[e + f*x])/(105 * c^4 * f * (1 - \sec[e + f*x])^2) - (167 * a^3 * \tan[e + f*x])/(105 * c^4 * f * (1 - \sec[e + f*x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3799

Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),

$x] - \text{Dist}[1/(a^2(2m + 1)), \text{Int}[\text{Csc}[e + fx](a + b\text{Csc}[e + fx])^{m+1}(a^m - b(2m + 1)\text{Csc}[e + fx]), x], x] /;$ FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^{(m_.)}(\text{csc}[(e_.) + (f_.)(x_.)](d_.) + (c_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[c^n, \text{Int}[\text{ExpandTrig}[(1 + (d\text{csc}[e + fx])/c)^n, (a + b\text{csc}[e + fx])^m, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rule 3919

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](d_.) + (c_.))/(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.)), x_Symbol] \rightarrow \text{Simp}[(c*x)/a, x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\text{Csc}[e + fx]/(a + b\text{Csc}[e + fx]), x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^{(m_.)}(\text{csc}[(e_.) + (f_.)(x_.)](d_.) + (c_.)), x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)\text{Cot}[e + fx](a + b\text{Csc}[e + fx])^m/(b*f*(2*m + 1)), x] + \text{Dist}[1/(a^2(2*m + 1)), \text{Int}[(a + b\text{Csc}[e + fx])^{m+1}\text{Simp}[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)\text{Csc}[e + fx], x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4000

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)](\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^{(m_.)}(\text{csc}[(e_.) + (f_.)(x_.)](B_.) + (A_.)), x_Symbol] \rightarrow \text{Simp}[(A*b - a*B)\text{Cot}[e + fx](a + b\text{Csc}[e + fx])^m/(a*f*(2*m + 1)), x] + \text{Dist}[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), \text{Int}[\text{Csc}[e + fx](a + b\text{Csc}[e + fx])^{m+1}, x], x] /;$ FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^3}{(c - c \sec(e + fx))^4} dx &= \frac{\int \left(\frac{a^3}{(1 - \sec(e + fx))^4} + \frac{3a^3 \sec(e + fx)}{(1 - \sec(e + fx))^4} + \frac{3a^3 \sec^2(e + fx)}{(1 - \sec(e + fx))^4} + \frac{a^3 \sec^3(e + fx)}{(1 - \sec(e + fx))^4} \right) dx}{c^4} \\
&= \frac{a^3 \int \frac{1}{(1 - \sec(e + fx))^4} dx}{c^4} + \frac{a^3 \int \frac{\sec^3(e + fx)}{(1 - \sec(e + fx))^4} dx}{c^4} + \frac{(3a^3) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^4} dx}{c^4} + \frac{(3a^3) \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^4} dx}{c^4} \\
&= -\frac{8a^3 \tan(e + fx)}{7c^4 f(1 - \sec(e + fx))^4} - \frac{a^3 \int \frac{-7-3 \sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{7c^4} + \frac{a^3 \int \frac{(-4-7 \sec(e + fx)) \sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{7c^4} \\
&= -\frac{8a^3 \tan(e + fx)}{7c^4 f(1 - \sec(e + fx))^4} + \frac{4a^3 \tan(e + fx)}{35c^4 f(1 - \sec(e + fx))^3} + \frac{a^3 \int \frac{35+20 \sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{35c^4} + \frac{a^3 \int \frac{35+20 \sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{35c^4} \\
&= -\frac{8a^3 \tan(e + fx)}{7c^4 f(1 - \sec(e + fx))^4} + \frac{4a^3 \tan(e + fx)}{35c^4 f(1 - \sec(e + fx))^3} - \frac{62a^3 \tan(e + fx)}{105c^4 f(1 - \sec(e + fx))^2} \\
&= \frac{a^3 x}{c^4} - \frac{8a^3 \tan(e + fx)}{7c^4 f(1 - \sec(e + fx))^4} + \frac{4a^3 \tan(e + fx)}{35c^4 f(1 - \sec(e + fx))^3} - \frac{62a^3 \tan(e + fx)}{105c^4 f(1 - \sec(e + fx))^2} \\
&= \frac{a^3 x}{c^4} - \frac{8a^3 \tan(e + fx)}{7c^4 f(1 - \sec(e + fx))^4} + \frac{4a^3 \tan(e + fx)}{35c^4 f(1 - \sec(e + fx))^3} - \frac{62a^3 \tan(e + fx)}{105c^4 f(1 - \sec(e + fx))^2}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 227, normalized size = 1.71

$$a^3 \csc\left(\frac{e}{2}\right) \csc^7\left(\frac{1}{2}(e + fx)\right) \left(-11270 \sin\left(e + \frac{fx}{2}\right) + 9114 \sin\left(e + \frac{3fx}{2}\right) + 5040 \sin\left(2e + \frac{3fx}{2}\right) - 3248 \sin\left(2e + \frac{5fx}{2}\right) + 1470 \sin\left(3e + \frac{5fx}{2}\right) - 674 \sin\left(3e + \frac{7fx}{2}\right)\right) / (13440 c^4 f)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]

[Out] (a^3*Csc[e/2]*Csc[(e + f*x)/2]^7*(3675*f*x*Cos[(f*x)/2] - 3675*f*x*Cos[e + (f*x)/2] - 2205*f*x*Cos[e + (3*f*x)/2] + 2205*f*x*Cos[2*e + (3*f*x)/2] + 735*f*x*Cos[2*e + (5*f*x)/2] - 735*f*x*Cos[3*e + (5*f*x)/2] - 105*f*x*Cos[3*e + (7*f*x)/2] + 105*f*x*Cos[4*e + (7*f*x)/2] - 12320*Sin[(f*x)/2] - 11270*Sin[e + (f*x)/2] + 9114*Sin[e + (3*f*x)/2] + 5040*Sin[2*e + (3*f*x)/2] - 3248*Sin[2*e + (5*f*x)/2] - 1470*Sin[3*e + (5*f*x)/2] + 674*Sin[3*e + (7*f*x)/2]))/(13440*c^4*f)

fricas [A] time = 0.45, size = 172, normalized size = 1.29

$$\frac{337 a^3 \cos(fx + e)^4 - 276 a^3 \cos(fx + e)^3 - 50 a^3 \cos(fx + e)^2 + 396 a^3 \cos(fx + e) - 167 a^3 + 105 (a^3 fx \cos(fx + e)^3 - 3 a^3 fx \cos(fx + e)^2 + 3 a^3 fx \cos(fx + e) - a^3 fx \sin(fx + e))}{105 (c^4 f \cos(fx + e)^3 - 3 c^4 f \cos(fx + e)^2 + 3 c^4 f \cos(fx + e) - c^4 f \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] 1/105*(337*a^3*cos(f*x + e)^4 - 276*a^3*cos(f*x + e)^3 - 50*a^3*cos(f*x + e)^2 + 396*a^3*cos(f*x + e) - 167*a^3 + 105*(a^3*f*x*cos(f*x + e)^3 - 3*a^3*f*x*cos(f*x + e)^2 + 3*a^3*f*x*cos(f*x + e) - a^3*f*x*sin(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f*sin(f*x + e))

giac [A] time = 0.59, size = 93, normalized size = 0.70

$$\frac{\frac{105(fx+e)a^3}{c^4} + \frac{210a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 70a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 42a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 15a^3}{c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7}}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")

[Out] 1/105*(105*(f*x + e)*a^3/c^4 + (210*a^3*tan(1/2*f*x + 1/2*e)^6 - 70*a^3*tan(1/2*f*x + 1/2*e)^4 + 42*a^3*tan(1/2*f*x + 1/2*e)^2 - 15*a^3)/(c^4*tan(1/2*f*x + 1/2*e)^7))/f

maple [A] time = 0.88, size = 111, normalized size = 0.83

$$-\frac{a^3}{7f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{2a^3}{5f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{2a^3}{3f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{2a^3}{f c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2a^3 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)

[Out] -1/7/f*a^3/c^4/tan(1/2*e+1/2*f*x)^7+2/5/f*a^3/c^4/tan(1/2*e+1/2*f*x)^5-2/3/f*a^3/c^4/tan(1/2*e+1/2*f*x)^3+2/f*a^3/c^4/tan(1/2*e+1/2*f*x)+2/f*a^3/c^4*a rctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.45, size = 383, normalized size = 2.88

$$5a^3 \left(\frac{336 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right)(\cos(fx+e)+1)^7}{c^4 \sin(fx+e)^7} \right) + \frac{3a^3 \left(\frac{21 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{105 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} + 15\right)}{c^4 \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

[Out] 1/840*(5*a^3*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7)) + 3*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 9*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f

mupad [B] time = 1.44, size = 122, normalized size = 0.92

$$\frac{a^3 \left(-\frac{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)^7}{7} + \frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{5} - \frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{3} + 2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + (e + fx) \sin\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}{c^4 f \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^4,x)`

[Out] $(a^3(2\cos(e/2 + (f*x)/2)\sin(e/2 + (f*x)/2)^6 - \cos(e/2 + (f*x)/2)^{7/7} + \sin(e/2 + (f*x)/2)^7(e + f*x) - (2\cos(e/2 + (f*x)/2)^3\sin(e/2 + (f*x)/2)^4)/3 + (2\cos(e/2 + (f*x)/2)^5\sin(e/2 + (f*x)/2)^2)/5)/(c^4*f*\sin(e/2 + (f*x)/2)^7)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{3 \sec(e+fx)}{\sec^4(e+fx) - 4 \sec^3(e+fx) + 6 \sec^2(e+fx) - 4 \sec(e+fx) + 1} dx + \int \frac{3 \sec^2(e+fx)}{\sec^4(e+fx) - 4 \sec^3(e+fx) + 6 \sec^2(e+fx) - 4 \sec(e+fx) + 1} dx + \int \frac{3 \sec^3(e+fx)}{\sec^4(e+fx) - 4 \sec^3(e+fx) + 6 \sec^2(e+fx) - 4 \sec(e+fx) + 1} dx \right) / c^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)`

[Out] $a^3 * (\text{Integral}(3*\sec(e + f*x)/(\sec(e + f*x)**4 - 4*\sec(e + f*x)**3 + 6*\sec(e + f*x)**2 - 4*\sec(e + f*x) + 1), x) + \text{Integral}(3*\sec(e + f*x)**2/(\sec(e + f*x)**4 - 4*\sec(e + f*x)**3 + 6*\sec(e + f*x)**2 - 4*\sec(e + f*x) + 1), x) + \text{Integral}(\sec(e + f*x)**3/(\sec(e + f*x)**4 - 4*\sec(e + f*x)**3 + 6*\sec(e + f*x)**2 - 4*\sec(e + f*x) + 1), x) + \text{Integral}(1/(\sec(e + f*x)**4 - 4*\sec(e + f*x)**3 + 6*\sec(e + f*x)**2 - 4*\sec(e + f*x) + 1), x))/c^4$

$$3.20 \quad \int \frac{(a+a \sec(e+fx))^3}{(c-c \sec(e+fx))^5} dx$$

Optimal. Leaf size=164

$$\frac{496a^3 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))} - \frac{181a^3 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))^2} - \frac{38a^3 \tan(e+fx)}{105c^5 f(1-\sec(e+fx))^3} + \frac{4a^3 \tan(e+fx)}{63c^5 f(1-\sec(e+fx))^4} - \frac{8a^3 \tan(e+fx)}{9c^5 f(1-\sec(e+fx))^5}$$

[Out] a^3*x/c^5-8/9*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5+4/63*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^4-38/105*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-181/315*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-496/315*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5

Rubi [A] time = 0.73, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$\frac{496a^3 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))} - \frac{181a^3 \tan(e+fx)}{315c^5 f(1-\sec(e+fx))^2} - \frac{38a^3 \tan(e+fx)}{105c^5 f(1-\sec(e+fx))^3} + \frac{4a^3 \tan(e+fx)}{63c^5 f(1-\sec(e+fx))^4} - \frac{8a^3 \tan(e+fx)}{9c^5 f(1-\sec(e+fx))^5}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]

[Out] (a^3*x)/c^5 - (8*a^3*Tan[e + f*x])/(9*c^5*f*(1 - Sec[e + f*x])^5) + (4*a^3*Tan[e + f*x])/(63*c^5*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*c^5*f*(1 - Sec[e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x])^2) - (496*a^3*Tan[e + f*x])/(315*c^5*f*(1 - Sec[e + f*x]))

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_),
x_Symbol] := Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d
_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[((A*b - a*B)*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^3}{(c - c \sec(e + fx))^5} dx &= \frac{\int \left(\frac{a^3}{(1 - \sec(e + fx))^5} + \frac{3a^3 \sec(e + fx)}{(1 - \sec(e + fx))^5} + \frac{3a^3 \sec^2(e + fx)}{(1 - \sec(e + fx))^5} + \frac{a^3 \sec^3(e + fx)}{(1 - \sec(e + fx))^5} \right) dx}{c^5} \\
&= \frac{a^3 \int \frac{1}{(1 - \sec(e + fx))^5} dx}{c^5} + \frac{a^3 \int \frac{\sec^3(e + fx)}{(1 - \sec(e + fx))^5} dx}{c^5} + \frac{(3a^3) \int \frac{\sec(e + fx)}{(1 - \sec(e + fx))^5} dx}{c^5} + \frac{(3a^3) \int \frac{\sec^2(e + fx)}{(1 - \sec(e + fx))^5} dx}{c^5} \\
&= -\frac{8a^3 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} - \frac{a^3 \int \frac{-9-4 \sec(e + fx)}{(1 - \sec(e + fx))^4} dx}{9c^5} + \frac{a^3 \int \frac{(-5-9 \sec(e + fx)) \sec(e + fx)}{(1 - \sec(e + fx))^4} dx}{9c^5} + \frac{a^3 \int \frac{(-1-9 \sec(e + fx)) \sec^2(e + fx)}{(1 - \sec(e + fx))^4} dx}{9c^5} \\
&= -\frac{8a^3 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} + \frac{4a^3 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} + \frac{a^3 \int \frac{63+39 \sec(e + fx)}{(1 - \sec(e + fx))^3} dx}{63c^5} + \frac{a^3 \int \frac{315+21 \sec(e + fx)}{(1 - \sec(e + fx))^2} dx}{63c^5} \\
&= -\frac{8a^3 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} + \frac{4a^3 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{38a^3 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} - \frac{a^3 \tan(e + fx)}{315c^5 f(1 - \sec(e + fx))^2} \\
&= -\frac{8a^3 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} + \frac{4a^3 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{38a^3 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} - \frac{a^3 \tan(e + fx)}{315c^5 f(1 - \sec(e + fx))^2} \\
&= \frac{a^3 x}{c^5} - \frac{8a^3 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} + \frac{4a^3 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{38a^3 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} - \frac{a^3 \tan(e + fx)}{315c^5 f(1 - \sec(e + fx))^2} \\
&= \frac{a^3 x}{c^5} - \frac{8a^3 \tan(e + fx)}{9c^5 f(1 - \sec(e + fx))^5} + \frac{4a^3 \tan(e + fx)}{63c^5 f(1 - \sec(e + fx))^4} - \frac{38a^3 \tan(e + fx)}{105c^5 f(1 - \sec(e + fx))^3} - \frac{a^3 \tan(e + fx)}{315c^5 f(1 - \sec(e + fx))^2}
\end{aligned}$$

Mathematica [A] time = 0.90, size = 283, normalized size = 1.73

$$a^3 \csc\left(\frac{e}{2}\right) \csc^9\left(\frac{1}{2}(e + fx)\right) \left(-122850 \sin\left(e + \frac{fx}{2}\right) + 103278 \sin\left(e + \frac{3fx}{2}\right) + 73290 \sin\left(2e + \frac{3fx}{2}\right) - 51102 \sin\left(2e + \frac{5fx}{2}\right) + 11340 \sin\left(2e + \frac{7fx}{2}\right) - 11340 \sin\left(3e + \frac{5fx}{2}\right) - 2835 \sin\left(3e + \frac{7fx}{2}\right) + 2835 \sin\left(4e + \frac{7fx}{2}\right) + 315 \sin\left(4e + \frac{9fx}{2}\right) - 315 \sin\left(5e + \frac{9fx}{2}\right) - 142002 \sin\left(\frac{fx}{2}\right) - 122850 \sin\left(e + \frac{fx}{2}\right) + 103278 \sin\left(e + \frac{3fx}{2}\right) + 73290 \sin\left(2e + \frac{3fx}{2}\right) - 51102 \sin\left(2e + \frac{5fx}{2}\right) - 24570 \sin\left(3e + \frac{5fx}{2}\right) + 13878 \sin\left(3e + \frac{7fx}{2}\right) + 5040 \sin\left(4e + \frac{7fx}{2}\right) - 2102 \sin\left(4e + \frac{9fx}{2}\right)\right) / (161280 c^5 f)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]

[Out] (a^3*Csc[e/2]*Csc[(e + f*x)/2]^9*(39690*f*x*Cos[(f*x)/2] - 39690*f*x*Cos[e + (f*x)/2] - 26460*f*x*Cos[e + (3*f*x)/2] + 26460*f*x*Cos[2*e + (3*f*x)/2] + 11340*f*x*Cos[2*e + (5*f*x)/2] - 11340*f*x*Cos[3*e + (5*f*x)/2] - 2835*f*x*Cos[3*e + (7*f*x)/2] + 2835*f*x*Cos[4*e + (7*f*x)/2] + 315*f*x*Cos[4*e + (9*f*x)/2] - 315*f*x*Cos[5*e + (9*f*x)/2] - 142002*Sin[(f*x)/2] - 122850*Sin[e + (f*x)/2] + 103278*Sin[e + (3*f*x)/2] + 73290*Sin[2*e + (3*f*x)/2] - 51102*Sin[2*e + (5*f*x)/2] - 24570*Sin[3*e + (5*f*x)/2] + 13878*Sin[3*e + (7*f*x)/2] + 5040*Sin[4*e + (7*f*x)/2] - 2102*Sin[4*e + (9*f*x)/2]))/(161280*c^5*f)

fricas [A] time = 0.44, size = 212, normalized size = 1.29

$$\frac{1051 a^3 \cos(fx + e)^5 - 1684 a^3 \cos(fx + e)^4 + 898 a^3 \cos(fx + e)^3 + 1468 a^3 \cos(fx + e)^2 - 1669 a^3 \cos(fx + e) + 496 a^3 + 315 (a^3 f \cos(fx + e)^4 - 4 c^5 f \cos(fx + e)^3)}{315 (c^5 f \cos(fx + e)^4 - 4 c^5 f \cos(fx + e)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/315*(1051*a^3*cos(f*x + e)^5 - 1684*a^3*cos(f*x + e)^4 + 898*a^3*cos(f*x + e)^3 + 1468*a^3*cos(f*x + e)^2 - 1669*a^3*cos(f*x + e) + 496*a^3 + 315*(a^3*f*cos(f*x + e)^4 - 4*a^3*f*cos(f*x + e)^3 + 6*a^3*f*cos(f*x + e)^2))

$$-4a^3 f x \cos(fx + e) + a^3 f x \sin(fx + e) / ((c^5 f \cos(fx + e))^4 - 4c^5 f \cos(fx + e)^3 + 6c^5 f \cos(fx + e)^2 - 4c^5 f \cos(fx + e) + c^5 f) \sin(fx + e)$$

giac [A] time = 0.56, size = 110, normalized size = 0.67

$$\frac{\frac{630(fx+e)a^3}{c^5} + \frac{1260a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 - 420a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 252a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 135a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 35a^3}{c^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^9}}{630f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")

[Out] 1/630*(630*(f*x + e)*a^3/c^5 + (1260*a^3*tan(1/2*f*x + 1/2*e)^8 - 420*a^3*tan(1/2*f*x + 1/2*e)^6 + 252*a^3*tan(1/2*f*x + 1/2*e)^4 - 135*a^3*tan(1/2*f*x + 1/2*e)^2 + 35*a^3)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f

maple [A] time = 0.89, size = 133, normalized size = 0.81

$$\frac{a^3}{18f c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9} - \frac{3a^3}{14f c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{2a^3}{5f c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{2a^3}{3f c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{2a^3}{f c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2a^3}{f c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)

[Out] 1/18/f*a^3/c^5/tan(1/2*e+1/2*f*x)^9-3/14/f*a^3/c^5/tan(1/2*e+1/2*f*x)^7+2/5/f*a^3/c^5/tan(1/2*e+1/2*f*x)^5-2/3/f*a^3/c^5/tan(1/2*e+1/2*f*x)^3+2/f*a^3/c^5/tan(1/2*e+1/2*f*x)+2/f*a^3/c^5*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.46, size = 403, normalized size = 2.46

$$a^3 \left(\frac{10080 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5} - \frac{\left(\frac{270 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1008 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{2730 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{9765 \sin(fx+e)^8}{(\cos(fx+e)+1)^8} - 35\right)(\cos(fx+e)+1)^9}{c^5 \sin(fx+e)^9} \right) - \frac{3a^3 \left(\frac{180 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{c^5 \sin(fx+e)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] 1/5040*(a^3*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 3*a^3*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 15*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 7*a^3*(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f

mupad [B] time = 1.46, size = 146, normalized size = 0.89

$$a^3 \left(\frac{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)^9}{18} - \frac{3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^7 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{14} + \frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{5} - \frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6}{3} + 2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{fx}{2}\right) \right) / c^5 f \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^5,x)`

[Out] $(a^3(\cos(e/2 + (f*x)/2)^{9/18} + 2\cos(e/2 + (f*x)/2)\sin(e/2 + (f*x)/2)^8 + \sin(e/2 + (f*x)/2)^9(e + f*x) - (2\cos(e/2 + (f*x)/2)^3\sin(e/2 + (f*x)/2)^6)/3 + (2\cos(e/2 + (f*x)/2)^5\sin(e/2 + (f*x)/2)^4/5 - (3\cos(e/2 + (f*x)/2)^7\sin(e/2 + (f*x)/2)^2/14))/(c^5f\sin(e/2 + (f*x)/2)^9)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{3 \sec(e+fx)}{\sec^5(e+fx) - 5 \sec^4(e+fx) + 10 \sec^3(e+fx) - 10 \sec^2(e+fx) + 5 \sec(e+fx) - 1} dx + \int \frac{3 \sec^2(e+fx)}{\sec^5(e+fx) - 5 \sec^4(e+fx) + 10 \sec^3(e+fx) - 10 \sec^2(e+fx) + 5 \sec(e+fx) - 1} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)`

[Out] $-a^3(\text{Integral}(3\sec(e + f*x)/(\sec(e + f*x)^5 - 5\sec(e + f*x)^4 + 10\sec(e + f*x)^3 - 10\sec(e + f*x)^2 + 5\sec(e + f*x) - 1), x) + \text{Integral}(3\sec(e + f*x)^2/(\sec(e + f*x)^5 - 5\sec(e + f*x)^4 + 10\sec(e + f*x)^3 - 10\sec(e + f*x)^2 + 5\sec(e + f*x) - 1), x) + \text{Integral}(\sec(e + f*x)^3/(\sec(e + f*x)^5 - 5\sec(e + f*x)^4 + 10\sec(e + f*x)^3 - 10\sec(e + f*x)^2 + 5\sec(e + f*x) - 1), x) + \text{Integral}(1/(\sec(e + f*x)^5 - 5\sec(e + f*x)^4 + 10\sec(e + f*x)^3 - 10\sec(e + f*x)^2 + 5\sec(e + f*x) - 1), x))/c^5$

$$3.21 \quad \int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^2} dx$$

Optimal. Leaf size=136

$$\frac{13c^5 \tan(e + fx)}{2a^2 f} - \frac{47c^5 \tanh^{-1}(\sin(e + fx))}{2a^2 f} + \frac{112c^5 \tan(e + fx)}{3a^2 f(\sec(e + fx) + 1)} + \frac{\tan(e + fx)(c^5 - c^5 \sec(e + fx))}{2a^2 f} + \frac{c^5 x}{a^2}$$

[Out] $c^5 x/a^2 - 47/2 * c^5 * \operatorname{arctanh}(\sin(f*x+e))/a^2/f + 13/2 * c^5 * \tan(f*x+e)/a^2/f + 112/3 * c^5 * \tan(f*x+e)/a^2/f / (1 + \sec(f*x+e)) - 32/3 * c^5 * \tan(f*x+e)/f / (a + a * \sec(f*x+e))^2 + 1/2 * (c^5 - c^5 * \sec(f*x+e)) * \tan(f*x+e)/a^2/f$

Rubi [A] time = 0.40, antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 26, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3904, 3886, 3473, 8, 2606, 2607, 30, 3767, 2621, 302, 207, 2620, 270, 288}

$$\frac{7c^5 \tan(e + fx)}{a^2 f} - \frac{64c^5 \cot^3(e + fx)}{3a^2 f} - \frac{48c^5 \cot(e + fx)}{a^2 f} + \frac{131c^5 \csc^3(e + fx)}{6a^2 f} + \frac{33c^5 \csc(e + fx)}{2a^2 f} - \frac{47c^5 \tanh^{-1}(\sin(e + fx))}{2a^2 f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c - c * \operatorname{Sec}[e + f*x])^5 / (a + a * \operatorname{Sec}[e + f*x])^2, x]$

[Out] $(c^5 x) / a^2 - (47 * c^5 * \operatorname{ArcTanh}[\operatorname{Sin}[e + f*x]]) / (2 * a^2 * f) - (48 * c^5 * \operatorname{Cot}[e + f*x]) / (a^2 * f) - (64 * c^5 * \operatorname{Cot}[e + f*x]^3) / (3 * a^2 * f) + (33 * c^5 * \operatorname{Csc}[e + f*x]) / (2 * a^2 * f) + (131 * c^5 * \operatorname{Csc}[e + f*x]^3) / (6 * a^2 * f) - (c^5 * \operatorname{Csc}[e + f*x]^3 * \operatorname{Sec}[e + f*x]^2) / (2 * a^2 * f) + (7 * c^5 * \operatorname{Tan}[e + f*x]) / (a^2 * f)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[x^{(m+1)} / (m+1), x] /; \operatorname{FreeQ}[m, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 207

$\operatorname{Int}[(a_ + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2] * x) / \operatorname{Rt}[-a, 2]] / (\operatorname{Rt}[-a, 2] * \operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}[a, b], x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

Rule 270

$\operatorname{Int}[(c_.) * (x_)^{(m_.)} * ((a_ + (b_.) * (x_)^{(n_.)})^{(p_.)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m * (a + b*x^n)^p, x], x] /; \operatorname{FreeQ}[a, b, c, m, n], x] \&\& \operatorname{IGtQ}[p, 0]$

Rule 288

$\operatorname{Int}[(c_.) * (x_)^{(m_.)} * ((a_ + (b_.) * (x_)^{(n_.)})^{(p_.)}), x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^n)^{(p+1)}) / (b*n*(p+1)), x] - \operatorname{Dist}[(c^n * (m-n+1)) / (b*n*(p+1)), \operatorname{Int}[(c*x)^{(m-n)} * (a + b*x^n)^{(p+1)}, x], x] /; \operatorname{FreeQ}[a, b, c], x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{GtQ}[m+1, n] \&\& \operatorname{!} \operatorname{LtQ}[(m+n*(p+1)+1)/n, 0] \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2606

```
Int[((a_)*sec[(e_) + (f_)*(x_)]^(m_))*((b_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 2620

```
Int[csc[(e_) + (f_)*(x_)]^(m_)*sec[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegerQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_) + (f_)*(x_)]*(a_))^(m_)*sec[(e_) + (f_)*(x_)]^(n_), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3473

```
Int[((b_)*tan[(c_) + (d_)*(x_)]^(n_)), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_) + (d_)*(x_)]*(e_))^(m_)*(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^2} dx &= \frac{\int \cot^4(e + fx)(c - c \sec(e + fx))^7 dx}{a^2 c^2} \\
&= \frac{\int (c^7 \cot^4(e + fx) - 7c^7 \cot^3(e + fx) \csc(e + fx) + 21c^7 \cot^2(e + fx) \csc^2(e + fx) - 35c^7 \cot(e + fx) \csc^3(e + fx) + 7c^7 \csc^4(e + fx)) dx}{a^2 c^2} \\
&= \frac{c^5 \int \cot^4(e + fx) dx}{a^2} - \frac{c^5 \int \csc^4(e + fx) \sec^3(e + fx) dx}{a^2} - \frac{(7c^5) \int \cot^3(e + fx) dx}{a^2} + \frac{35c^5 \int \cot^2(e + fx) dx}{a^2} - \frac{7c^5 \int \csc^2(e + fx) dx}{a^2} \\
&= -\frac{c^5 \cot^3(e + fx)}{3a^2 f} - \frac{c^5 \int \cot^2(e + fx) dx}{a^2} + \frac{c^5 \text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2} dx, x, \csc(e + fx)\right)}{a^2 f} \\
&= -\frac{34c^5 \cot(e + fx)}{a^2 f} - \frac{19c^5 \cot^3(e + fx)}{a^2 f} - \frac{7c^5 \csc(e + fx)}{a^2 f} + \frac{14c^5 \csc^3(e + fx)}{a^2 f} - \frac{7c^5 \csc^5(e + fx)}{a^2 f} \\
&= \frac{c^5 x}{a^2} - \frac{48c^5 \cot(e + fx)}{a^2 f} - \frac{64c^5 \cot^3(e + fx)}{3a^2 f} + \frac{14c^5 \csc(e + fx)}{a^2 f} + \frac{21c^5 \csc^3(e + fx)}{a^2 f} - \frac{7c^5 \csc^5(e + fx)}{a^2 f} \\
&= \frac{c^5 x}{a^2} - \frac{21c^5 \tanh^{-1}(\sin(e + fx))}{a^2 f} - \frac{48c^5 \cot(e + fx)}{a^2 f} - \frac{64c^5 \cot^3(e + fx)}{3a^2 f} + \frac{33c^5 \csc(e + fx)}{a^2 f} - \frac{7c^5 \csc^3(e + fx)}{a^2 f} \\
&= \frac{c^5 x}{a^2} - \frac{47c^5 \tanh^{-1}(\sin(e + fx))}{2a^2 f} - \frac{48c^5 \cot(e + fx)}{a^2 f} - \frac{64c^5 \cot^3(e + fx)}{3a^2 f} + \frac{33c^5 \csc(e + fx)}{a^2 f} - \frac{7c^5 \csc^3(e + fx)}{a^2 f}
\end{aligned}$$

Mathematica [B] time = 3.15, size = 384, normalized size = 2.82

$$\cos^3(e + fx) \cot\left(\frac{1}{2}(e + fx)\right) \csc^6\left(\frac{1}{2}(e + fx)\right) (c - c \sec(e + fx))^5 \left(-\frac{64 \tan\left(\frac{e}{2}\right) \cot\left(\frac{1}{2}(e + fx)\right) \csc^2\left(\frac{1}{2}(e + fx)\right)}{f} - \frac{64 \sec\left(\frac{e}{2}\right) \sin\left(\frac{1}{2}(e + fx)\right)}{f} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]

[Out] (Cos[e + f*x]^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^6*(c - c*Sec[e + f*x])^5*((-320*Cot[(e + f*x)/2]^2*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2])/f - (64*Csc[(e + f*x)/2]^3*Sec[e/2]*Sin[(f*x)/2])/f + 3*Cot[(e + f*x)/2]^3*(-4*x - (94*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f + (94*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])/f + 1/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) - 1/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) - (28*Sin[f*x])/(f*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) - (64*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*Tan[e/2])/f)/(96*a^2*(1 + Sec[e + f*x])^2)

fricas [A] time = 0.46, size = 242, normalized size = 1.78

$$12 c^5 f x \cos(fx + e)^4 + 24 c^5 f x \cos(fx + e)^3 + 12 c^5 f x \cos(fx + e)^2 - 141 (c^5 \cos(fx + e)^4 + 2 c^5 \cos(fx + e)^3 + c^5 \cos(fx + e)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/12*(12*c^5*f*x*cos(f*x + e)^4 + 24*c^5*f*x*cos(f*x + e)^3 + 12*c^5*f*x*cos(f*x + e)^2 - 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2))

+ e)^2)*log(sin(f*x + e) + 1) + 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(202*c^5*cos(f*x + e)^3 + 305*c^5*cos(f*x + e)^2 + 36*c^5*cos(f*x + e) - 3*c^5)*sin(f*x + e))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*((-8/3*tan((f*x+exp(1))/2)^3*c^5*a^4-16*tan((f*x+exp(1))/2)*c^5*a^4)/a^6-(-15*tan((f*x+exp(1))/2)^3*c^5+13*tan((f*x+exp(1))/2)*c^5)*1/2/a^2/(tan((f*x+exp(1))/2)-1)^2-2*c^5*1/2/a^2*(f*x+exp(1))/2-47*c^5*1/4/a^2*ln(abs(tan((f*x+exp(1))/2)-1))+47*c^5*1/4/a^2*ln(abs(tan((f*x+exp(1))/2)+1)))

maple [A] time = 0.71, size = 207, normalized size = 1.52

$$\frac{16c^5 \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{3f a^2} + \frac{32c^5 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{f a^2} - \frac{c^5}{2f a^2 \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) - 1 \right)^2} - \frac{15c^5}{2f a^2 \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) - 1 \right)} + \frac{47c^5 \ln \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{2f a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)

[Out] 16/3/f*c^5/a^2*tan(1/2*e+1/2*f*x)^3+32/f*c^5/a^2*tan(1/2*e+1/2*f*x)-1/2/f*c^5/a^2/(tan(1/2*e+1/2*f*x)-1)^2-15/2/f*c^5/a^2/(tan(1/2*e+1/2*f*x)-1)+47/2/f*c^5/a^2*ln(tan(1/2*e+1/2*f*x)-1)+1/2/f*c^5/a^2/(tan(1/2*e+1/2*f*x)+1)^2-15/2/f*c^5/a^2/(tan(1/2*e+1/2*f*x)+1)-47/2/f*c^5/a^2*ln(tan(1/2*e+1/2*f*x)+1)+2/f*c^5/a^2*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.44, size = 603, normalized size = 4.43

$$c^5 \left(\frac{6 \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} - \frac{5 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^2 - \frac{2a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} + \frac{21 \sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{21 \log \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1 \right)}{a^2} + \frac{21 \log \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1 \right)}{a^2} \right) + 5c^5 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] 1/6*(c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 5*c^5*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 - c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2))

$1)^3/a^2 - 12*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^2) + 10*c^5*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2 - 5*c^5*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2)/f$

mupad [B] time = 1.50, size = 145, normalized size = 1.07

$$\frac{c^5 x}{a^2} - \frac{15c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 - 13c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f \left(a^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 2a^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + a^2\right)} + \frac{32c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f} + \frac{16c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3a^2 f} - \frac{47c^5 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^2,x)

[Out] (c^5*x)/a^2 - (15*c^5*tan(e/2 + (f*x)/2)^3 - 13*c^5*tan(e/2 + (f*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) + (32*c^5*tan(e/2 + (f*x)/2))/(a^2*f) + (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) - (47*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^2*f)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^5 \left(\int \frac{5 \sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} dx + \int \left(-\frac{10 \sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} \right) dx + \int \frac{10 \sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} dx + \int \left(-\frac{10 \sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} \right) dx + \int \frac{10 \sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} dx + \int \left(-\frac{10 \sec^6(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} \right) dx \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)

[Out] -c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-5*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2

$$3.22 \quad \int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^2} dx$$

Optimal. Leaf size=102

$$\frac{c^4 \tan(e + fx)}{a^2 f} - \frac{32c^4 \cot^3(e + fx)}{3a^2 f} - \frac{16c^4 \cot(e + fx)}{a^2 f} + \frac{32c^4 \csc^3(e + fx)}{3a^2 f} - \frac{6c^4 \tanh^{-1}(\sin(e + fx))}{a^2 f} + \frac{c^4 x}{a^2}$$

[Out] $c^4 x/a^2 - 6c^4 \operatorname{arctanh}(\sin(fx+e))/a^2/f - 16c^4 \cot(fx+e)/a^2/f - 32/3c^4 \cot(fx+e)^3/a^2/f + 32/3c^4 \csc(fx+e)^3/a^2/f + c^4 \tan(fx+e)/a^2/f$

Rubi [A] time = 0.31, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3904, 3886, 3473, 8, 2606, 2607, 30, 3767, 2621, 302, 207, 2620, 270}

$$\frac{c^4 \tan(e + fx)}{a^2 f} - \frac{32c^4 \cot^3(e + fx)}{3a^2 f} - \frac{16c^4 \cot(e + fx)}{a^2 f} + \frac{32c^4 \csc^3(e + fx)}{3a^2 f} - \frac{6c^4 \tanh^{-1}(\sin(e + fx))}{a^2 f} + \frac{c^4 x}{a^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c \operatorname{Sec}[e + fx])^4/(a + a \operatorname{Sec}[e + fx])^2, x]$

[Out] $(c^4 x)/a^2 - (6c^4 \operatorname{ArcTanh}[\operatorname{Sin}[e + fx]])/(a^2 f) - (16c^4 \operatorname{Cot}[e + fx])/(a^2 f) - (32c^4 \operatorname{Cot}[e + fx]^3)/(3a^2 f) + (32c^4 \operatorname{Csc}[e + fx]^3)/(3a^2 f) + (c^4 \operatorname{Tan}[e + fx])/(a^2 f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rule 207

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 270

$\text{Int}[(c_)*(x_)^{(m_.)}*((a_ + (b_)*(x_)^{(n_.)})^{(p_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

Rule 302

$\text{Int}[(x_)^{(m_.)}/((a_ + (b_)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, 2*n - 1]$

Rule 2606

$\text{Int}[(a_)*\sec[(e_ + (f_)*(x_))]^{(m_.)}*((b_)*\tan[(e_ + (f_)*(x_))]^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)}*(-1 + x^2)^{((n-1)/2)}, x], x, \operatorname{Sec}[e + fx]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n-1)/2] \&\& !(\text{IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n + 1])$

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(b*x)^(n*(1 + x^2)^(m/2 - 1)), x], x, Tan[e + f*x]], x]
;/; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x]
;/; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x]
;/; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol]
:> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x]
;/; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol]
:> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x]
;/; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol]
:> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x]
;/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol]
:> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x]
;/; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^2} dx &= \frac{\int \cot^4(e + fx)(c - c \sec(e + fx))^6 dx}{a^2 c^2} \\
&= \frac{\int (c^6 \cot^4(e + fx) - 6c^6 \cot^3(e + fx) \csc(e + fx) + 15c^6 \cot^2(e + fx) \csc^2(e + fx) - \dots)}{a^2 c^2} \\
&= \frac{c^4 \int \cot^4(e + fx) dx}{a^2} + \frac{c^4 \int \csc^4(e + fx) \sec^2(e + fx) dx}{a^2} - \frac{(6c^4) \int \cot^3(e + fx) \csc(e + fx) dx}{a^2} \\
&= -\frac{c^4 \cot^3(e + fx)}{3a^2 f} - \frac{c^4 \int \cot^2(e + fx) dx}{a^2} + \frac{c^4 \text{Subst}\left(\int \frac{(1+x^2)^2}{x^4} dx, x, \tan(e + fx)\right)}{a^2 f} + \dots \\
&= -\frac{14c^4 \cot(e + fx)}{a^2 f} - \frac{31c^4 \cot^3(e + fx)}{3a^2 f} - \frac{6c^4 \csc(e + fx)}{a^2 f} + \frac{26c^4 \csc^3(e + fx)}{3a^2 f} + \frac{c^4 \int \dots}{a^2} \\
&= \frac{c^4 x}{a^2} - \frac{16c^4 \cot(e + fx)}{a^2 f} - \frac{32c^4 \cot^3(e + fx)}{3a^2 f} + \frac{32c^4 \csc^3(e + fx)}{3a^2 f} + \frac{c^4 \tan(e + fx)}{a^2 f} + \dots \\
&= \frac{c^4 x}{a^2} - \frac{6c^4 \tanh^{-1}(\sin(e + fx))}{a^2 f} - \frac{16c^4 \cot(e + fx)}{a^2 f} - \frac{32c^4 \cot^3(e + fx)}{3a^2 f} + \frac{32c^4 \csc^3(e + fx)}{3a^2 f} + \dots
\end{aligned}$$

Mathematica [B] time = 6.26, size = 753, normalized size = 7.38

$$\frac{x \cos^2(e + fx) \cot^4\left(\frac{e}{2} + \frac{fx}{2}\right) \csc^4\left(\frac{e}{2} + \frac{fx}{2}\right) (c - c \sec(e + fx))^4}{4(a \sec(e + fx) + a)^2} + \frac{\sin\left(\frac{fx}{2}\right) \cos^2(e + fx) \cot^4\left(\frac{e}{2} + \frac{fx}{2}\right) \csc^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{4f \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right)\right) (a \sec(e + fx) + a)^2 \left(\cos\left(\frac{e}{2} + \frac{fx}{2}\right) - \sin\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]

[Out] (x*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4)/(4*(a + a*Sec[e + f*x])^2) + (3*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x])^2) - (3*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*Log[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x])^2) + (4*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^3*Csc[e/2 + (f*x)/2]^5*Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(3*f*(a + a*Sec[e + f*x])^2) + (2*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]^7*Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(3*f*(a + a*Sec[e + f*x])^2) + (Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(4*f*(a + a*Sec[e + f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) + (Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(4*f*(a + a*Sec[e + f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])) + (2*Cos[e + f*x]^2*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*Tan[e/2])/(3*f*(a + a*Sec[e + f*x])^2)

fricas [B] time = 0.49, size = 220, normalized size = 2.16

$$3c^4fx \cos(fx + e)^3 + 6c^4fx \cos(fx + e)^2 + 3c^4fx \cos(fx + e) - 9\left(c^4 \cos(fx + e)^3 + 2c^4 \cos(fx + e)^2 + c^4 \cos(fx + e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] $\frac{1}{3}(3c^4f^3x^3\cos(fx+e)^3 + 6c^4f^2x^2\cos(fx+e)^2 + 3c^4fx\cos(fx+e) - 9(c^4\cos(fx+e)^3 + 2c^4\cos(fx+e)^2 + c^4\cos(fx+e))\log(\sin(fx+e)+1) + 9(c^4\cos(fx+e)^3 + 2c^4\cos(fx+e)^2 + c^4\cos(fx+e))\log(-\sin(fx+e)+1) + (19c^4\cos(fx+e)^2 + 38c^4\cos(fx+e) + 3c^4)\sin(fx+e))/(a^2f^3\cos(fx+e)^3 + 2a^2f^2\cos(fx+e)^2 + a^2f\cos(fx+e))$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*((4/3*tan((f*x+exp(1))/2)^3*c^4*a^4+4*tan((f*x+exp(1))/2)*c^4*a^4)/a^6-tan((f*x+exp(1))/2)*c^4/a^2/(tan((f*x+exp(1))/2)^2-1)+2*c^4*1/2/a^2*(f*x+exp(1))/2+3*c^4/a^2*ln(abs(tan((f*x+exp(1))/2)-1))-3*c^4/a^2*ln(abs(tan((f*x+exp(1))/2)+1)))

maple [A] time = 0.68, size = 159, normalized size = 1.56

$$\frac{8c^4 \left(\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}{3fa^2} + \frac{8c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{fa^2} - \frac{c^4}{fa^2 \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1 \right)} + \frac{6c^4 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1 \right)}{fa^2} - \frac{c^4}{fa^2 \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)

[Out] $\frac{8}{3}f^3c^4/a^2\tan(1/2*e+1/2*f*x)^3+8/f^2c^4/a^2\tan(1/2*e+1/2*f*x)-1/f^2c^4/a^2/(\tan(1/2*e+1/2*f*x)-1)+6/f^2c^4/a^2*\ln(\tan(1/2*e+1/2*f*x)-1)-1/f^2c^4/a^2/(\tan(1/2*e+1/2*f*x)+1)-6/f^2c^4/a^2*\ln(\tan(1/2*e+1/2*f*x)+1)+2/f^2c^4/a^2*\arctan(\tan(1/2*e+1/2*f*x))$

maxima [B] time = 0.44, size = 413, normalized size = 4.05

$$c^4 \left(\frac{\frac{15 \sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{a^2} + \frac{12 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{a^2} + \frac{12 \sin(fx+e)}{\left(a^2 - \frac{a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right) (\cos(fx+e)+1)} \right) + 4c^4 \left(\frac{9 \sin(fx+e)}{\cos(fx+e)+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] $\frac{1}{6}(c^4((15\sin(fx+e)/(\cos(fx+e)+1) + \sin(fx+e)^3/(\cos(fx+e)+1)^3)/a^2 - 12*\log(\sin(fx+e)/(\cos(fx+e)+1) + 1)/a^2 + 12*\log(\sin(fx+e)/(\cos(fx+e)+1) - 1)/a^2 + 12*\sin(fx+e)/((a^2 - a^2*\sin(fx+e)^2/(\cos(fx+e)+1)^2)*(\cos(fx+e)+1))) + 4*c^4*((9*\sin(fx+e)/(\cos(fx+e)+1) + \sin(fx+e)^3/(\cos(fx+e)+1)^3)/a^2 - 6*\log(\sin(fx+e)/(\cos(fx+e)+1) + 1)/a^2 + 6*\log(\sin(fx+e)/(\cos(fx+e)+1) - 1)/a^2) - c^4*((9*\sin(fx+e)/(\cos(fx+e)+1) - \sin(fx+e)^3/(\cos(fx+e)+1)^3)/a^2 - 12*\arctan(\sin(fx+e)/(\cos(fx+e)+1))/a^2) + 6*c^4*(3*\sin(fx+e)/(\cos(fx+e)+1) + \sin(fx+e)^3/(\cos(fx+e)+1)^3)/a^2 - 4*c^4*(3*\sin(fx+e)/(\cos(fx+e)+1) - \sin(fx+e)^3/(\cos(fx+e)+1)^3)/a^2)/f$

mupad [B] time = 1.47, size = 112, normalized size = 1.10

$$\frac{c^4 x}{a^2} + \frac{8c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f} + \frac{8c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3a^2 f} - \frac{12c^4 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2 f} - \frac{2c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f \left(a^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^2,x)`

[Out] $(c^4 x)/a^2 + (8c^4 \tan(e/2 + (f*x)/2))/(a^2 f) + (8c^4 \tan(e/2 + (f*x)/2)^3)/(3a^2 f) - (12c^4 \operatorname{atanh}(\tan(e/2 + (f*x)/2)))/(a^2 f) - (2c^4 \tan(e/2 + (f*x)/2))/(f(a^2 \tan(e/2 + (f*x)/2)^2 - a^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4 \left(\int \left(-\frac{4 \sec(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} \right) dx + \int \frac{6 \sec^2(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx + \int \left(-\frac{4 \sec^3(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} \right) dx + \int \frac{\sec(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)`

[Out] $c**4*(\operatorname{Integral}(-4*\sec(e + f*x)/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x) + \operatorname{Integral}(6*\sec(e + f*x)**2/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x) + \operatorname{Integral}(-4*\sec(e + f*x)**3/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x) + \operatorname{Integral}(\sec(e + f*x)**4/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x) + \operatorname{Integral}(1/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x))/a**2$

$$3.23 \quad \int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^2} dx$$

Optimal. Leaf size=85

$$-\frac{c^3 \tanh^{-1}(\sin(e + fx))}{a^2 f} + \frac{4c^3 \tan(e + fx)}{3a^2 f(\sec(e + fx) + 1)} - \frac{8c^3 \tan(e + fx)}{3a^2 f(\sec(e + fx) + 1)^2} + \frac{c^3 x}{a^2}$$

[Out] $c^3 x/a^2 - c^3 \operatorname{arctanh}(\sin(fx+e))/a^2/f - 8/3 c^3 \tan(fx+e)/a^2/f/(1+\sec(fx+e))^2 + 4/3 c^3 \tan(fx+e)/a^2/f/(1+\sec(fx+e))$

Rubi [A] time = 0.33, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3903, 3777, 3919, 3794, 3796, 3797, 3799, 3998, 3770}

$$-\frac{c^3 \tanh^{-1}(\sin(e + fx))}{a^2 f} + \frac{4c^3 \tan(e + fx)}{3a^2 f(\sec(e + fx) + 1)} - \frac{8c^3 \tan(e + fx)}{3a^2 f(\sec(e + fx) + 1)^2} + \frac{c^3 x}{a^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c \operatorname{Sec}[e + f*x])^3/(a + a \operatorname{Sec}[e + f*x])^2, x]$

[Out] $(c^3 x)/a^2 - (c^3 \operatorname{ArcTanh}[\operatorname{Sin}[e + f*x]])/(a^2 f) - (8 c^3 \operatorname{Tan}[e + f*x])/(3 a^2 f (1 + \operatorname{Sec}[e + f*x])^2) + (4 c^3 \operatorname{Tan}[e + f*x])/(3 a^2 f (1 + \operatorname{Sec}[e + f*x]))$

Rule 3770

$\text{Int}[\operatorname{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3777

$\text{Int}[(\operatorname{csc}[(c_.) + (d_.)(x_.)]*(b_.) + (a_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(\operatorname{Cot}[c + d*x]*(a + b \operatorname{Csc}[c + d*x])^n)/(d*(2*n + 1)), x] + \operatorname{Dist}[1/(a^2*(2*n + 1)), \text{Int}[(a + b \operatorname{Csc}[c + d*x])^{(n + 1)}*(a*(2*n + 1) - b*(n + 1)*\operatorname{Csc}[c + d*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{LeQ}[n, -1] \&\& \operatorname{IntegerQ}[2*n]$

Rule 3794

$\text{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]/(\operatorname{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Cot}[e + f*x]/(f*(b + a \operatorname{Csc}[e + f*x])), x] /; \operatorname{FreeQ}\{a, b, e, f\}, x \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3796

$\text{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]*(\operatorname{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b \operatorname{Cot}[e + f*x]*(a + b \operatorname{Csc}[e + f*x])^m)/(a*f*(2*m + 1)), x] + \operatorname{Dist}[(m + 1)/(a*(2*m + 1)), \text{Int}[\operatorname{Csc}[e + f*x]*(a + b \operatorname{Csc}[e + f*x])^{(m + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{LtQ}[m, -2^{(-1)}] \&\& \operatorname{IntegerQ}[2*m]$

Rule 3797

$\text{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]^2*(\operatorname{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(\operatorname{Cot}[e + f*x]*(a + b \operatorname{Csc}[e + f*x])^m)/(f*(2*m + 1)), x] + \operatorname{Dist}[m/(b*(2*m + 1)), \text{Int}[\operatorname{Csc}[e + f*x]*(a + b \operatorname{Csc}[e + f*x])^{(m + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{LtQ}[m, -2^{(-1)}]$

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_.)]^3*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_),
x_Symbol] := Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*
x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
&& LtQ[m + n, 2]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/(csc[(
e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^2} dx &= \frac{\int \left(\frac{c^3}{(1 + \sec(e + fx))^2} - \frac{3c^3 \sec(e + fx)}{(1 + \sec(e + fx))^2} + \frac{3c^3 \sec^2(e + fx)}{(1 + \sec(e + fx))^2} - \frac{c^3 \sec^3(e + fx)}{(1 + \sec(e + fx))^2} \right) dx}{a^2} \\ &= \frac{c^3 \int \frac{1}{(1 + \sec(e + fx))^2} dx}{a^2} - \frac{c^3 \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{a^2} - \frac{(3c^3) \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{a^2} + \frac{(3c^3) \int \frac{\sec^2(e + fx)}{(1 + \sec(e + fx))^2} dx}{a^2} \\ &= -\frac{8c^3 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{c^3 \int \frac{-3 + \sec(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} - \frac{c^3 \int \frac{\sec(e + fx)(-2 + 3 \sec(e + fx))}{1 + \sec(e + fx)} dx}{3a^2} - \frac{c^3 \int \frac{\sec^2(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} \\ &= \frac{c^3 x}{a^2} - \frac{8c^3 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} + \frac{c^3 \tan(e + fx)}{a^2 f (1 + \sec(e + fx))} - \frac{c^3 \int \sec(e + fx) dx}{a^2} - \frac{(4c^3)}{3a^2 f (1 + \sec(e + fx))} \\ &= \frac{c^3 x}{a^2} - \frac{c^3 \tanh^{-1}(\sin(e + fx))}{a^2 f} - \frac{8c^3 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} + \frac{4c^3 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))} \end{aligned}$$

Mathematica [B] time = 1.10, size = 216, normalized size = 2.54

$$\frac{c^3 (\cos(e + fx) - 1)^3 \cot\left(\frac{1}{2}(e + fx)\right) \csc^2\left(\frac{1}{2}(e + fx)\right) \left(4 \tan\left(\frac{e}{2}\right) \cot\left(\frac{1}{2}(e + fx)\right) \csc^2\left(\frac{1}{2}(e + fx)\right) + 4 \sec\left(\frac{e}{2}\right) \sin\left(\frac{e}{2}\right)\right)}{3a^2 f (1 + \sec(e + fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

[Out]
$$-1/6*(c^3*(-1 + \text{Cos}[e + f*x])^3*\text{Cot}[(e + f*x)/2]*\text{Csc}[(e + f*x)/2]^2*(3*\text{Cot}[(e + f*x)/2]^3*(f*x + \text{Log}[\text{Cos}[(e + f*x)/2] - \text{Sin}[(e + f*x)/2]] - \text{Log}[\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2]])) - 4*\text{Cot}[(e + f*x)/2]^2*\text{Csc}[(e + f*x)/2]*\text{Sec}[e/2]*\text{Sin}[(f*x)/2] + 4*\text{Csc}[(e + f*x)/2]^3*\text{Sec}[e/2]*\text{Sin}[(f*x)/2] + 4*\text{Cot}[(e + f*x)/2]*\text{Csc}[(e + f*x)/2]^2*\text{Tan}[e/2]))/(a^2*f*(1 + \text{Cos}[e + f*x])^2)$$

fricas [B] time = 0.46, size = 173, normalized size = 2.04

$$\frac{6c^3fx \cos(fx + e)^2 + 12c^3fx \cos(fx + e) + 6c^3fx - 3\left(c^3 \cos(fx + e)^2 + 2c^3 \cos(fx + e) + c^3\right) \log(\sin(fx + e) + 1) + 3\left(c^3 \cos(fx + e)^2 + 2c^3 \cos(fx + e) + c^3\right) \log(-\sin(fx + e) + 1) - 8\left(c^3 \cos(fx + e) - c^3\right) \sin(fx + e)}{6\left(a^2f \cos(fx + e) + a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out]
$$1/6*(6*c^3*f*x*\cos(f*x + e)^2 + 12*c^3*f*x*\cos(f*x + e) + 6*c^3*f*x - 3*(c^3*\cos(f*x + e)^2 + 2*c^3*\cos(f*x + e) + c^3)*\log(\sin(f*x + e) + 1) + 3*(c^3*\cos(f*x + e)^2 + 2*c^3*\cos(f*x + e) + c^3)*\log(-\sin(f*x + e) + 1) - 8*(c^3*\cos(f*x + e) - c^3)*\sin(f*x + e))/(a^2*f*\cos(f*x + e)^2 + 2*a^2*f*\cos(f*x + e) + a^2*f)$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*(-c^3*1/2/a^2*ln(abs(tan((f*x+exp(1))/2)-1))+c^3*1/2/a^2*ln(abs(tan((f*x+exp(1))/2)+1))-2*c^3*1/2/a^2*(f*x+exp(1))/2-2/3*tan((f*x+exp(1))/2)^3*c^3*a^4/a^6)

maple [A] time = 0.82, size = 90, normalized size = 1.06

$$\frac{4c^3 \left(\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}{3fa^2} + \frac{c^3 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1\right)}{fa^2} - \frac{c^3 \ln\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}{fa^2} + \frac{2c^3 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{fa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)

[Out]
$$4/3/f*c^3/a^2*\tan(1/2*e+1/2*f*x)^3+1/f*c^3/a^2*\ln(\tan(1/2*e+1/2*f*x)-1)-1/f*c^3/a^2*\ln(\tan(1/2*e+1/2*f*x)+1)+2/f*c^3/a^2*\arctan(\tan(1/2*e+1/2*f*x))$$

maxima [B] time = 0.43, size = 268, normalized size = 3.15

$$\frac{c^3 \left(\frac{9 \sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{6 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{a^2} + \frac{6 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{a^2} \right)}{6f} - c^3 \left(\frac{9 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{12 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out]
$$1/6*(c^3*((9*\sin(f*x + e))/(\cos(f*x + e) + 1) + \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2 - 6*\log(\sin(f*x + e)/(\cos(f*x + e) + 1) + 1)/a^2 + 6*\log(\sin(f$$

$*x + e)/(\cos(f*x + e) + 1) - 1)/a^2) - c^3*((9*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2 - 12*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^2) + 3*c^3*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2 - 3*c^3*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2)/f$

mupad [B] time = 1.41, size = 46, normalized size = 0.54

$$\frac{c^3 x}{a^2} - \frac{c^3 \left(2 \operatorname{atanh} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right) - \frac{4 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)^3}{3} \right)}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^2,x)

[Out] (c^3*x)/a^2 - (c^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*tan(e/2 + (f*x)/2)^3)/3))/(a^2*f)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3 \left(\int \frac{3 \sec(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx + \int \left(-\frac{3 \sec^2(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} \right) dx + \int \frac{\sec^3(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx + \int \left(-\frac{1}{\sec^2(e+fx)+2 \sec(e+fx)+1} \right) dx \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)

[Out] -c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2

$$3.24 \quad \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^2} dx$$

Optimal. Leaf size=67

$$-\frac{4c^2 \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)} - \frac{4c^2 \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)^2} + \frac{c^2 x}{a^2}$$

[Out] $c^2 x / a^2 - 4/3 c^2 \tan(f x + e) / a^2 / f / (1 + \sec(f x + e))^{-2} - 4/3 c^2 \tan(f x + e) / a^2 / f / (1 + \sec(f x + e))$

Rubi [A] time = 0.23, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3903, 3777, 3919, 3794, 3796, 3797}

$$-\frac{4c^2 \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)} - \frac{4c^2 \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)^2} + \frac{c^2 x}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2, x]

[Out] $(c^2 x) / a^2 - (4 c^2 \tan[e + f x]) / (3 a^2 f (1 + \sec[e + f x])^2) - (4 c^2 \tan[e + f x]) / (3 a^2 f (1 + \sec[e + f x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]

&& LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^2} dx &= \frac{\int \left(\frac{c^2}{(1 + \sec(e + fx))^2} - \frac{2c^2 \sec(e + fx)}{(1 + \sec(e + fx))^2} + \frac{c^2 \sec^2(e + fx)}{(1 + \sec(e + fx))^2} \right) dx}{a^2} \\ &= \frac{c^2 \int \frac{1}{(1 + \sec(e + fx))^2} dx}{a^2} + \frac{c^2 \int \frac{\sec^2(e + fx)}{(1 + \sec(e + fx))^2} dx}{a^2} - \frac{(2c^2) \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{a^2} \\ &= \frac{4c^2 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{c^2 \int \frac{-3 + \sec(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} \\ &= \frac{c^2 x}{a^2} - \frac{4c^2 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{(4c^2) \int \frac{\sec(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} \\ &= \frac{c^2 x}{a^2} - \frac{4c^2 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{4c^2 \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 1.00

$$\frac{c^2 \left(\frac{2 \tan^{-1} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{f} + \frac{2 \tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right)}{3f} - \frac{2 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{f} \right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]

[Out] (c^2*((2*ArcTan[Tan[e/2 + (f*x)/2]])/f - (2*Tan[e/2 + (f*x)/2])/f + (2*Tan[e/2 + (f*x)/2]^3)/(3*f)))/a^2

fricas [A] time = 0.45, size = 94, normalized size = 1.40

$$\frac{3c^2fx \cos(fx + e)^2 + 6c^2fx \cos(fx + e) + 3c^2fx - 4(2c^2 \cos(fx + e) + c^2) \sin(fx + e)}{3(a^2f \cos(fx + e)^2 + 2a^2f \cos(fx + e) + a^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*(3*c^2*f*x*cos(f*x + e)^2 + 6*c^2*f*x*cos(f*x + e) + 3*c^2*f*x - 4*(2*c^2*cos(f*x + e) + c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)

giac [A] time = 0.39, size = 63, normalized size = 0.94

$$\frac{\frac{3(fx+e)c^2}{a^2} + \frac{2\left(a^4c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 3a^4c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^6}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{3}*(3*(f*x + e)*c^2/a^2 + 2*(a^4*c^2*\tan(1/2*f*x + 1/2*e))^3 - 3*a^4*c^2*\tan(1/2*f*x + 1/2*e))/a^6)/f$

maple [A] time = 0.86, size = 65, normalized size = 0.97

$$\frac{2c^2 \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{3fa^2} - \frac{2c^2 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{fa^2} + \frac{2c^2 \arctan \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{fa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)

[Out] $\frac{2}{3}/f*c^2/a^2*\tan(1/2*e+1/2*f*x)^3-2/f*c^2/a^2*\tan(1/2*e+1/2*f*x)+2/f*c^2/a^2*\arctan(\tan(1/2*e+1/2*f*x))$

maxima [B] time = 0.43, size = 170, normalized size = 2.54

$$\frac{c^2 \left(\frac{9 \sin(fx+e) - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12 \arctan \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} \right)}{a^2} \right) - \frac{c^2 \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^2} + \frac{2c^2 \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] $-1/6*(c^2*((9*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2 - 12*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^2) - c^2*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2 + 2*c^2*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/a^2)/f$

mupad [B] time = 1.38, size = 38, normalized size = 0.57

$$\frac{2c^2 \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right)^3 - 3 \tan \left(\frac{e}{2} + \frac{fx}{2} \right) + \frac{3fx}{2} \right)}{3a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^2,x)

[Out] $(2*c^2*(\tan(e/2 + (f*x)/2)^3 - 3*\tan(e/2 + (f*x)/2) + (3*f*x)/2))/(3*a^2*f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2 \left(\int \left(-\frac{2 \sec(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} \right) dx + \int \frac{\sec^2(e+fx)}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx + \int \frac{1}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)

[Out] $c**2*(\text{Integral}(-2*\sec(e + f*x)/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x) + \text{Integral}(\sec(e + f*x)**2/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x) + \text{Integral}(1/(\sec(e + f*x)**2 + 2*\sec(e + f*x) + 1), x))/a**2$

$$3.25 \quad \int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^2} dx$$

Optimal. Leaf size=61

$$-\frac{5c \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)} - \frac{2c \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)^2} + \frac{cx}{a^2}$$

[Out] c*x/a^2-2/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-5/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))

Rubi [A] time = 0.15, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3903, 3777, 3919, 3794, 3796}

$$-\frac{5c \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)} - \frac{2c \tan(e + fx)}{3a^2 f (\sec(e + fx) + 1)^2} + \frac{cx}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]

[Out] (c*x)/a^2 - (2*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

a*d, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^2} dx &= \frac{\int \left(\frac{c}{(1 + \sec(e + fx))^2} - \frac{c \sec(e + fx)}{(1 + \sec(e + fx))^2} \right) dx}{a^2} \\
&= \frac{c \int \frac{1}{(1 + \sec(e + fx))^2} dx}{a^2} - \frac{c \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{a^2} \\
&= \frac{2c \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{c \int \frac{-3 + \sec(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} - \frac{c \int \frac{\sec(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} \\
&= \frac{cx}{a^2} - \frac{2c \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{c \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))} - \frac{(4c) \int \frac{\sec(e + fx)}{1 + \sec(e + fx)} dx}{3a^2} \\
&= \frac{cx}{a^2} - \frac{2c \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))^2} - \frac{5c \tan(e + fx)}{3a^2 f (1 + \sec(e + fx))}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 113, normalized size = 1.85

$$\frac{c \sec\left(\frac{e}{2}\right) \sec^3\left(\frac{1}{2}(e + fx)\right) \left(18 \sin\left(e + \frac{fx}{2}\right) - 14 \sin\left(e + \frac{3fx}{2}\right) + 9fx \cos\left(e + \frac{fx}{2}\right) + 3fx \cos\left(e + \frac{3fx}{2}\right) + 3fx \cos\left(e + \frac{3fx}{2}\right)\right)}{24a^2 f}$$

Antiderivative was successfully verified.

`[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]`

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^3*(9*f*x*Cos[(f*x)/2] + 9*f*x*Cos[e + (f*x)/2]
+ 3*f*x*Cos[e + (3*f*x)/2] + 3*f*x*Cos[2*e + (3*f*x)/2] - 24*Sin[(f*x)/2]
+ 18*Sin[e + (f*x)/2] - 14*Sin[e + (3*f*x)/2]))/(24*a^2*f)
```

fricas [A] time = 0.47, size = 86, normalized size = 1.41

$$\frac{3cfx \cos(fx + e)^2 + 6cfx \cos(fx + e) + 3cfx - (7c \cos(fx + e) + 5c) \sin(fx + e)}{3(a^2 f \cos(fx + e)^2 + 2a^2 f \cos(fx + e) + a^2 f)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fricas")`

```
[Out] 1/3*(3*c*f*x*cos(f*x + e)^2 + 6*c*f*x*cos(f*x + e) + 3*c*f*x - (7*c*cos(f*x
+ e) + 5*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a
^2*f)
```

giac [A] time = 0.28, size = 56, normalized size = 0.92

$$\frac{\frac{3(fx+e)c}{a^2} + \frac{a^4 c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 6a^4 c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^6}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")`

```
[Out] 1/3*(3*(f*x + e)*c/a^2 + (a^4*c*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*tan(1/2*f*
x + 1/2*e))/a^6)/f
```

maple [A] time = 0.90, size = 59, normalized size = 0.97

$$\frac{c \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{3fa^2} - \frac{2c \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{fa^2} + \frac{2c \arctan \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{fa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)`

[Out] `1/3/f*c/a^2*tan(1/2*e+1/2*f*x)^3-2/f*c/a^2*tan(1/2*e+1/2*f*x)+2/f*c/a^2*arctan(tan(1/2*e+1/2*f*x))`

maxima [B] time = 0.42, size = 119, normalized size = 1.95

$$\frac{c \left(\frac{9 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{12 \arctan \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} \right)}{a^2} \right) + \frac{c \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxima")`

[Out] `-1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f`

mupad [B] time = 1.34, size = 41, normalized size = 0.67

$$\frac{cx}{a^2} - \frac{c \left(6 \tan \left(\frac{e}{2} + \frac{fx}{2} \right) - \tan \left(\frac{e}{2} + \frac{fx}{2} \right)^3 \right)}{3a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^2,x)`

[Out] `(c*x)/a^2 - (c*(6*tan(e/2 + (f*x)/2) - tan(e/2 + (f*x)/2)^3))/(3*a^2*f)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c \left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1} dx + \int \left(-\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1} \right) dx \right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)`

[Out] `-c*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2`

$$3.26 \quad \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))} dx$$

Optimal. Leaf size=69

$$-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} + \frac{x}{a^2c}$$

[Out] x/a^2/c+1/3*cot(f*x+e)*(3-2*sec(f*x+e))/a^2/c/f-1/3*cot(f*x+e)^3*(1-sec(f*x+e))/a^2/c/f

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3882, 8}

$$-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} + \frac{x}{a^2c}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]

[Out] x/(a^2*c) + (Cot[e + f*x]*(3 - 2*Sec[e + f*x]))/(3*a^2*c*f) - (Cot[e + f*x]^3*(1 - Sec[e + f*x]))/(3*a^2*c*f)

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3882

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^m_*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[m, -1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m_*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^n_., x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))} dx &= \frac{\int \cot^4(e+fx)(c-c \sec(e+fx)) dx}{a^2c^2} \\ &= -\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} + \frac{\int \cot^2(e+fx)(-3c+2c \sec(e+fx)) dx}{3a^2c^2} \\ &= \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} \\ &= \frac{x}{a^2c} + \frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf} - \frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf} \end{aligned}$$

Mathematica [A] time = 0.55, size = 135, normalized size = 1.96

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)-8\sin(e+2fx))}{96a^2cf}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]

[Out] (Csc[e/2]*Csc[(e + f*x)/2]*Sec[e/2]*Sec[(e + f*x)/2]^3*(6*f*x*Cos[f*x] - 6*f*x*Cos[2*e + f*x] + 3*f*x*Cos[e + 2*f*x] - 3*f*x*Cos[3*e + 2*f*x] - 10*Sin[f*x] + 10*Sin[e + f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] - 8*Sin[e + 2*f*x]))/(96*a^2*c*f)

fricas [A] time = 0.43, size = 70, normalized size = 1.01

$$\frac{4\cos(fx+e)^2 + 3(fx\cos(fx+e) + fx)\sin(fx+e) + \cos(fx+e) - 2}{3(a^2cf\cos(fx+e) + a^2cf)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] 1/3*(4*cos(f*x + e)^2 + 3*(f*x*cos(f*x + e) + f*x)*sin(f*x + e) + cos(f*x + e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))

giac [A] time = 0.40, size = 85, normalized size = 1.23

$$\frac{\frac{12(fx+e)}{a^2c} + \frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)} + \frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 12a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] 1/12*(12*(f*x + e)/(a^2*c) + 3/(a^2*c*tan(1/2*f*x + 1/2*e)) + (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 12*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f

maple [A] time = 0.84, size = 87, normalized size = 1.26

$$\frac{\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{12fa^2c} - \frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{fa^2c} + \frac{1}{4fa^2c\tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2\arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{fa^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)

[Out] 1/12/f/a^2/c*tan(1/2*e+1/2*f*x)^3-1/f/a^2/c*tan(1/2*e+1/2*f*x)+1/4/f/a^2/c/tan(1/2*e+1/2*f*x)+2/f/a^2/c*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.43, size = 102, normalized size = 1.48

$$\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c} - \frac{24\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c} - \frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}$$

12f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] -1/12*((12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*c) - 24*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c) - 3*(cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f

mupad [B] time = 1.42, size = 69, normalized size = 1.00

$$\frac{x}{a^2 c} + \frac{\frac{4 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^4}{3} - \frac{7 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^2}{6} + \frac{1}{12}}{a^2 c f \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{f x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)

[Out] x/(a^2*c) + ((4*cos(e/2 + (f*x)/2)^4)/3 - (7*cos(e/2 + (f*x)/2)^2)/6 + 1/12)/(a^2*c*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1} dx}{a^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)

[Out] -Integral(1/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1), x)/(a**2*c)

$$3.27 \quad \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=46

$$-\frac{\cot^3(e+fx)}{3a^2c^2f} + \frac{\cot(e+fx)}{a^2c^2f} + \frac{x}{a^2c^2}$$

[Out] x/a^2/c^2+cot(f*x+e)/a^2/c^2/f-1/3*cot(f*x+e)^3/a^2/c^2/f

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3473, 8}

$$-\frac{\cot^3(e+fx)}{3a^2c^2f} + \frac{\cot(e+fx)}{a^2c^2f} + \frac{x}{a^2c^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2), x]

[Out] x/(a^2*c^2) + Cot[e + f*x]/(a^2*c^2*f) - Cot[e + f*x]^3/(3*a^2*c^2*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))^2} dx &= \frac{\int \cot^4(e+fx) dx}{a^2c^2} \\ &= \frac{\cot^3(e+fx)}{3a^2c^2f} - \frac{\int \cot^2(e+fx) dx}{a^2c^2} \\ &= \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f} + \frac{\int 1 dx}{a^2c^2} \\ &= \frac{x}{a^2c^2} + \frac{\cot(e+fx)}{a^2c^2f} - \frac{\cot^3(e+fx)}{3a^2c^2f} \end{aligned}$$

Mathematica [C] time = 0.05, size = 39, normalized size = 0.85

$$-\frac{\cot^3(e+fx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\tan^2(e+fx)\right)}{3a^2c^2f}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]

[Out] -1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(a^2*c^2*f)

fricas [A] time = 0.41, size = 81, normalized size = 1.76

$$\frac{4 \cos(fx + e)^3 + 3 \left(fx \cos(fx + e)^2 - fx \right) \sin(fx + e) - 3 \cos(fx + e)}{3 \left(a^2 c^2 f \cos(fx + e)^2 - a^2 c^2 f \right) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*(4*cos(f*x + e)^3 + 3*(f*x*cos(f*x + e)^2 - f*x)*sin(f*x + e) - 3*cos(f*x + e))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))

giac [B] time = 0.29, size = 100, normalized size = 2.17

$$\frac{\frac{24(fx+e)}{a^2c^2} + \frac{15 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1}{a^2c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3} + \frac{a^4c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 15a^4c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^6c^6}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] 1/24*(24*(f*x + e)/(a^2*c^2) + (15*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^2*c^2*tan(1/2*f*x + 1/2*e)^3) + (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 15*a^4*c^4*tan(1/2*f*x + 1/2*e))/(a^6*c^6))/f

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a \sec(fx + e))^2 (c - c \sec(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)

[Out] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)

maxima [A] time = 0.42, size = 46, normalized size = 1.00

$$\frac{\frac{3(fx+e)}{a^2c^2} + \frac{3 \tan(fx+e)^2 - 1}{a^2c^2 \tan(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] 1/3*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^2 - 1)/(a^2*c^2*tan(f*x + e)^3))/f

mupad [B] time = 1.47, size = 58, normalized size = 1.26

$$\frac{\cos(3e + 3fx) + \frac{3 \sin(3e + 3fx)(e + fx)}{4} - \frac{9 \sin(e + fx)(e + fx)}{4}}{3a^2c^2f \sin(e + fx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)`

[Out] $-(\cos(3e + 3fx) + (3\sin(3e + 3fx)(e + fx))/4 - (9\sin(e + fx)(e + fx))/4)/(3a^2c^2f\sin(e + fx)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^4(e+fx)-2\sec^2(e+fx)+1} dx$$

$$\frac{\int \frac{1}{\sec^4(e+fx)-2\sec^2(e+fx)+1} dx}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)`

[Out] `Integral(1/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c**2)`

$$3.28 \quad \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=98

$$\frac{\cot^5(e+fx)(\sec(e+fx)+1)}{5a^2c^3f} - \frac{\cot^3(e+fx)(4\sec(e+fx)+5)}{15a^2c^3f} + \frac{\cot(e+fx)(8\sec(e+fx)+15)}{15a^2c^3f} + \frac{x}{a^2c^3}$$

[Out] x/a^2/c^3+1/5*cot(f*x+e)^5*(1+sec(f*x+e))/a^2/c^3/f-1/15*cot(f*x+e)^3*(5+4*sec(f*x+e))/a^2/c^3/f+1/15*cot(f*x+e)*(15+8*sec(f*x+e))/a^2/c^3/f

Rubi [A] time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3882, 8}

$$\frac{\cot^5(e+fx)(\sec(e+fx)+1)}{5a^2c^3f} - \frac{\cot^3(e+fx)(4\sec(e+fx)+5)}{15a^2c^3f} + \frac{\cot(e+fx)(8\sec(e+fx)+15)}{15a^2c^3f} + \frac{x}{a^2c^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3), x]

[Out] x/(a^2*c^3) + (Cot[e + f*x]^5*(1 + Sec[e + f*x]))/(5*a^2*c^3*f) - (Cot[e + f*x]^3*(5 + 4*Sec[e + f*x]))/(15*a^2*c^3*f) + (Cot[e + f*x]*(15 + 8*Sec[e + f*x]))/(15*a^2*c^3*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3882

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[m, -1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))^3} dx &= -\frac{\int \cot^6(e+fx)(a+a \sec(e+fx)) dx}{a^3c^3} \\ &= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\int \cot^4(e+fx)(-5a-4a \sec(e+fx)) dx}{5a^3c^3} \\ &= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} \\ &= \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} \\ &= \frac{x}{a^2c^3} + \frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f} \end{aligned}$$

Mathematica [B] time = 1.41, size = 257, normalized size = 2.62

$$\frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc^5\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)(-534\sin(e+fx)+178\sin(2(e+fx))+178\sin(3(e+fx))-8\cos(e+fx))}{(30720a^2c^3f^5)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3), x]

[Out] (Csc[e/2]*Csc[(e + f*x)/2]^5*Sec[e/2]*Sec[(e + f*x)/2]^3*(360*f*x*Cos[f*x] - 360*f*x*Cos[2*e + f*x] - 120*f*x*Cos[e + 2*f*x] + 120*f*x*Cos[3*e + 2*f*x] - 120*f*x*Cos[2*e + 3*f*x] + 120*f*x*Cos[4*e + 3*f*x] + 60*f*x*Cos[3*e + 4*f*x] - 60*f*x*Cos[5*e + 4*f*x] + 200*Sin[e] - 584*Sin[f*x] - 534*Sin[e + f*x] + 178*Sin[2*(e + f*x)] + 178*Sin[3*(e + f*x)] - 89*Sin[4*(e + f*x)] - 520*Sin[2*e + f*x] + 248*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 248*Sin[2*e + 3*f*x] + 120*Sin[4*e + 3*f*x] - 184*Sin[3*e + 4*f*x]))/(30720*a^2*c^3*f^5)

fricas [A] time = 0.42, size = 154, normalized size = 1.57

$$\frac{23 \cos^4(fx + e) - 8 \cos^3(fx + e) - 27 \cos^2(fx + e) + 15 \left(fx \cos^3(fx + e) - fx \cos^2(fx + e) - fx \cos(fx + e) + 8 \right)}{15 \left(a^2 c^3 f \cos^3(fx + e) - a^2 c^3 f \cos^2(fx + e) - a^2 c^3 f \cos(fx + e) + a^2 c^3 f \sin(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(23*cos(f*x + e)^4 - 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) + f*x)*sin(f*x + e) + 7*cos(f*x + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))

giac [A] time = 0.43, size = 116, normalized size = 1.18

$$\frac{\frac{240(fx+e)}{a^2c^3} + \frac{3\left(80 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 10 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^2c^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5} + \frac{5\left(a^4c^6 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 18a^4c^6 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^6c^9}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] 1/240*(240*(f*x + e)/(a^2*c^3) + 3*(80*tan(1/2*f*x + 1/2*e)^4 - 10*tan(1/2*f*x + 1/2*e)^2 + 1)/(a^2*c^3*tan(1/2*f*x + 1/2*e)^5) + 5*(a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 18*a^4*c^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f

maple [A] time = 1.10, size = 130, normalized size = 1.33

$$\frac{\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{48fa^2c^3} - \frac{3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{8fa^2c^3} + \frac{1}{80fa^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{1}{8fa^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{1}{fa^2c^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)

[Out] 1/48/f/a^2/c^3*tan(1/2*e+1/2*f*x)^3-3/8/f/a^2/c^3*tan(1/2*e+1/2*f*x)+1/80/f/a^2/c^3/tan(1/2*e+1/2*f*x)^5-1/8/f/a^2/c^3/tan(1/2*e+1/2*f*x)^3+1/f/a^2/c^3/tan(1/2*e+1/2*f*x)+2/f/a^2/c^3*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.43, size = 147, normalized size = 1.50

$$\frac{5 \left(\frac{18 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right) - \frac{480 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2 c^3} + \frac{3 \left(\frac{10 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{80 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 1 \right) (\cos(fx+e)+1)^5}{a^2 c^3 \sin(fx+e)^5}}{240 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -1/240*(5*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*c^3) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^3) + 3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 80*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^5))/f

mupad [B] time = 1.54, size = 161, normalized size = 1.64

$$\frac{3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^8 + 5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 90 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 240 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 30 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 240 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{240 a^2 c^3 f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)

[Out] (3*cos(e/2 + (f*x)/2)^8 + 5*sin(e/2 + (f*x)/2)^8 - 90*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 30*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 + 240*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^5*(e + f*x))/(240*a^2*c^3*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^5(e+fx) - \sec^4(e+fx) - 2\sec^3(e+fx) + 2\sec^2(e+fx) + \sec(e+fx) - 1} dx}{a^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)

[Out] -Integral(1/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)

$$3.29 \quad \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=166

$$-\frac{2 \cot^7(e+fx)}{7a^2c^4f} + \frac{\cot^5(e+fx)}{5a^2c^4f} - \frac{\cot^3(e+fx)}{3a^2c^4f} + \frac{\cot(e+fx)}{a^2c^4f} - \frac{2 \csc^7(e+fx)}{7a^2c^4f} + \frac{6 \csc^5(e+fx)}{5a^2c^4f} - \frac{2 \csc^3(e+fx)}{a^2c^4f} + \frac{2 \csc(e+fx)}{a^2c^4f}$$

[Out] $x/a^2/c^4 + \cot(f*x+e)/a^2/c^4/f - 1/3*\cot(f*x+e)^3/a^2/c^4/f + 1/5*\cot(f*x+e)^5/a^2/c^4/f - 2/7*\cot(f*x+e)^7/a^2/c^4/f + 2*csc(f*x+e)/a^2/c^4/f - 2*csc(f*x+e)^3/a^2/c^4/f + 6/5*csc(f*x+e)^5/a^2/c^4/f - 2/7*csc(f*x+e)^7/a^2/c^4/f$

Rubi [A] time = 0.21, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30}

$$-\frac{2 \cot^7(e+fx)}{7a^2c^4f} + \frac{\cot^5(e+fx)}{5a^2c^4f} - \frac{\cot^3(e+fx)}{3a^2c^4f} + \frac{\cot(e+fx)}{a^2c^4f} - \frac{2 \csc^7(e+fx)}{7a^2c^4f} + \frac{6 \csc^5(e+fx)}{5a^2c^4f} - \frac{2 \csc^3(e+fx)}{a^2c^4f} + \frac{2 \csc(e+fx)}{a^2c^4f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]

[Out] $x/(a^2*c^4) + \text{Cot}[e + f*x]/(a^2*c^4*f) - \text{Cot}[e + f*x]^3/(3*a^2*c^4*f) + \text{Cot}[e + f*x]^5/(5*a^2*c^4*f) - (2*\text{Cot}[e + f*x]^7)/(7*a^2*c^4*f) + (2*\text{Csc}[e + f*x])/(a^2*c^4*f) - (2*\text{Csc}[e + f*x]^3)/(a^2*c^4*f) + (6*\text{Csc}[e + f*x]^5)/(5*a^2*c^4*f) - (2*\text{Csc}[e + f*x]^7)/(7*a^2*c^4*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^(n*(1 + x^2)^(m/2 - 1)), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],

$x] /; \text{FreeQ}\{b, c, d\}, x\} \&\& \text{GtQ}[n, 1]$

Rule 3886

$\text{Int}[(\cot[(c_.) + (d_.)(x_.)]*(e_.))^m*(\csc[(c_.) + (d_.)(x_.)]*(b_.) + (a_.))^n], x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(e*\cot[c + d*x])^m, (a + b*\csc[c + d*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x\} \&\& \text{IGtQ}[n, 0]$

Rule 3904

$\text{Int}[(\csc[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^m*(\csc[(e_.) + (f_.)(x_.)]*(d_.) + (c_.))^n], x_Symbol] :> \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{2*m}*(c + d*\csc[e + f*x])^{n-m}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \sec(e + fx))^2 (c - c \sec(e + fx))^4} dx &= \frac{\int \cot^8(e + fx) (a + a \sec(e + fx))^2 dx}{a^4 c^4} \\ &= \frac{\int (a^2 \cot^8(e + fx) + 2a^2 \cot^7(e + fx) \csc(e + fx) + a^2 \cot^6(e + fx) \csc^2(e + fx)) dx}{a^4 c^4} \\ &= \frac{\int \cot^8(e + fx) dx}{a^2 c^4} + \frac{\int \cot^6(e + fx) \csc^2(e + fx) dx}{a^2 c^4} + \frac{2 \int \cot^7(e + fx) \csc(e + fx) dx}{a^2 c^4} \\ &= -\frac{\cot^7(e + fx)}{7a^2 c^4 f} - \frac{\int \cot^6(e + fx) dx}{a^2 c^4} + \frac{\text{Subst}\left(\int x^6 dx, x, -\cot(e + fx)\right)}{a^2 c^4 f} \\ &= \frac{\cot^5(e + fx)}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)}{7a^2 c^4 f} + \frac{\int \cot^4(e + fx) dx}{a^2 c^4} - \frac{2 \text{Subst}\left(\int x^4 dx, x, -\cot(e + fx)\right)}{a^2 c^4 f} \\ &= -\frac{\cot^3(e + fx)}{3a^2 c^4 f} + \frac{\cot^5(e + fx)}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)}{7a^2 c^4 f} + \frac{2 \csc(e + fx)}{a^2 c^4 f} \\ &= \frac{\cot(e + fx)}{a^2 c^4 f} - \frac{\cot^3(e + fx)}{3a^2 c^4 f} + \frac{\cot^5(e + fx)}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)}{7a^2 c^4 f} + \frac{2 \csc(e + fx)}{a^2 c^4 f} \\ &= \frac{x}{a^2 c^4} + \frac{\cot(e + fx)}{a^2 c^4 f} - \frac{\cot^3(e + fx)}{3a^2 c^4 f} + \frac{\cot^5(e + fx)}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)}{7a^2 c^4 f} + \frac{2 \csc(e + fx)}{a^2 c^4 f} \end{aligned}$$

Mathematica [A] time = 1.27, size = 315, normalized size = 1.90

$$\frac{\csc\left(\frac{e}{2}\right) \sec\left(\frac{e}{2}\right) \csc^7\left(\frac{1}{2}(e + fx)\right) \sec^3\left(\frac{1}{2}(e + fx)\right) (-16002 \sin(e + fx) + 9144 \sin(2(e + fx)) + 3429 \sin(3(e + fx)))}{a^2 c^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]

[Out] (Csc[e/2]*Csc[(e + f*x)/2]^7*Sec[e/2]*Sec[(e + f*x)/2]^3*(5880*f*x*Cos[f*x] - 5880*f*x*Cos[2*e + f*x] - 3360*f*x*Cos[e + 2*f*x] + 3360*f*x*Cos[3*e + 2*f*x] - 1260*f*x*Cos[2*e + 3*f*x] + 1260*f*x*Cos[4*e + 3*f*x] + 1680*f*x*Cos[3*e + 4*f*x] - 1680*f*x*Cos[5*e + 4*f*x] - 420*f*x*Cos[4*e + 5*f*x] + 420*f*x*Cos[6*e + 5*f*x] + 4032*Sin[e] - 9632*Sin[f*x] - 16002*Sin[e + f*x] + 9144*Sin[2*(e + f*x)] + 3429*Sin[3*(e + f*x)] - 4572*Sin[4*(e + f*x)] + 1143*Sin[5*(e + f*x)] - 11760*Sin[2*e + f*x] + 8864*Sin[e + 2*f*x] + 3360*Sin[3*e + 2*f*x] + 2064*Sin[2*e + 3*f*x] + 2520*Sin[4*e + 3*f*x] - 4432*Sin[3*e

+ 4*f*x] - 1680*Sin[5*e + 4*f*x] + 1528*Sin[4*e + 5*f*x]))/(860160*a^2*c^4*f)

fricas [A] time = 0.49, size = 166, normalized size = 1.00

$$\frac{191 \cos(fx + e)^5 - 172 \cos(fx + e)^4 - 253 \cos(fx + e)^3 + 258 \cos(fx + e)^2 + 105 \left(fx \cos(fx + e) - 2 fx \cos(fx + e) \right)}{105 \left(a^2 c^4 f \cos(fx + e)^4 - 2 a^2 c^4 f \cos(fx + e)^3 + 2 a^2 c^4 f \cos(fx + e)^2 - a^2 c^4 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] 1/105*(191*cos(f*x + e)^5 - 172*cos(f*x + e)^4 - 253*cos(f*x + e)^3 + 258*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 87*cos(f*x + e) - 96)/((a^2*c^4*f*cos(f*x + e)^4 - 2*a^2*c^4*f*cos(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*sin(f*x + e))

giac [A] time = 0.95, size = 129, normalized size = 0.78

$$\frac{\frac{3360(fx+e)}{a^2c^4} + \frac{4410 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 770 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 147 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 15}{a^2c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7} + \frac{35 \left(a^4c^8 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 21a^4c^8 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}{a^6c^{12}}}{3360f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")

[Out] 1/3360*(3360*(f*x + e)/(a^2*c^4) + (4410*tan(1/2*f*x + 1/2*e)^6 - 770*tan(1/2*f*x + 1/2*e)^4 + 147*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^2*c^4*tan(1/2*f*x + 1/2*e)^7) + 35*(a^4*c^8*tan(1/2*f*x + 1/2*e)^3 - 21*a^4*c^8*tan(1/2*f*x + 1/2*e))/(a^6*c^12))/f

maple [A] time = 0.95, size = 153, normalized size = 0.92

$$\frac{\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{96f a^2c^4} - \frac{7 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{32f a^2c^4} - \frac{1}{224f a^2c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{7}{160f a^2c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{11}{48f a^2c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{1}{16f a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)

[Out] 1/96/f/a^2/c^4*tan(1/2*e+1/2*f*x)^3-7/32/f/a^2/c^4*tan(1/2*e+1/2*f*x)-1/224/f/a^2/c^4/tan(1/2*e+1/2*f*x)^7+7/160/f/a^2/c^4/tan(1/2*e+1/2*f*x)^5-11/48/f/a^2/c^4/tan(1/2*e+1/2*f*x)^3+21/16/f/a^2/c^4/tan(1/2*e+1/2*f*x)+2/f/a^2/c^4*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.43, size = 167, normalized size = 1.01

$$\frac{35 \left(\frac{21 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^2c^4} - \frac{6720 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^4} - \frac{\left(\frac{147 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{770 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4410 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 15 \right) (\cos(fx+e)+1)^7}{a^2c^4 \sin(fx+e)^7}$$

3360 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

[Out] $-1/3360*(35*(21*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)/(a^2*c^4) - 6720*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/(a^2*c^4) - (147*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 770*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 4410*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 15*(\cos(f*x + e) + 1)^7/(a^2*c^4*\sin(f*x + e)^7))/f$

mupad [B] time = 1.64, size = 185, normalized size = 1.11

$$\frac{35 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^{10} - 15 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^{10} - 735 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 + 4410 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 - 770 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 + 147 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 + 3360 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^7 (e + f*x)}{3360 a^2 c^4 f \cos\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{f*x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)`

[Out] $(35*\sin(e/2 + (f*x)/2)^{10} - 15*\cos(e/2 + (f*x)/2)^{10} - 735*\cos(e/2 + (f*x)/2)^2*\sin(e/2 + (f*x)/2)^8 + 4410*\cos(e/2 + (f*x)/2)^4*\sin(e/2 + (f*x)/2)^6 - 770*\cos(e/2 + (f*x)/2)^6*\sin(e/2 + (f*x)/2)^4 + 147*\cos(e/2 + (f*x)/2)^8*\sin(e/2 + (f*x)/2)^2 + 3360*\cos(e/2 + (f*x)/2)^3*\sin(e/2 + (f*x)/2)^7*(e + f*x))/(3360*a^2*c^4*f*\cos(e/2 + (f*x)/2)^3*\sin(e/2 + (f*x)/2)^7)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{\sec^6(e+fx) - 2\sec^5(e+fx) - \sec^4(e+fx) + 4\sec^3(e+fx) - \sec^2(e+fx) - 2\sec(e+fx) + 1}{a^2 c^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)`

[Out] `Integral(1/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**4 + 4*sec(e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**4)`

$$3.30 \quad \int \frac{1}{(a+a \sec(e+fx))^2(c-c \sec(e+fx))^5} dx$$

Optimal. Leaf size=210

$$\frac{4 \cot^9(e+fx)}{9a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot(e+fx)}{a^2c^5f} + \frac{4 \csc^9(e+fx)}{9a^2c^5f} - \frac{15 \csc^7(e+fx)}{7a^2c^5f} + \frac{21 \csc^5(e+fx)}{5a^2c^5f} - \frac{15 \csc^3(e+fx)}{3a^2c^5f} + \frac{6 \csc(e+fx)}{a^2c^5f}$$

[Out] x/a^2/c^5+cot(f*x+e)/a^2/c^5/f-1/3*cot(f*x+e)^3/a^2/c^5/f+1/5*cot(f*x+e)^5/a^2/c^5/f-1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+3*csc(f*x+e)/a^2/c^5/f-13/3*csc(f*x+e)^3/a^2/c^5/f+21/5*csc(f*x+e)^5/a^2/c^5/f-15/7*csc(f*x+e)^7/a^2/c^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f

Rubi [A] time = 0.29, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$\frac{4 \cot^9(e+fx)}{9a^2c^5f} - \frac{\cot^7(e+fx)}{7a^2c^5f} + \frac{\cot^5(e+fx)}{5a^2c^5f} - \frac{\cot^3(e+fx)}{3a^2c^5f} + \frac{\cot(e+fx)}{a^2c^5f} + \frac{4 \csc^9(e+fx)}{9a^2c^5f} - \frac{15 \csc^7(e+fx)}{7a^2c^5f} + \frac{21 \csc^5(e+fx)}{5a^2c^5f} - \frac{15 \csc^3(e+fx)}{3a^2c^5f} + \frac{6 \csc(e+fx)}{a^2c^5f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]

[Out] x/(a^2*c^5) + Cot[e + f*x]/(a^2*c^5*f) - Cot[e + f*x]^3/(3*a^2*c^5*f) + Cot[e + f*x]^5/(5*a^2*c^5*f) - Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^9)/(9*a^2*c^5*f) + (3*Csc[e + f*x])/(a^2*c^5*f) - (13*Csc[e + f*x]^3)/(3*a^2*c^5*f) + (21*Csc[e + f*x]^5)/(5*a^2*c^5*f) - (15*Csc[e + f*x]^7)/(7*a^2*c^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^5*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]]

*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3886

Int[(cot[(c_.) + (d_.)*(x_)])*(e_.)^(m_)*(csc[(c_.) + (d_.)*(x_)])*(b_.) + (a_.))^(n_), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_.))^(n_), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \sec(e + fx))^2 (c - c \sec(e + fx))^5} dx &= -\frac{\int \cot^{10}(e + fx)(a + a \sec(e + fx))^3 dx}{a^5 c^5} \\ &= -\frac{\int (a^3 \cot^{10}(e + fx) + 3a^3 \cot^9(e + fx) \csc(e + fx) + 3a^3 \cot^8(e + fx) \csc^2(e + fx) + 3a^3 \cot^7(e + fx) \csc^3(e + fx) + 3a^3 \cot^6(e + fx) \csc^4(e + fx) + 3a^3 \cot^5(e + fx) \csc^5(e + fx)) dx}{a^5 c^5} \\ &= -\frac{\int \cot^{10}(e + fx) dx}{a^2 c^5} - \frac{\int \cot^7(e + fx) \csc^3(e + fx) dx}{a^2 c^5} - \frac{3 \int \cot^4(e + fx) \csc^5(e + fx) dx}{a^2 c^5} \\ &= \frac{\cot^9(e + fx)}{9a^2 c^5 f} + \frac{\int \cot^8(e + fx) dx}{a^2 c^5} + \frac{\text{Subst}\left(\int x^2 (-1 + x^2)^3 dx\right)}{a^2 c^5 f} \\ &= -\frac{\cot^7(e + fx)}{7a^2 c^5 f} + \frac{4 \cot^9(e + fx)}{9a^2 c^5 f} - \frac{\int \cot^6(e + fx) dx}{a^2 c^5} + \frac{\text{Subst}\left(\int x^2 (-1 + x^2)^3 dx\right)}{a^2 c^5 f} \\ &= \frac{\cot^5(e + fx)}{5a^2 c^5 f} - \frac{\cot^7(e + fx)}{7a^2 c^5 f} + \frac{4 \cot^9(e + fx)}{9a^2 c^5 f} + \frac{3 \csc(e + fx)}{a^2 c^5 f} \\ &= -\frac{\cot^3(e + fx)}{3a^2 c^5 f} + \frac{\cot^5(e + fx)}{5a^2 c^5 f} - \frac{\cot^7(e + fx)}{7a^2 c^5 f} + \frac{4 \cot^9(e + fx)}{9a^2 c^5 f} \\ &= \frac{\cot(e + fx)}{a^2 c^5 f} - \frac{\cot^3(e + fx)}{3a^2 c^5 f} + \frac{\cot^5(e + fx)}{5a^2 c^5 f} - \frac{\cot^7(e + fx)}{7a^2 c^5 f} + \frac{4 \cot^9(e + fx)}{9a^2 c^5 f} \\ &= \frac{x}{a^2 c^5} + \frac{\cot(e + fx)}{a^2 c^5 f} - \frac{\cot^3(e + fx)}{3a^2 c^5 f} + \frac{\cot^5(e + fx)}{5a^2 c^5 f} - \frac{\cot^7(e + fx)}{7a^2 c^5 f} + \frac{4 \cot^9(e + fx)}{9a^2 c^5 f} \end{aligned}$$

Mathematica [A] time = 1.20, size = 383, normalized size = 1.82

$$\csc\left(\frac{e}{2}\right) \sec\left(\frac{e}{2}\right) \tan(e + fx) \sec^6(e + fx) (-675036 \sin(e + fx) + 506277 \sin(2(e + fx)) + 37502 \sin(3(e + fx)))$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]

[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^6*(181440*f*x*Cos[f*x] - 181440*f*x*Cos[2*e + f*x] - 136080*f*x*Cos[e + 2*f*x] + 136080*f*x*Cos[3*e + 2*f*x] - 10080*f*x*Cos[2*e + 3*f*x] + 10080*f*x*Cos[4*e + 3*f*x] + 60480*f*x*Cos[3*e + 4*f*x] - 60480*f*x*Cos[5*e + 4*f*x] - 30240*f*x*Cos[4*e + 5*f*x] + 30240*f*x*Cos[6*e + 5*f*x] + 5040*f*x*Cos[5*e + 6*f*x] - 5040*f*x*Cos[7*e + 6*f*x] + 169344*Sin[e] - 338112*Sin[f*x] - 675036*Sin[e + f*x] + 506277*Sin[2*(e + f*x)] + 37502*Sin[3*(e + f*x)] - 225012*Sin[4*(e + f*x)] + 112506*Sin[5*(e + f*x)] - 18751*Sin[6*(e + f*x)] - 431424*Sin[2*e + f*x] + 375552*Sin[e + 2*f*x] + 201600*Sin[3*e + 2*f*x] - 41248*Sin[2*e + 3*f*x] + 84000*Sin[4*e + 3*f*x] - 155712*Sin[3*e + 4*f*x] - 100800*Sin[5*e + 4*f*x] + 98016*Sin[4*e + 5*f*x] + 30240*Sin[6*e + 5*f*x] - 21376*Sin[5*e + 6*f*x])*Tan[e + f*x])/(645120*a^2*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^2)

fricas [A] time = 0.42, size = 232, normalized size = 1.10

$$\frac{668 \cos(fx + e)^6 - 1059 \cos(fx + e)^5 - 573 \cos(fx + e)^4 + 1813 \cos(fx + e)^3 - 393 \cos(fx + e)^2 + 315 \left(fx \cos(fx + e)^5 - 3a^2c^5f \cos(fx + e)^4 + 2a^2c^5f \cos(fx + e)^3 - 3f^2 \cos(fx + e)^2 + 2fx \cos(fx + e) - 789 \cos(fx + e) + 368 \right)}{a^2c^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^9} + \frac{105 \left(a^4c^{10} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 24a^4c^{10} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}{a^6c^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/315*(668*cos(f*x + e)^6 - 1059*cos(f*x + e)^5 - 573*cos(f*x + e)^4 + 1813*cos(f*x + e)^3 - 393*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^5 - 3*f*x*cos(f*x + e)^4 + 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 - 3*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 789*cos(f*x + e) + 368)/((a^2*c^5*f*cos(f*x + e)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))

giac [A] time = 0.48, size = 143, normalized size = 0.68

$$\frac{20160(fx+e)}{a^2c^5} + \frac{31185 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 - 6720 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 1827 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 360 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 35}{a^2c^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^9} + \frac{105 \left(a^4c^{10} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 24a^4c^{10} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}{a^6c^{15}}$$

20160 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")

[Out] 1/20160*(20160*(f*x + e)/(a^2*c^5) + (31185*tan(1/2*f*x + 1/2*e)^8 - 6720*tan(1/2*f*x + 1/2*e)^6 + 1827*tan(1/2*f*x + 1/2*e)^4 - 360*tan(1/2*f*x + 1/2*e)^2 + 35)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) + 105*(a^4*c^10*tan(1/2*f*x + 1/2*e)^3 - 24*a^4*c^10*tan(1/2*f*x + 1/2*e)))/(a^6*c^15)/f

maple [A] time = 0.94, size = 175, normalized size = 0.83

$$\frac{\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{192f a^2c^5} - \frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{8f a^2c^5} + \frac{1}{576f a^2c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9} - \frac{1}{56f a^2c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{29}{320f a^2c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{1}{3f a^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)

[Out] 1/192/f/a^2/c^5*tan(1/2*e+1/2*f*x)^3-1/8/f/a^2/c^5*tan(1/2*e+1/2*f*x)+1/576/f/a^2/c^5/tan(1/2*e+1/2*f*x)^9-1/56/f/a^2/c^5/tan(1/2*e+1/2*f*x)^7+29/320/f/a^2/c^5/tan(1/2*e+1/2*f*x)^5-1/3/f/a^2/c^5/tan(1/2*e+1/2*f*x)^3+99/64/f/a^2/c^5/tan(1/2*e+1/2*f*x)+2/f/a^2/c^5*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.43, size = 186, normalized size = 0.89

$$\frac{105 \left(\frac{24 \sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^2 c^5} - \frac{40320 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2 c^5} + \frac{\left(\frac{360 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1827 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{6720 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{31185 \sin(fx+e)^8}{(\cos(fx+e)+1)^8} - 35 \right) (\cos(fx+e)+1)^9}{a^2 c^5 \sin(fx+e)^9}$$

$$20160 f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] -1/20160*(105*(24*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*c^5) - 40320*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^5) + (360*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1827*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6720*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 31185*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(f*x + e)^9))/f

mupad [B] time = 1.78, size = 209, normalized size = 1.00

$$\frac{35 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} + 105 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} - 2520 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} + 31185 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 6720 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 1827 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 360 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 20160 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^9 (e + fx)}{20160 a^2 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)

[Out] (35*cos(e/2 + (f*x)/2)^12 + 105*sin(e/2 + (f*x)/2)^12 - 2520*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^10 + 31185*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1827*cos(e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^4 - 360*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^2 + 20160*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9*(e + f*x))/(20160*a^2*c^5*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^7(e+fx) - 3\sec^6(e+fx) + \sec^5(e+fx) + 5\sec^4(e+fx) - 5\sec^3(e+fx) - \sec^2(e+fx) + 3\sec(e+fx) - 1} dx}{a^2 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)

[Out] -Integral(1/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)/(a**2*c**5)

$$3.31 \quad \int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^3} dx$$

Optimal. Leaf size=162

$$-\frac{c^5 \tan(e + fx)}{a^3 f} + \frac{128c^5 \cot^5(e + fx)}{5a^3 f} + \frac{128c^5 \cot^3(e + fx)}{3a^3 f} + \frac{32c^5 \cot(e + fx)}{a^3 f} - \frac{128c^5 \csc^5(e + fx)}{5a^3 f} + \frac{64c^5 \csc^3(e + fx)}{3a^3 f}$$

[Out] $c^5 x/a^3 + 8c^5 \operatorname{arctanh}(\sin(fx+e))/a^3/f + 32c^5 \cot(fx+e)/a^3/f + 128/3c^5 \cot(fx+e)^3/a^3/f + 128/5c^5 \cot(fx+e)^5/a^3/f - 16c^5 \csc(fx+e)/a^3/f + 64/3c^5 \csc(fx+e)^3/a^3/f - 128/5c^5 \csc(fx+e)^5/a^3/f - c^5 \tan(fx+e)/a^3/f$

Rubi [A] time = 0.44, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30, 14, 3767, 2621, 302, 207, 2620, 270}

$$-\frac{c^5 \tan(e + fx)}{a^3 f} + \frac{128c^5 \cot^5(e + fx)}{5a^3 f} + \frac{128c^5 \cot^3(e + fx)}{3a^3 f} + \frac{32c^5 \cot(e + fx)}{a^3 f} - \frac{128c^5 \csc^5(e + fx)}{5a^3 f} + \frac{64c^5 \csc^3(e + fx)}{3a^3 f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c \operatorname{Sec}[e + fx])^5/(a + a \operatorname{Sec}[e + fx])^3, x]$

[Out] $(c^5 x)/a^3 + (8c^5 \operatorname{ArcTanh}[\operatorname{Sin}[e + fx]])/(a^3 f) + (32c^5 \operatorname{Cot}[e + fx])/(a^3 f) + (128c^5 \operatorname{Cot}[e + fx]^3)/(3a^3 f) + (128c^5 \operatorname{Cot}[e + fx]^5)/(5a^3 f) - (16c^5 \operatorname{Csc}[e + fx])/(a^3 f) + (64c^5 \operatorname{Csc}[e + fx]^3)/(3a^3 f) - (128c^5 \operatorname{Csc}[e + fx]^5)/(5a^3 f) - (c^5 \operatorname{Tan}[e + fx])/(a^3 f)$

Rule 8

$\text{Int}[a_, x_Symbol] := \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 14

$\text{Int}[(u_)*((c_)*(x_))^{(m_)}], x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}[\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_ + (b_)*(v_))] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 30

$\text{Int}[(x_)^{(m_)}], x_Symbol] := \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 194

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}], x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 207

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}], x_Symbol] := -\text{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 270

$\text{Int}[(c_)*(x_))^{(m_)*((a_ + (b_)*(x_)^{(n_)})^{(p_)}], x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 302

$\text{Int}[(x_)^m / ((a_) + (b_)*(x_)^n), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 2606

$\text{Int}[(a_)*\text{sec}(e_ + (f_)*(x_))^{m_} * ((b_)*\text{tan}(e_ + (f_)*(x_)))^{n_}, x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{m-1} * (-1 + x^2)^{(n-1)/2}, x], x, \text{Sec}[e + f*x]], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

$\text{Int}[\text{sec}(e_ + (f_)*(x_))^{m_} * ((b_)*\text{tan}(e_ + (f_)*(x_)))^{n_}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(b*x)^n * (1 + x^2)^{m/2 - 1}, x], x, \text{Tan}[e + f*x]], x] /;$ FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 2620

$\text{Int}[\text{csc}(e_ + (f_)*(x_))^{m_} * \text{sec}(e_ + (f_)*(x_))^{n_}, x_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(1 + x^2)^{(m+n)/2 - 1} / x^m, x], x, \text{Tan}[e + f*x]], x] /;$ FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rule 2621

$\text{Int}[(\text{csc}(e_ + (f_)*(x_)) * (a_))^{m_} * \text{sec}(e_ + (f_)*(x_))^{n_}, x_Symbol] \rightarrow -\text{Dist}[(f*a^n)^{-1}, \text{Subst}[\text{Int}[x^{m+n-1} / (-1 + x^2/a^2)^{(n+1)/2}, x], x, a*\text{Csc}[e + f*x]], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 3473

$\text{Int}[(b_)*\text{tan}(c_ + (d_)*(x_))^{n_}, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{n-1}) / (d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3767

$\text{Int}[\text{csc}(c_ + (d_)*(x_))^{n_}, x_Symbol] \rightarrow -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3886

$\text{Int}[(\text{cot}(c_ + (d_)*(x_)) * (e_))^{m_} * (\text{csc}(c_ + (d_)*(x_)) * (b_ + (a_)))^{n_}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e*\text{Cot}[c + d*x])^m, (a + b*\text{Csc}[c + d*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]

Rule 3904

$\text{Int}[(\text{csc}(e_ + (f_)*(x_)) * (b_ + (a_)))^{m_} * (\text{csc}(e_ + (f_)*(x_)) * (d_ + (c_)))^{n_}, x_Symbol] \rightarrow \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{2*m} * (c + d*\text{Csc}[e + f*x])^{n-m}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^3} dx &= -\frac{\int \cot^6(e + fx)(c - c \sec(e + fx))^8 dx}{a^3 c^3} \\
&= -\frac{\int (c^8 \cot^6(e + fx) - 8c^8 \cot^5(e + fx) \csc(e + fx) + 28c^8 \cot^4(e + fx) \csc^2(e + fx) - \dots)}{a^3 c^3} \\
&= -\frac{c^5 \int \cot^6(e + fx) dx}{a^3} - \frac{c^5 \int \csc^6(e + fx) \sec^2(e + fx) dx}{a^3} + \frac{(8c^5) \int \cot^5(e + fx) \csc(e + fx) dx}{a^3} \\
&= \frac{c^5 \cot^5(e + fx)}{5a^3 f} + \frac{c^5 \int \cot^4(e + fx) dx}{a^3} - \frac{c^5 \text{Subst}\left(\int \frac{(1+x^2)^3}{x^6} dx, x, \tan(e + fx)\right)}{a^3 f} - \frac{c^5 \int \csc^4(e + fx) dx}{a^3} \\
&= \frac{28c^5 \cot(e + fx)}{a^3 f} + \frac{55c^5 \cot^3(e + fx)}{3a^3 f} + \frac{57c^5 \cot^5(e + fx)}{5a^3 f} - \frac{56c^5 \csc^5(e + fx)}{5a^3 f} - \frac{c^5 \int \csc^2(e + fx) dx}{a^3} \\
&= \frac{32c^5 \cot(e + fx)}{a^3 f} + \frac{128c^5 \cot^3(e + fx)}{3a^3 f} + \frac{128c^5 \cot^5(e + fx)}{5a^3 f} - \frac{16c^5 \csc(e + fx)}{a^3 f} + \frac{6c^5 \int \csc^2(e + fx) dx}{a^3} \\
&= \frac{c^5 x}{a^3} + \frac{8c^5 \tanh^{-1}(\sin(e + fx))}{a^3 f} + \frac{32c^5 \cot(e + fx)}{a^3 f} + \frac{128c^5 \cot^3(e + fx)}{3a^3 f} + \frac{128c^5 \cot^5(e + fx)}{5a^3 f}
\end{aligned}$$

Mathematica [B] time = 5.88, size = 557, normalized size = 3.44

$$\frac{c^5 \sec\left(\frac{e}{2}\right) (\cos(e + fx) - 1)^5 \cot\left(\frac{1}{2}(e + fx)\right) \csc^4\left(\frac{1}{2}(e + fx)\right) \left(1016 \sin\left(\frac{fx}{2}\right) \cot^6\left(\frac{1}{2}(e + fx)\right) \csc\left(\frac{1}{2}(e + fx)\right) + \dots\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]

[Out] -1/240*(c^5*(-1 + Cos[e + f*x])^5*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Sec[e/2]*(-60*Cos[e]*Cot[(e + f*x)/2]^7*(f*x - 8*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sec[e/2] + 48*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Sec[e/2]^2*(Sin[e/2] - Sin[(3*e)/2]) + 8*(-7 + Cos[e + f*x])*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2]^4*Sec[e/2]^2*(Sin[e/2] - Sin[(3*e)/2]) + 1016*Cot[(e + f*x)/2]^6*Csc[(e + f*x)/2]*Sin[(f*x)/2] + (-140 + 76*Cos[e] + 131*Cos[f*x] - 210*Cos[e + f*x] - 84*Cos[2*(e + f*x)] - 14*Cos[3*(e + f*x)] + 131*Cos[2*e + f*x] + 66*Cos[e + 2*f*x] + 66*Cos[3*e + 2*f*x] + 21*Cos[2*e + 3*f*x] + 21*Cos[4*e + 3*f*x])*Csc[(e + f*x)/2]^7*Sec[e/2]^2*Sin[(f*x)/2] + 2*Cot[(e + f*x)/2]^5*Sec[e/2]*(30*Cos[e]*(f*x - 8*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - (Cos[e] + 15*(-1 + Cos[f*x] + Cos[e + f*x]))*Csc[(e + f*x)/2]^2*Tan[e/2]))/(a^3*f*(1 + Cos[e + f*x])^3*(-1 + Cot[(e + f*x)/2])*(1 + Cot[(e + f*x)/2])*(-1 + Tan[e/2])*(1 + Tan[e/2]))

fricas [A] time = 0.45, size = 289, normalized size = 1.78

$$\frac{15c^5 fx \cos(fx + e)^4 + 45c^5 fx \cos(fx + e)^3 + 45c^5 fx \cos(fx + e)^2 + 15c^5 fx \cos(fx + e) + 60(c^5 \cos(fx + e) + \dots)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fricas")


```
[Out] 1/15*(15*c^5*f*x*cos(f*x + e)^4 + 45*c^5*f*x*cos(f*x + e)^3 + 45*c^5*f*x*cos(f*x + e)^2 + 15*c^5*f*x*cos(f*x + e) + 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(sin(f*x + e) + 1) - 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(-sin(f*x + e) + 1) - (239*c^5*cos(f*x + e)^3 + 477*c^5*cos(f*x + e)^2 + 349*c^5*cos(f*x + e) + 15*c^5)*sin(f*x + e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f*((4/5*tan((f*x+exp(1))/2)^5*c^5*a^12+4/3*tan((f*x+exp(1))/2)^3*c^5*a^12+8*tan((f*x+exp(1))/2)*c^5*a^12)/a^15-tan((f*x+exp(1))/2)*c^5/a^3/(tan((f*x+exp(1))/2)^2-1)-2*c^5*1/2/a^3*(f*x+exp(1))/2+4*c^5/a^3*ln(abs(tan((f*x+exp(1))/2)-1))-4*c^5/a^3*ln(abs(tan((f*x+exp(1))/2)+1)))
```

maple [A] time = 0.64, size = 179, normalized size = 1.10

$$\frac{8c^5 \left(\tan^5 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{5fa^3} - \frac{8c^5 \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{3fa^3} - \frac{16c^5 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{fa^3} + \frac{c^5}{fa^3 \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) - 1 \right)} - \frac{8c^5 \ln \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -8/5/f*c^5/a^3*tan(1/2*e+1/2*f*x)^5-8/3/f*c^5/a^3*tan(1/2*e+1/2*f*x)^3-16/f*c^5/a^3*tan(1/2*e+1/2*f*x)+1/f*c^5/a^3/(tan(1/2*e+1/2*f*x)-1)-8/f*c^5/a^3*ln(tan(1/2*e+1/2*f*x)-1)+8/f*c^5/a^3*ln(tan(1/2*e+1/2*f*x)+1)+1/f*c^5/a^3/(tan(1/2*e+1/2*f*x)+1)+2/f*c^5/a^3*arctan(tan(1/2*e+1/2*f*x))
```

maxima [B] time = 0.44, size = 562, normalized size = 3.47

$$3c^5 \left(\frac{40 \sin(fx+e)}{\left(a^3 - \frac{a^3 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} \right) (\cos(fx+e)+1)} + \frac{\frac{85 \sin(fx+e)}{\cos(fx+e)+1} + \frac{10 \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{\sin^5(fx+e)}{(\cos(fx+e)+1)^5}}{a^3} - \frac{60 \log \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1 \right)}{a^3} + \frac{60 \log \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1 \right)}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(3*c^5*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3 + 5*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3 + c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3 + 10*c^5*(15*s
```

$\sin(fx + e)/(\cos(fx + e) + 1) + 10\sin(fx + e)^3/(\cos(fx + e) + 1)^3 + 3\sin(fx + e)^5/(\cos(fx + e) + 1)^5/a^3 + 5c^5(15\sin(fx + e)/(\cos(fx + e) + 1) - 10\sin(fx + e)^3/(\cos(fx + e) + 1)^3 + 3\sin(fx + e)^5/(\cos(fx + e) + 1)^5)/a^3 - 30c^5(5\sin(fx + e)/(\cos(fx + e) + 1) - \sin(fx + e)^5/(\cos(fx + e) + 1)^5)/a^3)/f$

mupad [B] time = 1.49, size = 134, normalized size = 0.83

$$\frac{c^5 x}{a^3} - \frac{16c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^3 f} - \frac{8c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{3a^3 f} - \frac{8c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5}{5a^3 f} + \frac{16c^5 \operatorname{atanh}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^3 f} + \frac{2c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f \left(a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^3,x)`

[Out] $(c^5x)/a^3 - (16c^5\tan(e/2 + (fx)/2))/(a^3f) - (8c^5\tan(e/2 + (fx)/2)^3)/(3a^3f) - (8c^5\tan(e/2 + (fx)/2)^5)/(5a^3f) + (16c^5\operatorname{atanh}(\tan(e/2 + (fx)/2)))/(a^3f) + (2c^5\tan(e/2 + (fx)/2))/(f(a^3\tan(e/2 + (fx)/2)^2 - a^3))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^5 \left(\int \frac{5 \sec(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} dx + \int \left(-\frac{10 \sec^2(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} \right) dx + \int \frac{10 \sec^3(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)`

[Out] $-c**5*(\operatorname{Integral}(5*\sec(e + f*x)/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(-10*\sec(e + f*x)**2/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(10*\sec(e + f*x)**3/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(-5*\sec(e + f*x)**4/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(\sec(e + f*x)**5/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(-1/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x))/a**3$

$$3.32 \quad \int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^3} dx$$

Optimal. Leaf size=148

$$\frac{c^4 \tanh^{-1}(\sin(e + fx))}{a^3 f} - \frac{c^4 \tan(e + fx) \sec^2(e + fx)}{5a^3 f (\sec(e + fx) + 1)^3} - \frac{23c^4 \tan(e + fx)}{5a^3 f (\sec(e + fx) + 1)} + \frac{14c^4 \tan(e + fx)}{5a^3 f (\sec(e + fx) + 1)^2} - \frac{3c^4}{a^3 f (\sec(e + fx) + 1)}$$

[Out] $c^4 x/a^3 + c^4 \operatorname{arctanh}(\sin(fx+e))/a^3/f - 3c^4 \tan(fx+e)/a^3/f/(1+\sec(fx+e))^3 - 1/5 c^4 \sec(fx+e)^2 \tan(fx+e)/a^3/f/(1+\sec(fx+e))^3 + 14/5 c^4 \tan(fx+e)/a^3/f/(1+\sec(fx+e))^2 - 23/5 c^4 \tan(fx+e)/a^3/f/(1+\sec(fx+e))$

Rubi [A] time = 0.61, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000, 3816, 4008, 3998, 3770}

$$\frac{c^4 \tanh^{-1}(\sin(e + fx))}{a^3 f} - \frac{c^4 \tan(e + fx) \sec^2(e + fx)}{5a^3 f (\sec(e + fx) + 1)^3} - \frac{23c^4 \tan(e + fx)}{5a^3 f (\sec(e + fx) + 1)} + \frac{14c^4 \tan(e + fx)}{5a^3 f (\sec(e + fx) + 1)^2} - \frac{3c^4}{a^3 f (\sec(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c \operatorname{Sec}[e + fx])^4/(a + a \operatorname{Sec}[e + fx])^3, x]$

[Out] $(c^4 x)/a^3 + (c^4 \operatorname{ArcTanh}[\operatorname{Sin}[e + fx]])/(a^3 f) - (3c^4 \operatorname{Tan}[e + fx])/(a^3 f (1 + \operatorname{Sec}[e + fx])^3) - (c^4 \operatorname{Sec}[e + fx]^2 \operatorname{Tan}[e + fx])/(5a^3 f (1 + \operatorname{Sec}[e + fx])^3) + (14c^4 \operatorname{Tan}[e + fx])/(5a^3 f (1 + \operatorname{Sec}[e + fx])^2) - (23c^4 \operatorname{Tan}[e + fx])/(5a^3 f (1 + \operatorname{Sec}[e + fx]))$

Rule 3770

$\text{Int}[\operatorname{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3777

$\text{Int}[(\operatorname{csc}[(c_.) + (d_.)(x_.)]*(b_.) + (a_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(\operatorname{Cot}[c + dx]*(a + b \operatorname{Csc}[c + dx])^n)/(d*(2*n + 1)), x] + \operatorname{Dist}[1/(a^2*(2*n + 1)), \text{Int}[(a + b \operatorname{Csc}[c + dx])^{(n + 1)}*(a*(2*n + 1) - b*(n + 1)*\operatorname{Csc}[c + dx]), x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{LeQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 3794

$\text{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]/(\operatorname{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Cot}[e + fx]/(f*(b + a \operatorname{Csc}[e + fx])), x] /; \operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3796

$\text{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]*(\operatorname{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b \operatorname{Cot}[e + fx]*(a + b \operatorname{Csc}[e + fx])^m)/(a*f*(2*m + 1)), x] + \operatorname{Dist}[(m + 1)/(a*(2*m + 1)), \text{Int}[\operatorname{Csc}[e + fx]*(a + b \operatorname{Csc}[e + fx])^{(m + 1)}], x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{LtQ}[m, -2] \ \&\& \operatorname{IntegerQ}[2*m]$

Rule 3797

$\text{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]^2*(\operatorname{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(\operatorname{Cot}[e + fx]*(a + b \operatorname{Csc}[e + fx])^m)/(f*(2*m + 1)), x] + \operatorname{Dist}[m/(b*(2*m + 1)), \text{Int}[\operatorname{Csc}[e + fx]*(a + b \operatorname{Csc}[e + fx])^{(m + 1)}], x],$

x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3799

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3816

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]

Rule 4008

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^3} dx &= \frac{\int \left(\frac{c^4}{(1+\sec(e+fx))^3} - \frac{4c^4 \sec(e+fx)}{(1+\sec(e+fx))^3} + \frac{6c^4 \sec^2(e+fx)}{(1+\sec(e+fx))^3} - \frac{4c^4 \sec^3(e+fx)}{(1+\sec(e+fx))^3} + \frac{c^4 \sec^4(e+fx)}{(1+\sec(e+fx))^3} \right) dx}{a^3} \\ &= \frac{c^4 \int \frac{1}{(1+\sec(e+fx))^3} dx}{a^3} + \frac{c^4 \int \frac{\sec^4(e+fx)}{(1+\sec(e+fx))^3} dx}{a^3} - \frac{(4c^4) \int \frac{\sec(e+fx)}{(1+\sec(e+fx))^3} dx}{a^3} - \frac{(4c^4) \int \frac{\sec^2(e+fx)}{(1+\sec(e+fx))^3} dx}{a^3} \\ &= -\frac{3c^4 \tan(e + fx)}{a^3 f(1 + \sec(e + fx))^3} - \frac{c^4 \sec^2(e + fx) \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} - \frac{c^4 \int \frac{(2-5 \sec(e+fx)) \sec^2(e+fx)}{(1+\sec(e+fx))^2} dx}{5a^3} \\ &= -\frac{3c^4 \tan(e + fx)}{a^3 f(1 + \sec(e + fx))^3} - \frac{c^4 \sec^2(e + fx) \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} + \frac{14c^4 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^2} \\ &= \frac{c^4 x}{a^3} - \frac{3c^4 \tan(e + fx)}{a^3 f(1 + \sec(e + fx))^3} - \frac{c^4 \sec^2(e + fx) \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} + \frac{14c^4 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^2} \\ &= \frac{c^4 x}{a^3} + \frac{c^4 \tanh^{-1}(\sin(e + fx))}{a^3 f} - \frac{3c^4 \tan(e + fx)}{a^3 f(1 + \sec(e + fx))^3} - \frac{c^4 \sec^2(e + fx) \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} \end{aligned}$$

Mathematica [A] time = 1.20, size = 231, normalized size = 1.56

$$\frac{c^4(\cos(e + fx) - 1)^4 \cot\left(\frac{1}{2}(e + fx)\right) \csc^2\left(\frac{1}{2}(e + fx)\right) \left(8 \tan\left(\frac{e}{2}\right) \cot^3\left(\frac{1}{2}(e + fx)\right) \csc^2\left(\frac{1}{2}(e + fx)\right) - 4 \tan\left(\frac{e}{2}\right) \csc^2\left(\frac{1}{2}(e + fx)\right)\right)}{a^3 f(1 + \sec(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]

[Out] (c^4*(-1 + Cos[e + f*x])^4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(5*Cot[(e + f*x)/2]^5*(f*x - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - (9 + 8*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]^5*Sec[e/2]*Sin[(f*x)/2] + 8*Cot[(e + f*x)/2]^3*Csc[(e + f*x)/2]^2*Tan[e/2] - 4*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*Tan[e/2))/(10*a^3*f*(1 + Cos[e + f*x])^3)

fricas [A] time = 0.48, size = 242, normalized size = 1.64

$$10c^4fx \cos(fx + e)^3 + 30c^4fx \cos(fx + e)^2 + 30c^4fx \cos(fx + e) + 10c^4fx + 5\left(c^4 \cos(fx + e)^3 + 3c^4 \cos(fx + e)^2 + 3c^4 \cos(fx + e) + 10c^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/10*(10*c^4*f*x*cos(f*x + e)^3 + 30*c^4*f*x*cos(f*x + e)^2 + 30*c^4*f*x*cos(f*x + e) + 10*c^4*f*x + 5*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + 10*c^4))

$c^4 \cos(fx + e) + c^4 \log(\sin(fx + e) + 1) - 5(c^4 \cos(fx + e)^3 + 3c^4 \cos(fx + e)^2 + 3c^4 \cos(fx + e) + c^4) \log(-\sin(fx + e) + 1) - 16(3c^4 \cos(fx + e)^2 + 4c^4 \cos(fx + e) + 3c^4) \sin(fx + e) / (a^3 f \cos(fx + e)^3 + 3a^3 f \cos(fx + e)^2 + 3a^3 f \cos(fx + e) + a^3 f)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(-c^4*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)-1))+c^4*1/2/a^3*ln(abs(tan((f*x+exp(1))/2)+1))+2*c^4*1/2/a^3*(f*x+exp(1))/2+(-2/5*tan((f*x+exp(1))/2))^5*c^4*a^12-2*tan((f*x+exp(1))/2)*c^4*a^12)/a^15)

maple [A] time = 0.77, size = 110, normalized size = 0.74

$$\frac{4c^4 \left(\tan^5 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{5fa^3} - \frac{4c^4 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{fa^3} - \frac{c^4 \ln \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) - 1 \right)}{fa^3} + \frac{c^4 \ln \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) + 1 \right)}{fa^3} + \frac{2c^4 \arctan \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)

[Out] -4/5/f*c^4/a^3*tan(1/2*e+1/2*f*x)^5-4/f*c^4/a^3*tan(1/2*e+1/2*f*x)-1/f*c^4/a^3*ln(tan(1/2*e+1/2*f*x)-1)+1/f*c^4/a^3*ln(tan(1/2*e+1/2*f*x)+1)+2/f*c^4/a^3*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.44, size = 396, normalized size = 2.68

$$c^4 \left(\frac{\frac{105 \sin(fx+e)}{\cos(fx+e)+1} + \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{60 \log \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1 \right)}{a^3} + \frac{60 \log \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1 \right)}{a^3} \right) + c^4 \left(\frac{\frac{105 \sin(fx+e)}{\cos(fx+e)+1} - \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="maxima")

[Out] -1/60*(c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 18*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f

mupad [B] time = 1.42, size = 50, normalized size = 0.34

$$\frac{c^4 \left(2 \operatorname{atanh} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right) - 4 \tan \left(\frac{e}{2} + \frac{fx}{2} \right) - \frac{4 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)^5}{5} + fx \right)}{a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^3,x)`

[Out] $(c^4*(2*\operatorname{atanh}(\tan(e/2 + (f*x)/2)) - 4*\tan(e/2 + (f*x)/2) - (4*\tan(e/2 + (f*x)/2)^5)/5 + f*x))/(a^3*f)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4 \left(\int \left(-\frac{4 \sec(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} \right) dx + \int \frac{6 \sec^2(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} dx + \int \left(-\frac{4 \sec(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} \right) dx \right) / a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)`

[Out] $c**4*(\operatorname{Integral}(-4*\sec(e + f*x)/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(6*\sec(e + f*x)**2/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(-4*\sec(e + f*x)**3/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(\sec(e + f*x)**4/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \operatorname{Integral}(1/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x))/a**3$

$$3.33 \quad \int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^3} dx$$

Optimal. Leaf size=96

$$-\frac{26c^3 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)} + \frac{4c^3 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)^2} - \frac{8c^3 \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^3} + \frac{c^3 x}{a^3}$$

[Out] $c^3 x/a^3 - 8/5 * c^3 * \tan(f*x+e)/a^3 / f / (1 + \sec(f*x+e))^3 + 4/15 * c^3 * \tan(f*x+e)/a^3 / f / (1 + \sec(f*x+e))^2 - 26/15 * c^3 * \tan(f*x+e)/a^3 / f / (1 + \sec(f*x+e))$

Rubi [A] time = 0.42, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797, 3799, 4000}

$$-\frac{26c^3 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)} + \frac{4c^3 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)^2} - \frac{8c^3 \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^3} + \frac{c^3 x}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3, x]

[Out] $(c^3 x)/a^3 - (8 * c^3 * \tan[e + f*x]) / (5 * a^3 * f * (1 + \sec[e + f*x])^3) + (4 * c^3 * \tan[e + f*x]) / (15 * a^3 * f * (1 + \sec[e + f*x])^2) - (26 * c^3 * \tan[e + f*x]) / (15 * a^3 * f * (1 + \sec[e + f*x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3799

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a

$^2 - b^2, 0]$ && LtQ[m, $-2^{(-1)}$]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4000

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, $-2^{(-1)}$]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^3} dx &= \frac{\int \left(\frac{c^3}{(1 + \sec(e + fx))^3} - \frac{3c^3 \sec(e + fx)}{(1 + \sec(e + fx))^3} + \frac{3c^3 \sec^2(e + fx)}{(1 + \sec(e + fx))^3} - \frac{c^3 \sec^3(e + fx)}{(1 + \sec(e + fx))^3} \right) dx}{a^3} \\ &= \frac{c^3 \int \frac{1}{(1 + \sec(e + fx))^3} dx}{a^3} - \frac{c^3 \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^3} dx}{a^3} - \frac{(3c^3) \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^3} dx}{a^3} + \frac{(3c^3) \int \frac{\sec^2(e + fx)}{(1 + \sec(e + fx))^3} dx}{a^3} \\ &= -\frac{8c^3 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} - \frac{c^3 \int \frac{-5 + 2\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{5a^3} - \frac{c^3 \int \frac{\sec(e + fx)(-3 + 5\sec(e + fx))}{(1 + \sec(e + fx))^2} dx}{5a^3} \\ &= -\frac{8c^3 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} + \frac{4c^3 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))^2} + \frac{c^3 \int \frac{15 - 7\sec(e + fx)}{1 + \sec(e + fx)} dx}{15a^3} - \frac{c^3 \int \frac{\sec^2(e + fx)}{1 + \sec(e + fx)} dx}{15a^3} \\ &= \frac{c^3 x}{a^3} - \frac{8c^3 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} + \frac{4c^3 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))^2} - \frac{4c^3 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))} \\ &= \frac{c^3 x}{a^3} - \frac{8c^3 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} + \frac{4c^3 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))^2} - \frac{26c^3 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.94

$$\frac{c^3 \left(-\frac{2 \tan^{-1} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{f} + \frac{2 \tan^5 \left(\frac{e}{2} + \frac{fx}{2} \right)}{5f} - \frac{2 \tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right)}{3f} + \frac{2 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{f} \right)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]

[Out] -((c^3*((-2*ArcTan[Tan[e/2 + (f*x)/2]])/f + (2*Tan[e/2 + (f*x)/2])/f - (2*Tan[e/2 + (f*x)/2]^3)/(3*f) + (2*Tan[e/2 + (f*x)/2]^5)/(5*f)))/a^3

fricas [A] time = 0.44, size = 138, normalized size = 1.44

$$\frac{15c^3fx \cos(fx + e)^3 + 45c^3fx \cos(fx + e)^2 + 45c^3fx \cos(fx + e) + 15c^3fx - 2(23c^3 \cos(fx + e)^2 + 24c^3 \cos(fx + e) + 13c^3 \sin(fx + e))}{15(a^3f \cos(fx + e)^3 + 3a^3f \cos(fx + e)^2 + 3a^3f \cos(fx + e) + a^3f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(15*c^3*f*x*cos(f*x + e)^3 + 45*c^3*f*x*cos(f*x + e)^2 + 45*c^3*f*x*cos(f*x + e) + 15*c^3*f*x - 2*(23*c^3*cos(f*x + e)^2 + 24*c^3*cos(f*x + e) + 13*c^3*sin(f*x + e)))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)

giac [A] time = 0.37, size = 84, normalized size = 0.88

$$\frac{\frac{15(fx+e)c^3}{a^3} - \frac{2\left(3a^{12}c^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 5a^{12}c^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15a^{12}c^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^{15}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")

[Out] 1/15*(15*(f*x + e)*c^3/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f

maple [A] time = 0.79, size = 87, normalized size = 0.91

$$-\frac{2c^3 \left(\tan^5 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{5fa^3} + \frac{2c^3 \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{3fa^3} - \frac{2c^3 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{fa^3} + \frac{2c^3 \arctan \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)

[Out] -2/5/f*c^3/a^3*tan(1/2*e+1/2*f*x)^5+2/3/f*c^3/a^3*tan(1/2*e+1/2*f*x)^3-2/f*c^3/a^3*tan(1/2*e+1/2*f*x)+2/f*c^3/a^3*arctan(tan(1/2*e+1/2*f*x))

maxima [B] time = 0.43, size = 277, normalized size = 2.89

$$c^3 \left(\frac{\frac{105 \sin(fx+e)}{\cos(fx+e)+1} - \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3} \right) + \frac{c^3 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1} + \frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right)}{a^3} + \frac{3c^3 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1} \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="maxima")

[Out]
$$-1/60*(c^3*((105*\sin(f*x + e))/(\cos(f*x + e) + 1) - 20*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)/a^3 - 120*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^3 + c^3*(15*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)/a^3 + 3*c^3*(15*\sin(f*x + e)/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)/a^3 - 9*c^3*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)/a^3)/f$$

mupad [B] time = 1.40, size = 93, normalized size = 0.97

$$\frac{c^3 x}{a^3} - \frac{\frac{46 \sin\left(\frac{e}{2} + \frac{fx}{2}\right) c^3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{15} - \frac{22 \sin\left(\frac{e}{2} + \frac{fx}{2}\right) c^3 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{15} + \frac{2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right) c^3}{5}}{a^3 f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^3,x)

[Out]
$$(c^3*x)/a^3 - ((2*c^3*\sin(e/2 + (f*x)/2))/5 - (22*c^3*\cos(e/2 + (f*x)/2)^2*\sin(e/2 + (f*x)/2))/15 + (46*c^3*\cos(e/2 + (f*x)/2)^4*\sin(e/2 + (f*x)/2))/15)/(a^3*f*\cos(e/2 + (f*x)/2)^5)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3 \left(\int \frac{3 \sec(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} dx + \int \left(-\frac{3 \sec^2(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} \right) dx + \int \frac{\sec^3}{\sec^3(e+fx)+3 \sec^2} \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)

[Out]
$$-c**3*(\text{Integral}(3*\sec(e + f*x)/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \text{Integral}(-3*\sec(e + f*x)**2/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \text{Integral}(\sec(e + f*x)**3/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x) + \text{Integral}(-1/(\sec(e + f*x)**3 + 3*\sec(e + f*x)**2 + 3*\sec(e + f*x) + 1), x))/a**3$$

$$3.34 \quad \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^3} dx$$

Optimal. Leaf size=96

$$-\frac{23c^2 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)} - \frac{8c^2 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)^2} - \frac{4c^2 \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^3} + \frac{c^2 x}{a^3}$$

[Out] $c^2 x / a^3 - 4/5 c^2 \tan(fx + e) / a^3 / f / (1 + \sec(fx + e))^3 - 8/15 c^2 \tan(fx + e) / a^3 / f / (1 + \sec(fx + e))^2 - 23/15 c^2 \tan(fx + e) / a^3 / f / (1 + \sec(fx + e))$

Rubi [A] time = 0.30, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796, 3797}

$$-\frac{23c^2 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)} - \frac{8c^2 \tan(e + fx)}{15a^3 f(\sec(e + fx) + 1)^2} - \frac{4c^2 \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^3} + \frac{c^2 x}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3, x]

[Out] $(c^2 x) / a^3 - (4 c^2 \tan[e + f x]) / (5 a^3 f (1 + \sec[e + f x])^3) - (8 c^2 \tan[e + f x]) / (15 a^3 f (1 + \sec[e + f x])^2) - (23 c^2 \tan[e + f x]) / (15 a^3 f (1 + \sec[e + f x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_), x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3797

Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_), x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c)^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]

&& LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^3} dx &= \frac{\int \left(\frac{c^2}{(1 + \sec(e + fx))^3} - \frac{2c^2 \sec(e + fx)}{(1 + \sec(e + fx))^3} + \frac{c^2 \sec^2(e + fx)}{(1 + \sec(e + fx))^3} \right) dx}{a^3} \\ &= \frac{c^2 \int \frac{1}{(1 + \sec(e + fx))^3} dx}{a^3} + \frac{c^2 \int \frac{\sec^2(e + fx)}{(1 + \sec(e + fx))^3} dx}{a^3} - \frac{(2c^2) \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^3} dx}{a^3} \\ &= -\frac{4c^2 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} - \frac{c^2 \int \frac{-5 + 2 \sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{5a^3} + \frac{(3c^2) \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{5a^3} - \frac{(4c^2) \int \frac{\sec^2(e + fx)}{(1 + \sec(e + fx))^2} dx}{5a^3} \\ &= -\frac{4c^2 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} - \frac{8c^2 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))^2} + \frac{c^2 \int \frac{15 - 7 \sec(e + fx)}{1 + \sec(e + fx)} dx}{15a^3} + \frac{c^2 \int \frac{\sec^2(e + fx)}{1 + \sec(e + fx)} dx}{15a^3} \\ &= \frac{c^2 x}{a^3} - \frac{4c^2 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} - \frac{8c^2 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))^2} - \frac{c^2 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))} \\ &= \frac{c^2 x}{a^3} - \frac{4c^2 \tan(e + fx)}{5a^3 f(1 + \sec(e + fx))^3} - \frac{8c^2 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))^2} - \frac{23c^2 \tan(e + fx)}{15a^3 f(1 + \sec(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.47, size = 171, normalized size = 1.78

$$\frac{c^2 \sec\left(\frac{e}{2}\right) \sec^5\left(\frac{1}{2}(e + fx)\right) \left(360 \sin\left(e + \frac{fx}{2}\right) - 280 \sin\left(e + \frac{3fx}{2}\right) + 150 \sin\left(2e + \frac{3fx}{2}\right) - 86 \sin\left(2e + \frac{5fx}{2}\right) + 15 \sin\left(2e + \frac{7fx}{2}\right)\right)}{(480 a^3 f)}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3, x]

[Out] (c^2*Sec[e/2]*Sec[(e + f*x)/2]^5*(150*f*x*Cos[(f*x)/2] + 150*f*x*Cos[e + (f*x)/2] + 75*f*x*Cos[e + (3*f*x)/2] + 75*f*x*Cos[2*e + (3*f*x)/2] + 15*f*x*Cos[2*e + (5*f*x)/2] + 15*f*x*Cos[3*e + (5*f*x)/2] - 500*Sin[(f*x)/2] + 360*Sin[e + (f*x)/2] - 280*Sin[e + (3*f*x)/2] + 150*Sin[2*e + (3*f*x)/2] - 86*Sin[2*e + (5*f*x)/2]))/(480*a^3*f)

fricas [A] time = 0.44, size = 138, normalized size = 1.44

$$\frac{15 c^2 f x \cos (f x + e)^3 + 45 c^2 f x \cos (f x + e)^2 + 45 c^2 f x \cos (f x + e) + 15 c^2 f x - \left(43 c^2 \cos (f x + e)^2 + 54 c^2 \cos (f x + e) + 15\right)}{15 \left(a^3 f \cos (f x + e)^3 + 3 a^3 f \cos (f x + e)^2 + 3 a^3 f \cos (f x + e) + a^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{15} \cdot (15 \cdot c^2 \cdot f \cdot x \cdot \cos(f \cdot x + e)^3 + 45 \cdot c^2 \cdot f \cdot x \cdot \cos(f \cdot x + e)^2 + 45 \cdot c^2 \cdot f \cdot x \cdot \cos(f \cdot x + e) + 15 \cdot c^2 \cdot f \cdot x - (43 \cdot c^2 \cdot \cos(f \cdot x + e)^2 + 54 \cdot c^2 \cdot \cos(f \cdot x + e) + 23 \cdot c^2) \cdot \sin(f \cdot x + e)) / (a^3 \cdot f \cdot \cos(f \cdot x + e)^3 + 3 \cdot a^3 \cdot f \cdot \cos(f \cdot x + e)^2 + 3 \cdot a^3 \cdot f \cdot \cos(f \cdot x + e) + a^3 \cdot f)$

giac [A] time = 0.33, size = 84, normalized size = 0.88

$$\frac{\frac{15(fx+e)c^2}{a^3} - \frac{3a^{12}c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 10a^{12}c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 30a^{12}c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^{15}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="giac")

[Out] $\frac{1}{15} \cdot (15 \cdot (f \cdot x + e) \cdot c^2 / a^3 - (3 \cdot a^{12} \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 10 \cdot a^{12} \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 30 \cdot a^{12} \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)) / a^{15}) / f$

maple [A] time = 0.77, size = 87, normalized size = 0.91

$$-\frac{c^2 \left(\tan^5 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{5f a^3} + \frac{2c^2 \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{3f a^3} - \frac{2c^2 \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{f a^3} + \frac{2c^2 \arctan \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{f a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)

[Out] $-1/5/f \cdot c^2/a^3 \cdot \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^5 + 2/3/f \cdot c^2/a^3 \cdot \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^3 - 2/f \cdot c^2/a^3 \cdot \tan(1/2 \cdot e + 1/2 \cdot f \cdot x) + 2/f \cdot c^2/a^3 \cdot \arctan(\tan(1/2 \cdot e + 1/2 \cdot f \cdot x))$

maxima [B] time = 0.43, size = 211, normalized size = 2.20

$$\frac{c^2 \left(\frac{105 \sin(fx+e)}{\cos(fx+e)+1} - \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3} \right)}{a^3} + \frac{2c^2 \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1} - \frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right)}{a^3} - \frac{3c^2 \left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1} \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="maxima")

[Out] $-1/60 \cdot (c^2 \cdot ((105 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) - 20 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) / a^3 - 120 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1)) / a^3) + 2 \cdot c^2 \cdot (15 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) - 10 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) / a^3 - 3 \cdot c^2 \cdot (5 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) - \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) / a^3) / f$

mupad [B] time = 1.40, size = 93, normalized size = 0.97

$$\frac{c^2 x}{a^3} - \frac{\frac{43 \sin\left(\frac{e}{2} + \frac{fx}{2}\right) c^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{15} - \frac{16 \sin\left(\frac{e}{2} + \frac{fx}{2}\right) c^2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{15} + \frac{\sin\left(\frac{e}{2} + \frac{fx}{2}\right) c^2}{5}}{a^3 f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^3,x)

[Out] (c^2*x)/a^3 - ((c^2*sin(e/2 + (f*x)/2))/5 - (16*c^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2))/15 + (43*c^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/15)/(a^3*f*cos(e/2 + (f*x)/2)^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int \left(-\frac{2 \sec(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} \right) dx + \int \frac{\sec^2(e+fx)}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} dx + \int \frac{1}{\sec^3(e+fx)+3 \sec^2(e+fx)+3 \sec(e+fx)+1} dx \right) / a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)

[Out] c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3

$$3.35 \quad \int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^3} dx$$

Optimal. Leaf size=88

$$-\frac{8c \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)} - \frac{3c \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^2} - \frac{2c \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^3} + \frac{cx}{a^3}$$

[Out] $c*x/a^3 - 2/5*c*\tan(f*x+e)/a^3/f/(1+\sec(f*x+e))^3 - 3/5*c*\tan(f*x+e)/a^3/f/(1+\sec(f*x+e))^2 - 8/5*c*\tan(f*x+e)/a^3/f/(1+\sec(f*x+e))$

Rubi [A] time = 0.20, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3903, 3777, 3922, 3919, 3794, 3796}

$$-\frac{8c \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)} - \frac{3c \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^2} - \frac{2c \tan(e + fx)}{5a^3 f(\sec(e + fx) + 1)^3} + \frac{cx}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]

[Out] $(c*x)/a^3 - (2*c*\tan[e + f*x])/(5*a^3*f*(1 + \sec[e + f*x])^3) - (3*c*\tan[e + f*x])/(5*a^3*f*(1 + \sec[e + f*x])^2) - (8*c*\tan[e + f*x])/(5*a^3*f*(1 + \sec[e + f*x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^n, x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3796

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^n, x_Symbol] :> Dist[c^n, Int[ExpandTrig[(1 + (d*csc[e + f*x])/c]^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0] && LtQ[m + n, 2]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

a*d, 0]

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^3} dx &= \frac{\int \left(\frac{c}{(1 + \sec(e + fx))^3} - \frac{c \sec(e + fx)}{(1 + \sec(e + fx))^3} \right) dx}{a^3} \\ &= \frac{c \int \frac{1}{(1 + \sec(e + fx))^3} dx}{a^3} - \frac{c \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^3} dx}{a^3} \\ &= \frac{2c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^3} - \frac{c \int \frac{-5 + 2 \sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{5a^3} - \frac{(2c) \int \frac{\sec(e + fx)}{(1 + \sec(e + fx))^2} dx}{5a^3} \\ &= \frac{2c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^3} - \frac{3c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^2} + \frac{c \int \frac{15 - 7 \sec(e + fx)}{1 + \sec(e + fx)} dx}{15a^3} - \frac{(2c)}{15a^3} \\ &= \frac{cx}{a^3} - \frac{2c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^3} - \frac{3c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^2} - \frac{2c \tan(e + fx)}{15a^3 f (1 + \sec(e + fx))} \\ &= \frac{cx}{a^3} - \frac{2c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^3} - \frac{3c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))^2} - \frac{8c \tan(e + fx)}{5a^3 f (1 + \sec(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.46, size = 169, normalized size = 1.92

$$\frac{c \sec\left(\frac{e}{2}\right) \sec^5\left(\frac{1}{2}(e + fx)\right) \left(110 \sin\left(e + \frac{fx}{2}\right) - 90 \sin\left(e + \frac{3fx}{2}\right) + 40 \sin\left(2e + \frac{3fx}{2}\right) - 26 \sin\left(2e + \frac{5fx}{2}\right) + 50fx \cos\left(2e + \frac{5fx}{2}\right)\right)}{(160a^3 f)}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]

[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^5*(50*f*x*Cos[(f*x)/2] + 50*f*x*Cos[e + (f*x)/2] + 25*f*x*Cos[e + (3*f*x)/2] + 25*f*x*Cos[2*e + (3*f*x)/2] + 5*f*x*Cos[2*e + (5*f*x)/2] + 5*f*x*Cos[3*e + (5*f*x)/2] - 150*Sin[(f*x)/2] + 110*Sin[e + (f*x)/2] - 90*Sin[e + (3*f*x)/2] + 40*Sin[2*e + (3*f*x)/2] - 26*Sin[2*e + (5*f*x)/2]))/(160*a^3*f)

fricas [A] time = 0.43, size = 124, normalized size = 1.41

$$\frac{5cfx \cos^3(fx + e) + 15cfx \cos^2(fx + e) + 15cfx \cos(fx + e) + 5cfx - \left(13c \cos^2(fx + e) + 19c \cos(fx + e) + 5\right)}{5\left(a^3 f \cos^3(fx + e) + 3a^3 f \cos^2(fx + e) + 3a^3 f \cos(fx + e) + a^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{5} * (5 * c * f * x * \cos(f * x + e)^3 + 15 * c * f * x * \cos(f * x + e)^2 + 15 * c * f * x * \cos(f * x + e) + 5 * c * f * x - (13 * c * \cos(f * x + e)^2 + 19 * c * \cos(f * x + e) + 8 * c) * \sin(f * x + e)) / (a^3 * f * \cos(f * x + e)^3 + 3 * a^3 * f * \cos(f * x + e)^2 + 3 * a^3 * f * \cos(f * x + e) + a^3 * f)$

giac [A] time = 1.19, size = 75, normalized size = 0.85

$$\frac{\frac{10(fx+e)c}{a^3} - \frac{a^{12}c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 5a^{12}c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 20a^{12}c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^{15}}}{10f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac")`

[Out] $\frac{1}{10} * (10 * (f * x + e) * c / a^3 - (a^{12} * c * \tan(1/2 * f * x + 1/2 * e)^5 - 5 * a^{12} * c * \tan(1/2 * f * x + 1/2 * e)^3 + 20 * a^{12} * c * \tan(1/2 * f * x + 1/2 * e)) / a^{15} / f$

maple [A] time = 0.75, size = 79, normalized size = 0.90

$$-\frac{c \left(\tan^5 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{10f a^3} + \frac{c \left(\tan^3 \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{2f a^3} - \frac{2c \tan \left(\frac{e}{2} + \frac{fx}{2} \right)}{f a^3} + \frac{2c \arctan \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right)}{f a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)`

[Out] $-1/10/f*c/a^3*\tan(1/2*e+1/2*f*x)^5+1/2/f*c/a^3*\tan(1/2*e+1/2*f*x)^3-2/f*c/a^3*\tan(1/2*e+1/2*f*x)+2/f*c/a^3*\arctan(\tan(1/2*e+1/2*f*x))$

maxima [A] time = 0.43, size = 159, normalized size = 1.81

$$\frac{c \left(\frac{105 \sin(fx+e)}{\cos(fx+e)+1} - \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{120 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3} \right) + \frac{c \left(\frac{15 \sin(fx+e)}{\cos(fx+e)+1} - \frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right)}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxima")`

[Out] $-1/60 * (c * ((105 * \sin(f * x + e) / (\cos(f * x + e) + 1) - 20 * \sin(f * x + e)^3 / (\cos(f * x + e) + 1)^3 + 3 * \sin(f * x + e)^5 / (\cos(f * x + e) + 1)^5) / a^3 - 120 * \arctan(\sin(f * x + e) / (\cos(f * x + e) + 1)) / a^3) + c * (15 * \sin(f * x + e) / (\cos(f * x + e) + 1) - 10 * \sin(f * x + e)^3 / (\cos(f * x + e) + 1)^3 + 3 * \sin(f * x + e)^5 / (\cos(f * x + e) + 1)^5) / a^3) / f$

mupad [B] time = 1.38, size = 85, normalized size = 0.97

$$\frac{\frac{c x}{a^3} - \frac{\frac{13 c \sin\left(\frac{e}{2} + \frac{fx}{2}\right) \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{5} - \frac{7 c \sin\left(\frac{e}{2} + \frac{fx}{2}\right) \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{10} + \frac{c \sin\left(\frac{e}{2} + \frac{fx}{2}\right)}{10}}{a^3 f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^3,x)`

[Out] $(c * x) / a^3 - ((c * \sin(e/2 + (f * x) / 2)) / 10 - (7 * c * \cos(e/2 + (f * x) / 2)^2 * \sin(e/2 + (f * x) / 2)) / 10 + (13 * c * \cos(e/2 + (f * x) / 2)^4 * \sin(e/2 + (f * x) / 2)) / 5) / (a^3 * f * \cos(e/2 + (f * x) / 2)^5)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c \left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1} dx + \int \left(-\frac{1}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1} \right) dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)

[Out] -c*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3

$$3.36 \quad \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))} dx$$

Optimal. Leaf size=126

$$\frac{2 \cot^5(e+fx)}{5a^3cf} - \frac{\cot^3(e+fx)}{3a^3cf} + \frac{\cot(e+fx)}{a^3cf} - \frac{2 \csc^5(e+fx)}{5a^3cf} + \frac{4 \csc^3(e+fx)}{3a^3cf} - \frac{2 \csc(e+fx)}{a^3cf} + \frac{x}{a^3c}$$

[Out] $x/a^3/c + \cot(f*x+e)/a^3/c/f - 1/3*\cot(f*x+e)^3/a^3/c/f + 2/5*\cot(f*x+e)^5/a^3/c/f - 2*csc(f*x+e)/a^3/c/f + 4/3*csc(f*x+e)^3/a^3/c/f - 2/5*csc(f*x+e)^5/a^3/c/f$

Rubi [A] time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30}

$$\frac{2 \cot^5(e+fx)}{5a^3cf} - \frac{\cot^3(e+fx)}{3a^3cf} + \frac{\cot(e+fx)}{a^3cf} - \frac{2 \csc^5(e+fx)}{5a^3cf} + \frac{4 \csc^3(e+fx)}{3a^3cf} - \frac{2 \csc(e+fx)}{a^3cf} + \frac{x}{a^3c}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]

[Out] $x/(a^3*c) + \text{Cot}[e + f*x]/(a^3*c*f) - \text{Cot}[e + f*x]^3/(3*a^3*c*f) + (2*\text{Cot}[e + f*x]^5)/(5*a^3*c*f) - (2*\text{Csc}[e + f*x])/(a^3*c*f) + (4*\text{Csc}[e + f*x]^3)/(3*a^3*c*f) - (2*\text{Csc}[e + f*x]^5)/(5*a^3*c*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)^(m_)]*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^(n*(1 + x^2)^(m/2 - 1)), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3886

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \sec(e + fx))^3 (c - c \sec(e + fx))} dx &= -\frac{\int \cot^6(e + fx) (c - c \sec(e + fx))^2 dx}{a^3 c^3} \\ &= -\frac{\int (c^2 \cot^6(e + fx) - 2c^2 \cot^5(e + fx) \csc(e + fx) + c^2 \cot^4(e + fx) \csc^2(e + fx)) dx}{a^3 c^3} \\ &= -\frac{\int \cot^6(e + fx) dx}{a^3 c} - \frac{\int \cot^4(e + fx) \csc^2(e + fx) dx}{a^3 c} + \frac{2 \int \cot^2(e + fx) \csc^2(e + fx) dx}{a^3 c} \\ &= \frac{\cot^5(e + fx)}{5a^3 cf} + \frac{\int \cot^4(e + fx) dx}{a^3 c} - \frac{\text{Subst}\left(\int x^4 dx, x, -\cot(e + fx)\right)}{a^3 cf} \\ &= -\frac{\cot^3(e + fx)}{3a^3 cf} + \frac{2 \cot^5(e + fx)}{5a^3 cf} - \frac{\int \cot^2(e + fx) dx}{a^3 c} - \frac{2 \text{Subst}\left(\int x^2 dx, x, -\cot(e + fx)\right)}{a^3 cf} \\ &= \frac{\cot(e + fx)}{a^3 cf} - \frac{\cot^3(e + fx)}{3a^3 cf} + \frac{2 \cot^5(e + fx)}{5a^3 cf} - \frac{2 \csc(e + fx)}{a^3 cf} + \frac{2 \int \cot^2(e + fx) dx}{a^3 c} \\ &= \frac{x}{a^3 c} + \frac{\cot(e + fx)}{a^3 cf} - \frac{\cot^3(e + fx)}{3a^3 cf} + \frac{2 \cot^5(e + fx)}{5a^3 cf} - \frac{2 \csc(e + fx)}{a^3 cf} \end{aligned}$$

Mathematica [A] time = 0.99, size = 197, normalized size = 1.56

$$\frac{\csc\left(\frac{e}{2}\right) \sec\left(\frac{e}{2}\right) \csc\left(\frac{1}{2}(e + fx)\right) \sec^5\left(\frac{1}{2}(e + fx)\right) (-445 \sin(e + fx) - 356 \sin(2(e + fx)) - 89 \sin(3(e + fx)) + \dots)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]

[Out] -1/3840*(Csc[e/2]*Csc[(e + f*x)/2]*Sec[e/2]*Sec[(e + f*x)/2]^5*(-150*f*x*Cos[f*x] + 150*f*x*Cos[2*e + f*x] - 120*f*x*Cos[e + 2*f*x] + 120*f*x*Cos[3*e + 2*f*x] - 30*f*x*Cos[2*e + 3*f*x] + 30*f*x*Cos[4*e + 3*f*x] + 80*Sin[e] + 280*Sin[f*x] - 445*Sin[e + f*x] - 356*Sin[2*(e + f*x)] - 89*Sin[3*(e + f*x)] + 240*Sin[2*e + f*x] + 296*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 104*Sin[2*e + 3*f*x]))/(a^3*c*f)

fricas [A] time = 0.43, size = 109, normalized size = 0.87

$$\frac{26 \cos(fx + e)^3 + 22 \cos(fx + e)^2 + 15 \left(fx \cos(fx + e)^2 + 2 fx \cos(fx + e) + fx \right) \sin(fx + e) - 17 \cos(fx + e)}{15 \left(a^3 cf \cos(fx + e)^2 + 2 a^3 cf \cos(fx + e) + a^3 cf \right) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] $1/15*(26*\cos(f*x + e)^3 + 22*\cos(f*x + e)^2 + 15*(f*x*\cos(f*x + e)^2 + 2*f*x*\cos(f*x + e) + f*x)*\sin(f*x + e) - 17*\cos(f*x + e) - 16)/((a^3*c*f*\cos(f*x + e)^2 + 2*a^3*c*f*\cos(f*x + e) + a^3*c*f)*\sin(f*x + e))$

giac [A] time = 0.33, size = 107, normalized size = 0.85

$$\frac{\frac{120(fx+e)}{a^3c} + \frac{15}{a^3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} - \frac{3a^{12}c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 25a^{12}c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 165a^{12}c^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^{15}c^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] $1/120*(120*(f*x + e)/(a^3*c) + 15/(a^3*c*\tan(1/2*f*x + 1/2*e)) - (3*a^12*c^4*\tan(1/2*f*x + 1/2*e)^5 - 25*a^12*c^4*\tan(1/2*f*x + 1/2*e)^3 + 165*a^12*c^4*\tan(1/2*f*x + 1/2*e))/(a^15*c^5))/f$

maple [A] time = 0.98, size = 109, normalized size = 0.87

$$-\frac{\tan^5\left(\frac{e}{2} + \frac{fx}{2}\right)}{40f a^3c} + \frac{5\left(\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{24f a^3c} - \frac{11 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{8f a^3c} + \frac{1}{8f a^3c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)

[Out] $-1/40/f/a^3/c*\tan(1/2*e+1/2*f*x)^5+5/24/f/a^3/c*\tan(1/2*e+1/2*f*x)^3-11/8/f/a^3/c*\tan(1/2*e+1/2*f*x)+1/8/f/a^3/c/\tan(1/2*e+1/2*f*x)+2/f/a^3/c*\arctan(\tan(1/2*e+1/2*f*x))$

maxima [A] time = 0.42, size = 122, normalized size = 0.97

$$\frac{\frac{165 \sin(fx+e)}{\cos(fx+e)+1} - \frac{25 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{240 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c} - \frac{15(\cos(fx+e)+1)}{a^3c \sin(fx+e)}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] $-1/120*((165*\sin(f*x + e)/(\cos(f*x + e) + 1) - 25*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)/(a^3*c) - 240*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/(a^3*c) - 15*(\cos(f*x + e) + 1)/(a^3*c*\sin(f*x + e)))/f$

mupad [B] time = 1.43, size = 82, normalized size = 0.65

$$\frac{\frac{x}{a^3c} + \frac{\frac{26 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6}{15} - \frac{28 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4}{15} + \frac{17 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2}{60} - \frac{1}{40}}{a^3c f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)

[Out] x/(a^3*c) + ((17*cos(e/2 + (f*x)/2)^2)/60 - (28*cos(e/2 + (f*x)/2)^4)/15 + (26*cos(e/2 + (f*x)/2)^6)/15 - 1/40)/(a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1} dx}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)

[Out] -Integral(1/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1), x)/(a**3*c)

$$3.37 \quad \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=100

$$\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} + \frac{x}{a^3c^2}$$

[Out] x/a^3/c^2+1/15*cot(f*x+e)*(15-8*sec(f*x+e))/a^3/c^2/f-1/15*cot(f*x+e)^3*(5-4*sec(f*x+e))/a^3/c^2/f+1/5*cot(f*x+e)^5*(1-sec(f*x+e))/a^3/c^2/f

Rubi [A] time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3882, 8}

$$\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} + \frac{x}{a^3c^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2), x]

[Out] x/(a^3*c^2) + (Cot[e + f*x]*(15 - 8*Sec[e + f*x]))/(15*a^3*c^2*f) - (Cot[e + f*x]^3*(5 - 4*Sec[e + f*x]))/(15*a^3*c^2*f) + (Cot[e + f*x]^5*(1 - Sec[e + f*x]))/(5*a^3*c^2*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3882

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[m, -1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^2} dx &= -\frac{\int \cot^6(e+fx)(c-c \sec(e+fx)) dx}{a^3c^3} \\ &= \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} - \frac{\int \cot^4(e+fx)(-5c+4c \sec(e+fx)) dx}{5a^3c^3} \\ &= -\frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} + \frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f} \\ &= \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} \\ &= \frac{x}{a^3c^2} + \frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f} \end{aligned}$$

Mathematica [B] time = 1.01, size = 257, normalized size = 2.57

$$\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\csc^3\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)(534\sin(e+fx)+178\sin(2(e+fx))-178\sin(3(e+fx))-$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]

[Out] (Csc[e/2]*Csc[(e + f*x)/2]^3*Sec[e/2]*Sec[(e + f*x)/2]^5*(360*f*x*Cos[f*x] - 360*f*x*Cos[2*e + f*x] + 120*f*x*Cos[e + 2*f*x] - 120*f*x*Cos[3*e + 2*f*x] - 120*f*x*Cos[2*e + 3*f*x] + 120*f*x*Cos[4*e + 3*f*x] - 60*f*x*Cos[3*e + 4*f*x] + 60*f*x*Cos[5*e + 4*f*x] - 200*Sin[e] - 584*Sin[f*x] + 534*Sin[e + f*x] + 178*Sin[2*(e + f*x)] - 178*Sin[3*(e + f*x)] - 89*Sin[4*(e + f*x)] - 520*Sin[2*e + f*x] - 248*Sin[e + 2*f*x] - 120*Sin[3*e + 2*f*x] + 248*Sin[2*e + 3*f*x] + 120*Sin[4*e + 3*f*x] + 184*Sin[3*e + 4*f*x]))/(30720*a^3*c^2*f)

fricas [A] time = 0.43, size = 154, normalized size = 1.54

$$\frac{23 \cos(fx + e)^4 + 8 \cos(fx + e)^3 - 27 \cos(fx + e)^2 + 15 \left(fx \cos(fx + e)^3 + fx \cos(fx + e)^2 - fx \cos(fx + e) \right)}{15 \left(a^3 c^2 f \cos(fx + e)^3 + a^3 c^2 f \cos(fx + e)^2 - a^3 c^2 f \cos(fx + e) - a^3 c^2 f \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/15*(23*cos(f*x + e)^4 + 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^3 + f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) - f*x)*sin(f*x + e) - 7*cos(f*x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))

giac [A] time = 0.35, size = 122, normalized size = 1.22

$$\frac{\frac{240(fx+e)}{a^3c^2} + \frac{5\left(18 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)}{a^3c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3} - \frac{3\left(a^{12}c^8 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 10a^{12}c^8 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 80a^{12}c^8 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^{15}c^{10}}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] 1/240*(240*(f*x + e)/(a^3*c^2) + 5*(18*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2*tan(1/2*f*x + 1/2*e)^3) - 3*(a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8*tan(1/2*f*x + 1/2*e)^3 + 80*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f

maple [A] time = 1.08, size = 131, normalized size = 1.31

$$-\frac{\tan^5\left(\frac{e}{2} + \frac{fx}{2}\right)}{80f a^3 c^2} + \frac{\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{8f a^3 c^2} - \frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f a^3 c^2} - \frac{1}{48f a^3 c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3} + \frac{3}{8f a^3 c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)} + \frac{2 \arctan\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f a^3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)

[Out] -1/80/f/a^3/c^2*tan(1/2*e+1/2*f*x)^5+1/8/f/a^3/c^2*tan(1/2*e+1/2*f*x)^3-1/f/a^3/c^2*tan(1/2*e+1/2*f*x)-1/48/f/a^3/c^2/tan(1/2*e+1/2*f*x)^3+3/8/f/a^3/c^2/tan(1/2*e+1/2*f*x)+2/f/a^3/c^2*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.43, size = 146, normalized size = 1.46

$$\frac{3 \left(\frac{80 \sin(fx+e)}{\cos(fx+e)+1} - \frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right) - \frac{480 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3 c^2} - \frac{5 \left(\frac{18 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 1 \right) (\cos(fx+e)+1)^3}{a^3 c^2 \sin(fx+e)^3}}{240 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] -1/240*(3*(80*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^2) - 5*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e)^3))/f

mupad [B] time = 1.50, size = 161, normalized size = 1.61

$$\frac{5 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^8 + 3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 30 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 240 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 90 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 240 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^3}{240 a^3 c^2 f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)

[Out] -(5*cos(e/2 + (f*x)/2)^8 + 3*sin(e/2 + (f*x)/2)^8 - 30*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 90*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 - 240*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^3*(e + f*x))/(240*a^3*c^2*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^5(e+fx)+\sec^4(e+fx)-2\sec^3(e+fx)-2\sec^2(e+fx)+\sec(e+fx)+1} dx}{a^3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)

[Out] Integral(1/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)

$$3.38 \quad \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=67

$$\frac{\cot^5(e+fx)}{5a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot(e+fx)}{a^3c^3f} + \frac{x}{a^3c^3}$$

[Out] $x/a^3/c^3 + \cot(f*x+e)/a^3/c^3/f - 1/3*\cot(f*x+e)^3/a^3/c^3/f + 1/5*\cot(f*x+e)^5/a^3/c^3/f$

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3473, 8}

$$\frac{\cot^5(e+fx)}{5a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot(e+fx)}{a^3c^3f} + \frac{x}{a^3c^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]

[Out] $x/(a^3*c^3) + \text{Cot}[e + f*x]/(a^3*c^3*f) - \text{Cot}[e + f*x]^3/(3*a^3*c^3*f) + \text{Cot}[e + f*x]^5/(5*a^3*c^3*f)$

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^3} dx &= -\frac{\int \cot^6(e+fx) dx}{a^3c^3} \\ &= \frac{\cot^5(e+fx)}{5a^3c^3f} + \frac{\int \cot^4(e+fx) dx}{a^3c^3} \\ &= -\frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f} - \frac{\int \cot^2(e+fx) dx}{a^3c^3} \\ &= \frac{\cot(e+fx)}{a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f} + \frac{\int 1 dx}{a^3c^3} \\ &= \frac{x}{a^3c^3} + \frac{\cot(e+fx)}{a^3c^3f} - \frac{\cot^3(e+fx)}{3a^3c^3f} + \frac{\cot^5(e+fx)}{5a^3c^3f} \end{aligned}$$

Mathematica [C] time = 0.07, size = 39, normalized size = 0.58

$$\frac{\cot^5(e + fx) {}_2F_1\left(-\frac{5}{2}, 1; -\frac{3}{2}; -\tan^2(e + fx)\right)}{5a^3c^3f}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]

[Out] (Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*a^3*c^3*f)

fricas [A] time = 0.45, size = 118, normalized size = 1.76

$$\frac{23 \cos(fx + e)^5 - 35 \cos(fx + e)^3 + 15 \left(fx \cos(fx + e)^4 - 2 fx \cos(fx + e)^2 + fx \right) \sin(fx + e) + 15 \cos(fx + e)}{15 \left(a^3 c^3 f \cos(fx + e)^4 - 2 a^3 c^3 f \cos(fx + e)^2 + a^3 c^3 f \right) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(23*cos(f*x + e)^5 - 35*cos(f*x + e)^3 + 15*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^2 + f*x)*sin(f*x + e) + 15*cos(f*x + e))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))

giac [B] time = 0.46, size = 136, normalized size = 2.03

$$\frac{\frac{480(fx+e)}{a^3c^3} + \frac{330 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 35 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3}{a^3c^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5} - \frac{3a^{12}c^{12} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 35a^{12}c^{12} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 330a^{12}c^{12} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^{15}c^{15}}}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] 1/480*(480*(f*x + e)/(a^3*c^3) + (330*tan(1/2*f*x + 1/2*e)^4 - 35*tan(1/2*f*x + 1/2*e)^2 + 3)/(a^3*c^3*tan(1/2*f*x + 1/2*e)^5) - (3*a^12*c^12*tan(1/2*f*x + 1/2*e)^5 - 35*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 330*a^12*c^12*tan(1/2*f*x + 1/2*e))/(a^15*c^15))/f

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + a \sec(fx + e))^3 (c - c \sec(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)

[Out] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)

maxima [A] time = 0.43, size = 56, normalized size = 0.84

$$\frac{\frac{15(fx+e)}{a^3c^3} + \frac{15 \tan(fx+e)^4 - 5 \tan(fx+e)^2 + 3}{a^3c^3 \tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] 1/15*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(a^3*c^3*tan(f*x + e)^5))/f

mupad [B] time = 1.57, size = 94, normalized size = 1.40

$$\frac{\frac{5 \cos(e+fx)}{24} - \frac{5 \cos(3e+3fx)}{48} + \frac{23 \cos(5e+5fx)}{240} - \frac{5 \sin(3e+3fx)(e+fx)}{16} + \frac{\sin(5e+5fx)(e+fx)}{16} + \frac{5 \sin(e+fx)(e+fx)}{8}}{a^3 c^3 f \sin(e+fx)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)

[Out] ((5*cos(e + f*x))/24 - (5*cos(3*e + 3*f*x))/48 + (23*cos(5*e + 5*f*x))/240 - (5*sin(3*e + 3*f*x)*(e + f*x))/16 + (sin(5*e + 5*f*x)*(e + f*x))/16 + (5*sin(e + f*x)*(e + f*x))/8)/(a^3*c^3*f*sin(e + f*x)^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^6(e+fx)-3\sec^4(e+fx)+3\sec^2(e+fx)-1} dx}{a^3 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)

[Out] -Integral(1/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1), x)/(a**3*c**3)

$$3.39 \quad \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=129

$$-\frac{\cot^7(e+fx)(\sec(e+fx)+1)}{7a^3c^4f} + \frac{\cot^5(e+fx)(6\sec(e+fx)+7)}{35a^3c^4f} - \frac{\cot^3(e+fx)(24\sec(e+fx)+35)}{105a^3c^4f} + \frac{\cot(e+fx)}{a^3c^4f}$$

[Out] x/a^3/c^4-1/7*cot(f*x+e)^7*(1+sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)^5*(7+6*sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)*(35+16*sec(f*x+e))/a^3/c^4/f-1/105*cot(f*x+e)^3*(35+24*sec(f*x+e))/a^3/c^4/f

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3904, 3882, 8}

$$-\frac{\cot^7(e+fx)(\sec(e+fx)+1)}{7a^3c^4f} + \frac{\cot^5(e+fx)(6\sec(e+fx)+7)}{35a^3c^4f} - \frac{\cot^3(e+fx)(24\sec(e+fx)+35)}{105a^3c^4f} + \frac{\cot(e+fx)}{a^3c^4f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4), x]

[Out] x/(a^3*c^4) - (Cot[e + f*x]^7*(1 + Sec[e + f*x]))/(7*a^3*c^4*f) + (Cot[e + f*x]^5*(7 + 6*Sec[e + f*x]))/(35*a^3*c^4*f) + (Cot[e + f*x]*(35 + 16*Sec[e + f*x]))/(35*a^3*c^4*f) - (Cot[e + f*x]^3*(35 + 24*Sec[e + f*x]))/(105*a^3*c^4*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3882

Int[(cot[(c_.) + (d_.)*(x_.)]*(e_.))^m*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[m, -1]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^n, x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))^3 (c - c \sec(e + fx))^4} dx = \frac{\int \cot^8(e + fx)(a + a \sec(e + fx)) dx}{a^4 c^4}$$

$$= -\frac{\cot^7(e + fx)(1 + \sec(e + fx))}{7a^3 c^4 f} + \frac{\int \cot^6(e + fx)(-7a - 6a \sec(e + fx)) dx}{7a^4 c^4}$$

$$= -\frac{\cot^7(e + fx)(1 + \sec(e + fx))}{7a^3 c^4 f} + \frac{\cot^5(e + fx)(7 + 6 \sec(e + fx))}{35a^3 c^4 f}$$

$$= -\frac{\cot^7(e + fx)(1 + \sec(e + fx))}{7a^3 c^4 f} + \frac{\cot^5(e + fx)(7 + 6 \sec(e + fx))}{35a^3 c^4 f}$$

$$= -\frac{\cot^7(e + fx)(1 + \sec(e + fx))}{7a^3 c^4 f} + \frac{\cot^5(e + fx)(7 + 6 \sec(e + fx))}{35a^3 c^4 f}$$

$$= \frac{x}{a^3 c^4} - \frac{\cot^7(e + fx)(1 + \sec(e + fx))}{7a^3 c^4 f} + \frac{\cot^5(e + fx)(7 + 6 \sec(e + fx))}{35a^3 c^4 f}$$

Mathematica [B] time = 1.41, size = 362, normalized size = 2.81

$$\frac{\csc\left(\frac{e}{2}\right) \sec\left(\frac{e}{2}\right) \csc^7\left(\frac{1}{2}(e + fx)\right) \sec^5\left(\frac{1}{2}(e + fx)\right) (-22860 \sin(e + fx) + 5715 \sin(2(e + fx)) + 11430 \sin(3(e + fx)))}{a^3 c^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]

[Out] (Csc[e/2]*Csc[(e + f*x)/2]^7*Sec[e/2]*Sec[(e + f*x)/2]^5*(16800*f*x*Cos[f*x] - 16800*f*x*Cos[2*e + f*x] - 4200*f*x*Cos[e + 2*f*x] + 4200*f*x*Cos[3*e + 2*f*x] - 8400*f*x*Cos[2*e + 3*f*x] + 8400*f*x*Cos[4*e + 3*f*x] + 3360*f*x*Cos[3*e + 4*f*x] - 3360*f*x*Cos[5*e + 4*f*x] + 1680*f*x*Cos[4*e + 5*f*x] - 1680*f*x*Cos[6*e + 5*f*x] - 840*f*x*Cos[5*e + 6*f*x] + 840*f*x*Cos[7*e + 6*f*x] + 3136*Sin[e] - 30112*Sin[f*x] - 22860*Sin[e + f*x] + 5715*Sin[2*(e + f*x)] + 11430*Sin[3*(e + f*x)] - 4572*Sin[4*(e + f*x)] - 2286*Sin[5*(e + f*x)] + 1143*Sin[6*(e + f*x)] - 26208*Sin[2*e + f*x] + 14080*Sin[e + 2*f*x] + 16400*Sin[2*e + 3*f*x] + 11760*Sin[4*e + 3*f*x] - 7904*Sin[3*e + 4*f*x] - 3360*Sin[5*e + 4*f*x] - 3952*Sin[4*e + 5*f*x] - 1680*Sin[6*e + 5*f*x] + 2816*Sin[5*e + 6*f*x]))/(6881280*a^3*c^4*f)

fricas [A] time = 0.45, size = 232, normalized size = 1.80

$$\frac{176 \cos(fx + e)^6 - 71 \cos(fx + e)^5 - 335 \cos(fx + e)^4 + 125 \cos(fx + e)^3 + 225 \cos(fx + e)^2 + 105 (fx + e) \cos(fx + e) - 105 (a^3 c^4 f \cos(fx + e)^5 - a^3 c^4 f \cos(fx + e)^4 - 2 a^3 c^4 f \cos(fx + e)^3 + 2 a^3 c^4 f \cos(fx + e)^2 + a^3 c^4 f \cos(fx + e) - a^3 c^4 f) \sin(fx + e)}{a^3 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] 1/105*(176*cos(f*x + e)^6 - 71*cos(f*x + e)^5 - 335*cos(f*x + e)^4 + 125*cos(f*x + e)^3 + 225*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 + f*x*cos(f*x + e) - f*x)*sin(f*x + e) - 57*cos(f*x + e) - 48)/((a^3*c^4*f*cos(f*x + e)^5 - a^3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*cos(f*x + e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))

giac [A] time = 0.44, size = 150, normalized size = 1.16

$$\frac{6720 (fx+e)}{a^3 c^4} + \frac{6720 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 - 1015 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 168 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 15}{a^3 c^4 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^7} - \frac{7 \left(3 a^{12} c^{16} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 - 40 a^{12} c^{16} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 15 a^{12} c^{16} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + 15 a^{12} c^{16}\right) \sin\left(\frac{1}{2} fx + \frac{1}{2} e\right)}{a^{15} c^{20}}$$

6720 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")

[Out] $\frac{1}{6720} \cdot \frac{6720 \cdot (f \cdot x + e)}{a^3 \cdot c^4} + \frac{6720 \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right)^6 - 1015 \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right)^4 + 168 \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right)^2 - 15}{a^3 \cdot c^4 \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right)^7} - \frac{7 \cdot (3 \cdot a^{12} \cdot c^{16} \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right)^5 - 40 \cdot a^{12} \cdot c^{16} \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right)^3 + 435 \cdot a^{12} \cdot c^{16} \cdot \tan\left(\frac{1}{2} \cdot f \cdot x + \frac{1}{2} \cdot e\right))}{a^{15} \cdot c^{20}} \cdot f$

maple [A] time = 0.98, size = 174, normalized size = 1.35

$$-\frac{\tan^5\left(\frac{e}{2} + \frac{fx}{2}\right)}{320f a^3 c^4} + \frac{\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{24f a^3 c^4} - \frac{29 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{64f a^3 c^4} - \frac{1}{448f a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{1}{40f a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5} - \frac{1}{192f a^3 c^4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)

[Out] $-\frac{1}{320} \cdot \frac{f}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right)^5 + \frac{1}{24} \cdot \frac{f}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right)^3 - \frac{9}{64} \cdot \frac{f}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right) - \frac{1}{448} \cdot \frac{f}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right)^7 + \frac{1}{40} \cdot \frac{f}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right)^5 - \frac{29}{192} \cdot \frac{f}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right)^3 + \frac{1}{f} \cdot \frac{1}{a^3 \cdot c^4} \cdot \tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right) + \frac{2}{f} \cdot \frac{1}{a^3 \cdot c^4} \cdot \arctan\left(\tan\left(\frac{1}{2} \cdot e + \frac{1}{2} \cdot f \cdot x\right)\right)$

maxima [A] time = 0.44, size = 187, normalized size = 1.45

$$\frac{7 \left(\frac{435 \sin(fx+e)}{\cos(fx+e)+1} - \frac{40 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right) - \frac{13440 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3 c^4} - \frac{\left(\frac{168 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1015 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{6720 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 15 \right) (\cos(fx+e)+1)^7}{a^3 c^4 \sin(fx+e)^7}}{6720 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

[Out] $-\frac{1}{6720} \cdot \left(7 \cdot \frac{435 \cdot \sin(f \cdot x + e)}{\cos(f \cdot x + e) + 1} - 40 \cdot \frac{\sin(f \cdot x + e)^3}{(\cos(f \cdot x + e) + 1)^3} + 3 \cdot \frac{\sin(f \cdot x + e)^5}{(\cos(f \cdot x + e) + 1)^5} \right) / (a^3 \cdot c^4) - \frac{13440 \cdot \arctan\left(\frac{\sin(f \cdot x + e)}{\cos(f \cdot x + e) + 1}\right)}{a^3 \cdot c^4} - \frac{(168 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 - 1015 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 6720 \cdot \sin(f \cdot x + e)^6 / (\cos(f \cdot x + e) + 1)^6 - 15) \cdot (\cos(f \cdot x + e) + 1)^7}{a^3 \cdot c^4 \cdot \sin(f \cdot x + e)^7} \right) / f$

mupad [B] time = 1.85, size = 209, normalized size = 1.62

$$\frac{15 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} + 21 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} - 280 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} + 3045 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 6720 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 1015 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 168 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 6720 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^7 \cdot (e + f \cdot x)}{(6720 \cdot a^3 \cdot c^4 \cdot f \cdot \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)

[Out] $-\frac{(15 \cdot \cos(e/2 + (f \cdot x)/2)^{12} + 21 \cdot \sin(e/2 + (f \cdot x)/2)^{12} - 280 \cdot \cos(e/2 + (f \cdot x)/2)^2 \cdot \sin(e/2 + (f \cdot x)/2)^{10} + 3045 \cdot \cos(e/2 + (f \cdot x)/2)^4 \cdot \sin(e/2 + (f \cdot x)/2)^8 - 6720 \cdot \cos(e/2 + (f \cdot x)/2)^6 \cdot \sin(e/2 + (f \cdot x)/2)^6 + 1015 \cdot \cos(e/2 + (f \cdot x)/2)^8 \cdot \sin(e/2 + (f \cdot x)/2)^4 - 168 \cdot \cos(e/2 + (f \cdot x)/2)^{10} \cdot \sin(e/2 + (f \cdot x)/2)^2 - 6720 \cdot \cos(e/2 + (f \cdot x)/2)^5 \cdot \sin(e/2 + (f \cdot x)/2)^7 \cdot (e + f \cdot x))}{(6720 \cdot a^3 \cdot c^4 \cdot f \cdot \cos(e/2 + (f \cdot x)/2)^5 \cdot \sin(e/2 + (f \cdot x)/2)^7)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^7(e+fx) - \sec^6(e+fx) - 3\sec^5(e+fx) + 3\sec^4(e+fx) + 3\sec^3(e+fx) - 3\sec^2(e+fx) - \sec(e+fx) + 1} dx$$

$$a^3 c^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)

[Out] Integral(1/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**5 + 3*sec(e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x) + 1), x)/
(a**3*c**4)

$$3.40 \quad \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^5} dx$$

Optimal. Leaf size=210

$$\frac{2 \cot^9(e+fx)}{9a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot(e+fx)}{a^3c^5f} + \frac{2 \csc^9(e+fx)}{9a^3c^5f} - \frac{8 \csc^7(e+fx)}{7a^3c^5f} + \frac{12 \csc^5(e+fx)}{5a^3c^5f}$$

[Out] $x/a^3/c^5 + \cot(f*x+e)/a^3/c^5/f - 1/3*\cot(f*x+e)^3/a^3/c^5/f + 1/5*\cot(f*x+e)^5/a^3/c^5/f - 1/7*\cot(f*x+e)^7/a^3/c^5/f + 2/9*\cot(f*x+e)^9/a^3/c^5/f + 2*\csc(f*x+e)/a^3/c^5/f - 8/3*\csc(f*x+e)^3/a^3/c^5/f + 12/5*\csc(f*x+e)^5/a^3/c^5/f - 8/7*\csc(f*x+e)^7/a^3/c^5/f + 2/9*\csc(f*x+e)^9/a^3/c^5/f$

Rubi [A] time = 0.24, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30}

$$\frac{2 \cot^9(e+fx)}{9a^3c^5f} - \frac{\cot^7(e+fx)}{7a^3c^5f} + \frac{\cot^5(e+fx)}{5a^3c^5f} - \frac{\cot^3(e+fx)}{3a^3c^5f} + \frac{\cot(e+fx)}{a^3c^5f} + \frac{2 \csc^9(e+fx)}{9a^3c^5f} - \frac{8 \csc^7(e+fx)}{7a^3c^5f} + \frac{12 \csc^5(e+fx)}{5a^3c^5f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5), x]

[Out] $x/(a^3*c^5) + \text{Cot}[e + f*x]/(a^3*c^5*f) - \text{Cot}[e + f*x]^3/(3*a^3*c^5*f) + \text{Cot}[e + f*x]^5/(5*a^3*c^5*f) - \text{Cot}[e + f*x]^7/(7*a^3*c^5*f) + (2*\text{Cot}[e + f*x]^9)/(9*a^3*c^5*f) + (2*\text{Csc}[e + f*x])/(a^3*c^5*f) - (8*\text{Csc}[e + f*x]^3)/(3*a^3*c^5*f) + (12*\text{Csc}[e + f*x]^5)/(5*a^3*c^5*f) - (8*\text{Csc}[e + f*x]^7)/(7*a^3*c^5*f) + (2*\text{Csc}[e + f*x]^9)/(9*a^3*c^5*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^(n*(1 + x^2)^(m/2 - 1)), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 3473

$$x*\cos[6*e + 5*f*x] - 40320*f*x*\cos[5*e + 6*f*x] + 40320*f*x*\cos[7*e + 6*f*x] + 10080*f*x*\cos[6*e + 7*f*x] - 10080*f*x*\cos[8*e + 7*f*x] + 259584*\sin[e] - 897024*\sin[f*x] - 1152405*\sin[e + f*x] + 512180*\sin[2*(e + f*x)] + 486571*\sin[3*(e + f*x)] - 409744*\sin[4*(e + f*x)] - 25609*\sin[5*(e + f*x)] + 102436*\sin[6*(e + f*x)] - 25609*\sin[7*(e + f*x)] - 825216*\sin[2*e + f*x] + 622976*\sin[e + 2*f*x] + 142464*\sin[3*e + 2*f*x] + 297088*\sin[2*e + 3*f*x] + 430080*\sin[4*e + 3*f*x] - 424192*\sin[3*e + 4*f*x] - 188160*\sin[5*e + 4*f*x] + 2048*\sin[4*e + 5*f*x] - 40320*\sin[6*e + 5*f*x] + 112768*\sin[5*e + 6*f*x] + 40320*\sin[7*e + 6*f*x] - 38272*\sin[6*e + 7*f*x])*Tan[e + f*x]/(2580480*a^3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)$$

fricas [A] time = 0.46, size = 271, normalized size = 1.29

$$\frac{598 \cos(fx + e)^7 - 566 \cos(fx + e)^6 - 1212 \cos(fx + e)^5 + 1310 \cos(fx + e)^4 + 860 \cos(fx + e)^3 - 1014 \cos(fx + e)^2 + 315 \left(a^3 c^5 f \cos(fx + e)^6 - 2 a^3 c^5 f \cos(fx + e)^5 - a^3 c^5 f \cos(fx + e)^4 + 4 a^3 c^5 f \cos(fx + e)^3 - a^3 c^5 f \cos(fx + e)^2 - 2 a^3 c^5 f \cos(fx + e) + a^3 c^5 f \sin(fx + e) - 197 \cos(fx + e) + 256 \right)}{a^3 c^5 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^9} - \frac{63 \left(a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 - 15 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 185 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 1152 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 896 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - 640 a^{12} c^{20} \right)}{40320 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/315*(598*cos(f*x + e)^7 - 566*cos(f*x + e)^6 - 1212*cos(f*x + e)^5 + 1310*cos(f*x + e)^4 + 860*cos(f*x + e)^3 - 1014*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^6 - 2*f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 + 4*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2 - 2*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 197*cos(f*x + e) + 256)/((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^3*c^5*f*cos(f*x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e)^2 - 2*a^3*c^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))

giac [A] time = 0.51, size = 163, normalized size = 0.78

$$\frac{40320(fx+e)}{a^3c^5} + \frac{51345 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^8 - 9765 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 + 2331 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 - 405 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 35}{a^3c^5 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^9} - \frac{63 \left(a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 - 15 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 185 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 1152 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 896 a^{12} c^{20} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - 640 a^{12} c^{20} \right)}{40320 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")

[Out] 1/40320*(40320*(f*x + e)/(a^3*c^5) + (51345*tan(1/2*f*x + 1/2*e)^8 - 9765*tan(1/2*f*x + 1/2*e)^6 + 2331*tan(1/2*f*x + 1/2*e)^4 - 405*tan(1/2*f*x + 1/2*e)^2 + 35)/(a^3*c^5*tan(1/2*f*x + 1/2*e)^9) - 63*(a^12*c^20*tan(1/2*f*x + 1/2*e)^5 - 15*a^12*c^20*tan(1/2*f*x + 1/2*e)^3 + 185*a^12*c^20*tan(1/2*f*x + 1/2*e)))/(a^15*c^25))/f

maple [A] time = 0.98, size = 197, normalized size = 0.94

$$-\frac{\tan^5\left(\frac{e}{2} + \frac{fx}{2}\right)}{640 f a^3 c^5} + \frac{3 \left(\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}{128 f a^3 c^5} - \frac{37 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{128 f a^3 c^5} + \frac{1}{1152 f a^3 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9} - \frac{9}{896 f a^3 c^5 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7} + \frac{1}{640 f a^3 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)

[Out] -1/640/f/a^3/c^5*tan(1/2*e+1/2*f*x)^5+3/128/f/a^3/c^5*tan(1/2*e+1/2*f*x)^3-37/128/f/a^3/c^5*tan(1/2*e+1/2*f*x)+1/1152/f/a^3/c^5/tan(1/2*e+1/2*f*x)^9-9/896/f/a^3/c^5/tan(1/2*e+1/2*f*x)^7+37/640/f/a^3/c^5/tan(1/2*e+1/2*f*x)^5-3/128/f/a^3/c^5/tan(1/2*e+1/2*f*x)^3+163/128/f/a^3/c^5/tan(1/2*e+1/2*f*x)+2/f/a^3/c^5*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.44, size = 205, normalized size = 0.98

$$\frac{63 \left(\frac{185 \sin(fx+e)}{\cos(fx+e)+1} - \frac{15 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right) - \frac{80640 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3 c^5} + \frac{\left(\frac{405 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{2331 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{9765 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{51345 \sin(fx+e)^8}{(\cos(fx+e)+1)^8} + \frac{35 \sin(fx+e)^9}{(\cos(fx+e)+1)^9} \right)}{a^3 c^5 \sin(fx+e)^9}}{40320 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] -1/40320*(63*(185*sin(f*x + e)/(cos(f*x + e) + 1) - 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - 80640*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^5) + (405*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2331*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9765*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 51345*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(a^3*c^5*sin(f*x + e)^9)/f

mupad [B] time = 2.04, size = 233, normalized size = 1.11

$$\frac{35 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} - 63 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} + 945 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} - 11655 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} + 51345 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 9765 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 2331 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 405 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 40320 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^9 (e + fx)}{40320 a^3 c^5 f \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)

[Out] (35*cos(e/2 + (f*x)/2)^14 - 63*sin(e/2 + (f*x)/2)^14 + 945*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^12 - 11655*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^10 + 51345*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^8 - 9765*cos(e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^6 + 2331*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^4 - 405*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^2 + 40320*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^9*(e + f*x))/(40320*a^3*c^5*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^9)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1} dx}{a^3 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)

[Out] -Integral(1/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x)**6 + 6*sec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e + f*x) - 1), x)/(a**3*c**5)

$$3.41 \quad \int \frac{1}{(a+a \sec(e+fx))^3(c-c \sec(e+fx))^6} dx$$

Optimal. Leaf size=252

$$-\frac{4 \cot^{11}(e+fx)}{11a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot(e+fx)}{a^3c^6f} - \frac{4 \csc^{11}(e+fx)}{11a^3c^6f} + \frac{19 \csc^9(e+fx)}{9a^3c^6f} - \frac{4 \csc^7(e+fx)}{7a^3c^6f} + \frac{19 \csc^5(e+fx)}{5a^3c^6f} - \frac{4 \csc^3(e+fx)}{3a^3c^6f} + \frac{19 \csc(e+fx)}{a^3c^6f}$$

[Out] x/a^3/c^6+cot(f*x+e)/a^3/c^6/f-1/3*cot(f*x+e)^3/a^3/c^6/f+1/5*cot(f*x+e)^5/a^3/c^6/f-1/7*cot(f*x+e)^7/a^3/c^6/f+1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*x+e)^11/a^3/c^6/f+3*csc(f*x+e)/a^3/c^6/f-16/3*csc(f*x+e)^3/a^3/c^6/f+34/5*csc(f*x+e)^5/a^3/c^6/f-36/7*csc(f*x+e)^7/a^3/c^6/f+19/9*csc(f*x+e)^9/a^3/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f

Rubi [A] time = 0.30, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3904, 3886, 3473, 8, 2606, 194, 2607, 30, 270}

$$-\frac{4 \cot^{11}(e+fx)}{11a^3c^6f} + \frac{\cot^9(e+fx)}{9a^3c^6f} - \frac{\cot^7(e+fx)}{7a^3c^6f} + \frac{\cot^5(e+fx)}{5a^3c^6f} - \frac{\cot^3(e+fx)}{3a^3c^6f} + \frac{\cot(e+fx)}{a^3c^6f} - \frac{4 \csc^{11}(e+fx)}{11a^3c^6f} + \frac{19 \csc^9(e+fx)}{9a^3c^6f} - \frac{4 \csc^7(e+fx)}{7a^3c^6f} + \frac{19 \csc^5(e+fx)}{5a^3c^6f} - \frac{4 \csc^3(e+fx)}{3a^3c^6f} + \frac{19 \csc(e+fx)}{a^3c^6f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6), x]

[Out] x/(a^3*c^6) + Cot[e + f*x]/(a^3*c^6*f) - Cot[e + f*x]^3/(3*a^3*c^6*f) + Cot[e + f*x]^5/(5*a^3*c^6*f) - Cot[e + f*x]^7/(7*a^3*c^6*f) + Cot[e + f*x]^9/(9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + (3*Csc[e + f*x])/(a^3*c^6*f) - (16*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (34*Csc[e + f*x]^5)/(5*a^3*c^6*f) - (36*Csc[e + f*x]^7)/(7*a^3*c^6*f) + (19*Csc[e + f*x]^9)/(9*a^3*c^6*f) - (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x]
;/; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x]
;/; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3886

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol]
:> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[c + d*x])^n, x], x]
;/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol]
:> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x]
;/; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \sec(e + fx))^3 (c - c \sec(e + fx))^6} dx &= \frac{\int \cot^{12}(e + fx)(a + a \sec(e + fx))^3 dx}{a^6 c^6} \\ &= \frac{\int (a^3 \cot^{12}(e + fx) + 3a^3 \cot^{11}(e + fx) \csc(e + fx) + 3a^3 \cot^{10}(e + fx) \csc^2(e + fx) + \dots)}{a^6 c^6} \\ &= \frac{\int \cot^{12}(e + fx) dx}{a^3 c^6} + \frac{\int \cot^9(e + fx) \csc^3(e + fx) dx}{a^3 c^6} + \frac{3 \int \cot^{10}(e + fx) \csc^2(e + fx) dx}{a^3 c^6} \\ &= -\frac{\cot^{11}(e + fx)}{11a^3 c^6 f} - \frac{\int \cot^{10}(e + fx) dx}{a^3 c^6} - \frac{\text{Subst}\left(\int x^2 (-1 + x^2) dx, x, \cot(e + fx)\right)}{a^3 c^6} \\ &= \frac{\cot^9(e + fx)}{9a^3 c^6 f} - \frac{4 \cot^{11}(e + fx)}{11a^3 c^6 f} + \frac{\int \cot^8(e + fx) dx}{a^3 c^6} - \frac{\text{Subst}\left(\int x^2 (-1 + x^2) dx, x, \cot(e + fx)\right)}{a^3 c^6} \\ &= -\frac{\cot^7(e + fx)}{7a^3 c^6 f} + \frac{\cot^9(e + fx)}{9a^3 c^6 f} - \frac{4 \cot^{11}(e + fx)}{11a^3 c^6 f} + \frac{3 \csc(e + fx)}{a^3 c^6 f} \\ &= \frac{\cot^5(e + fx)}{5a^3 c^6 f} - \frac{\cot^7(e + fx)}{7a^3 c^6 f} + \frac{\cot^9(e + fx)}{9a^3 c^6 f} - \frac{4 \cot^{11}(e + fx)}{11a^3 c^6 f} \\ &= -\frac{\cot^3(e + fx)}{3a^3 c^6 f} + \frac{\cot^5(e + fx)}{5a^3 c^6 f} - \frac{\cot^7(e + fx)}{7a^3 c^6 f} + \frac{\cot^9(e + fx)}{9a^3 c^6 f} \\ &= \frac{\cot(e + fx)}{a^3 c^6 f} - \frac{\cot^3(e + fx)}{3a^3 c^6 f} + \frac{\cot^5(e + fx)}{5a^3 c^6 f} - \frac{\cot^7(e + fx)}{7a^3 c^6 f} + \frac{\cot^9(e + fx)}{9a^3 c^6 f} \\ &= \frac{x}{a^3 c^6} + \frac{\cot(e + fx)}{a^3 c^6 f} - \frac{\cot^3(e + fx)}{3a^3 c^6 f} + \frac{\cot^5(e + fx)}{5a^3 c^6 f} - \frac{\cot^7(e + fx)}{7a^3 c^6 f} + \frac{\cot^9(e + fx)}{9a^3 c^6 f} \end{aligned}$$

Mathematica [A] time = 2.36, size = 499, normalized size = 1.98

$\csc\left(\frac{e}{2}\right) \sec\left(\frac{e}{2}\right) \tan(e + fx) \sec^8(e + fx) (-86058610 \sin(e + fx) + 51635166 \sin(2(e + fx)) + 26599934 \sin(3(e + fx)))$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]

[Out] (Csc[e/2]*Sec[e/2]*Sec[e + f*x]^8*(24393600*f*x*Cos[f*x] - 24393600*f*x*Cos[2*e + f*x] - 14636160*f*x*Cos[e + 2*f*x] + 14636160*f*x*Cos[3*e + 2*f*x] - 7539840*f*x*Cos[2*e + 3*f*x] + 7539840*f*x*Cos[4*e + 3*f*x] + 11088000*f*x*Cos[3*e + 4*f*x] - 11088000*f*x*Cos[5*e + 4*f*x] - 2217600*f*x*Cos[4*e + 5*f*x] + 2217600*f*x*Cos[6*e + 5*f*x] - 2217600*f*x*Cos[5*e + 6*f*x] + 2217600*f*x*Cos[7*e + 6*f*x] + 1330560*f*x*Cos[6*e + 7*f*x] - 1330560*f*x*Cos[8*e + 7*f*x] - 221760*f*x*Cos[7*e + 8*f*x] + 221760*f*x*Cos[9*e + 8*f*x] + 17677440*Sin[e] - 49287040*Sin[f*x] - 86058610*Sin[e + f*x] + 51635166*Sin[2*(e + f*x)] + 26599934*Sin[3*(e + f*x)] - 39117550*Sin[4*(e + f*x)] + 7823510*Sin[5*(e + f*x)] + 7823510*Sin[6*(e + f*x)] - 4694106*Sin[7*(e + f*x)] + 782351*Sin[8*(e + f*x)] - 55651200*Sin[2*e + f*x] + 47971968*Sin[e + 2*f*x] + 14990976*Sin[3*e + 2*f*x] + 8100992*Sin[2*e + 3*f*x] + 24334464*Sin[4*e + 3*f*x] - 28627840*Sin[3*e + 4*f*x] - 19071360*Sin[5*e + 4*f*x] + 9687680*Sin[4*e + 5*f*x] - 147840*Sin[6*e + 5*f*x] + 5548160*Sin[5*e + 6*f*x] + 3991680*Sin[7*e + 6*f*x] - 4393344*Sin[6*e + 7*f*x] - 1330560*Sin[8*e + 7*f*x] + 953984*Sin[7*e + 8*f*x])*Tan[e + f*x])/((113541120*a^3*c^6*f*(-1 + Sec[e + f*x])^6*(1 + Sec[e + f*x])^3)

fricas [A] time = 0.45, size = 310, normalized size = 1.23

$$\frac{7453 \cos^8(fx + e) - 11964 \cos^7(fx + e) - 11866 \cos^6(fx + e) + 30542 \cos^5(fx + e) + 90 \cos^4(fx + e) - 26438 \cos^3(fx + e) + 8539 \cos^2(fx + e) + 3465 f \cos^7(fx + e) - 3 f^2 \cos^6(fx + e) + f^2 \cos^5(fx + e) + 5 f^3 \cos^4(fx + e) - 5 f^4 \cos^3(fx + e) - f^4 \cos^2(fx + e) + 3 f^5 \cos(fx + e) - f^5 \sin(fx + e) + 7671 \cos(fx + e) - 3712}{a^3 c^6 f \cos^7(fx + e) - 3 a^3 c^6 f \cos^6(fx + e) + a^3 c^6 f \cos^5(fx + e) + 5 a^3 c^6 f \cos^4(fx + e) - 5 a^3 c^6 f \cos^3(fx + e) - a^3 c^6 f \cos^2(fx + e) + 3 a^3 c^6 f \cos(fx + e) - a^3 c^6 f \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fricas")

[Out] 1/3465*(7453*cos(f*x + e)^8 - 11964*cos(f*x + e)^7 - 11866*cos(f*x + e)^6 + 30542*cos(f*x + e)^5 + 90*cos(f*x + e)^4 - 26438*cos(f*x + e)^3 + 8539*cos(f*x + e)^2 + 3465*(f*x*cos(f*x + e)^7 - 3*f*x*cos(f*x + e)^6 + f*x*cos(f*x + e)^5 + 5*f*x*cos(f*x + e)^4 - 5*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e)^2 + 3*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 7671*cos(f*x + e) - 3712)/((a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x + e)^6 + a^3*c^6*f*cos(f*x + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6*f*cos(f*x + e)^3 - a^3*c^6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a^3*c^6*f)*sin(f*x + e))

giac [A] time = 0.98, size = 179, normalized size = 0.71

$$\frac{887040 (fx+e)}{a^3 c^6} + \frac{5 \left(264726 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^{10} - 59136 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^8 + 18018 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 - 4554 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 770 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 63 \right)}{a^3 c^6 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^{11}}$$

887040 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="giac")

[Out] 1/887040*(887040*(f*x + e)/(a^3*c^6) + 5*(264726*tan(1/2*f*x + 1/2*e)^10 - 59136*tan(1/2*f*x + 1/2*e)^8 + 18018*tan(1/2*f*x + 1/2*e)^6 - 4554*tan(1/2*f*x + 1/2*e)^4 + 770*tan(1/2*f*x + 1/2*e)^2 - 63)/(a^3*c^6*tan(1/2*f*x + 1/2*e)^11) - 231*(3*a^12*c^24*tan(1/2*f*x + 1/2*e)^5 - 50*a^12*c^24*tan(1/2*f*x + 1/2*e)^3 + 690*a^12*c^24*tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f

maple [A] time = 1.00, size = 219, normalized size = 0.87

$$\frac{\tan^5\left(\frac{e}{2} + \frac{fx}{2}\right)}{1280f a^3c^6} + \frac{5\left(\tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{384f a^3c^6} - \frac{23 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{128f a^3c^6} - \frac{1}{2816f a^3c^6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{11}} + \frac{5}{1152f a^3c^6 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)

[Out] -1/1280/f/a^3/c^6*tan(1/2*e+1/2*f*x)^5+5/384/f/a^3/c^6*tan(1/2*e+1/2*f*x)^3-23/128/f/a^3/c^6*tan(1/2*e+1/2*f*x)-1/2816/f/a^3/c^6/tan(1/2*e+1/2*f*x)^11+5/1152/f/a^3/c^6/tan(1/2*e+1/2*f*x)^9-23/896/f/a^3/c^6/tan(1/2*e+1/2*f*x)^7+13/128/f/a^3/c^6/tan(1/2*e+1/2*f*x)^5-1/3/f/a^3/c^6/tan(1/2*e+1/2*f*x)^3+191/128/f/a^3/c^6/tan(1/2*e+1/2*f*x)+2/f/a^3/c^6*arctan(tan(1/2*e+1/2*f*x))

maxima [A] time = 0.45, size = 227, normalized size = 0.90

$$\frac{231\left(\frac{690 \sin(fx+e)}{\cos(fx+e)+1} - \frac{50 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6} - \frac{1774080 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^6} - \frac{5\left(\frac{770 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{4554 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{18018 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{59136 \sin(fx+e)^8}{(\cos(fx+e)+1)^8} + \frac{264726 \sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}} - 63(\cos(fx+e)+1)^{11}\right)}{a^3c^6 \sin(fx+e)}$$

887040 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="maxima")

[Out] -1/887040*(231*(690*sin(f*x + e)/(cos(f*x + e) + 1) - 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) - 1774080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^6) - 5*(770*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4554*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 18018*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 59136*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 264726*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*(cos(f*x + e) + 1)^11)/(a^3*c^6*sin(f*x + e)^11)/f

mupad [B] time = 2.33, size = 257, normalized size = 1.02

$$\frac{315 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{16} + 693 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{16} - 11550 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} + 159390 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} - 1323630 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^6 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} + 295680 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^8 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^8 - 90090 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 22770 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 3850 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 887040 \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{11} (e + fx)}{(887040 * a^3 * c^6 * f * \cos\left(\frac{e}{2} + \frac{fx}{2}\right)^5 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)

[Out] -(315*cos(e/2 + (f*x)/2)^16 + 693*sin(e/2 + (f*x)/2)^16 - 11550*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^14 + 159390*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^12 - 1323630*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^10 + 295680*cos(e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^8 - 90090*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)^6 + 22770*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^4 - 3850*cos(e/2 + (f*x)/2)^14*sin(e/2 + (f*x)/2)^2 - 887040*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^11*(e + f*x))/(887040*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^11)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^9(e+fx)-3\sec^8(e+fx)+8\sec^6(e+fx)-6\sec^5(e+fx)-6\sec^4(e+fx)+8\sec^3(e+fx)-3\sec(e+fx)+1} dx$$

a^3c^6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Integral(1/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)**6 - 6*sec
(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e + f*x) + 1),
x)/(a**3*c**6)
```

3.42 $\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^4 dx$

Optimal. Leaf size=175

$$\frac{2a^4c^4 \tan^7(e + fx)}{7f(a \sec(e + fx) + a)^{7/2}} - \frac{2a^3c^4 \tan^5(e + fx)}{5f(a \sec(e + fx) + a)^{5/2}} + \frac{2a^2c^4 \tan^3(e + fx)}{3f(a \sec(e + fx) + a)^{3/2}} + \frac{2\sqrt{a}c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a \sec(e + fx) + a}}\right)}{f}$$

[Out] $2*c^4*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/f-2*a*c^4*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/3*a^2*c^4*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}-2/5*a^3*c^4*\tan(f*x+e)^5/f/(a+a*\sec(f*x+e))^{(5/2)}+2/7*a^4*c^4*\tan(f*x+e)^7/f/(a+a*\sec(f*x+e))^{(7/2)}$

Rubi [A] time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 302, 203}

$$\frac{2a^4c^4 \tan^7(e + fx)}{7f(a \sec(e + fx) + a)^{7/2}} - \frac{2a^3c^4 \tan^5(e + fx)}{5f(a \sec(e + fx) + a)^{5/2}} + \frac{2a^2c^4 \tan^3(e + fx)}{3f(a \sec(e + fx) + a)^{3/2}} + \frac{2\sqrt{a}c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a \sec(e + fx) + a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^4, x]$

[Out] $(2*\text{Sqrt}[a]*c^4*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/f - (2*a*c^4*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^2*c^4*\text{Tan}[e + f*x]^3)/(3*f*(a + a*\text{Sec}[e + f*x])^{(3/2)}) - (2*a^3*c^4*\text{Tan}[e + f*x]^5)/(5*f*(a + a*\text{Sec}[e + f*x])^{(5/2)}) + (2*a^4*c^4*\text{Tan}[e + f*x]^7)/(7*f*(a + a*\text{Sec}[e + f*x])^{(7/2)})$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 302

$\text{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^{(m_)} / (a + b*x^{(n_)}), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 3887

$\text{Int}[\text{cot}[(c_ + (d_)*(x_))]^{(m_)}*(\text{csc}[(c_ + (d_)*(x_))]*(b_ + (a_))^{(n_)}), x_Symbol] \rightarrow \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^{(m*(2 + a*x^2))^{(m/2 + n - 1/2)}})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]]], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\text{csc}[(e_ + (f_)*(x_))]*(b_ + (a_))^{(m_)}*(\text{csc}[(e_ + (f_)*(x_))]*(d_ + (c_))^{(n_)}), x_Symbol] \rightarrow \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n - m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{RationalQ}[n] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^4 dx &= (a^4 c^4) \int \frac{\tan^8(e + fx)}{(a + a \sec(e + fx))^{7/2}} dx \\
&= -\frac{(2a^5 c^4) \operatorname{Subst}\left(\int \frac{x^8}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{(2a^5 c^4) \operatorname{Subst}\left(\int \left(-\frac{1}{a^4} + \frac{x^2}{a^3} - \frac{x^4}{a^2} + \frac{x^6}{a} + \frac{1}{a^4(1+ax^2)}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{2ac^4 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2 c^4 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{2a^3 c^4 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2\sqrt{a} c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} - \frac{2ac^4 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2 c^4 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.05, size = 121, normalized size = 0.69

$$\frac{2c^4 \tan\left(\frac{1}{2}(e + fx)\right) \sec^3(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((-198 \cos(e + fx) + 61 \cos(2(e + fx)) - 44 \cos(3(e + fx)))\right)}{105f\sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]

[Out] (2*c^4*(105*ArcTan[Sqrt[-1 + Sec[e + f*x]]])*Cos[e + f*x]^3 + (76 - 198*Cos[e + f*x] + 61*Cos[2*(e + f*x)] - 44*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]/(105*f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.51, size = 373, normalized size = 2.13

$$\frac{105 \left(c^4 \cos^4(fx + e) + c^4 \cos^3(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right)}{105 \left(f \cos^4(fx + e) + f \cos^3(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \left(c - \frac{c}{\cos(e + fx)} \right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4,x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4 \left(\int \left(-4\sqrt{a \sec(e + fx) + a} \sec(e + fx) \right) dx + \int 6\sqrt{a \sec(e + fx) + a} \sec^2(e + fx) dx + \int \left(-4\sqrt{a \sec(e + fx) + a} \sec^3(e + fx) \right) dx + \int \sqrt{a \sec(e + fx) + a} \sec^4(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)

[Out] c**4*(Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(6*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x) + Integral(sqrt(a*sec(e + f*x) + a), x))

3.43 $\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^3 dx$

Optimal. Leaf size=140

$$\frac{2a^3c^3 \tan^5(e + fx)}{5f(a \sec(e + fx) + a)^{5/2}} + \frac{2a^2c^3 \tan^3(e + fx)}{3f(a \sec(e + fx) + a)^{3/2}} + \frac{2\sqrt{a}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2ac^3 \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}}$$

[Out] $2*c^3*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/f-2*a*c^3*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/3*a^2*c^3*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}-2/5*a^3*c^3*\tan(f*x+e)^5/f/(a+a*\sec(f*x+e))^{(5/2)}$

Rubi [A] time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 302, 203}

$$\frac{2a^3c^3 \tan^5(e + fx)}{5f(a \sec(e + fx) + a)^{5/2}} + \frac{2a^2c^3 \tan^3(e + fx)}{3f(a \sec(e + fx) + a)^{3/2}} + \frac{2\sqrt{a}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2ac^3 \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]

[Out] $(2*\text{Sqrt}[a]*c^3*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/f - (2*a*c^3*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^2*c^3*\text{Tan}[e + f*x]^3)/(3*f*(a + a*\text{Sec}[e + f*x])^{(3/2)}) - (2*a^3*c^3*\text{Tan}[e + f*x]^5)/(5*f*(a + a*\text{Sec}[e + f*x])^{(5/2)})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^3 dx &= - \left((a^3 c^3) \int \frac{\tan^6(e + fx)}{(a + a \sec(e + fx))^{5/2}} dx \right) \\
&= \frac{(2a^4 c^3) \operatorname{Subst} \left(\int \frac{x^6}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= \frac{(2a^4 c^3) \operatorname{Subst} \left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \frac{x^4}{a} - \frac{1}{a^3(1+ax^2)} \right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= -\frac{2ac^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^2 c^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{2a^3 c^3 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2\sqrt{a} c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} - \frac{2ac^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^2 c^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.19, size = 111, normalized size = 0.79

$$\frac{c^3 \tan\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((22 \cos(e + fx) - 23 \cos(2(e + fx)) - 29) \sqrt{\sec(e + fx) - 1} \right)}{15f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]

[Out] (c^3*(30*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (-29 + 22*Cos[e + f*x] - 23*Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(15*f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.48, size = 347, normalized size = 2.48

$$\frac{15 \left(c^3 \cos^3(fx + e) + c^3 \cos^2(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right)}{15 \left(f \cos^3(fx + e) + f \cos^2(fx + e) \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(23*c^3*cos(f*x + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (23*c^3*cos(f*x + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

[In] `int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3,x)`

[Out] `int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int 3\sqrt{a \sec(e + fx) + a} \sec(e + fx) dx + \int \left(-3\sqrt{a \sec(e + fx) + a} \sec^2(e + fx) \right) dx + \int \sqrt{a \sec(e + fx) + a} \sec^3(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))*3*(a+a*sec(f*x+e))^(1/2),x)`

[Out] `-c**3*(Integral(3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x) + Integral(-sqrt(a*sec(e + f*x) + a), x))`

3.44 $\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^2 dx$

Optimal. Leaf size=105

$$\frac{2a^2c^2 \tan^3(e + fx)}{3f(a \sec(e + fx) + a)^{3/2}} + \frac{2\sqrt{a}c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2ac^2 \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}}$$

[Out] $2*c^2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/f-2*a*c^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/3*a^2*c^2*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}$

Rubi [A] time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 302, 203}

$$\frac{2a^2c^2 \tan^3(e + fx)}{3f(a \sec(e + fx) + a)^{3/2}} + \frac{2\sqrt{a}c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2ac^2 \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^2, x]$

[Out] $(2*\text{Sqrt}[a]*c^2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/f - (2*a*c^2*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^2*c^2*\text{Tan}[e + f*x]^3)/(3*f*(a + a*\text{Sec}[e + f*x])^{(3/2)})$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 302

$\text{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^{(n_)}), x_Symbol] :> \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 3887

$\text{Int}[\cot[(c_ + (d_)*(x_))]^{(m_)}*(\csc[(c_ + (d_)*(x_)]*(b_ + (a_))^{(n_)}), x_Symbol] :> \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2 + a*x^2)^{(m/2 + n - 1/2)})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]]], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\csc[(e_ + (f_)*(x_)]*(b_ + (a_))^{(m_)}*(\csc[(e_ + (f_)*(x_)]*(d_ + (c_))^{(n_)}), x_Symbol] :> \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n - m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{RationalQ}[n] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^2 dx &= (a^2 c^2) \int \frac{\tan^4(e + fx)}{(a + a \sec(e + fx))^{3/2}} dx \\
&= -\frac{(2a^3 c^2) \operatorname{Subst}\left(\int \frac{x^4}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{(2a^3 c^2) \operatorname{Subst}\left(\int \left(-\frac{1}{a^2} + \frac{x^2}{a} + \frac{1}{a^2(1+ax^2)}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{2ac^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2 c^2 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{(2ac^2) \operatorname{Subst}}{f} \\
&= \frac{2\sqrt{a} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} - \frac{2ac^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2 c^2 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.89, size = 97, normalized size = 0.92

$$\frac{2c^2 \tan\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((4 \cos(e + fx) - 1) \sqrt{\sec(e + fx) - 1} - 3 \cos(e + fx) \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right) \right)}{3f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]

[Out] (-2*c^2*(-3*ArcTan[Sqrt[-1 + Sec[e + f*x]]])*Cos[e + f*x] + (-1 + 4*Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]/(3*f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.49, size = 313, normalized size = 2.98

$$\frac{3 \left(c^2 \cos^2(fx + e) + c^2 \cos(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos(fx + e)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right) - 2 \left(4c^2 \cos(fx + e) - c^2 \right) \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \sin(fx + e)}{3 \left(f \cos^2(fx + e) + f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) + a), x))
```

3.45 $\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx)) dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2ac \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

[Out] $2*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/f-2*a*c*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3904, 3887, 321, 203}

$$\frac{2\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2ac \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]

[Out] $(2*\text{Sqrt}[a]*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/f - (2*a*c*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 321

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx)) dx &= - \left((ac) \int \frac{\tan^2(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx \right) \\
&= \frac{(2a^2c) \operatorname{Subst} \left(\int \frac{x^2}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= -\frac{2ac \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} - \frac{(2ac) \operatorname{Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= \frac{2\sqrt{a}c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} - \frac{2ac \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 70, normalized size = 1.06

$$\frac{2c \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(\sqrt{\sec(e + fx) - 1} - \tan^{-1}\left(\sqrt{\sec(e + fx) - 1}\right)\right)}{f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]

[Out] (-2*c*(-ArcTan[Sqrt[-1 + Sec[e + f*x]]] + Sqrt[-1 + Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x]))*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.44, size = 234, normalized size = 3.55

$$\frac{\left((c \cos(fx + e) + c) \sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e) - a}{\cos(fx+e)+1} \right) - 2c \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) \right)}{f \cos(fx + e) + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] (((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e) + f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -c*(Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-sqrt(a*s  
ec(e + f*x) + a), x))
```

$$3.46 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{c-c \sec(e+fx)} dx$$

Optimal. Leaf size=69

$$\frac{2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{cf} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf}$$

[Out] $2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/c/f+2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c/f$

Rubi [A] time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 325, 203}

$$\frac{2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{cf} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c*f) + (2*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(e + fx)}}{c - c \sec(e + fx)} dx &= -\frac{\int \cot^2(e + fx)(a + a \sec(e + fx))^{3/2} dx}{ac} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{cf} - \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} + \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{cf}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 90, normalized size = 1.30

$$\frac{2a \tan(e + fx) \sec(e + fx) \left(\cos(e + fx) \sqrt{\sec(e + fx) - 1} - (\cos(e + fx) - 1) \tan^{-1} \left(\sqrt{\sec(e + fx) - 1} \right) \right)}{cf (\sec(e + fx) - 1)^{3/2} \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]

[Out] (2*a*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x])) + Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]]*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x]))^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.59, size = 266, normalized size = 3.86

$$\left[\frac{\sqrt{-a} \log \left(\frac{8a \cos(fx+e)^3 - 4(2 \cos(fx+e)^2 - \cos(fx+e)) \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) - 7a \cos(fx+e)+a}{\cos(fx+e)+1} \right) \sin(fx+e) + 4 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{2cf \sin(fx+e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(sqrt(-a)*log(-(8*a*cos(f*x + e))^3 - 4*(2*cos(f*x + e))^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(c*f*sin(f*x + e)), (sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(c*f*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x)/c
```

$$3.47 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=104

$$-\frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3ac^2f} + \frac{2 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^2f} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^2f}$$

[Out] $-2/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{3/2}/a/c^2/f+2*\arctan(a^{1/2}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{1/2})*a^{1/2}/c^2/f+2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{1/2}/c^2/f$

Rubi [A] time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 325, 203}

$$-\frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3ac^2f} + \frac{2 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^2f} + \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^2f}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]`

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c^2*f) + (2*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c^2*f) - (2*\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^{3/2})/(3*a*c^2*f)$

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 325

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]`

Rule 3887

`Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]`

Rule 3904

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n-m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])`

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^2} dx &= \frac{\int \cot^4(e + fx)(a + a \sec(e + fx))^{5/2} dx}{a^2 c^2} \\
&= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^2 f} \\
&= -\frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3ac^2 f} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} \\
&= \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^2 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3ac^2 f} - \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} \\
&= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} + \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^2 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3ac^2 f}
\end{aligned}$$

Mathematica [C] time = 0.24, size = 78, normalized size = 0.75

$$-\frac{2\sqrt{\cos(e + fx)} \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} {}_2F_1\left(-\frac{3}{2}, -\frac{3}{2}, -\frac{1}{2}; 2 \sin^2\left(\frac{1}{2}(e + fx)\right)\right)}{3c^2 f (\cos(e + fx) - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]

[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-3/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(3*c^2*f*(-1 + Cos[e + f*x])^2)

fricas [A] time = 0.55, size = 339, normalized size = 3.26

$$\left[\frac{3\sqrt{-a}(\cos(fx + e) - 1) \log\left(\frac{8a \cos(fx+e)^3 - 4(2 \cos(fx+e)^2 - \cos(fx+e))\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) - 7a \cos(fx+e)+a}{\cos(fx+e)+1}\right)}{6(c^2 f \cos(fx + e) - c^2 f) \sin(fx + e)} \right] \sin(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/6*(3*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(4*cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*sqrt(a)*(cos(f*x + e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(4*cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

[In] `int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2, x)`

[Out] `int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a \sec(e+fx)+a}}{\sec^2(e+fx)-2 \sec(e+fx)+1} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2, x)`

[Out] `Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/c**2`

$$3.48 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=139

$$\frac{2 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5a^2c^3f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3ac^3f} + \frac{2 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^3f} + \dots$$

[Out] $-2/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^(3/2)/a/c^3/f+2/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^(5/2)/a^2/c^3/f+2*\arctan(a^(1/2)*\tan(f*x+e)/(a+a*\sec(f*x+e))^(1/2))*a^(1/2)/c^3/f+2*\cot(f*x+e)*(a+a*\sec(f*x+e))^(1/2)/c^3/f$

Rubi [A] time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 325, 203}

$$\frac{2 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5a^2c^3f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3ac^3f} + \frac{2 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^3f} + \dots$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c^3*f) + (2*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c^3*f) - (2*\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^(3/2))/(3*a*c^3*f) + (2*\text{Cot}[e + f*x]^5*(a + a*\text{Sec}[e + f*x])^(5/2))/(5*a^2*c^3*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 325

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2+n+1/2))/d, Subst[Int[(x^m*(2+a*x^2)^(m/2+n-1/2))/(1+a*x^2), x], x, Cot[c+d*x]/Sqrt[a+b*Csc[c+d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2-b^2, 0] && IntegerQ[m/2] && IntegerQ[n-1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^(m/2), Int[Cot[e+f*x]^(2*m)*(c+d*Csc[e+f*x])^(n-m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c+a*d, 0] && EqQ[a^2-b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m-n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^3} dx &= -\frac{\int \cot^6(e + fx)(a + a \sec(e + fx))^{7/2} dx}{a^3 c^3} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^6(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2 c^3 f} \\
&= \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^3 f} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^3 f} \\
&= -\frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3ac^3 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^3 f} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^3 f} \\
&= \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^3 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3ac^3 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^3 f} \\
&= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^3 f} + \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^3 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3ac^3 f}
\end{aligned}$$

Mathematica [C] time = 0.26, size = 78, normalized size = 0.56

$$\frac{2\sqrt{\cos(e + fx)} \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} {}_2F_1\left(-\frac{5}{2}, -\frac{5}{2}, -\frac{3}{2}; 2 \sin^2\left(\frac{1}{2}(e + fx)\right)\right)}{5c^3 f (\cos(e + fx) - 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]

[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-5/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(5*c^3*f*(-1 + Cos[e + f*x])^3)

fricas [A] time = 0.58, size = 405, normalized size = 2.91

$$\frac{15 \left(\cos(fx + e)^2 - 2 \cos(fx + e) + 1 \right) \sqrt{-a} \log \left(-\frac{8a \cos(fx+e)^3 - 4 \left(2 \cos(fx+e)^2 - \cos(fx+e) \right) \sqrt{-a} \sqrt{\frac{a \cos(fx+e) + a}{\cos(fx+e)}} \sin(fx+e) - 7a \cos(fx+e) + a}{\cos(fx+e) + 1} \right)}{30 \left(c^3 f \cos(fx + e)^2 - 2c^3 f \cos(fx + e) + c^3 f \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/30*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e)]

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c - \frac{c}{\cos(e+fx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a \sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1} dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)

[Out] -Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)/c**3

$$3.49 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=174

$$\frac{2 \cot^7(e+fx)(a \sec(e+fx)+a)^{7/2}}{7a^3c^4f} + \frac{2 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5a^2c^4f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^3}{3ac^4f}$$

[Out] $-2/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/a/c^4/f+2/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/a^2/c^4/f-2/7*\cot(f*x+e)^7*(a+a*\sec(f*x+e))^{(7/2)}/a^3/c^4/f+2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/c^4/f+2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^4/f$

Rubi [A] time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 325, 203}

$$\frac{2 \cot^7(e+fx)(a \sec(e+fx)+a)^{7/2}}{7a^3c^4f} + \frac{2 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5a^2c^4f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^3}{3ac^4f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c^4*f) + (2*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c^4*f) - (2*\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^{(3/2)})/(3*a*c^4*f) + (2*\text{Cot}[e + f*x]^5*(a + a*\text{Sec}[e + f*x])^{(5/2)})/(5*a^2*c^4*f) - (2*\text{Cot}[e + f*x]^7*(a + a*\text{Sec}[e + f*x])^{(7/2)})/(7*a^3*c^4*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n-m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I

ntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^4} dx &= \frac{\int \cot^8(e + fx)(a + a \sec(e + fx))^{9/2} dx}{a^4 c^4} \\
 &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{x^8(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^3 c^4 f} \\
 &= -\frac{2 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^3 c^4 f} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^6(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2 c^4 f} \\
 &= \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^3 c^4 f} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a c^4 f} \\
 &= -\frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3a c^4 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^3 c^4 f} \\
 &= \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^4 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3a c^4 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^3 c^4 f} \\
 &= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^4 f} + \frac{2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^4 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3a c^4 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^2 c^4 f} - \frac{2 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^3 c^4 f}
 \end{aligned}$$

Mathematica [C] time = 0.24, size = 78, normalized size = 0.45

$$\frac{2\sqrt{\cos(e + fx)} \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} {}_2F_1\left(-\frac{7}{2}, -\frac{7}{2}; -\frac{5}{2}; 2 \sin^2\left(\frac{1}{2}(e + fx)\right)\right)}{7c^4 f (\cos(e + fx) - 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4, x]

[Out] (-2*Sqrt[Cos[e + f*x]]*Hypergeometric2F1[-7/2, -7/2, -5/2, 2*Sin[(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(7*c^4*f*(-1 + Cos[e + f*x])^4)

fricas [A] time = 0.64, size = 475, normalized size = 2.73

$$\frac{105 \left(\cos(fx + e)^3 - 3 \cos(fx + e)^2 + 3 \cos(fx + e) - 1 \right) \sqrt{-a} \log \left(\frac{8a \cos(fx + e)^3 - 4(2 \cos(fx + e)^2 - \cos(fx + e)) \sqrt{-a} \sqrt{\frac{a}{\cos(fx + e) + 1}}}{\cos(fx + e) + 1} \right)}{210 \left(c^4 f \cos(fx + e)^3 - 3 \cos(fx + e)^2 + 3 \cos(fx + e) - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")

[Out] [1/210*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a))

$$h(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2))+3}$$

$$52*\cos(f*x+e)^4+105*2^{(1/2)*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2))}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)-812*\cos(f*x+e)^3+700*\cos(f*x+e)^2-210*\cos(f*x+e))/\sin(f*x+e)^5/(-1+\cos(f*x+e))$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c - \frac{c}{\cos(e+fx)}\right)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4,x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a \sec(e+fx)+a}}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1} dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**4,x)

[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)/c**4

3.50 $\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^3 dx$

Optimal. Leaf size=177

$$\frac{2a^{3/2}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^5c^3 \tan^7(e+fx)}{7f(a \sec(e+fx)+a)^{7/2}} - \frac{2a^4c^3 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^3c^3 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} - \frac{2a^2c^3 \tan(e+fx)}{f}$$

[Out] $2a^{3/2}c^3 \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/f - 2a^2c^3 \tan(fx+e)/f/(a+a \sec(fx+e))^{1/2} + 2/3a^3c^3 \tan(fx+e)^3/f/(a+a \sec(fx+e))^{3/2} - 2/5a^4c^3 \tan(fx+e)^5/f/(a+a \sec(fx+e))^{5/2} - 2/7a^5c^3 \tan(fx+e)^7/f/(a+a \sec(fx+e))^{7/2}$

Rubi [A] time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 459, 302, 203}

$$-\frac{2a^5c^3 \tan^7(e+fx)}{7f(a \sec(e+fx)+a)^{7/2}} - \frac{2a^4c^3 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^3c^3 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{3/2}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]

[Out] $(2a^{3/2}c^3 \text{ArcTan}[(\text{Sqrt}[a] \text{Tan}[e + fx])/\text{Sqrt}[a + a \text{Sec}[e + fx]])]/f - (2a^2c^3 \text{Tan}[e + fx])/(f \text{Sqrt}[a + a \text{Sec}[e + fx]]) + (2a^3c^3 \text{Tan}[e + fx]^3)/(3f(a + a \text{Sec}[e + fx])^{3/2}) - (2a^4c^3 \text{Tan}[e + fx]^5)/(5f(a + a \text{Sec}[e + fx])^{5/2}) - (2a^5c^3 \text{Tan}[e + fx]^7)/(7f(a + a \text{Sec}[e + fx])^{7/2})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(b*e*(m+n*(p+1)+1)), x] - Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(b*(m+n*(p+1)+1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p+1) + 1, 0]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^3 dx &= - \left((a^3 c^3) \int \frac{\tan^6(e + fx)}{(a + a \sec(e + fx))^{3/2}} dx \right) \\ &= \frac{(2a^5 c^3) \operatorname{Subst} \left(\int \frac{x^6(2+ax^2)}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{2a^5 c^3 \tan^7(e + fx)}{7f(a + a \sec(e + fx))^{7/2}} + \frac{(2a^5 c^3) \operatorname{Subst} \left(\int \frac{x^6}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{2a^5 c^3 \tan^7(e + fx)}{7f(a + a \sec(e + fx))^{7/2}} + \frac{(2a^5 c^3) \operatorname{Subst} \left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \frac{x^4}{a} - \frac{x^6}{a^3} \right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{2a^2 c^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^3 c^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{2a^4 c^3 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} \\ &= -\frac{2a^{3/2} c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} - \frac{2a^2 c^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^3 c^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.95, size = 122, normalized size = 0.69

$$\frac{ac^3 \tan\left(\frac{1}{2}(e + fx)\right) \sec^3(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((-171 \cos(e + fx) + 32 \cos(2(e + fx)) - 73 \cos(3(e + fx))) \right)}{105f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a*c^3*(210*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3 + (2 - 171*Cos[e + f*x] + 32*Cos[2*(e + f*x)] - 73*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*Sqrt[-1 + Sec[e + f*x]])
```

fricas [A] time = 0.48, size = 385, normalized size = 2.18

$$\frac{105 \left(ac^3 \cos^4(fx + e) + ac^3 \cos^3(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e) - a}{\cos(fx+e)+1} \right)}{105 \left(f \cos(fx + e) \right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```


$2*\cos(f*x+e)/(1+\cos(f*x+e))^{7/2}*\sin(f*x+e)+2336*\cos(f*x+e)^4-2848*\cos(f*x+e)^3+128*\cos(f*x+e)^2+624*\cos(f*x+e)-240)/\cos(f*x+e)^3/\sin(f*x+e)*a$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{3/2} \left(c - \frac{c}{\cos(e + fx)} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int \left(-a\sqrt{a \sec(e + fx) + a} \right) dx + \int 2a\sqrt{a \sec(e + fx) + a} \sec(e + fx) dx + \int \left(-2a\sqrt{a \sec(e + fx) + a} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**3,x)

[Out] -c**3*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x))

3.51 $\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^2 dx$

Optimal. Leaf size=142

$$\frac{2a^{3/2}c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2a^4c^2 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^3c^2 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} - \frac{2a^2c^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

[Out] $2a^{(3/2)}c^2 \arctan(a^{(1/2)} \tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f - 2a^2c^2 \tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)} + 2/3a^3c^2 \tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)} + 2/5a^4c^2 \tan(f*x+e)^5/f/(a+a*\sec(f*x+e))^{(5/2)}$

Rubi [A] time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 459, 302, 203}

$$\frac{2a^4c^2 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^3c^2 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{3/2}c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^2c^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(3/2)}*(c - c*\text{Sec}[e + f*x])^2, x]$

[Out] $(2*a^{(3/2)}*c^2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/f - (2*a^2*c^2*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^3*c^2*\text{Tan}[e + f*x]^3)/(3*f*(a + a*\text{Sec}[e + f*x])^{(3/2)}) + (2*a^4*c^2*\text{Tan}[e + f*x]^5)/(5*f*(a + a*\text{Sec}[e + f*x])^{(5/2)})$

Rule 203

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 302

$\text{Int}[x^m/(a + b*x^n), x_Symbol] := \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 459

$\text{Int}[(e*x)^m*(a + b*x^n)^p*(c + d*x^n), x_Symbol] := \text{Simp}[(d*(e*x)^{m+1}*(a + b*x^n)^{p+1})/(b*e*(m + n*(p + 1) + 1)), x] - \text{Dist}[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), \text{Int}[(e*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m + n*(p + 1) + 1, 0]$

Rule 3887

$\text{Int}[\cot[(c + d*x)^m]*(\csc[(c + d*x)]*(b + a))^{(n)}, x_Symbol] := \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[x^m*(2 + a*x^2)^{(m/2 + n - 1/2)}]/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]]] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\csc[(e + f*x)]*(b + a))^{(m)}*(\csc[(e + f*x)]*(d + c))^{(n)}, x_Symbol] := \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c$

+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^2 dx &= (a^2 c^2) \int \frac{\tan^4(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx \\ &= \frac{(2a^4 c^2) \text{Subst}\left(\int \frac{x^4(2+ax^2)}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\ &= \frac{2a^4 c^2 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} - \frac{(2a^4 c^2) \text{Subst}\left(\int \frac{x^4}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\ &= \frac{2a^4 c^2 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} - \frac{(2a^4 c^2) \text{Subst}\left(\int \left(-\frac{1}{a^2} + \frac{x^2}{a} + \frac{1}{a^2(1+ax^2)}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\ &= -\frac{2a^2 c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^3 c^2 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} + \frac{2a^4 c^2 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} \\ &= \frac{2a^{3/2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} - \frac{2a^2 c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^3 c^2 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.91, size = 112, normalized size = 0.79

$$\frac{ac^2 \tan\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((2 \cos(e + fx) + 17 \cos(2(e + fx)) + 11) \sqrt{\sec(e + fx) - 1}\right)}{15f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]

[Out] -1/15*(a*c^2*(-30*ArcTan[Sqrt[-1 + Sec[e + f*x]]])*Cos[e + f*x]^2 + (11 + 2*Cos[e + f*x] + 17*Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]]*Sec[e + f*x]^2 *Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.46, size = 355, normalized size = 2.50

$$\frac{15 \left(ac^2 \cos^3(fx + e) + ac^2 \cos^2(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right)}{15 \left(f \cos^3(fx + e) + f \cos^2(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{3/2} \left(c - \frac{c}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int a \sqrt{a \sec(e + fx) + a} dx + \int \left(-a \sqrt{a \sec(e + fx) + a} \sec(e + fx) \right) dx + \int \left(-a \sqrt{a \sec(e + fx) + a} \sec^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**2,x)

[Out] c**2*(Integral(a*sqrt(a*sec(e + f*x) + a), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))

3.52 $\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx)) dx$

Optimal. Leaf size=101

$$\frac{2a^{3/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^3c \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} - \frac{2a^2c \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

[Out] $2*a^{(3/2)}*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f-2*a^2*c*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2/3*a^3*c*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}$

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3904, 3887, 459, 321, 203}

$$-\frac{2a^3c \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{3/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^2c \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(3/2)}*(c - c*\text{Sec}[e + f*x]),x]$

[Out] $(2*a^{(3/2)}*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/f - (2*a^2*c*\text{Tan}[e + f*x])/f/\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*a^3*c*\text{Tan}[e + f*x]^3)/(3*f*(a + a*\text{Sec}[e + f*x])^{(3/2)})$

Rule 203

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 321

$\text{Int}[(c*x)^m*(a + b*x^n)^p, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{n*(m-n+1)})/(b*(m+n*p+1)), \text{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 459

$\text{Int}[(e*x)^m*(a + b*x^n)^p*((c + d*x)^n), x_Symbol] \rightarrow \text{Simp}[(d*(e*x)^{m+1}*(a + b*x^n)^{p+1})/(b*e*(m+n*(p+1)+1)), x] - \text{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(b*(m+n*(p+1)+1)), \text{Int}[(e*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m+n*(p+1)+1, 0]$

Rule 3887

$\text{Int}[\cot[(c + d*x)^m*(\csc[(c + d*x])*(b + a))]^n, x_Symbol] \rightarrow \text{Dist}[(-2*a^{(m/2+n+1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2+a*x^2)^{(m/2+n-1/2)})/(1+a*x^2), x], x, \text{Cot}[c+d*x]/\text{Sqrt}[a+b*\text{Csc}[c+d*x]]], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[n-1/2]$

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-(a*c))^(m), Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx)) dx &= - \left((ac) \int \sqrt{a + a \sec(e + fx)} \tan^2(e + fx) dx \right) \\ &= \frac{(2a^3c) \operatorname{Subst} \left(\int \frac{x^2(2+ax^2)}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{2a^3c \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} + \frac{(2a^3c) \operatorname{Subst} \left(\int \frac{x^2}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{2a^2c \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2a^3c \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{(2a^2c) \operatorname{Subst} \left(\int \frac{x^2}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= \frac{2a^{3/2}c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} - \frac{2a^2c \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2a^3c \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.68, size = 96, normalized size = 0.95

$$\frac{2ac \tan\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((2 \cos(e + fx) + 1) \sqrt{\sec(e + fx) - 1} - 3 \cos(e + fx) \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right) \right)}{3f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

```
[Out] (-2*a*c*(-3*ArcTan[Sqrt[-1 + Sec[e + f*x]])*Cos[e + f*x] + (1 + 2*Cos[e + f
*x])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(
e + f*x)/2])/(3*f*Sqrt[-1 + Sec[e + f*x]])
```

fricas [A] time = 0.46, size = 303, normalized size = 3.00

$$\frac{3 \left(ac \cos^2(fx + e) + ac \cos(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e) - a}{\cos(fx+e)+1} \right) - 2 \left(f \cos^2(fx + e) + f \cos(fx + e) \right)}{3 \left(f \cos^2(fx + e) + f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x +
e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(2*a*c*cos(f*x + e)
+ a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
```

$e)^2 + f \cos(fx + e)), -2/3*(3*(a*c*\cos(f*x + e))^2 + a*c*\cos(f*x + e))*\sqrt{a}*\arctan(\sqrt{(a*\cos(f*x + e) + a)/\cos(f*x + e)}*\cos(f*x + e)/(\sqrt{a}*\sin(f*x + e))) + (2*a*c*\cos(f*x + e) + a*c)*\sqrt{(a*\cos(f*x + e) + a)/\cos(f*x + e)}*\sin(f*x + e))/(f*\cos(f*x + e)^2 + f*\cos(f*x + e))]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)-2*(2*(1/2*sqrt(2)*a^3*c*sign(cos(f*x+exp(1)))-1/6*sqrt(2)*a^3*c*sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2/sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)/(-a*tan(1/2*(f*x+exp(1)))^2+a)*tan(1/2*(f*x+exp(1)))+1/2*a^2*sqrt(-a)*c*sign(cos(f*x+exp(1)))*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a)/abs(a))/f

maple [B] time = 1.43, size = 212, normalized size = 2.10

$$c \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(3 \sin(fx+e) \cos(fx+e) \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{3}{2}} \sqrt{2} + 3\sqrt{2} a \right) \\ 6f \cos(fx+e) s$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x)

[Out] $1/6*c/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*(3*\sin(f*x+e)*\cos(f*x+e)*\arctan(\frac{1}{2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{3/2}*2^{1/2}+3*2^{1/2}*\operatorname{arctanh}(\frac{1}{2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{3/2}*\sin(f*x+e)+8*\cos(f*x+e)^2-4*\cos(f*x+e)-4)/\cos(f*x+e)/\sin(f*x+e)*a$

maxima [B] time = 0.82, size = 998, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] $1/2*((a*\arctan2((\cos(2*f*x + 2*e))^2 + \sin(2*f*x + 2*e))^2 + 2*\cos(2*f*x + 2*e) + 1)^{1/4}*(\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1)) - \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1)))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))$

```

*x + 2*e))))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
+ 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*((cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{3/2} \left(c - \frac{c}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \left(\int \left(-a\sqrt{a \sec(e + fx) + a} \right) dx + \int a\sqrt{a \sec(e + fx) + a} \sec^2(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e)),x)

[Out] -c*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x))

$$3.53 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{c-c \sec(e+fx)} dx$$

Optimal. Leaf size=70

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} + \frac{4a \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{cf}$$

[Out] $2*a^{(3/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c/f+4*a*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c/f$

Rubi [A] time = 0.16, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 453, 203}

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} + \frac{4a \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{cf}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(3/2)}/(c - c*\text{Sec}[e + f*x]),x]$

[Out] $(2*a^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(c*f) + (4*a*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c*f)$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/(\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[a, b], x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 453

$\text{Int}[(e_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] := \text{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*e^{(m+1)}), x] + \text{Dist}[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^{(m+1)}), \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[a, b, c, d, e, p], x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{IntegerQ}[n] \parallel \text{GtQ}[e, 0]) \&\& ((\text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]) \parallel (\text{LtQ}[n, 0] \&\& \text{GtQ}[m+n, -1])) \&\& !\text{ILtQ}[p, -1]$

Rule 3887

$\text{Int}[\cot[(c_ + (d_)*(x_))]^{(m_)}*(\csc[(c_ + (d_)*(x_))]*(b_ + (a_))^{(n_)}), x_Symbol] := \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2 + a*x^2)^{(m/2 + n - 1/2)})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]], x] /; \text{FreeQ}[a, b, c, d], x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\csc[(e_ + (f_)*(x_))]*(b_ + (a_))^{(m_)}*(\csc[(e_ + (f_)*(x_))]*(d_ + (c_))^{(n_)}), x_Symbol] := \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n-m)}, x], x] /; \text{FreeQ}[a, b, c, d, e, f, n], x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{3/2}}{c - c \sec(e + fx)} dx &= -\frac{\int \cot^2(e + fx)(a + a \sec(e + fx))^{5/2} dx}{ac} \\
&= \frac{(2a) \text{Subst}\left(\int \frac{2+ax^2}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{4a \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{cf} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} + \frac{4a \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{cf}
\end{aligned}$$

Mathematica [A] time = 0.58, size = 93, normalized size = 1.33

$$\frac{2a^2 \tan(e + fx) \sec(e + fx) \left(2 \cos(e + fx) \sqrt{\sec(e + fx) - 1} - (\cos(e + fx) - 1) \tan^{-1} \left(\sqrt{\sec(e + fx) - 1}\right)\right)}{cf (\sec(e + fx) - 1)^{3/2} \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]

[Out] (2*a^2*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x]))) + 2*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]]*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.54, size = 269, normalized size = 3.84

$$\left[\frac{\sqrt{-a} a \log \left(\frac{8 a \cos(fx+e)^3 - 4 \left(2 \cos(fx+e)^2 - \cos(fx+e) \right) \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) - 7 a \cos(fx+e)+a}{\cos(fx+e)+1} \right) \sin(fx+e) + 8 a \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{2 c f \sin(fx+e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(sqrt(-a)*a*log(-(8*a*cos(f*x + e))^3 - 4*(2*cos(f*x + e))^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(c*f*sin(f*x + e)), (a^(3/2)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 4*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(c*f*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x))/c
```

$$3.54 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=102

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^2 f} - \frac{4 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3c^2 f} + \frac{2a \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^2 f}$$

[Out] $2*a^{(3/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^2/f-4/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/c^2/f+2*a*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^2/f$

Rubi [A] time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 453, 325, 203}

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^2 f} - \frac{4 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3c^2 f} + \frac{2a \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^2 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]

[Out] $(2*a^{(3/2)}*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^2*f) + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^2*f) - (4*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^{(3/2)})/(3*c^2*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2+n+1/2))/d, Subst[Int[(x^m*(2+a*x^2)^(m/2+n-1/2))/(1+a*x^2), x], x, Cot[c+d*x]/Sqrt[a+b*Csc[c+d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^2} dx &= \frac{\int \cot^4(e + fx)(a + a \sec(e + fx))^{7/2} dx}{a^2 c^2} \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{2+ax^2}{x^4(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} \\ &= -\frac{4 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^2 f} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} \\ &= \frac{2a \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^2 f} - \frac{4 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^2 f} - \frac{(2a^2) \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} \\ &= \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^2 f} + \frac{2a \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^2 f} - \frac{4 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^2 f} \end{aligned}$$

Mathematica [A] time = 0.64, size = 113, normalized size = 1.11

$$\frac{2a\sqrt{\cos(e+fx)} \tan\left(\frac{1}{2}(e+fx)\right) \sqrt{a(\sec(e+fx)+1)} \left(\sqrt{\cos(e+fx)}(5\cos(e+fx)-3) - 6\sqrt{2} \sin^{-1}\left(\sqrt{2} \sin\left(\frac{e+fx}{2}\right)\right)\right)}{3c^2 f(\cos(e+fx)-1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2, x]

[Out] (-2*a*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[Cos[e + f*x]]*(-3 + 5*Cos[e + f*x]) - 6*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^3)*Tan[(e + f*x)/2])/(3*c^2*f*(-1 + Cos[e + f*x])^2)

fricas [A] time = 0.55, size = 351, normalized size = 3.44

$$\frac{3(a \cos(fx + e) - a)\sqrt{-a} \log\left(-\frac{8a \cos(fx+e)^3 - 4(2 \cos(fx+e)^2 - \cos(fx+e))\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) - 7a \cos(fx+e)+a}{\cos(fx+e)+1}\right) \sin(fx+e)}{6(c^2 f \cos(fx + e) - c^2 f) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/6*(3*(a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))]

```
e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*(a*cos(f*x + e) -
a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e)
+ 2*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)4*(1/196608*(-262144*a^4*sqrt(-a)*sign
(cos(f*x+exp(1))))-393216*a^2*sqrt(-a)*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-s
qrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(f*x+exp(1)))+393216*a^3*sqrt(-a)*
(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(
cos(f*x+exp(1)))/sqrt(2)/c^2/((sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)
*tan(1/2*(f*x+exp(1))))^2-a)^3-1/4*a^2*sqrt(-a)*sign(cos(f*x+exp(1)))*ln(ab
s(2*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4
*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan
(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a)/c^2/abs(a))/f
```

maple [B] time = 1.55, size = 215, normalized size = 2.11

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(-3 \cos(fx+e) \sin(fx+e) \sqrt{2} \sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2 \cos(fx+e)} \right) + 3 \sqrt{2} \sin(fx+e) \right)}{3c^2 f \sin(fx+e) (-1 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/3/c^2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-3*cos(f*x+e)*sin(f*x+e)*2^(
1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+3*2^(1/2)*sin(f*x+e)*arctanh
(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+10*cos(f*x+e)^2-6*cos(f*x+e))/sin(f*x+e)
/(-1+cos(f*x+e))*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1} dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)

[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2

$$3.55 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=137

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^3 f} + \frac{4 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5ac^3 f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3c^3 f} + \frac{2a \cot^2(e+fx)(a \sec(e+fx)+a)^{1/2}}{c^3 f}$$

[Out] $2*a^{(3/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^3/f-2/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/c^3/f+4/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/a/c^3/f+2*a*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^3/f$

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 453, 325, 203}

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^3 f} + \frac{4 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5ac^3 f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3c^3 f} + \frac{2a \cot^2(e+fx)(a \sec(e+fx)+a)^{1/2}}{c^3 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]

[Out] $(2*a^{(3/2)}*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(c^3*f) + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) - (2*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^{(3/2)})/(3*c^3*f) + (4*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^{(5/2)})/(5*a*c^3*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^3} dx &= -\frac{\int \cot^6(e + fx)(a + a \sec(e + fx))^{9/2} dx}{a^3 c^3} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{2+ax^2}{x^6(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^3 f} \\ &= \frac{4 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^3 f} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^3 f} \\ &= -\frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^3 f} + \frac{4 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^3 f} + \frac{(2a)}{c^3 f} \\ &= \frac{2a \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^3 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^3 f} + \frac{4 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^3 f} \\ &= \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^3 f} + \frac{2a \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^3 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^3 f} \end{aligned}$$

Mathematica [C] time = 0.80, size = 102, normalized size = 0.74

$$\frac{2a \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(5(\cos(e + fx) - 1) {}_2F_1\left(-\frac{3}{2}, -\frac{3}{2}; -\frac{1}{2}; 2 \sin^2\left(\frac{1}{2}(e + fx)\right)\right) + 6 \cos^{\frac{5}{2}}(e + fx)\right)}{15c^3 f \cos^{\frac{5}{2}}(e + fx)(\sec(e + fx) - 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]

[Out] (2*a*(6*Cos[e + f*x]^(5/2) + 5*(-1 + Cos[e + f*x])*Hypergeometric2F1[-3/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2])*Sqrt[a*(1 + Sec[e + f*x])*Tan[(e + f*x)/2])/(15*c^3*f*Cos[e + f*x]^(5/2)*(-1 + Sec[e + f*x])^3)

fricas [A] time = 0.56, size = 417, normalized size = 3.04

$$\frac{15 \left(a \cos^2(fx + e) - 2a \cos(fx + e) + a \right) \sqrt{-a} \log \left(\frac{8a \cos^3(fx+e) - 4 \left(2 \cos^2(fx+e) - \cos(fx+e) \right) \sqrt{-a} \sqrt{\frac{a \cos(fx+e) + a}{\cos(fx+e)}} \sin(fx+e)}{\cos(fx+e) + 1} \right)}{30 \left(c^3 f \cos^2(fx + e) - 2c^3 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.


```
+cos(f*x+e))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+30*cos(f*x+e)*sin(f*x+e)
*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))-15*2^(1/2)*sin(f*x+e)*ar
ctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2
))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+52*cos(f*x+e)^3-70*cos(f*x+e)^2+30*
cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^2*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3,x)
```

[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1} dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1} dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2
+ 3*sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

$$3.56 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=172

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^4 f} - \frac{4 \cot^7(e+fx)(a \sec(e+fx)+a)^{7/2}}{7a^2 c^4 f} + \frac{2 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5ac^4 f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3a^2 c^4 f}$$

[Out] $2*a^{(3/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^4/f-2/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/c^4/f+2/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/a/c^4/f-4/7*\cot(f*x+e)^7*(a+a*\sec(f*x+e))^{(7/2)}/a^2/c^4/f+2*a*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^4/f$

Rubi [A] time = 0.19, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 453, 325, 203}

$$-\frac{4 \cot^7(e+fx)(a \sec(e+fx)+a)^{7/2}}{7a^2 c^4 f} + \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^4 f} + \frac{2 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5ac^4 f} - \frac{2 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3a^2 c^4 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4, x]

[Out] $(2*a^{(3/2)}*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(c^4*f) + (2*a*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^{(3/2)})/(3*c^4*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^{(5/2)})/(5*a*c^4*f) - (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^{(7/2)})/(7*a^2*c^4*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In

tegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^4} dx = \frac{\int \cot^8(e + fx)(a + a \sec(e + fx))^{11/2} dx}{a^4 c^4}$$

$$= -\frac{2 \operatorname{Subst}\left(\int \frac{2+ax^2}{x^8(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2 c^4 f}$$

$$= -\frac{4 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^2 c^4 f} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^6(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^4 f}$$

$$= \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^4 f} - \frac{4 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^2 c^4 f} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^6(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^4 f}$$

$$= -\frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^4 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^4 f} - \frac{4 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^2 c^4 f}$$

$$= \frac{2a \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{c^4 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^4 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^4 f} - \frac{4 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^2 c^4 f}$$

$$= \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^4 f} + \frac{2a \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{c^4 f} - \frac{2 \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^4 f} + \frac{2 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5ac^4 f} - \frac{4 \cot^7(e + fx)(a + a \sec(e + fx))^{7/2}}{7a^2 c^4 f}$$

Mathematica [C] time = 1.29, size = 102, normalized size = 0.59

$$\frac{2a \sqrt{\cos(e + fx)} \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(7(\cos(e + fx) - 1) {}_2F_1\left(-\frac{5}{2}, -\frac{5}{2}; -\frac{3}{2}; 2 \sin^2\left(\frac{1}{2}(e + fx)\right)\right)\right) + \dots}{35c^4 f (\cos(e + fx) - 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]

```
[Out] (-2*a*Sqrt[Cos[e + f*x]]*(10*Cos[e + f*x]^(7/2) + 7*(-1 + Cos[e + f*x])*Hyp
ergeometric2F1[-5/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2])*Sqrt[a*(1 + Sec[e +
f*x]])*Tan[(e + f*x)/2])/(35*c^4*f*(-1 + Cos[e + f*x])^4)
```

fricas [A] time = 0.60, size = 495, normalized size = 2.88

$$\frac{105 \left(a \cos(fx + e)^3 - 3a \cos(fx + e)^2 + 3a \cos(fx + e) - a \right) \sqrt{-a} \log \left(\frac{8a \cos(fx + e)^3 - 4 \left(2 \cos(fx + e)^2 - \cos(fx + e) \right) \sqrt{-a}}{\cos(fx + e)} \right)}{210 \left(c^4 f \cos(fx + e) \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*
sqrt(-a)*log(-8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt
(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x +
e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(191*a*cos(f*x + e)^4 - 406*a*
cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(a*cos(f*x + e)^3
- 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x +
e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(191*a*cos(f*x + e)^4 - 406*a
*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
+ 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)4*(-1/389134
563384311844517523670949001658364619902060465887004069926833254234155915914
772480000*(2436723575477952740669255368085415146426072243854822101954056922
78918722816680679964672000*sqrt(2)*a^8*sqrt(-a)*sign(cos(f*x+exp(1)))+38913
456338431184451752367094900165836461990206046588700406992683325423415591591
4772480000*sqrt(2)*a^2*sqrt(-a)*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a
)*tan(1/2*(f*x+exp(1))))^12*sign(cos(f*x+exp(1)))-1653821894383325339199475
601533257048049634583756980019767297189041330495162642637783040000*sqrt(2)*
a^3*sqrt(-a)*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(
1))))^10*sign(cos(f*x+exp(1)))+39237735141251444322183636820691000551765840
12443031027291038428901980194405485473955840000*sqrt(2)*a^4*sqrt(-a)*(sqrt(
-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^8*sign(cos(f*
x+exp(1)))-4604759000047690160124030106229852957314668841048846329548160800
860175104178338324807680000*sqrt(2)*a^5*sqrt(-a)*(sqrt(-a*tan(1/2*(f*x+exp(
1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^6*sign(cos(f*x+exp(1)))+346329761
412037541620596067144611475944511712833814639433622234881596268398765164147
5072000*sqrt(2)*a^6*sqrt(-a)*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*t
an(1/2*(f*x+exp(1))))^4*sign(cos(f*x+exp(1)))-13165719394502550739509550867
10788944133630668637909584363769919119176825560848844980224000*sqrt(2)*a^7*
sqrt(-a)*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1)))
)^2*sign(cos(f*x+exp(1)))/c^4/((sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a
)*tan(1/2*(f*x+exp(1))))^2-a)^7-1/4*a^2*sqrt(-a)*sign(cos(f*x+exp(1)))*ln(a
```

$$\frac{\sqrt{2 \cdot (\sqrt{-a \cdot \tan(1/2 \cdot (f \cdot x + \exp(1)))})^2 + a} - \sqrt{-a} \cdot \tan(1/2 \cdot (f \cdot x + \exp(1)))})^2 - 4 \cdot \sqrt{2} \cdot \text{abs}(a - 6 \cdot a) / \text{abs}(2 \cdot (\sqrt{-a \cdot \tan(1/2 \cdot (f \cdot x + \exp(1)))})^2 + a) - \sqrt{-a} \cdot \tan(1/2 \cdot (f \cdot x + \exp(1)))})^2 + 4 \cdot \sqrt{2} \cdot \text{abs}(a - 6 \cdot a)}{c^4 / \text{abs}(a)} / f$$

maple [B] time = 1.77, size = 401, normalized size = 2.33

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (1 + \cos(fx + e)) \left(105 (\cos^3(fx + e)) \sin(fx + e) \sqrt{2} \sqrt{-\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh}\left(\frac{\sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e)}{2 \cos(fx+e)}\right) \right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x)`

[Out] $\frac{1}{105 c^4 f} \left(a \left(\frac{1 + \cos(fx + e)}{\cos(fx + e)} \right)^{1/2} (1 + \cos(fx + e)) (105 \cos(fx + e)^3 \sin(fx + e) 2^{1/2} (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \operatorname{arctanh}(1/2 (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \sin(fx + e) / \cos(fx + e) 2^{1/2}) - 315 \sin(fx + e) \cos(fx + e)^2 2^{1/2} (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \operatorname{arctanh}(1/2 (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \sin(fx + e) / \cos(fx + e) 2^{1/2}) + 315 \cos(fx + e) \sin(fx + e) 2^{1/2} (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \operatorname{arctanh}(1/2 (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \sin(fx + e) / \cos(fx + e) 2^{1/2}) - 105 2^{1/2} \sin(fx + e) \operatorname{arctanh}(1/2 (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} \sin(fx + e) / \cos(fx + e) 2^{1/2}) * (-2 \cos(fx + e) / (1 + \cos(fx + e)))^{1/2} - 382 \cos(fx + e)^4 + 812 \cos(fx + e)^3 - 700 \cos(fx + e)^2 + 210 \cos(fx + e)) / \sin(fx + e)^3 / (-1 + \cos(fx + e))^2 a \right)$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4,x)`

[Out] `int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a \sqrt{a \sec(e+fx)+a}}{\sec^4(e+fx)-4 \sec^3(e+fx)+6 \sec^2(e+fx)-4 \sec(e+fx)+1} dx + \int \frac{a \sqrt{a \sec(e+fx)+a} \sec(e+fx)}{\sec^4(e+fx)-4 \sec^3(e+fx)+6 \sec^2(e+fx)-4 \sec(e+fx)+1} dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**4,x)`

```
[Out] (Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 +
6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)*
*2 - 4*sec(e + f*x) + 1), x))/c**4
```

3.57 $\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^3 dx$

Optimal. Leaf size=212

$$\frac{2a^{5/2}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^7c^3 \tan^9(e+fx)}{9f(a \sec(e+fx)+a)^{9/2}} - \frac{6a^6c^3 \tan^7(e+fx)}{7f(a \sec(e+fx)+a)^{7/2}} - \frac{2a^5c^3 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^4c^3 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{5/2}c^3}{3f(a \sec(e+fx)+a)^{3/2}}$$

[Out] $2a^{5/2}c^3 \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/f - 2a^7c^3 \tan^9(fx+e)/f/(a+a \sec(fx+e))^{9/2} - 6a^6c^3 \tan^7(fx+e)/f/(a+a \sec(fx+e))^{7/2} - 2a^5c^3 \tan^5(fx+e)/f/(a+a \sec(fx+e))^{5/2} + 2a^4c^3 \tan^3(fx+e)/f/(a+a \sec(fx+e))^{3/2} + 2a^{5/2}c^3/(3f(a \sec(fx+e)+a)^{3/2})$

Rubi [A] time = 0.19, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^7c^3 \tan^9(e+fx)}{9f(a \sec(e+fx)+a)^{9/2}} - \frac{6a^6c^3 \tan^7(e+fx)}{7f(a \sec(e+fx)+a)^{7/2}} - \frac{2a^5c^3 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^4c^3 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{5/2}c^3}{3f(a \sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]

[Out] $(2a^{5/2}c^3 \arctan(\sqrt{a} \tan(e+fx)/\sqrt{a+a \sec(e+fx)}))/f - (2a^7c^3 \tan^9(e+fx)/f/(a+a \sec(e+fx))^{9/2}) + (2a^6c^3 \tan^7(e+fx)/f/(a+a \sec(e+fx))^{7/2}) - (2a^5c^3 \tan^5(e+fx)/f/(a+a \sec(e+fx))^{5/2}) + (2a^4c^3 \tan^3(e+fx)/f/(a+a \sec(e+fx))^{3/2}) + (2a^{5/2}c^3)/(3f(a \sec(e+fx)+a)^{3/2})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.))/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I

nIntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
 \int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^3 dx &= - \left((a^3 c^3) \int \frac{\tan^6(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx \right) \\
 &= \frac{(2a^6 c^3) \operatorname{Subst} \left(\int \frac{x^6 (2+ax^2)^2}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
 &= \frac{(2a^6 c^3) \operatorname{Subst} \left(\int \left(\frac{1}{a^3} - \frac{x^2}{a^2} + \frac{x^4}{a} + 3x^6 + ax^8 - \frac{1}{a^3(1+ax^2)} \right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
 &= -\frac{2a^3 c^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^4 c^3 \tan^3(e + fx)}{3f (a + a \sec(e + fx))^{3/2}} - \frac{2a^5 c^3}{5f (a + a \sec(e + fx))^{5/2}} \\
 &= \frac{2a^{5/2} c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} - \frac{2a^3 c^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^5 c^3}{5f (a + a \sec(e + fx))^{5/2}}
 \end{aligned}$$

Mathematica [A] time = 1.31, size = 134, normalized size = 0.63

$$\frac{a^2 c^3 \tan\left(\frac{1}{2}(e + fx)\right) \sec^4(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((164 \cos(e + fx) + 1004 \cos(2(e + fx)) + 68 \cos(3(e + fx)) + 383 \cos(4(e + fx))) \sqrt{-1 + \sec(e + fx)} \right) \sec[e + fx]^4 \sqrt{a(1 + \sec[e + fx])} \tan[(e + fx)/2]}{1260 f \sqrt{\sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]

[Out] -1/1260*(a^2*c^3*(-2520*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^4 + (901 + 164*Cos[e + f*x] + 1004*Cos[2*(e + f*x)] + 68*Cos[3*(e + f*x)] + 383*Cos[4*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.53, size = 441, normalized size = 2.08

$$\frac{315 \left(a^2 c^3 \cos^5(fx + e) + a^2 c^3 \cos^4(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx+e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e) + a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e)}{\cos(fx+e) + 1} \right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(383*a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*x + e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(315*

$168\cos(f*x+e)^4-5312\cos(f*x+e)^3+4064\cos(f*x+e)^2+1280\cos(f*x+e)-1120)/\cos(f*x+e)^4/\sin(f*x+e)*a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c - \frac{c}{\cos(e + fx)} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int \left(-a^2 \sqrt{a \sec(e + fx) + a} \right) dx + \int a^2 \sqrt{a \sec(e + fx) + a} \sec(e + fx) dx + \int 2a^2 \sqrt{a \sec(e + fx) + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**3,x)

[Out] -c**3*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x) + Integral(-a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**5, x))

3.58 $\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^2 dx$

Optimal. Leaf size=177

$$\frac{2a^{5/2}c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2a^6c^2 \tan^7(e+fx)}{7f(a \sec(e+fx)+a)^{7/2}} + \frac{6a^5c^2 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^4c^2 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} - \frac{2a^3c^2 \tan(e+fx)}{f\sqrt{a}}$$

[Out] $2a^{5/2}c^2 \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/f - 2a^3c^2 \tan(fx+e)/f/(a+a \sec(fx+e))^{1/2} + 2/3 a^4c^2 \tan^3(fx+e)/f/(a+a \sec(fx+e))^{3/2} + 6/5 a^5c^2 \tan^5(fx+e)/f/(a+a \sec(fx+e))^{5/2} + 2/7 a^6c^2 \tan^7(fx+e)/f/(a+a \sec(fx+e))^{7/2}$

Rubi [A] time = 0.17, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^6c^2 \tan^7(e+fx)}{7f(a \sec(e+fx)+a)^{7/2}} + \frac{6a^5c^2 \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} + \frac{2a^4c^2 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{5/2}c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^3c^2 \tan(e+fx)}{f\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]

[Out] $(2a^{5/2}c^2 \text{ArcTan}[\frac{\sqrt{a} \tan[e + f*x]}{\sqrt{a + a \sec[e + f*x]}}])/f - (2a^3c^2 \tan[e + f*x])/(f \sqrt{a + a \sec[e + f*x]}) + (2a^4c^2 \tan^3[e + f*x])/(3f(a + a \sec[e + f*x])^{3/2}) + (6a^5c^2 \tan^5[e + f*x])/(5f(a + a \sec[e + f*x])^{5/2}) + (2a^6c^2 \tan^7[e + f*x])/(7f(a + a \sec[e + f*x])^{7/2})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.))/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^2 dx &= (a^2 c^2) \int \sqrt{a + a \sec(e + fx)} \tan^4(e + fx) dx \\
&= \frac{(2a^5 c^2) \operatorname{Subst}\left(\int \frac{x^4(2+ax^2)^2}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= \frac{(2a^5 c^2) \operatorname{Subst}\left(\int \left(-\frac{1}{a^2} + \frac{x^2}{a} + 3x^4 + ax^6 + \frac{1}{a^2(1+ax^2)}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{2a^3 c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^4 c^2 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} + \frac{6a^5 c^2}{5f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2a^{5/2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} - \frac{2a^3 c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{2a^5 c^2}{3f(a + a \sec(e + fx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.92, size = 124, normalized size = 0.70

$$\frac{2a^2 c^2 \tan\left(\frac{1}{2}(e + fx)\right) \sec^3(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((51 \cos(e + fx) + 23 \cos(2(e + fx)) + 23 \cos(3(e + fx))) \right)}{105f \sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]

[Out] (-2*a^2*c^2*(-105*ArcTan[Sqrt[-1 + Sec[e + f*x]]])*Cos[e + f*x]^3 + (8 + 51*Cos[e + f*x] + 23*Cos[2*(e + f*x)] + 23*Cos[3*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(105*f*Sqrt[-1 + Sec[e + f*x]])

fricas [A] time = 0.49, size = 409, normalized size = 2.31

$$\frac{105 \left(a^2 c^2 \cos^4(fx + e) + a^2 c^2 \cos^3(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e) + a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e) - a}{\cos(fx+e) + 1} \right)}{105 \left(f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(92*a^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (92*a^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c - \frac{c}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int a^2 \sqrt{a \sec(e + fx) + a} dx + \int \left(-2a^2 \sqrt{a \sec(e + fx) + a} \sec^2(e + fx) \right) dx + \int a^2 \sqrt{a \sec(e + fx) + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**2,x)

[Out] c**2*(Integral(a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x))

3.59 $\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx)) dx$

Optimal. Leaf size=132

$$\frac{2a^{5/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^5c \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} - \frac{2a^4c \tan^3(e+fx)}{f(a \sec(e+fx)+a)^{3/2}} - \frac{2a^3c \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

[Out] $2*a^{(5/2)}*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f-2*a^3*c*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2*a^4*c*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}-2/5*a^5*c*\tan(f*x+e)^5/f/(a+a*\sec(f*x+e))^{(5/2)}$

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3904, 3887, 461, 203}

$$-\frac{2a^5c \tan^5(e+fx)}{5f(a \sec(e+fx)+a)^{5/2}} - \frac{2a^4c \tan^3(e+fx)}{f(a \sec(e+fx)+a)^{3/2}} + \frac{2a^{5/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} - \frac{2a^3c \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] `Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]`

[Out] $(2*a^{(5/2)}*c*\text{ArcTan}[\text{Sqrt}[a]*\text{Tan}[e + f*x]]/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/f - (2*a^3*c*\text{Tan}[e + f*x]/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*a^4*c*\text{Tan}[e + f*x]^3)/(f*(a + a*\text{Sec}[e + f*x])^{(3/2)}) - (2*a^5*c*\text{Tan}[e + f*x]^5)/(5*f*(a + a*\text{Sec}[e + f*x])^{(5/2)})$

Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

Rule 461

`Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])`

Rule 3887

`Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]`

Rule 3904

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])`

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx)) dx &= - \left((ac) \int (a + a \sec(e + fx))^{3/2} \tan^2(e + fx) dx \right) \\
&= \frac{(2a^4c) \operatorname{Subst} \left(\int \frac{x^2(2+ax^2)^2}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= \frac{(2a^4c) \operatorname{Subst} \left(\int \left(\frac{1}{a} + 3x^2 + ax^4 - \frac{1}{a(1+ax^2)} \right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= -\frac{2a^3c \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2a^4c \tan^3(e + fx)}{f(a + a \sec(e + fx))^{3/2}} - \frac{2a^5c \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2a^{5/2}c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} - \frac{2a^3c \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2a^4c \tan^3(e + fx)}{f(a + a \sec(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 110, normalized size = 0.83

$$\frac{a^2c \tan\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left((6 \cos(e + fx) + \cos(2(e + fx)) + 3) \sqrt{\sec(e + fx) - 1} \right)}{5f\sqrt{\sec(e + fx) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]

[Out] -1/5*(a^2*c*(-10*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2 + (3 + 6*Cos[e + f*x] + Cos[2*(e + f*x)])*Sqrt[-1 + Sec[e + f*x]]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[-1 + Sec[e + f*x]]))

fricas [A] time = 0.50, size = 353, normalized size = 2.67

$$\frac{5 \left(a^2c \cos^3(fx + e) + a^2c \cos^2(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right)}{5 \left(f \cos^3(fx + e) + f \cos^2(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(a^2*c*cos(f*x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c*cos(f*x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

$$\begin{aligned} &^2 \cos(2fx + 2e)^2 + a^2 \sin(2fx + 2e)^2 + 2a^2 \cos(2fx + 2e) + a \\ &^2 \arctan2((\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + \\ &1)^{1/4} (\cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))), \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) - \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) \\ &), (\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) + \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) \\ &+ 1) - (a^2 \cos(2fx + 2e)^2 + a^2 \sin(2fx + 2e)^2 + 2a^2 \cos(2fx + 2e) + a^2 \arctan2((\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) - \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) \\ &), (\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) + \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) \\ &- 1) - (a^2 \cos(2fx + 2e)^2 + a^2 \sin(2fx + 2e)^2 + 2a^2 \cos(2fx + 2e) + a^2 \arctan2((\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))), (\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) + 1) + (a^2 \cos(2fx + 2e)^2 + a^2 \sin(2fx + 2e)^2 + 2a^2 \cos(2fx + 2e) + a^2 \arctan2((\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))), (\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) - 1) \sqrt{a} \\ &) * c / ((\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1) * f) \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c - \frac{c}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \left(\int \left(-a^2 \sqrt{a \sec(e + fx) + a} \right) dx + \int \left(-a^2 \sqrt{a \sec(e + fx) + a} \sec(e + fx) \right) dx + \int a^2 \sqrt{a \sec(e + fx) + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e)),x)

[Out] -c*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))

$$3.60 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{c-c \sec(e+fx)} dx$$

Optimal. Leaf size=103

$$\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} - \frac{2a^3 \tan(e+fx)}{cf \sqrt{a \sec(e+fx)+a}} + \frac{8a^2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{cf}$$

[Out] $2a^{5/2} \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/c/f+8a^2 \cot(fx+e) \sqrt{a \sec(fx+e)+a}/c/f-2a^3 \tan(fx+e)/c/f/(a+a \sec(fx+e))^{1/2}$

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{8a^2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{cf} + \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} - \frac{2a^3 \tan(e+fx)}{cf \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]

[Out] $(2a^{5/2} \text{ArcTan}[\text{Sqrt}[a] \text{Tan}[e + fx]]/\text{Sqrt}[a + a \text{Sec}[e + fx]])/(c*f) + (8a^2 \text{Cot}[e + fx] \text{Sqrt}[a + a \text{Sec}[e + fx]])/(c*f) - (2a^3 \text{Tan}[e + fx])/(c*f \text{Sqrt}[a + a \text{Sec}[e + fx]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^(m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{c - c \sec(e + fx)} dx &= -\frac{\int \cot^2(e + fx)(a + a \sec(e + fx))^{7/2} dx}{ac} \\
&= \frac{(2a^2) \text{Subst}\left(\int \frac{(2+ax^2)^2}{x^2(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{(2a^2) \text{Subst}\left(\int \left(a + \frac{4}{x^2} - \frac{a}{1+ax^2}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{8a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{cf} - \frac{2a^3 \tan(e + fx)}{cf\sqrt{a + a \sec(e + fx)}} - \frac{(2a^3) \text{Subst}\left(\int \frac{1}{x} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} + \frac{8a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{cf} - \frac{2a^3 \tan(e + fx)}{cf\sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 0.72, size = 96, normalized size = 0.93

$$\frac{2a^3 \tan(e + fx) \sec(e + fx) \left((5 \cos(e + fx) - 1) \sqrt{\sec(e + fx) - 1} - (\cos(e + fx) - 1) \tan^{-1} \left(\sqrt{\sec(e + fx) - 1} \right) \right)}{cf (\sec(e + fx) - 1)^{3/2} \sqrt{a (\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]

[Out] (2*a^3*(-(ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(-1 + Cos[e + f*x]))) + (-1 + 5*Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]*Sec[e + f*x]*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.58, size = 291, normalized size = 2.83

$$\frac{\sqrt{-a} a^2 \log \left(\frac{8 a \cos(fx+e)^3 - 4 \left(2 \cos(fx+e)^2 - \cos(fx+e) \right) \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) - 7 a \cos(fx+e) + a}{\cos(fx+e)+1} \right) \sin(fx+e) + 4 \left(5 a^2 \right)}{2 c f \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(sqrt(-a)*a^2*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a^2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e)), (a^(5/2))*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(5*a^2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

[Out] `int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2 \sqrt{a \sec(e+fx)+a}}{\sec(e+fx)-1} dx + \int \frac{2a^2 \sqrt{a \sec(e+fx)+a} \sec(e+fx)}{\sec(e+fx)-1} dx + \int \frac{a^2 \sqrt{a \sec(e+fx)+a} \sec^2(e+fx)}{\sec(e+fx)-1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)`

[Out] `-(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x) - 1), x))/c`

$$3.61 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=74

$$\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^2 f} - \frac{8a \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3c^2 f}$$

[Out] $2*a^{(5/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^2/f-8/3*a*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/c^2/f$

Rubi [A] time = 0.17, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^2 f} - \frac{8a \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3c^2 f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(5/2)}/(c - c*\text{Sec}[e + f*x])^2, x]$

[Out] $(2*a^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]]])/(c^2*f) - (8*a*\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^{(3/2)})/(3*c^2*f)$

Rule 203

$\text{Int}[(a + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

$\text{Int}[(e_*)*(x_)^{(m_*)}*((a + (b_*)*(x_)^{(n_*)})^{(p_*)})/((c_*) + (d_*)*(x_)^{(n_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p/(c + d*x^n), x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

$\text{Int}[\cot[(c_*) + (d_*)*(x_)]^{(m_*)}*(\csc[(c_*) + (d_*)*(x_)]*(b_*) + (a_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2 + a*x^2)^{(m/2 + n - 1/2)})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]]], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

$\text{Int}[(\csc[(e_*) + (f_*)*(x_)]*(b_*) + (a_*)^{(m_*)}*(\csc[(e_*) + (f_*)*(x_)]*(d_*) + (c_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n - m)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^2} dx &= \frac{\int \cot^4(e + fx)(a + a \sec(e + fx))^{9/2} dx}{a^2 c^2} \\
&= \frac{(2a) \text{ Subst} \left(\int \frac{(2+ax^2)^2}{x^4(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{c^2 f} \\
&= \frac{(2a) \text{ Subst} \left(\int \left(\frac{4}{x^4} + \frac{a^2}{1+ax^2} \right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{c^2 f} \\
&= -\frac{8a \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^2 f} - \frac{(2a^3) \text{ Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{c^2 f} \\
&= \frac{2a^{5/2} \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{c^2 f} - \frac{8a \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^2 f}
\end{aligned}$$

Mathematica [A] time = 4.22, size = 102, normalized size = 1.38

$$\frac{\cos^{\frac{5}{2}}(e + fx) \csc^3\left(\frac{1}{2}(e + fx)\right) \sec^5\left(\frac{1}{2}(e + fx)\right) (a(\sec(e + fx) + 1))^{5/2} \left(4 \cos^{\frac{3}{2}}(e + fx) - 3(1 - \cos(e + fx))\right)^{3/2}}{24c^2 f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]

[Out] -1/24*(Cos[e + f*x]^(5/2)*(-3*ArcSin[Sqrt[1 - Cos[e + f*x]]])*(1 - Cos[e + f*x])^(3/2) + 4*Cos[e + f*x]^(3/2))*Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2))/(c^2*f)

fricas [B] time = 0.57, size = 339, normalized size = 4.58

$$\frac{16 a^2 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e)^2 + 3 (a^2 \cos(fx+e) - a^2) \sqrt{-a} \log \left(-\frac{8 a \cos(fx+e)^3 - 4 (2 \cos(fx+e)^2 - \cos(fx+e)) \sqrt{-a}}{\cos(fx+e)} \right)}{6 (c^2 f \cos(fx+e) - c^2 f) \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/6*(16*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(8*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError


```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2,x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

$$3.62 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=104

$$\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^3 f} + \frac{2a^2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{c^3 f} + \frac{8 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5c^3 f}$$

[Out] $2*a^{(5/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^3/f+8/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/c^3/f+2*a^2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^3/f$

Rubi [A] time = 0.18, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{c^3 f} + \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^3 f} + \frac{8 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5c^3 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]

[Out] $(2*a^{(5/2)}*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c^3*f) + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^3*f) + (8*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^{(5/2)})/(5*c^3*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.))/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^3} dx &= -\frac{\int \cot^6(e + fx)(a + a \sec(e + fx))^{11/2} dx}{a^3 c^3} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{(2+ax^2)^2}{x^6(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^3 f} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(\frac{4}{x^6} + \frac{a^2}{x^2} - \frac{a^3}{1+ax^2}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^3 f} \\
&= \frac{2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^3 f} + \frac{8 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5c^3 f} - \frac{(2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)})^2}{c^3 f} \\
&= \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^3 f} + \frac{2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^3 f} + \frac{8 \cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5c^3 f}
\end{aligned}$$

Mathematica [C] time = 5.33, size = 196, normalized size = 1.88

$$a^2 \sqrt{\cos(e + fx)} \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(4(20 \cos(e + fx) - 15 \cos(2(e + fx)) - 29) {}_2F_1\left(-\frac{5}{2}, -\frac{1}{2}, \frac{1}{2}, 2 \sin\left(\frac{e + fx}{2}\right)^2\right) + 5 \sqrt{\cos(e + fx)} (11 \cos(e + fx) + 3 \cos(2(e + fx))) \sin(e + fx)^2 \tan\left(\frac{e + fx}{2}\right)\right) / (60 c^3 f (-1 + \cos(e + fx))^3)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]

[Out] (a^2*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*(30*ArcSin[Sqrt[1 - Cos[e + f*x]]]*Cos[(e + f*x)/2]^2*Sqrt[1 - Cos[e + f*x]]*(-1 + 7*Cos[e + f*x]) + 4*(-29 + 20*Cos[e + f*x] - 15*Cos[2*(e + f*x)])*Hypergeometric2F1[-5/2, -1/2, 1/2, 2*Sin[(e + f*x)/2]^2] + 5*Sqrt[Cos[e + f*x]]*(11*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Sin[e + f*x]^2*Tan[(e + f*x)/2])/(60*c^3*f*(-1 + Cos[e + f*x])^3)

fricas [B] time = 0.57, size = 441, normalized size = 4.24

$$\frac{5 \left(a^2 \cos^2(fx + e) - 2a^2 \cos(fx + e) + a^2 \right) \sqrt{-a} \log \left(-\frac{8a \cos^3(fx+e) - 4(2 \cos^2(fx+e) - \cos(fx+e)) \sqrt{-a} \sqrt{\frac{a \cos(fx+e) + a}{\cos(fx+e)}} \sin(fx+e) - 7a \cos(fx+e) + a}{\cos(fx+e) + 1} \right)}{10 \left(c^3 f \cos^2(fx + e) - 2c^3 f \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/10*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/5*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^2 + 5*a^2*cos(f*x + e))*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e)]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**3,x)

[Out] Timed out

$$3.63 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^4} dx$$

Optimal. Leaf size=140

$$\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^4 f} + \frac{2a^2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{c^4 f} - \frac{8 \cot^7(e+fx)(a \sec(e+fx)+a)^{7/2}}{7ac^4 f} - \frac{2a \cot^3(e+fx)}{7ac^4 f}$$

[Out] $2*a^{(5/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^4/f-2/3*a*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/c^4/f-8/7*\cot(f*x+e)^7*(a+a*\sec(f*x+e))^{(7/2)}/a/c^4/f+2*a^2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^4/f$

Rubi [A] time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{2a^2 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{c^4 f} + \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^4 f} - \frac{8 \cot^7(e+fx)(a \sec(e+fx)+a)^{7/2}}{7ac^4 f} - \frac{2a \cot^3(e+fx)}{7ac^4 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]

[Out] $(2*a^{(5/2)}*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(c^4*f) + (2*a^2*\cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^4*f) - (2*a*\cot[e + f*x]^3*(a + a*Sec[e + f*x])^{(3/2)})/(3*c^4*f) - (8*\cot[e + f*x]^7*(a + a*Sec[e + f*x])^{(7/2)})/(7*a*c^4*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^4} dx &= \frac{\int \cot^8(e + fx)(a + a \sec(e + fx))^{13/2} dx}{a^4 c^4} \\
&= -\frac{2 \operatorname{Subst}\left(\int \frac{(2+ax^2)^2}{x^8(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^4 f} \\
&= -\frac{2 \operatorname{Subst}\left(\int \left(\frac{4}{x^8} + \frac{a^2}{x^4} - \frac{a^3}{x^2} + \frac{a^4}{1+ax^2}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{ac^4 f} \\
&= \frac{2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^4 f} - \frac{2a \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^4 f} - \frac{8a^2 \cot^5(e + fx)(a + a \sec(e + fx))^{1/2}}{15c^4 f} \\
&= \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^4 f} + \frac{2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^4 f} - \frac{2a \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^4 f}
\end{aligned}$$

Mathematica [C] time = 7.98, size = 361, normalized size = 2.58

$$\sin^8\left(\frac{e}{2} + \frac{fx}{2}\right) \sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}} \sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)} \csc^7\left(\frac{1}{2}(e+fx)\right) \sec^5\left(\frac{1}{2}(e+fx)\right) \sec^{\frac{3}{2}}(e+fx)(a \sec(e+fx))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
[Out] -1/210*(Csc[(e + f*x)/2]^7*Sec[(e + f*x)/2]^5*Sec[e + f*x]^(3/2)*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2]^8*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(336*Hypergeometric2F1[-5/2, -1/2, 1/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2*(3 - 8*Sin[(e + f*x)/2]^2 + 5*Sin[(e + f*x)/2]^4) + 4*Hypergeometric2F1[-7/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2]*(15 - 42*Sin[(e + f*x)/2]^2 + 35*Sin[(e + f*x)/2]^4) - 105*Cos[(e + f*x)/2]^4*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[2*Sin[(e + f*x)/2]^2]]*(Sin[(e + f*x)/2]^2)^(3/2) + 2*Sin[(e + f*x)/2]^4*(5 - 4*Sin[(e + f*x)/2]^2)*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/(f*(c - c*Sec[e + f*x])^4)
```

fricas [B] time = 0.57, size = 527, normalized size = 3.76

$$\left[\frac{21 \left(a^2 \cos^3(fx + e) - 3a^2 \cos^2(fx + e) + 3a^2 \cos(fx + e) - a^2 \right) \sqrt{-a} \log \left(\frac{8a \cos^3(fx+e) - 4(2 \cos^2(fx+e) - \cos(fx+e)) \sqrt{-a}}{42 \left(c^4 f \cos(fx + e) \right)} \right)}{42 \left(c^4 f \cos(fx + e) \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
[Out] [1/42*(21*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e)))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(40*a^2*cos(f*x + e)^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x + e))*sqrt(-a)]
```


maple [B] time = 1.63, size = 395, normalized size = 2.82

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(-21 (\cos^3(fx+e)) \sin(fx+e) \sqrt{2} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \right) + 63$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x)

[Out] 1/21/c^4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-21*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+63*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))-63*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+80*cos(f*x+e)^4+21*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-154*cos(f*x+e)^3+140*cos(f*x+e)^2-42*cos(f*x+e))/sin(f*x+e)/(-1+cos(f*x+e))^3*a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4,x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**4,x)

[Out] Timed out

$$3.64 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^5} dx$$

Optimal. Leaf size=172

$$\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^5 f} + \frac{8 \cot^9(e+fx)(a \sec(e+fx)+a)^{9/2}}{9a^2 c^5 f} + \frac{2a^2 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^5 f} + \frac{2 \cot^5(e+fx)}{c^5 f}$$

[Out] $2*a^{(5/2)}*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^5/f-2/3*a*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/c^5/f+2/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/c^5/f+8/9*\cot(f*x+e)^9*(a+a*\sec(f*x+e))^{(9/2)}/a^2/c^5/f+2*a^2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/c^5/f$

Rubi [A] time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3904, 3887, 461, 203}

$$\frac{8 \cot^9(e+fx)(a \sec(e+fx)+a)^{9/2}}{9a^2 c^5 f} + \frac{2a^2 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{c^5 f} + \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{c^5 f} + \frac{2 \cot^5(e+fx)}{c^5 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5, x]

[Out] $(2*a^{(5/2)}*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(c^5*f) + (2*a^2*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c^5*f) - (2*a*Cot[e + f*x])^3*(a + a*Sec[e + f*x])^{(3/2)}/(3*c^5*f) + (2*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^{(5/2)})/(5*c^5*f) + (8*Cot[e + f*x]^9*(a + a*Sec[e + f*x])^{(9/2)})/(9*a^2*c^5*f)$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3887

Int[cot[(c_) + (d_)*(x_)]^(m_)*(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^5} dx &= -\frac{\int \cot^{10}(e + fx)(a + a \sec(e + fx))^{15/2} dx}{a^5 c^5} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{(2+ax^2)^2}{x^{10}(1+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2 c^5 f} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(\frac{4}{x^{10}} + \frac{a^2}{x^6} - \frac{a^3}{x^4} + \frac{a^4}{x^2} - \frac{a^5}{1+ax^2}\right) dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2 c^5 f} \\
&= \frac{2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^5 f} - \frac{2a \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^5 f} + \frac{2}{3} \\
&= \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{c^5 f} + \frac{2a^2 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{c^5 f} - \frac{2a \cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3c^5 f}
\end{aligned}$$

Mathematica [C] time = 3.56, size = 205, normalized size = 1.19

$$a^2 \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(-15(\cos(e + fx) - 2 \cos(2(e + fx))) - \cos(3(e + fx)) + 2\right) {}_3F_2\left(-\frac{7}{2}, -\frac{3}{2}, -\frac{1}{2}, 2, 2\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]

[Out] (a^2*Sqrt[a*(1 + Sec[e + f*x])]*((109 + 108*Cos[e + f*x] + 63*Cos[2*(e + f*x)])*Hypergeometric2F1[-9/2, -5/2, -3/2, 2*Sin[(e + f*x)/2]^2] - 15*(2 + Cos[e + f*x] - 2*Cos[2*(e + f*x)] - Cos[3*(e + f*x)])*HypergeometricPFQ[{-7/2, -3/2, 2}, {-1/2, 1}, 2*Sin[(e + f*x)/2]^2] + 240*(1 + 2*Cos[e + f*x])*Hypergeometric2F1[-7/2, -3/2, -1/2, 2*Sin[(e + f*x)/2]^2]*Sin[e + f*x]^2)*Tan[(e + f*x)/2])/(315*c^5*f*Cos[e + f*x]^(9/2)*(-1 + Sec[e + f*x])^5)

fricas [A] time = 0.58, size = 601, normalized size = 3.49

$$\frac{45 \left(a^2 \cos^4(fx + e) - 4 a^2 \cos^3(fx + e) + 6 a^2 \cos^2(fx + e) - 4 a^2 \cos(fx + e) + a^2 \right) \sqrt{-a} \log\left(-\frac{8 a \cos(fx + e)}{90 \left(a^2 \cos^4(fx + e) - 4 a^2 \cos^3(fx + e) + 6 a^2 \cos^2(fx + e) - 4 a^2 \cos(fx + e) + a^2 \right)} \right)}{90 \left(a^2 \cos^4(fx + e) - 4 a^2 \cos^3(fx + e) + 6 a^2 \cos^2(fx + e) - 4 a^2 \cos(fx + e) + a^2 \right) \sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] [1/90*(45*(a^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2 - 4*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x + e)^3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f*cos(f*x + e))]

$(\sqrt{-a \tan(1/2 * (f * x + \exp(1)))^2 + a} - \sqrt{-a} * \tan(1/2 * (f * x + \exp(1))))^2 * \text{sign}(\cos(f * x + \exp(1))) / c^5 / ((\sqrt{-a \tan(1/2 * (f * x + \exp(1)))^2 + a} - \sqrt{-a} * \tan(1/2 * (f * x + \exp(1))))^2 - a)^{9/4} * a^3 * \sqrt{-a} * \text{sign}(\cos(f * x + \exp(1))) * \ln(\text{abs}(2 * (\sqrt{-a \tan(1/2 * (f * x + \exp(1)))^2 + a} - \sqrt{-a} * \tan(1/2 * (f * x + \exp(1))))^2 - 4 * \sqrt{2} * \text{abs}(a) - 6 * a) / \text{abs}(2 * (\sqrt{-a \tan(1/2 * (f * x + \exp(1)))^2 + a} - \sqrt{-a} * \tan(1/2 * (f * x + \exp(1))))^2 + 4 * \sqrt{2} * \text{abs}(a) - 6 * a)) / c^5 / \text{abs}(a)) / f$

maple [B] time = 1.90, size = 492, normalized size = 2.86

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (1 + \cos(fx + e)) \left(-45 \sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}} (\cos^4(fx + e)) \sin(fx + e) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}}}{2\cos(fx+e)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x)

[Out] $-1/45/c^5/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*(1+\cos(f*x+e))*(-45*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\cos(f*x+e)^4*\sin(f*x+e)*2^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2}))+180*\cos(f*x+e)^3*\sin(f*x+e)*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2}))-270*\sin(f*x+e)*\cos(f*x+e)^2*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2}))+178*\cos(f*x+e)^5+180*\cos(f*x+e)*\sin(f*x+e)*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2}))-486*\cos(f*x+e)^4-45*2^{1/2}*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2}))*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}+648*\cos(f*x+e)^3-390*\cos(f*x+e)^2+90*\cos(f*x+e))/\sin(f*x+e)^3/(-1+\cos(f*x+e))^3*a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5,x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**5,x)
```

```
[Out] Timed out
```


$$3.65 \quad \int \frac{(c - c \sec(e + fx))^4}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=185

$$\frac{2a^2c^4 \tan^5(e + fx)}{5f(a \sec(e + fx) + a)^{5/2}} + \frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a \sec(e + fx) + a}}\right)}{\sqrt{a} f} - \frac{16\sqrt{2}c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{2} \sqrt{a \sec(e + fx) + a}}\right)}{\sqrt{a} f} - \frac{2ac^4 \tan^3(e + fx)}{f(a \sec(e + fx) + a)^{3/2}}$$

[Out] $2*c^4*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f/a^{(1/2)}-16*c^4*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/f/a^{(1/2)}+14*c^4*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2*a*c^4*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}+2/5*a^2*c^4*\tan(f*x+e)^5/f/(a+a*\sec(f*x+e))^{(5/2)}$

Rubi [A] time = 0.28, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 479, 582, 522, 203}

$$\frac{2a^2c^4 \tan^5(e + fx)}{5f(a \sec(e + fx) + a)^{5/2}} - \frac{2ac^4 \tan^3(e + fx)}{f(a \sec(e + fx) + a)^{3/2}} + \frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a \sec(e + fx) + a}}\right)}{\sqrt{a} f} - \frac{16\sqrt{2}c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{2} \sqrt{a \sec(e + fx) + a}}\right)}{\sqrt{a} f}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]], x]

[Out] $(2*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f) - (16*Sqrt[2]*c^4*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*f) + (14*c^4*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) - (2*a*c^4*Tan[e + f*x]^3)/(f*(a + a*Sec[e + f*x])^{(3/2)}) + (2*a^2*c^4*Tan[e + f*x]^5)/(5*f*(a + a*Sec[e + f*x])^{(5/2)})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 479

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1))

```
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))]*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n
_.), x_Symbol] :=> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_.))^(n_.), x_Symbol] :=> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c - c \sec(e + fx))^4}{\sqrt{a + a \sec(e + fx)}} dx = (a^4 c^4) \int \frac{\tan^8(e + fx)}{(a + a \sec(e + fx))^{9/2}} dx$$

$$= -\frac{(2a^4 c^4) \operatorname{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f}$$

$$= \frac{2a^2 c^4 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} + \frac{(2a^2 c^4) \operatorname{Subst}\left(\int \frac{x^4(10+15ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{5f}$$

$$= -\frac{2ac^4 \tan^3(e + fx)}{f(a + a \sec(e + fx))^{3/2}} + \frac{2a^2 c^4 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} - \frac{(2c^4) \operatorname{Subst}\left(\int \frac{x^2(90a+105a^2x^2)}{(1+ax^2)(2+ax^2)}\right)}{15f}$$

$$= \frac{14c^4 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2ac^4 \tan^3(e + fx)}{f(a + a \sec(e + fx))^{3/2}} + \frac{2a^2 c^4 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} + \frac{(2c^4) \operatorname{Subst}\left(\int \frac{x^2(90a+105a^2x^2)}{(1+ax^2)(2+ax^2)}\right)}{15f}$$

$$= \frac{14c^4 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2ac^4 \tan^3(e + fx)}{f(a + a \sec(e + fx))^{3/2}} + \frac{2a^2 c^4 \tan^5(e + fx)}{5f(a + a \sec(e + fx))^{5/2}} - \frac{(2c^4) \operatorname{Subst}\left(\int \frac{x^2(90a+105a^2x^2)}{(1+ax^2)(2+ax^2)}\right)}{15f}$$

$$= \frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} f} - \frac{16\sqrt{2} c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} f} + \frac{14c^4 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}}$$

Mathematica [A] time = 1.45, size = 153, normalized size = 0.83

$$\frac{c^4 \cot\left(\frac{1}{2}(e + fx)\right) \sec^3(e + fx) \left(-155 \cos(e + fx) + 96 \cos(2(e + fx)) - 41 \cos(3(e + fx)) + 20 \cos^3(e + fx)\right) \sqrt{\sec(e + fx)}}{10f\sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```


-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.97index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.97, size = 544, normalized size = 2.94

$$c^4 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(5 \sin(fx+e) \sqrt{2} (\cos^2(fx+e)) \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{5}{2}} + 10 \sin(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x)
[Out] -1/20*c^4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(5*sin(f*x+e)*2^(1/2)*cos(f*x+e)^2*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+10*sin(f*x+e)*2^(1/2)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+80*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x+e)*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e)*cos(f*x+e)^2+5*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x+e)+160*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x+e)*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e)*cos(f*x+e)+80*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*sin(f*x+e)+328*cos(f*x+e)^3-384*cos(f*x+e)^2+64*cos(f*x+e)-8)/cos(f*x+e)^2/sin(f*x+e)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)} \right)^4}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2),x)
```

[Out] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4 \left(\int \left(-\frac{4 \sec(e + fx)}{\sqrt{a \sec(e + fx) + a}} \right) dx + \int \frac{6 \sec^2(e + fx)}{\sqrt{a \sec(e + fx) + a}} dx + \int \left(-\frac{4 \sec^3(e + fx)}{\sqrt{a \sec(e + fx) + a}} \right) dx + \int \frac{\sec(e + fx)}{\sqrt{a \sec(e + fx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(1/2),x)`

[Out] `c**4*(Integral(-4*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(6*sec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(-4*sec(e + f*x)**3/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**4/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))`

$$3.66 \quad \int \frac{(c - c \sec(e + fx))^3}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=152

$$\frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{8\sqrt{2} c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{2ac^3 \tan^3(e+fx)}{3f(a \sec(e+fx) + a)^{3/2}} + \frac{6c^3 \tan(e+fx)}{f\sqrt{a \sec(e+fx) + a}}$$

[Out] $2c^3 \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/f/a^{1/2} - 8c^3 \arctan(1/2 a^{1/2} \tan(fx+e) 2^{1/2}/(a+a \sec(fx+e))^{1/2}) 2^{1/2}/f/a^{1/2} + 6c^3 \tan(fx+e)/f/(a+a \sec(fx+e))^{1/2} - 2/3 a c^3 \tan(fx+e)^3/f/(a+a \sec(fx+e))^{3/2}$

Rubi [A] time = 0.23, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 479, 582, 522, 203}

$$-\frac{2ac^3 \tan^3(e+fx)}{3f(a \sec(e+fx) + a)^{3/2}} + \frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{8\sqrt{2} c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} + \frac{6c^3 \tan(e+fx)}{f\sqrt{a \sec(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]], x]

[Out] $(2c^3 \text{ArcTan}[(\text{Sqrt}[a] \text{Tan}[e + fx])/\text{Sqrt}[a + a \text{Sec}[e + fx]])/(\text{Sqrt}[a] f) - (8 \text{Sqrt}[2] c^3 \text{ArcTan}[(\text{Sqrt}[a] \text{Tan}[e + fx])/(\text{Sqrt}[2] \text{Sqrt}[a + a \text{Sec}[e + fx]])])/(\text{Sqrt}[a] f) + (6c^3 \text{Tan}[e + fx])/(f \text{Sqrt}[a + a \text{Sec}[e + fx]]) - (2a c^3 \text{Tan}[e + fx]^3)/(3f(a + a \text{Sec}[e + fx])^{3/2})$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 479

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IntQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +

$b*x^n)^p*(c + d*x^n)^q*\text{Simp}[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1]$

Rule 3887

$\text{Int}[\text{cot}[(c_.) + (d_.)*(x_.)]^{(m_.)}*(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2 + a*x^2)^{(m/2 + n - 1/2)})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n - m)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^3}{\sqrt{a + a \sec(e + fx)}} dx &= - \left((a^3 c^3) \int \frac{\tan^6(e + fx)}{(a + a \sec(e + fx))^{7/2}} dx \right) \\ &= \frac{(2a^3 c^3) \text{Subst} \left(\int \frac{x^6}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{2ac^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{(2ac^3) \text{Subst} \left(\int \frac{x^2(6+9ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{3f} \\ &= \frac{6c^3 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2ac^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} + \frac{(2c^3) \text{Subst} \left(\int \frac{18a+21a^2x^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{3af} \\ &= \frac{6c^3 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2ac^3 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{(2c^3) \text{Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= \frac{2c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{a} f} - \frac{8\sqrt{2} c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{a} f} + \frac{6c^3 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.33, size = 166, normalized size = 1.09

$$\frac{4c^3 \cos\left(\frac{e}{2}\right) \cos(e) \cot\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \left(11 \cos(e + fx) - 5 \cos(2(e + fx)) + 3 \cos^2(e + fx)\right) \sqrt{\sec(e + fx)}}{3f \left(\cos\left(\frac{e}{2}\right) + \cos\left(\frac{3e}{2}\right)\right) \sqrt{a \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]], x]

[Out] (4*c^3*Cos[e/2]*Cos[e]*Cot[(e + f*x)/2]*(-6 + 11*Cos[e + f*x] - 5*Cos[2*(e + f*x)] + 3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 12*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2

```
*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(3*f*(Cos[e/2] + Cos[(3*e)/2])*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 1.03, size = 518, normalized size = 3.41

$$12\sqrt{2}\left(ac^3\cos(fx+e)^2 + ac^3\cos(fx+e)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] [1/3*(12*sqrt(2)*(a*c^3*cos(f*x + e)^2 + a*c^3*cos(f*x + e))*sqrt(-1/a)*log
((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)
*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - 3*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(10*
c^3*cos(f*x + e) - c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -2/3*(3*(c^3*cos(f*x + e)^2 + c
^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)/(sqrt(a)*sin(f*x + e))) - (10*c^3*cos(f*x + e) - c^3)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 12*sqrt(2)*(a*c^3*cos(f*x + e)^
2 + a*c^3*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^2 + a*f
*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by interval
```


Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.67index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.92, size = 372, normalized size = 2.45

$$c^3 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(3 \sin(fx+e) \cos(fx+e) \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{3}{2}} \sqrt{2} + 3\sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)

[Out] 1/6*c^3/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(3*sin(f*x+e)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)+3*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)+24*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e)*cos(f*x+e)+24*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)-40*cos(f*x+e)^2+44*cos(f*x+e)-4)/sin(f*x+e)/cos(f*x+e)/a

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^3}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int \frac{3 \sec(e + fx)}{\sqrt{a \sec(e + fx) + a}} dx + \int \left(-\frac{3 \sec^2(e + fx)}{\sqrt{a \sec(e + fx) + a}} \right) dx + \int \frac{\sec^3(e + fx)}{\sqrt{a \sec(e + fx) + a}} dx + \int \left(-\frac{1}{\sqrt{a \sec(e + fx) + a}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)

[Out] -c**3*(Integral(3*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-3*sec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**3/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))

$$3.67 \quad \int \frac{(c - c \sec(e + fx))^2}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=119

$$\frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{4\sqrt{2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} + \frac{2c^2 \tan(e+fx)}{f \sqrt{a \sec(e+fx)+a}}$$

[Out] $2*c^2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f/a^{(1/2)}-4*c^2*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/f/a^{(1/2)}+2*c^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 479, 522, 203}

$$\frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{4\sqrt{2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} + \frac{2c^2 \tan(e+fx)}{f \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]

[Out] $(2*c^2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*f) - (4*\text{Sqrt}[2]*c^2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(\text{Sqrt}[a]*f) + (2*c^2*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 479

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{(c - c \sec(e + fx))^2}{\sqrt{a + a \sec(e + fx)}} dx = (a^2 c^2) \int \frac{\tan^4(e + fx)}{(a + a \sec(e + fx))^{5/2}} dx$$

$$= \frac{(2a^2 c^2) \text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + \frac{(2c^2) \text{Subst}\left(\int \frac{2+3ax^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} - \frac{(2c^2) \text{Subst}\left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} + \frac{(8c^2) \text{Subst}\left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f}$$

$$= \frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} f} - \frac{4\sqrt{2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} f} + \frac{2c^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}}$$

Mathematica [A] time = 0.51, size = 124, normalized size = 1.04

$$\frac{2c^2 \cot\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \left(-\cos(e + fx) + \cos(e + fx) \sqrt{\sec(e + fx) - 1} \tan^{-1}\left(\sqrt{\sec(e + fx) - 1}\right) - 2\sqrt{2} \cos(e + fx)\right)}{f \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
[Out] (2*c^2*Cot[(e + f*x)/2]*(1 - Cos[e + f*x] + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [A] time = 0.73, size = 438, normalized size = 3.68

$$2c^2 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx + e) + 2\sqrt{2} (ac^2 \cos(fx + e) + ac^2) \sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{-\frac{1}{a}} \cos(fx+e) \sin(fx+e) + \dots}{\cos(fx+e)^2 + 2 \cos(fx+e) \dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 2*sqrt(2)*(a
*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 +
2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - (c^2*cos(f*x
+ e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos
(f*x + e) + 1)))/(a*f*cos(f*x + e) + a*f), 2*(c^2*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2
*sqrt(2)*(a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*
x + e) + a*f)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct if
the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableUnabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
```



```
)^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)+4*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-2*sin(f*x+e))*((a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(1+cos(f*x+e)))/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
sign: argument cannot be imaginary; found %i
```

mapad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^2}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int \left(-\frac{2 \sec(e+fx)}{\sqrt{a \sec(e+fx) + a}} \right) dx + \int \frac{\sec^2(e+fx)}{\sqrt{a \sec(e+fx) + a}} dx + \int \frac{1}{\sqrt{a \sec(e+fx) + a}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))
```

$$3.68 \quad \int \frac{c - c \sec(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx$$

Optimal. Leaf size=87

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{2\sqrt{2} c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f}$$

[Out] $2*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f/a^{(1/2)}-2*c*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/f/a^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3904, 3887, 481, 203}

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{2\sqrt{2} c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]

[Out] $(2*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(\text{Sqrt}[a]*f) - (2*\text{Sqrt}[2]*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/(\text{Sqrt}[a]*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 481

Int[((e_.)*(x_)^(m_.))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned}
\int \frac{c - c \sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx &= - \left((ac) \int \frac{\tan^2(e + fx)}{(a + a \sec(e + fx))^{3/2}} dx \right) \\
&= \frac{(2ac) \operatorname{Subst} \left(\int \frac{x^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= - \frac{(2c) \operatorname{Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} + \frac{(4c) \operatorname{Subst} \left(\int \frac{1}{2+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= \frac{2c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{a} f} - \frac{2\sqrt{2} c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{a} f}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 82, normalized size = 0.94

$$\frac{2c \cot\left(\frac{1}{2}(e + fx)\right) \sqrt{\sec(e + fx) - 1} \left(\tan^{-1}\left(\sqrt{\sec(e + fx) - 1}\right) - \sqrt{2} \tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right) \right)}{f \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (2*c*(ArcTan[Sqrt[-1 + Sec[e + f*x]]] - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]])*Cot[(e + f*x)/2]*Sqrt[-1 + Sec[e + f*x]]/(f*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.60, size = 298, normalized size = 3.43

$$\frac{\sqrt{2} ac \sqrt{-\frac{1}{a}} \log \left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{-\frac{1}{a}} \cos(fx+e) \sin(fx+e) + 3 \cos(fx+e)^2 + 2 \cos(fx+e) - 1}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right) - \sqrt{-a} c \log \left(\frac{2a \cos(fx+e)^2 + 2 \sqrt{-a} \cos(fx+e) + a}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [(sqrt(2)*a*c*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a)*c*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), 2*(sqrt(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assump
tions on a variableWarning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableUnable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
```


able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.15index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [A] time = 1.52, size = 144, normalized size = 1.66

$$c \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \left(\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + 2 \ln \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) - \cos(fx+e)}{\sin(fx+e)} \right) \right) / a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)

[Out] -c/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+2*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e)))/a

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c - \frac{c}{\cos(e+fx)}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \left(\int \frac{\sec(e+fx)}{\sqrt{a \sec(e+fx) + a}} dx + \int \left(-\frac{1}{\sqrt{a \sec(e+fx) + a}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)

[Out] -c*(Integral(sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))

$$3.69 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))} dx$$

Optimal. Leaf size=121

$$\frac{\cot(e+fx)\sqrt{a \sec(e+fx)+a}}{acf} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} cf} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{2} \sqrt{a} cf}$$

[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)-1/2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f

Rubi [A] time = 0.19, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3904, 3887, 480, 522, 203}

$$\frac{\cot(e+fx)\sqrt{a \sec(e+fx)+a}}{acf} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} cf} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{2} \sqrt{a} cf}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]

[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(Sqrt[a]*c*f) - ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]*Sqrt[a]*c*f) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 480

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*e*(m+1)), x] - Dist[1/(a*c*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[(b*c+a*d)*(m+n+1)+n*(b*c*p+a*d*q)+b*d*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3887

Int[cot[(c_)+(d_)*(x_)]^(m_)*(csc[(c_)+(d_)*(x_)]*(b_)+(a_))^(n_), x_Symbol] := Dist[(-2*a^(m/2+n+1/2))/d, Subst[Int[(x^m*(2+a*x^2)^(m/2+n-1/2))/(1+a*x^2), x], x, Cot[c+d*x]/Sqrt[a+b*Csc[c+d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2-b^2, 0] && IntegerQ[m/2] && IntegerQ[n-1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))} dx &= -\frac{\int \cot^2(e + fx) \sqrt{a + a \sec(e + fx)} dx}{ac} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{acf} \\ &= \frac{\cot(e + fx) \sqrt{a + a \sec(e + fx)}}{acf} + \frac{\operatorname{Subst}\left(\int \frac{-3a - a^2 x^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{acf} \\ &= \frac{\cot(e + fx) \sqrt{a + a \sec(e + fx)}}{acf} + \frac{\operatorname{Subst}\left(\int \frac{1}{2+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} cf} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{2} \sqrt{a} cf} + \frac{\cot(e + fx)}{cf} \end{aligned}$$

Mathematica [A] time = 0.57, size = 101, normalized size = 0.83

$$\frac{\cot\left(\frac{1}{2}(e + fx)\right) \left(4\sqrt{\sec(e + fx) - 1} \tan^{-1}\left(\sqrt{\sec(e + fx) - 1}\right) - \sqrt{2} \sqrt{\sec(e + fx) - 1} \tan^{-1}\left(\frac{\sqrt{\sec(e + fx) - 1}}{\sqrt{2}}\right)\right) + 2}{2cf\sqrt{a}(\sec(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]

[Out] (Cot[(e + f*x)/2]*(2 + 4*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]] - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]])/(2*c*f*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.62, size = 436, normalized size = 3.60

$$\frac{\sqrt{2} a \sqrt{-\frac{1}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{-\frac{1}{a}} \cos(fx+e) \sin(fx+e) + 3 \cos(fx+e)^2 + 2 \cos(fx+e) - 1}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1}\right) \sin(fx+e) - 2 \sqrt{-a} \log\left(\frac{8a}{4acf}\right)}{4acf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x

```

+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) - 2*sqrt(-a)*
log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(
cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*
sin(f*x + e) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))
*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*
c*f*sin(f*x + e))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assump
tions on a variableWarning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableUnable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
```


mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c - \frac{c}{\cos(e+fx)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{a \sec(e+fx)+a} \sec(e+fx) - \sqrt{a \sec(e+fx)+a}} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)

[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) + a)), x)/c

$$3.70 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=161

$$-\frac{\cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3a^2c^2f} + \frac{3 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{2ac^2f} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}c^2f} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{2\sqrt{2} \sqrt{a}c^2f}$$

[Out] $-1/3*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{3/2}/a^2/c^2/f+2*\arctan(a^{1/2}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{1/2})/c^2/f/a^{1/2}-1/4*\arctan(1/2*a^{1/2}*\tan(f*x+e)*2^{1/2}/(a+a*\sec(f*x+e))^{1/2})/c^2/f*2^{1/2}/a^{1/2}+3/2*\cot(f*x+e)*(a+a*\sec(f*x+e))^{1/2}/a/c^2/f$

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 480, 583, 522, 203}

$$-\frac{\cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{3a^2c^2f} + \frac{3 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{2ac^2f} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}c^2f} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{2\sqrt{2} \sqrt{a}c^2f}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]

[Out] $(2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*c^2*f) - \text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(2*\text{Sqrt}[2]*\text{Sqrt}[a]*c^2*f) + (3*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(2*a*c^2*f) - (\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^{3/2})/(3*a^2*c^2*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 480

Int[((e_.)*(x_)^m)*((a_) + (b_.)*(x_)^n)^p*((c_) + (d_.)*(x_)^n)^q, x_Symbol] := Simp[((e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*e*(m+1)), x] - Dist[1/(a*c*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[(b*c+a*d)*(m+n+1)+n*(b*c*p+a*d*q)+b*d*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^n)/((a_) + (b_.)*(x_)^n)*((c_) + (d_.)*(x_)^n), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_.)*(x_)^m)*((a_) + (b_.)*(x_)^n)^p*((c_) + (d_.)*(x_)^n)^q*((e_) + (f_.)*(x_)^n), x_Symbol] := Simp[(e*(g*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*g*(m+1)), x] + Dist[1/(a*c*g^n*(m+1)), Int[(g*x)^(m+n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[a*f*c*(m+1) -


```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n
_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^2} dx &= \frac{\int \cot^4(e + fx)(a + a \sec(e + fx))^{3/2} dx}{a^2 c^2} \\ &= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2 c^2 f} \\ &= -\frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3a^2 c^2 f} - \frac{\operatorname{Subst}\left(\int \frac{-9a-3a^2x^2}{x^2(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{3a^2 c^2 f} \\ &= \frac{3 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{2ac^2 f} - \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3a^2 c^2 f} \\ &= \frac{3 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{2ac^2 f} - \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{3a^2 c^2 f} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} c^2 f} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{2\sqrt{2} \sqrt{a} c^2 f} + \frac{3 \cot(e + fx)\sqrt{a + a \sec(e + fx)}}{2ac^2 f} \end{aligned}$$

Mathematica [C] time = 24.12, size = 5576, normalized size = 34.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2), x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.60, size = 520, normalized size = 3.23

$$\left[\frac{3\sqrt{2}\sqrt{-a}(\cos(fx+e)-1)\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{\sin(fx+e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/24*(3*sqrt(2)*sqrt(-a)*(cos(f*x + e) - 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 12*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e), 1/12*(3*sqrt(2)*sqrt(a)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 12*sqrt(a)*(cos(f*x + e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
 2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
 g, integration of abs or sign assumes constant sign by intervals (correct if
 the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
 tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
 ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
 i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
 *pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
 able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
 n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep

nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
 pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
 4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
 nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
 gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
 p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
 tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
 heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
 t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
 pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
 le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
 /2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
 k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
 ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
 _nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
 2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
 ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
 ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
 step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
 /t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
 pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
 ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
 : (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
 2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
 p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
 ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
 nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
 t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discon
 tinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
 2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
 (-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
 tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
 ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*
 pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
 ck sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep^2-1)]Evaluation time: 1.03index.cc index_m i_lex_is_greater Error: Ba
 d Argument Value

maple [B] time = 1.95, size = 377, normalized size = 2.34

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(12 \sin(fx+e) (\cos^2(fx+e)) \sqrt{2} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + 3 \sqrt{\frac{2}{1+\cos(fx+e)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)`

[Out]
$$\frac{1}{12c^2f} \left(\frac{a(1+\cos(fx+e))}{\cos(fx+e)} \right)^{1/2} \left(12\sin(fx+e)\cos(fx+e)^2 \right)^{1/2} \left(\frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2} \frac{\sin(fx+e)}{\cos(fx+e)} 2^{1/2} + 3 \left(\frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2} \ln \left(\left(\frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2} \frac{\sin(fx+e) - \cos(fx+e) + 1}{\sin(fx+e)} \right) \frac{\cos(fx+e)^2 \sin(fx+e) - 12 \cdot 2^{1/2} \sin(fx+e) \operatorname{arctanh} \left(\frac{1}{2} \frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2} \frac{\sin(fx+e)}{\cos(fx+e)} 2^{1/2}}{\left(\frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2} - 3 \sin(fx+e) \left(\frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{1/2}} \right)^{1/2} \frac{\sin(fx+e) - \cos(fx+e) + 1}{\sin(fx+e)} - 22\cos(fx+e)^3 - 4\cos(fx+e)^2 + 18\cos(fx+e) \Big/ \sin(fx+e)^3/a$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec(fx+e) + a} (c \sec(fx+e) - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c - \frac{c}{\cos(e+fx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2),x)`

[Out] `int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{a \sec(e+fx)+a} \sec^2(e+fx) - 2\sqrt{a \sec(e+fx)+a} \sec(e+fx) + \sqrt{a \sec(e+fx)+a}} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)`

[Out] `Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - 2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + sqrt(a*sec(e + f*x) + a)), x)/c**2`

$$3.71 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=196

$$\frac{\cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5a^3c^3f} - \frac{\cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{2a^2c^3f} + \frac{7 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{4ac^3f} + \frac{2 \tan^{-1}}{}$$

[Out] $-1/2*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/a^2/c^3/f+1/5*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/a^3/c^3/f+2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c^3/f/a^{(1/2)}-1/8*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/c^3/f*2^{(1/2)}/a^{(1/2)}+7/4*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/a/c^3/f$

Rubi [A] time = 0.28, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 480, 583, 522, 203}

$$\frac{\cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{5a^3c^3f} - \frac{\cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{2a^2c^3f} + \frac{7 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{4ac^3f} + \frac{2 \tan^{-1}}{}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]

[Out] $(2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*c^3*f) - \text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(4*\text{Sqrt}[2]*\text{Sqrt}[a]*c^3*f) + (7*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(4*a*c^3*f) - (\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^{(3/2)})/(2*a^2*c^3*f) + (\text{Cot}[e + f*x]^5*(a + a*\text{Sec}[e + f*x])^{(5/2)})/(5*a^3*c^3*f)$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 480

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*e^(m+1)), x] - Dist[1/(a*c*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[(b*c+a*d)*(m+n+1)+n*(b*c*p+a*d*q)+b*d*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*g^(m+1)), x] + Dist[1/(a*c*g^n*(

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_.)]^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n
_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^3} dx = -\frac{\int \cot^6(e + fx)(a + a \sec(e + fx))^{5/2} dx}{a^3 c^3}$$

$$= \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^6(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^3 c^3 f}$$

$$= \frac{\cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^3 c^3 f} + \frac{\operatorname{Subst}\left(\int \frac{-15a - 5a^2 x^2}{x^4(1+ax^2)(2+ax^2)}\right)}{5a^3 c^3}$$

$$= -\frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{2a^2 c^3 f} + \frac{\cot^5(e + fx)(a + a \sec(e + fx))^{5/2}}{5a^3 c^3 f}$$

$$= \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{4ac^3 f} - \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{2a^2 c^3 f}$$

$$= \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{4ac^3 f} - \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{2a^2 c^3 f}$$

$$= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{a} c^3 f} - \frac{\tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{4\sqrt{2} \sqrt{a} c^3 f} + \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{4ac^3 f}$$

Mathematica [C] time = 23.74, size = 5592, normalized size = 28.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Result too large to show
```


tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.28index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 2.57, size = 545, normalized size = 2.78

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (1 + \cos(fx + e))^2 \left(40 \sin(fx + e) (\cos^2(fx + e)) \sqrt{2} \sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}}}{2 \cos} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)

[Out] -1/40/c^3/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1+cos(f*x+e))^2*(40*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))-80*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+40*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-10*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-98*cos(f*x+e)^3+5*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+160*cos(f*x+e)^2-70*cos(f*x+e)/sin(f*x+e)^5/a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{a \sec(fx + e) + a} (c \sec(fx + e) - c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c - \frac{c}{\cos(e+fx)} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{a \sec(e+fx)+a} \sec^3(e+fx)-3\sqrt{a \sec(e+fx)+a} \sec^2(e+fx)+3\sqrt{a \sec(e+fx)+a} \sec(e+fx)-\sqrt{a \sec(e+fx)+a}}{c^3} dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) + a)), x)/c**3
```

$$3.72 \quad \int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=203

$$\frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2}\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{8c^4 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} - \frac{14c^4 \tan(e+fx)}{af\sqrt{a \sec(e+fx)+a}}$$

[Out] $2*c^4*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/f+12*c^4*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/a^{(3/2)}/f-14*c^4*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}+8/3*c^4*\tan(f*x+e)^3/f/(a+a*\sec(f*x+e))^{(3/2)}-a*c^4*\sec(1/2*e+1/2*f*x)^2*\sin(f*x+e)*\tan(f*x+e)^4/f/(a+a*\sec(f*x+e))^{(5/2)}$

Rubi [A] time = 0.29, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 470, 582, 522, 203}

$$\frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2}\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{8c^4 \tan^3(e+fx)}{3f(a \sec(e+fx)+a)^{3/2}} - \frac{14c^4 \tan(e+fx)}{af\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $(2*c^4*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/(a^{(3/2)*f}) + (12*\text{Sqrt}[2]*c^4*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/(a^{(3/2)*f}) - (14*c^4*\text{Tan}[e + f*x])/(a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (8*c^4*\text{Tan}[e + f*x]^3)/(3*f*(a + a*\text{Sec}[e + f*x])^{(3/2)}) - (a*c^4*\text{Sec}[(e + f*x)/2]^2*\text{Sin}[e + f*x]*\text{Tan}[e + f*x]^4)/(f*(a + a*\text{Sec}[e + f*x])^{(5/2)})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m

```

- n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]

```

Rule 3887

```

Int[cot[(c_.) + (d_.)*(x_.)]^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n
_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]

```

Rule 3904

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^{3/2}} dx &= (a^4 c^4) \int \frac{\tan^8(e + fx)}{(a + a \sec(e + fx))^{11/2}} dx \\
&= -\frac{(2a^3 c^4) \operatorname{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{ac^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^4(e + fx)}{f(a + a \sec(e + fx))^{5/2}} - \frac{(ac^4) \operatorname{Subst}\left(\int \frac{x^4(10+8ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= \frac{8c^4 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{ac^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^4(e + fx)}{f(a + a \sec(e + fx))^{5/2}} + \frac{c^4 \operatorname{Subst}\left(\int \frac{x^2(10+8ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{14c^4 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} + \frac{8c^4 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{ac^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^4(e + fx)}{f(a + a \sec(e + fx))^{5/2}} \\
&= -\frac{14c^4 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} + \frac{8c^4 \tan^3(e + fx)}{3f(a + a \sec(e + fx))^{3/2}} - \frac{ac^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^4(e + fx)}{f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} f} + \frac{12\sqrt{2} c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} f} - \frac{14c^4 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 1.58, size = 196, normalized size = 0.97

$$\frac{c^4 \csc\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \left(20 \cos(e + fx) - 26 \cos(2(e + fx)) + 28 \cos(3(e + fx)) + 6 \left(\cos\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx)\right)\right)}{af\sqrt{a + a \sec(e + fx)}}$$

Antiderivative was successfully verified.

Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 2.04index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.97, size = 552, normalized size = 2.72

$$c^4 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (-1 + \cos(fx + e)) \left(3 \operatorname{arctanh} \left(\frac{\sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e)\sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{3}{2}} \sin(fx + e) (\cos^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2), x)

[Out] -1/6*c^4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(3*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)-36*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*sin(f*x+e)+6*sin(f*x+e)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)-72*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)+3*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)-36*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)+112*cos(f*x+e)^3-52*cos(f*x+e)^2-64*cos(f*x+e)+4)/cos(f*x+e)/sin(f*x+e)^3/a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^4}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2), x)`

[Out] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4 \left(\int \left(-\frac{4 \sec(e + fx)}{a \sqrt{a \sec(e + fx) + a} \sec(e + fx) + a \sqrt{a \sec(e + fx) + a}} \right) dx + \int \frac{6 \sec^2(e + fx)}{a \sqrt{a \sec(e + fx) + a} \sec(e + fx) + a \sqrt{a \sec(e + fx) + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(3/2), x)`

[Out] `c**4*(Integral(-4*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))`

$$3.73 \quad \int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=169

$$\frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{4c^3 \tan(e+fx)}{af\sqrt{a \sec(e+fx)+a}} + \frac{c^3 \sin(e+fx) \tan^2(e+fx)}{f(a \sec(e+fx)+a)}$$

[Out] $2c^3 \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/a^{3/2}/f + 2c^3 \arctan(1/2 a^{1/2} \tan(fx+e) 2^{1/2}/(a+a \sec(fx+e))^{1/2}) 2^{1/2}/a^{3/2}/f - 4c^3 \tan(fx+e)/af/(a+a \sec(fx+e))^{1/2} + c^3 \sec(1/2 e + 1/2 fx)^2 \sin(fx+e) \tan(fx+e)^2/f/(a+a \sec(fx+e))^{3/2}$

Rubi [A] time = 0.24, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 470, 582, 522, 203}

$$\frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{4c^3 \tan(e+fx)}{af\sqrt{a \sec(e+fx)+a}} + \frac{c^3 \sin(e+fx) \tan^2(e+fx)}{f(a \sec(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $(2c^3 \text{ArcTan}[\text{Sqrt}[a] \text{Tan}[e + fx]]/\text{Sqrt}[a + a \text{Sec}[e + fx]])/(a^{3/2}f) + (2\text{Sqrt}[2]c^3 \text{ArcTan}[\text{Sqrt}[a] \text{Tan}[e + fx]]/(\text{Sqrt}[2] \text{Sqrt}[a + a \text{Sec}[e + fx]]))/(a^{3/2}f) - (4c^3 \text{Tan}[e + fx])/(af \text{Sqrt}[a + a \text{Sec}[e + fx]]) + (c^3 \text{Sec}[(e + fx)/2]^2 \text{Sin}[e + fx] \text{Tan}[e + fx]^2)/(f(a + a \text{Sec}[e + fx])^{3/2})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m

$-n+1)(a+bx^n)^{p+1}(c+dx^n)^{q+1})/(b*d*(m+n*(p+q+1)+1)), x] - \text{Dist}[g^n/(b*d*(m+n*(p+q+1)+1)), \text{Int}[(g*x)^{m-n}(a+bx^n)^p(c+dx^n)^q \text{Simp}[a*f*c*(m-n+1)+(a*f*d*(m+n*q+1)+b*(f*c*(m+n*p+1)-e*d*(m+n*(p+q+1)+1))]x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1]$

Rule 3887

$\text{Int}[\cot[(c_.)+(d_.)*(x_.)]^{(m_.)}*(\csc[(c_.)+(d_.)*(x_.)]*(b_.)+(a_.))^{(n_.)}, x_Symbol] := \text{Dist}[(-2*a^{(m/2+n+1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2+a*x^2)^{(m/2+n-1/2)})/(1+a*x^2), x], x, \text{Cot}[c+d*x]/\text{Sqrt}[a+b*\text{Csc}[c+d*x]], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2-b^2, 0] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n-1/2]$

Rule 3904

$\text{Int}[(\csc[(e_.)+(f_.)*(x_.)]*(b_.)+(a_.))^{(m_.)}*(\csc[(e_.)+(f_.)*(x_.)]*(d_.)+(c_.))^{(n_.)}, x_Symbol] := \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e+f*x]^{(2*m)}*(c+d*\text{Csc}[e+f*x])^{(n-m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c+a*d, 0] \&\& \text{EqQ}[a^2-b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m-n, 0])$

Rubi steps

$$\begin{aligned} \int \frac{(c-c \sec(e+fx))^3}{(a+a \sec(e+fx))^{3/2}} dx &= -\left((a^3 c^3) \int \frac{\tan^6(e+fx)}{(a+a \sec(e+fx))^{9/2}} dx \right) \\ &= \frac{(2a^2 c^3) \text{Subst}\left(\int \frac{x^6}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= \frac{c^3 \sec^2\left(\frac{1}{2}(e+fx)\right) \sin(e+fx) \tan^2(e+fx)}{f(a+a \sec(e+fx))^{3/2}} + \frac{c^3 \text{Subst}\left(\int \frac{x^2(6+4ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{4c^3 \tan(e+fx)}{af\sqrt{a+a \sec(e+fx)}} + \frac{c^3 \sec^2\left(\frac{1}{2}(e+fx)\right) \sin(e+fx) \tan^2(e+fx)}{f(a+a \sec(e+fx))^{3/2}} - \frac{c^3 \text{Subst}\left(\int \frac{x^2(6+4ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= -\frac{4c^3 \tan(e+fx)}{af\sqrt{a+a \sec(e+fx)}} + \frac{c^3 \sec^2\left(\frac{1}{2}(e+fx)\right) \sin(e+fx) \tan^2(e+fx)}{f(a+a \sec(e+fx))^{3/2}} - \frac{(2c^3) \text{Subst}\left(\int \frac{x^2(6+4ax^2)}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\ &= \frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} f} + \frac{2\sqrt{2} c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} f} - \frac{4c^3 \tan(e+fx)}{af\sqrt{a+a \sec(e+fx)}} \end{aligned}$$

Mathematica [A] time = 1.77, size = 132, normalized size = 0.78

$$\frac{2c^3 \tan\left(\frac{1}{2}(e+fx)\right) \left(-\sec(e+fx) + \cot^2\left(\frac{1}{2}(e+fx)\right) \sqrt{\sec(e+fx)-1} \tan^{-1}\left(\sqrt{\sec(e+fx)-1}\right) + \sqrt{2} \cot^2\left(\frac{1}{2}(e+fx)\right) \right)}{af\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2), x]

[Out] (2*c^3*(-3 + ArcTan[Sqrt[-1 + Sec[e + f*x]]])*Cot[(e + f*x)/2]^2*Sqrt[-1 + Sec[e + f*x]] + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cot[(e + f*x)

) / 2] ^ 2 * Sqrt[-1 + Sec[e + f*x]] - Sec[e + f*x]) * Tan[(e + f*x) / 2]) / (a * f * Sqrt[a * (1 + Sec[e + f*x])])

fricas [A] time = 1.02, size = 550, normalized size = 3.25

$$\sqrt{2} \left(ac^3 \cos^2(fx + e) + 2ac^3 \cos(fx + e) + ac^3 \right) \sqrt{-\frac{1}{a}} \log \left(-\frac{2\sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{-\frac{1}{a}} \cos(fx+e) \sin(fx+e) - 3 \cos(fx+e)^2 - 2}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] [(sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*sqrt(-1/a)*
log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x
+ e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2
+ 2*cos(f*x + e) + 1)) - (c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sq
rt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
) - 2*(3*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*si
n(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -2*((c^3
*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*c^3*cos(f
*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(
2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*arctan(sqrt(2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/
sqrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
```


le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.75index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.99, size = 377, normalized size = 2.23

$$c^3 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(-\sin(fx+e) (\cos^2(fx+e)) \sqrt{2} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x)

[Out] $-c^3/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*(-\sin(f*x+e)*\cos(f*x+e)^{2*2^{1/2}}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})+2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(((2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))*\cos(f*x+e)^{2*\sin(f*x+e)+2^{1/2}}*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}+6*\cos(f*x+e)^3-2*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(((2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))-10*\cos(f*x+e)^{2+2*\cos(f*x+e)+2}/\sin(f*x+e)^3/a^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(c \sec(fx+e) - c)^3}{(a \sec(fx+e) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -integrate((c*sec(f*x + e) - c)^3/(a*sec(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^3}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)

[Out] `int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \left(\int \frac{3 \sec(e + fx)}{a \sqrt{a \sec(e + fx) + a} \sec(e + fx) + a} dx + \int \left(-\frac{3 \sec^2(e + fx)}{a \sqrt{a \sec(e + fx) + a} \sec(e + fx)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)`

[Out] `-c**3*(Integral(3*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))`

$$3.74 \quad \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=119

$$\frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2} f} - \frac{\sqrt{2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2} f} - \frac{2c^2 \tan(e+fx)}{f(a \sec(e+fx) + a)^{3/2}}$$

[Out] $2*c^2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/f-c^2*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/a^{(3/2)}/f-2*c^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(3/2)}$

Rubi [A] time = 0.19, antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 470, 12, 391, 203}

$$\frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2} f} - \frac{\sqrt{2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2} f} - \frac{c^2 \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{af \sqrt{a \sec(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $(2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^{(3/2)}*f) - (Sqrt[2]*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(a^{(3/2)}*f) - (c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]])$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^{3/2}} dx &= (a^2 c^2) \int \frac{\tan^4(e + fx)}{(a + a \sec(e + fx))^{7/2}} dx \\ &= -\frac{(2ac^2) \operatorname{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\ &= -\frac{c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{af \sqrt{a + a \sec(e + fx)}} - \frac{c^2 \operatorname{Subst}\left(\int \frac{2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{af} \\ &= -\frac{c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{af \sqrt{a + a \sec(e + fx)}} - \frac{(2c^2) \operatorname{Subst}\left(\int \frac{1}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{af} \\ &= -\frac{c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{af \sqrt{a + a \sec(e + fx)}} - \frac{(2c^2) \operatorname{Subst}\left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{af} + \frac{c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{af \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} f} - \frac{\sqrt{2} c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} f} - \frac{c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{af \sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.11, size = 128, normalized size = 1.08

$$\frac{c^2 \cot\left(\frac{1}{2}(e + fx)\right) \left(\sec^2\left(\frac{1}{2}(e + fx)\right) (\cos(e + fx) + (\cos(e + fx) + 1)\sqrt{\sec(e + fx) - 1}) \tan^{-1}\left(\sqrt{\sec(e + fx) - 1}\right)\right)}{af \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2), x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(Sec[(e + f*x)/2]^2*(-1 + Cos[e + f*x] + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]) - Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]])/(a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [B] time = 0.93, size = 542, normalized size = 4.55

$$4c^2 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - \sqrt{2} \left(ac^2 \cos(fx+e)^2 + 2ac^2 \cos(fx+e) + ac^2 \right) \sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{-\frac{1}{a}} \cos(fx+e) \sin(fx+e) + 3\cos(fx+e)^2 + 2\cos(fx+e) - 1}{(\cos(fx+e))^2 + 2\cos(fx+e) + 1} \right) + 2 \left(c^2 \cos(fx+e)^2 + 2c^2 \cos(fx+e) + c^2 \right) \sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 + 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e) - a}{(\cos(fx+e))^2 + 2\cos(fx+e) + 1} \right) / (a^2 f \cos(fx+e)^2 + 2a^2 f \cos(fx+e) + a^2 f), - \left(2c^2 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + 2(c^2 \cos(fx+e)^2 + 2c^2 \cos(fx+e) + c^2) \sqrt{a} \arctan \left(\frac{\sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e)}{\sqrt{a} \sin(fx+e)} \right) - \sqrt{2} \left(ac^2 \cos(fx+e)^2 + 2ac^2 \cos(fx+e) + ac^2 \right) \arctan \left(\frac{\sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e)}{\sqrt{a} \sin(fx+e)} \right) \right) / \sqrt{a} / (a^2 f \cos(fx+e)^2 + 2a^2 f \cos(fx+e) + a^2 f)]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
 2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
 ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
 step/2)Warning, integration of abs or sign assumes constant sign by interval
 s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
 : (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
 2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_

1.44index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.76, size = 369, normalized size = 3.10

$$c^2 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(\cos(fx+e) \sin(fx+e) \sqrt{2} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + \sqrt{2} \sin(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x)

[Out] $-c^2/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(\cos(f*x+e)*\sin(f*x+e)*2^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)}+2^{(1/2)}*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}+\cos(f*x+e)*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))+\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))-2*\cos(f*x+e)^2+2*\cos(f*x+e))/(1+\cos(f*x+e))/\sin(f*x+e)/a^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c \sec(fx+e) - c)^2}{(a \sec(fx+e) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((c*sec(f*x+e) - c)^2/(a*sec(f*x+e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^2}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int \left(\frac{2 \sec(e+fx)}{a \sqrt{a \sec(e+fx) + a} \sec(e+fx) + a \sqrt{a \sec(e+fx) + a}} \right) dx + \int \frac{\sec^2(e+fx)}{a \sqrt{a \sec(e+fx) + a} \sec(e+fx) + a \sec(e+fx)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)

[Out] $c**2*(\operatorname{Integral}(-2*\sec(e+f*x)/(a*\sqrt{a*\sec(e+f*x)+a})*\sec(e+f*x)+a*\sqrt{a*\sec(e+f*x)+a}),x)+\operatorname{Integral}(\sec(e+f*x)**2/(a*\sqrt{a*\sec(e+f*x)+a})*\sec(e+f*x)+a*\sqrt{a*\sec(e+f*x)+a}),x)+\operatorname{Integral}(1/(a*\sqrt{a*\sec(e+f*x)+a})*\sec(e+f*x)+a*\sqrt{a*\sec(e+f*x)+a}),x)$

$$3.75 \quad \int \frac{c - c \sec(e+fx)}{(a + a \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=113

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{3c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{2} a^{3/2}f} - \frac{c \tan(e+fx)}{f(a \sec(e+fx)+a)^{3/2}}$$

[Out] $2*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/f-3/2*c*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/f*2^{(1/2)}-c*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(3/2)}$

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3904, 3887, 471, 522, 203}

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}f} - \frac{3c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{2} a^{3/2}f} - \frac{c \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{2af \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $(2*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(a^{(3/2)}*f) - (3*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/(\text{Sqrt}[2]*a^{(3/2)}*f) - (c*\text{Sec}[(e + f*x)/2]^{2}*\text{Sin}[e + f*x])/(2*a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 471

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^{3/2}} dx = - \left((ac) \int \frac{\tan^2(e + fx)}{(a + a \sec(e + fx))^{5/2}} dx \right)$$

$$= \frac{(2c) \operatorname{Subst} \left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f}$$

$$= -\frac{c \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{2af \sqrt{a + a \sec(e + fx)}} - \frac{c \operatorname{Subst} \left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{af}$$

$$= -\frac{c \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{2af \sqrt{a + a \sec(e + fx)}} - \frac{(2c) \operatorname{Subst} \left(\int \frac{1}{1+ax^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{af} + \dots$$

$$= \frac{2c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{a^{3/2} f} - \frac{3c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{2} a^{3/2} f} - \frac{c \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{2af \sqrt{a + a \sec(e + fx)}}$$

Mathematica [A] time = 1.00, size = 130, normalized size = 1.15

$$\frac{c \cot \left(\frac{1}{2}(e + fx) \right) \left(\sec^2 \left(\frac{1}{2}(e + fx) \right) (\cos(e + fx) + 2(\cos(e + fx) + 1)\sqrt{\sec(e + fx) - 1}) \tan^{-1} \left(\sqrt{\sec(e + fx) - 1} \right) \right)}{2af \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2), x]
```

```
[Out] (c*Cot[(e + f*x)/2]*(Sec[(e + f*x)/2]^2*(-1 + Cos[e + f*x] + 2*ArcTan[Sqrt[-1 + Sec[e + f*x]])*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]]) - 3*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]])/(2*a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [B] time = 0.67, size = 505, normalized size = 4.47

$$\frac{4c \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + 3\sqrt{2} \left(c \cos(fx+e)^2 + 2c \cos(fx+e) + c \right) \sqrt{-a} \log \left(-\frac{2\sqrt{2}\sqrt{-a}}{\dots} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2), x, algorithm="fricas")
```

```
[Out] [-1/4*(4*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))]/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableUnabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
```



```

ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discon
tinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by int
ervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluatio
n time: 1.23index.cc index_m i_lex_is_greater Error: Bad Argument Value

```

maple [B] time = 1.87, size = 371, normalized size = 3.28

$$c \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(-2 \cos(fx+e) \sin(fx+e) \sqrt{2} \sqrt{-\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2 \cos(fx+e)} \right) - 3 \cos \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x)

```

[Out] 1/2*c/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)*sin(f*x+e)*2^(1/
2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))-3*cos(f*x+e)*sin(f*x+e)*(-2*cos
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln((( -2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-2*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)-3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(
((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+
2*cos(f*x+e)^2-2*cos(f*x+e))/(1+cos(f*x+e))/sin(f*x+e)/a^2

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c \sec(fx+e) - c}{(a \sec(fx+e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c - \frac{c}{\cos(e+fx)}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \left(\int \frac{\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}} dx + \int \left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)

[Out] -c*(Integral(sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))

$$3.76 \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))} dx$$

Optimal. Leaf size=177

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}cf} - \frac{7 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{4\sqrt{2} a^{3/2}cf} + \frac{\cot(e+fx)\sqrt{a \sec(e+fx)+a}}{4a^2cf} + \frac{\cos(e+fx) \cot(e+fx)}{4a^2cf}$$

[Out] $2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/c/f-7/8*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/c/f*2^{(1/2)}+1/4*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/a^2/c/f+1/4*\cos(f*x+e)*\cot(f*x+e)*\sec(1/2*e+1/2*f*x)^2*(a+a*\sec(f*x+e))^{(1/2)}/a^2/c/f$

Rubi [A] time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 472, 583, 522, 203}

$$\frac{\cot(e+fx)\sqrt{a \sec(e+fx)+a}}{4a^2cf} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}cf} - \frac{7 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{4\sqrt{2} a^{3/2}cf} + \frac{\cos(e+fx) \cot(e+fx)}{4a^2cf}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]

[Out] $(2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(a^{(3/2)}*c*f) - (7*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(4*\text{Sqrt}[2]*a^{(3/2)}*c*f) + (\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(4*a^2*c*f) + (\text{Cos}[e + f*x]*\text{Cot}[e + f*x]*\text{Sec}[(e + f*x)/2]^2*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(4*a^2*c*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(

$m + 1)), \text{Int}[(g*x)^{(m+n)}*(a + b*x^n)^p*(c + d*x^n)^q*\text{Simp}[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p, q\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

Rule 3887

$\text{Int}[\text{cot}[(c_.) + (d_.)*(x_.)]^{(m_.)}*(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^{(n_.)}, x_Symbol] :> \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2 + a*x^2)^{(m/2 + n - 1/2)})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}, x_Symbol] :> \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n - m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2}(c - c \sec(e + fx))} dx = -\frac{\int \frac{\cot^2(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx}{ac}$$

$$= \frac{2 \text{Subst}\left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^2cf}$$

$$= \frac{\cos(e + fx) \cot(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right) \sqrt{a + a \sec(e + fx)}}{4a^2cf} + \dots$$

$$= \frac{\cot(e + fx) \sqrt{a + a \sec(e + fx)}}{4a^2cf} + \frac{\cos(e + fx) \cot(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right)}{4a^2cf}$$

$$= \frac{\cot(e + fx) \sqrt{a + a \sec(e + fx)}}{4a^2cf} + \frac{\cos(e + fx) \cot(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right)}{4a^2cf}$$

$$= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{4\sqrt{2} a^{3/2}cf} + \frac{\cot(e + fx)}{a^{3/2}cf}$$

Mathematica [A] time = 1.18, size = 154, normalized size = 0.87

$$\frac{\sin^2\left(\frac{1}{2}(e + fx)\right) \tan\left(\frac{1}{2}(e + fx)\right) \left(3 \cos(e + fx) - 7\sqrt{2} \cos^2\left(\frac{1}{2}(e + fx)\right)\right) \sqrt{\sec(e + fx) - 1} \tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right) + 2acf(\cos(e + fx) - 1)^2 \sqrt{a(\sec(e + fx) + 1)}}{2acf(\cos(e + fx) - 1)^2 \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]

[Out] ((1 + 3*Cos[e + f*x] - 7*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x)/2]^2*Sqrt[-1 + Sec[e + f*x]] + 8*ArcTan[Sqrt[-1 + Sec[e + f*x]]])

]*(1 + Cos[e + f*x])*Sqrt[-1 + Sec[e + f*x]])*Sin[(e + f*x)/2]^2*Tan[(e + f*x)/2))/(2*a*c*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.63, size = 514, normalized size = 2.90

$$\frac{7\sqrt{2}\sqrt{-a}(\cos(fx+e)+1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [-1/16*(7*sqrt(2)*sqrt(-a)*(cos(f*x + e) + 1)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(-a)*(cos(f*x + e) + 1)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/8*(7*sqrt(2)*sqrt(a)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 8*sqrt(a)*(cos(f*x + e) + 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
 i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
 *pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
 able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
 n of abs or sign assumes constant sign by intervals (correct if the argumen
 t is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t
 _nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
 /t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
 >(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
 2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no

maple [B] time = 1.90, size = 377, normalized size = 2.13

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(8 \sin(fx+e) (\cos^2(fx+e)) \sqrt{2} \sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2 \cos(fx+e)} \right) + 7 \sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x)`

[Out] `1/8/c/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(8*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+7*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-8*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-7*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-6*cos(f*x+e)^3+4*cos(f*x+e)^2+2*cos(f*x+e))/sin(f*x+e)^3/a^2`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(a \sec(fx+e) + a)^{\frac{3}{2}} (c \sec(fx+e) - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")`

[Out] `-integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{\frac{3}{2}} \left(c - \frac{c}{\cos(e+fx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))),x)`

[Out] `int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a \sqrt{a \sec(e+fx)+a} \sec^2(e+fx) - a \sqrt{a \sec(e+fx)+a}} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)`

[Out] `-Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)), x)/c`

$$3.77 \quad \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=214

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}c^2f} - \frac{9 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{8\sqrt{2} a^{3/2}c^2f} + \frac{\cot^3(e+fx)(a \sec(e+fx)+a)^{3/2} \cos(e+fx) \cot^3(e+fx)}{12a^3c^2f}$$

[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f+1/12*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-1/4*cos(f*x+e)*cot(f*x+e)^3*sec(1/2*e+1/2*f*x)^2*(a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-9/16*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f*2^(1/2)+7/8*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c^2/f

Rubi [A] time = 0.28, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 472, 583, 522, 203}

$$\frac{\cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{12a^3c^2f} + \frac{7 \cot(e+fx) \sqrt{a \sec(e+fx)+a}}{8a^2c^2f} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}c^2f} - \frac{9 \tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{2} \sqrt{a}}\right)}{8\sqrt{2} a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]

[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(a^(3/2)*c^2*f) - (9*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(8*Sqrt[2]*a^(3/2)*c^2*f) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*a^2*c^2*f) + (Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(12*a^3*c^2*f) - (Cos[e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(3/2))/(4*a^3*c^2*f)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 472

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^(m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Simp[(e*(g*x)^(m+1)*(a +

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g^(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2}(c - c \sec(e + fx))^2} dx = \frac{\int \cot^4(e + fx) \sqrt{a + a \sec(e + fx)} dx}{a^2 c^2}$$

$$= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^3 c^2 f}$$

$$= -\frac{\cos(e + fx) \cot^3(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right) (a + a \sec(e + fx))^3}{4a^3 c^2 f}$$

$$= \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{12a^3 c^2 f} - \frac{\cos(e + fx) \cot^3(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right) (a + a \sec(e + fx))^3}{12a^3 c^2 f}$$

$$= \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{8a^2 c^2 f} + \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{12a^3 c^2 f}$$

$$= \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{8a^2 c^2 f} + \frac{\cot^3(e + fx)(a + a \sec(e + fx))^{3/2}}{12a^3 c^2 f}$$

$$= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2} c^2 f} - \frac{9 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{8\sqrt{2} a^{3/2} c^2 f} + \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{8a^2 c^2 f}$$

Mathematica [C] time = 24.03, size = 5612, normalized size = 26.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2), x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.65, size = 560, normalized size = 2.62

$$\frac{27\sqrt{2}\left(\cos(fx+e)^2-1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [-1/96*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), 1/48*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>
 (-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
 ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
 step/2)Warning, integration of abs or sign assumes constant sign by interval
 s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
 : (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
 2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/

maple [B] time = 2.50, size = 387, normalized size = 1.81

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(48 \sin(fx+e) (\cos^2(fx+e)) \sqrt{2} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + 27 \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x)`

[Out]
$$-1/48/c^2/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*(48*\sin(f*x+e)*\cos(f*x+e)^{2^{1/2}}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})+27*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)-48*2^{1/2}*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}-27*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))-62*\cos(f*x+e)^3+4*\cos(f*x+e)^2+42*\cos(f*x+e))/\sin(f*x+e)^5*(-1+\cos(f*x+e)^2)/a^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(fx+e) + a)^{3/2} (c \sec(fx+e) - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")`

[Out] `integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2} \left(c - \frac{c}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2),x)`

[Out] `int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a\sqrt{a \sec(e+fx)+a} \sec^3(e+fx)-a\sqrt{a \sec(e+fx)+a} \sec^2(e+fx)-a\sqrt{a \sec(e+fx)+a} \sec(e+fx)+a\sqrt{a \sec(e+fx)+a}}{c^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)`

[Out] `Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x)/c**2`

$$3.78 \quad \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^3} dx$$

Optimal. Leaf size=249

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2}c^3f} - \frac{11 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{16\sqrt{2}a^{3/2}c^3f} - \frac{3 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{20a^4c^3f} + \frac{\cos(e+fx) \cot^5(e+fx)}{16a^2c^3f}$$

[Out] $2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(3/2)}/c^3/f-5/24*\cot(f*x+e)^3*(a+a*\sec(f*x+e))^{(3/2)}/a^3/c^3/f-3/20*\cot(f*x+e)^5*(a+a*\sec(f*x+e))^{(5/2)}/a^4/c^3/f+1/4*\cos(f*x+e)*\cot(f*x+e)^5*\sec(1/2*e+1/2*f*x)^2*(a+a*\sec(f*x+e))^{(5/2)}/a^4/c^3/f-11/32*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)})/(a+a*\sec(f*x+e))^{(1/2)}/a^{(3/2)}/c^3/f*2^{(1/2)}+21/16*\cot(f*x+e)*(a+a*\sec(f*x+e))^{(1/2)}/a^2/c^3/f$

Rubi [A] time = 0.34, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 472, 583, 522, 203}

$$\frac{3 \cot^5(e+fx)(a \sec(e+fx)+a)^{5/2}}{20a^4c^3f} - \frac{5 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{24a^3c^3f} + \frac{21 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{16a^2c^3f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]

[Out] $(2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(a^{(3/2)}*c^3*f) - (11*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/(16*\text{Sqrt}[2]*a^{(3/2)}*c^3*f) + (21*\text{Cot}[e + f*x]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(16*a^2*c^3*f) - (5*\text{Cot}[e + f*x]^3*(a + a*\text{Sec}[e + f*x])^{(3/2)})/(24*a^3*c^3*f) - (3*\text{Cot}[e + f*x]^5*(a + a*\text{Sec}[e + f*x])^{(5/2)})/(20*a^4*c^3*f) + (\text{Cos}[e + f*x]*\text{Cot}[e + f*x]^5*\text{Sec}[(e + f*x)/2]^2*(a + a*\text{Sec}[e + f*x])^{(5/2)})/(4*a^4*c^3*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^(m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.)*((e_.) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^3} dx &= -\frac{\int \cot^6(e + fx) (a + a \sec(e + fx))^{3/2} dx}{a^3 c^3} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{x^6 (1 + ax^2) (2 + ax^2)^2} dx, x, -\frac{\tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{a^4 c^3 f} \\ &= \frac{\cos(e + fx) \cot^5(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right) (a + a \sec(e + fx))^{5/2}}{4a^4 c^3 f} \\ &= -\frac{3 \cot^5(e + fx) (a + a \sec(e + fx))^{5/2}}{20a^4 c^3 f} + \frac{\cos(e + fx) \cot^5(e + fx)}{20a^4 c^3 f} \\ &= -\frac{5 \cot^3(e + fx) (a + a \sec(e + fx))^{3/2}}{24a^3 c^3 f} - \frac{3 \cot^5(e + fx) (a + a \sec(e + fx))^{5/2}}{20a^4 c^3 f} \\ &= \frac{21 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{16a^2 c^3 f} - \frac{5 \cot^3(e + fx) (a + a \sec(e + fx))^{5/2}}{24a^3 c^3 f} \\ &= \frac{21 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{16a^2 c^3 f} - \frac{5 \cot^3(e + fx) (a + a \sec(e + fx))^{5/2}}{24a^3 c^3 f} \\ &= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{a^{3/2} c^3 f} - \frac{11 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{2} \sqrt{a + a \sec(e + fx)}}\right)}{16\sqrt{2} a^{3/2} c^3 f} + \frac{21 \cot(e + fx)}{20a^4 c^3 f} \end{aligned}$$

Mathematica [C] time = 23.98, size = 5629, normalized size = 22.61

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]

[Out] Result too large to show

fricas [A] time = 0.69, size = 714, normalized size = 2.87

$$165\sqrt{2}\left(\cos(fx+e)^3 - \cos(fx+e)^2 - \cos(fx+e) + 1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-}{\cos(fx+e)^2+2\cos(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [-1/960*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/480*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 480*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)]]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
 2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>
 (-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3),x)`

[Out] `int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)+2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)-a\sqrt{a\sec(e+fx)+a}}{c^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)`

[Out] `-Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - a*sqrt(a*sec(e + f*x) + a)), x)/c**3`

$$3.79 \quad \int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=260

$$\frac{2c^5 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} - \frac{23\sqrt{2} c^5 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} + \frac{21c^5 \tan(e+fx)}{a^2 f \sqrt{a \sec(e+fx)+a}} - \frac{19c^5 \tan^3(e+fx)}{6af(a \sec(e+fx)+a)^{3/2}}$$

[Out] $2*c^5*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f-23*c^5*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/a^{(5/2)}/f+21*c^5*\tan(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)}-19/6*c^5*\tan(f*x+e)^3/a/f/(a+a*\sec(f*x+e))^{(3/2)}+3/4*c^5*\sec(1/2*e+1/2*f*x)^2*\sin(f*x+e)*\tan(f*x+e)^4/f/(a+a*\sec(f*x+e))^{(5/2)}+1/4*a*c^5*\sec(1/2*e+1/2*f*x)^4*\sin(f*x+e)^2*\tan(f*x+e)^5/f/(a+a*\sec(f*x+e))^{(7/2)}$

Rubi [A] time = 0.34, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3904, 3887, 470, 578, 582, 522, 203}

$$\frac{2c^5 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} - \frac{23\sqrt{2} c^5 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} + \frac{21c^5 \tan(e+fx)}{a^2 f \sqrt{a \sec(e+fx)+a}} - \frac{19c^5 \tan^3(e+fx)}{6af(a \sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $(2*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(a^{(5/2)*f} - (23*Sqrt[2]*c^5*ArcTan[(Sqrt[a]*Tan[e + f*x])/((Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(a^{(5/2)*f} + (21*c^5*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]) - (19*c^5*Tan[e + f*x]^3)/(6*a*f*(a + a*Sec[e + f*x])^{(3/2)}) + (3*c^5*Sec[(e + f*x)/2]^2*Sin[e + f*x]*Tan[e + f*x]^4)/(4*f*(a + a*Sec[e + f*x])^{(5/2)}) + (a*c^5*Sec[(e + f*x)/2]^4*Sin[e + f*x]^2*Tan[e + f*x]^5)/(4*f*(a + a*Sec[e + f*x])^{(7/2)})$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^{5/2}} dx &= - \left((a^5 c^5) \int \frac{\tan^{10}(e + fx)}{(a + a \sec(e + fx))^{15/2}} dx \right) \\
&= \frac{(2a^3 c^5) \operatorname{Subst} \left(\int \frac{x^{10}}{(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= \frac{ac^5 \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx) \tan^5(e + fx)}{4f(a + a \sec(e + fx))^{7/2}} + \frac{(ac^5) \operatorname{Subst} \left(\int \frac{x^6(14+10ax^2)}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{2f} \\
&= \frac{3c^5 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx) \tan^4(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} + \frac{ac^5 \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx) \tan^5(e + fx)}{4f(a + a \sec(e + fx))^{7/2}} \\
&= -\frac{19c^5 \tan^3(e + fx)}{6af(a + a \sec(e + fx))^{3/2}} + \frac{3c^5 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx) \tan^4(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} + \frac{ac^5 \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx) \tan^5(e + fx)}{4f(a + a \sec(e + fx))^{7/2}} \\
&= \frac{21c^5 \tan(e + fx)}{a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{19c^5 \tan^3(e + fx)}{6af(a + a \sec(e + fx))^{3/2}} + \frac{3c^5 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx) \tan^4(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{21c^5 \tan(e + fx)}{a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{19c^5 \tan^3(e + fx)}{6af(a + a \sec(e + fx))^{3/2}} + \frac{3c^5 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx) \tan^4(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2c^5 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{a^{5/2} f} - \frac{23\sqrt{2} c^5 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{a^{5/2} f} + \frac{21c^5 \tan(e + fx)}{a^2 f \sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 3.64, size = 180, normalized size = 0.69

$$c^5 \cot \left(\frac{1}{2}(e + fx) \right) \sec^2(e + fx) \left((-30 \cos(e + fx) + 52 \cos(2(e + fx)) - 66 \cos(3(e + fx)) - 37 \cos(4(e + fx)) + 8 \cos(5(e + fx))) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2), x]

[Out] (c^5*Cot[(e + f*x)/2]*((81 - 30*Cos[e + f*x] + 52*Cos[2*(e + f*x)] - 66*Cos[3*(e + f*x)] - 37*Cos[4*(e + f*x)])*Sec[(e + f*x)/2]^4 + 96*ArcTan[Sqrt[-1 + Sec[e + f*x]])*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 1104*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(48*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 2.96, size = 742, normalized size = 2.85

$$\left[\frac{69\sqrt{2} \left(ac^5 \cos^4(fx + e) + 3ac^5 \cos^3(fx + e) + 3ac^5 \cos^2(fx + e) + ac^5 \cos(fx + e) \right) \sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(fx+e)}{\cos(fx+e)}}}{\cos(fx+e)} \right)}{\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] [1/6*(69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*c
os(f*x + e)^2 + a*c^5*cos(f*x + e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x
+ e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 6*(c
^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f
*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*
x + e) + 1)) + 4*(37*c^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*co
s(f*x + e) - c^5)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/(a^
3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*
f*cos(f*x + e)), -1/3*(6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5
*cos(f*x + e)^2 + c^5*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(37*c^5*cos(f*x +
e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*cos(f*x + e) - c^5)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sin(f*x + e) - 69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*
a*c^5*cos(f*x + e)^3 + 3*a*c^5*cos(f*x + e)^2 + a*c^5*cos(f*x + e))*arctan(
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e)))/sqrt(a)/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 + a^3*f*cos(f*x + e))]
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integra
tion of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
```



```
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assum
es constant sign by intervals (correct if the argument is real):Check [abs(
t_nostep^2-1)]Evaluation time: 3.46index.cc index_m i_lex_is_greater Error:
Bad Argument Value
```

maple [B] time = 2.55, size = 726, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6*c^5/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(-3*cos(f*
x+e)^3*sin(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-69*cos(
f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-9*arctanh(1/2*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(3/2)*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)-207*cos(f*x+e)
^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*sin(f*x+e)-9*sin(f*x+e)*cos(f*x+
e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2
^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*2^(1/2)-207*sin(f*x+e)*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(3/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)-3*2^(1/2)*arctanh(1/2*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(3/2)*sin(f*x+e)-69*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*
sin(f*x+e)+148*cos(f*x+e)^4+132*cos(f*x+e)^3-200*cos(f*x+e)^2-84*cos(f*x+e)
+4)/cos(f*x+e)/sin(f*x+e)^5/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^5}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$-c^5 \int \frac{5 \sec(e + fx)}{a^2 \sqrt{a \sec(e + fx) + a} \sec^2(e + fx) + 2a^2 \sqrt{a \sec(e + fx) + a} \sec(e + fx) + a^2 \sqrt{a \sec(e + fx) + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] -c**5*(Integral(5*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x)
) + a)), x) + Integral(-10*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a
*sec(e + f*x) + a)), x) + Integral(10*sec(e + f*x)**3/(a**2*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-5*sec(e + f*x)**4/(a**2*sqrt
(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(
e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**5/(a
**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) +
a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

$$3.80 \quad \int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=229

$$\frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} - \frac{11c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{2} a^{5/2} f} + \frac{7c^4 \tan(e+fx)}{2a^2 f \sqrt{a \sec(e+fx)+a}} - \frac{c^4 \sin^2(e+fx) \tan^3(e+fx)}{4f(a \sec(e+fx)+a)}$$

[Out] $2c^4 \arctan(a^{(1/2)} \tan(fx+e)/(a+a \sec(fx+e))^{(1/2)})/a^{(5/2)}/f - 11/2 c^4 \arctan(1/2 a^{(1/2)} \tan(fx+e) 2^{(1/2)}/(a+a \sec(fx+e))^{(1/2)})/a^{(5/2)}/f * 2^{(1/2)} + 7/2 c^4 \tan(fx+e)/a^2/f/(a+a \sec(fx+e))^{(1/2)} - 1/4 c^4 \sec(1/2 e + 1/2 fx)^2 \sin(fx+e) \tan(fx+e)^2/a/f/(a+a \sec(fx+e))^{(3/2)} - 1/4 c^4 \sec(1/2 e + 1/2 fx)^4 \sin(fx+e)^2 \tan(fx+e)^3/f/(a+a \sec(fx+e))^{(5/2)}$

Rubi [A] time = 0.30, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3904, 3887, 470, 578, 582, 522, 203}

$$\frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} - \frac{11c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{2} a^{5/2} f} + \frac{7c^4 \tan(e+fx)}{2a^2 f \sqrt{a \sec(e+fx)+a}} - \frac{c^4 \sin^2(e+fx) \tan^3(e+fx)}{4f(a \sec(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $(2c^4 \text{ArcTan}[(\text{Sqrt}[a] \text{Tan}[e + fx])/\text{Sqrt}[a + a \text{Sec}[e + fx]])]/(a^{(5/2)} f) - (11c^4 \text{ArcTan}[(\text{Sqrt}[a] \text{Tan}[e + fx])/(\text{Sqrt}[2] \text{Sqrt}[a + a \text{Sec}[e + fx]])])/(\text{Sqrt}[2] a^{(5/2)} f) + (7c^4 \text{Tan}[e + fx])/(2a^2 f \text{Sqrt}[a + a \text{Sec}[e + fx]]) - (c^4 \text{Sec}[(e + fx)/2]^2 \text{Sin}[e + fx] \text{Tan}[e + fx]^2)/(4a f (a + a \text{Sec}[e + fx])^{(3/2)}) - (c^4 \text{Sec}[(e + fx)/2]^4 \text{Sin}[e + fx]^2 \text{Tan}[e + fx]^3)/(4f (a + a \text{Sec}[e + fx])^{(5/2)})$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

```

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

```

Rule 582

```

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]

```

Rule 3887

```

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

```

Rule 3904

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^{5/2}} dx &= (a^4 c^4) \int \frac{\tan^8(e + fx)}{(a + a \sec(e + fx))^{13/2}} dx \\
&= -\frac{(2a^2 c^4) \operatorname{Subst}\left(\int \frac{x^8}{(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\
&= -\frac{c^4 \sec^4\left(\frac{1}{2}(e + fx)\right) \sin^2(e + fx) \tan^3(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} - \frac{c^4 \operatorname{Subst}\left(\int \frac{x^4(10+6ax^2)}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{2f} \\
&= -\frac{c^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^2(e + fx)}{4af(a + a \sec(e + fx))^{3/2}} - \frac{c^4 \sec^4\left(\frac{1}{2}(e + fx)\right) \sin^2(e + fx) \tan^2(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{7c^4 \tan(e + fx)}{2a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^2(e + fx)}{4af(a + a \sec(e + fx))^{3/2}} - \frac{c^4 \sec^4\left(\frac{1}{2}(e + fx)\right) \sin^2(e + fx) \tan^2(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{7c^4 \tan(e + fx)}{2a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c^4 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx) \tan^2(e + fx)}{4af(a + a \sec(e + fx))^{3/2}} - \frac{c^4 \sec^4\left(\frac{1}{2}(e + fx)\right) \sin^2(e + fx) \tan^2(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} \\
&= \frac{2c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{5/2} f} - \frac{11c^4 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{\sqrt{2} a^{5/2} f} + \frac{7c^4 \tan(e + fx)}{2a^2 f \sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 2.68, size = 164, normalized size = 0.72

$$\frac{c^4 \cot\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \left((19 \cos(e + fx) - 12 \cos(2(e + fx)) - 3 \cos(3(e + fx)) - 4) \sec^4\left(\frac{1}{2}(e + fx)\right) + 32 \right)}{16a^2 f \sqrt{a \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2), x]

[Out] (c^4*Cot[(e + f*x)/2]*((-4 + 19*Cos[e + f*x] - 12*Cos[2*(e + f*x)] - 3*Cos[3*(e + f*x)])*Sec[(e + f*x)/2]^4 + 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]] - 88*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]]/Sqrt[2])*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]]*Sec[e + f*x]/(16*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 2.48, size = 655, normalized size = 2.86

$$\frac{11 \sqrt{2} \left(c^4 \cos^3(fx + e) + 3c^4 \cos^2(fx + e) + 3c^4 \cos(fx + e) + c^4 \right) \sqrt{-a} \log \left(-\frac{2 \sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e)}{\cos(fx+e)^2 + \dots} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2), x, algorithm="fricas")

[Out] [-1/4*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c

$$\begin{aligned} & \cos(f*x + e))*\cos(f*x + e)*\sin(f*x + e) - 3*a*\cos(f*x + e)^2 - 2*a*\cos(f*x + \\ & e) + a)/(\cos(f*x + e)^2 + 2*\cos(f*x + e) + 1)) + 4*(c^4*\cos(f*x + e)^3 + 3 \\ & *c^4*\cos(f*x + e)^2 + 3*c^4*\cos(f*x + e) + c^4)*\sqrt{-a}*\log((2*a*\cos(f*x + \\ & e)^2 + 2*\sqrt{-a})*\sqrt{(a*\cos(f*x + e) + a)/\cos(f*x + e)}*\cos(f*x + e)*\sin \\ & (f*x + e) + a*\cos(f*x + e) - a)/(\cos(f*x + e) + 1)) - 4*(3*c^4*\cos(f*x + e) \\ & ^2 + 9*c^4*\cos(f*x + e) + 2*c^4)*\sqrt{(a*\cos(f*x + e) + a)/\cos(f*x + e)}* \sin \\ & (f*x + e))/ (a^3*f*\cos(f*x + e)^3 + 3*a^3*f*\cos(f*x + e)^2 + 3*a^3*f*\cos(f* \\ & x + e) + a^3*f), 1/2*(11*\sqrt{2})*(c^4*\cos(f*x + e)^3 + 3*c^4*\cos(f*x + e)^2 \\ & + 3*c^4*\cos(f*x + e) + c^4)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{(a*\cos(f*x + e) + \\ & a)/\cos(f*x + e)}*\cos(f*x + e)/(\sqrt{a}*\sin(f*x + e))) - 4*(c^4*\cos(f*x + e) \\ & ^3 + 3*c^4*\cos(f*x + e)^2 + 3*c^4*\cos(f*x + e) + c^4)*\sqrt{a}*\arctan(\sqrt{((\\ & a*\cos(f*x + e) + a)/\cos(f*x + e))*\cos(f*x + e)/(\sqrt{a}*\sin(f*x + e))}) + 2* \\ & (3*c^4*\cos(f*x + e)^2 + 9*c^4*\cos(f*x + e) + 2*c^4)*\sqrt{(a*\cos(f*x + e) + \\ & a)/\cos(f*x + e)}*\sin(f*x + e))/ (a^3*f*\cos(f*x + e)^3 + 3*a^3*f*\cos(f*x + e) \\ & ^2 + 3*a^3*f*\cos(f*x + e) + a^3*f)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
 /2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
 x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
 able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
 n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
 /2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
 ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
 eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs o
 r sign assumes constant sign by intervals (correct if the argument is real)
 :Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)
 >(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
 2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
 step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
 nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
 o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
 pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
 2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
 nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
 gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
 p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
 tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
 heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
 t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
 i/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warn
 ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
 umptions on a variableWarning, assuming -2*a+a is positive. Hint: run assum

maple [B] time = 2.06, size = 550, normalized size = 2.40

$$c^4 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(2 \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} (\cos^4(fx+e)) \sin(fx+e) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + 11 \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x)`

[Out]
$$-1/2*c^4/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\cos(f*x+e)^4*\sin(f*x+e)*2^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})+11*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))*\cos(f*x+e)^4*\sin(f*x+e)-4*\sin(f*x+e)*\cos(f*x+e)^2*2^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})-22*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)+6*\cos(f*x+e)^5*2^{(1/2)}*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}+11*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))-32*\cos(f*x+e)^3+36*\cos(f*x+e)^2-6*\cos(f*x+e)-4)/\sin(f*x+e)^{5/a^3}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^4}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2),x)`

[Out] `int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^4 \left(\int \left(\frac{4 \sec(e+fx)}{a^2 \sqrt{a \sec(e+fx) + a} \sec^2(e+fx) + 2a^2 \sqrt{a \sec(e+fx) + a} \sec(e+fx) + a^2 \sqrt{a \sec(e+fx) + a}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(5/2),x)`


```
[Out] c**4*(Integral(-4*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x)
) + a)), x) + Integral(6*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec
(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*s
ec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*
*2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a**2*sqrt(a*sec(e + f*x) + a)), x))
```

$$3.81 \quad \int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=191

$$\frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{7c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{2\sqrt{2} a^{5/2}f} - \frac{c^3 \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{4a^2f \sqrt{a \sec(e+fx)+a}} + \frac{c^3 \sin^2(e+fx) \tan(e+fx)}{4af(a \sec(e+fx)+a)}$$

[Out] $2*c^3*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f-7/4*c^3*a$
 $rctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f*2^{(1$
 $/2)-1/4*c^3*\sec(1/2*e+1/2*f*x)^2*\sin(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)+1/$
 $4*c^3*\sec(1/2*e+1/2*f*x)^4*\sin(f*x+e)^2*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(3/$
 $2)}$

Rubi [A] time = 0.25, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 470, 578, 522, 203}

$$-\frac{c^3 \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{4a^2f \sqrt{a \sec(e+fx)+a}} + \frac{2c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{7c^3 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{2\sqrt{2} a^{5/2}f} + \frac{c^3 \sin^2(e+fx) \tan(e+fx)}{4af(a \sec(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $(2*c^3*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(a^{(5/2)*f}$
 $- (7*c^3*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])]/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]))$
 $)/(2*\text{Sqrt}[2]*a^{(5/2)*f} - (c^3*\text{Sec}[(e + f*x)/2]^2*\text{Sin}[e + f*x])/(4*a^2*f*\text{Sqr}$
 $\text{rt}[a + a*\text{Sec}[e + f*x]]) + (c^3*\text{Sec}[(e + f*x)/2]^4*\text{Sin}[e + f*x]^2*\text{Tan}[e + f*$
 $x])/(4*a*f*(a + a*\text{Sec}[e + f*x])^{(3/2)})$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q)*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*

$(g*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)}/(b*n*(b*c-a*d)*$
 $(p+1), x] - \text{Dist}[g^n/(b*n*(b*c-a*d)*(p+1)), \text{Int}[(g*x)^{(m-n)}*(a+$
 $b*x^n)^{(p+1)}*(c+d*x^n)^q*\text{Simp}[c*(b*e-a*f)*(m-n+1) + (d*(b*e-a*f)$
 $)*(m+n*q+1) - b*n*(c*f-d*e)*(p+1)*x^n, x], x] /; \text{FreeQ}[\{a, b,$
 $c, d, e, f, g, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m-n+1, 0]$

Rule 3887

$\text{Int}[\text{cot}[(c_.) + (d_.)*(x_.)]^{(m_.)}*(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^{(n$
 $_.), x_Symbol] := \text{Dist}[(-2*a^{(m/2+n+1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2+a*x^2)$
 $^{(m/2+n-1/2)})/(1+a*x^2), x], x, \text{Cot}[c+d*x]/\text{Sqrt}[a+b*\text{Csc}[c+d*x]]$
 $], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2-b^2, 0] \&\& \text{IntegerQ}[m/2] \&\& \text{In$
 $tegerQ}[n-1/2]$

Rule 3904

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*($
 $d_.) + (c_.))^{(n_.), x_Symbol] := \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e+f*x]^{(2*m)}*(c$
 $+d*\text{Csc}[e+f*x]^{(n-m)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{Eq}$
 $\text{Q}[b*c+a*d, 0] \&\& \text{EqQ}[a^2-b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{I$
 $ntegerQ}[n] \&\& \text{GtQ}[m-n, 0])$

Rubi steps

$$\int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^{5/2}} dx = - \left((a^3 c^3) \int \frac{\tan^6(e + fx)}{(a + a \sec(e + fx))^{11/2}} dx \right)$$

$$= \frac{(2ac^3) \text{Subst} \left(\int \frac{x^6}{(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f}$$

$$= \frac{c^3 \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx) \tan(e + fx)}{4af(a + a \sec(e + fx))^{3/2}} + \frac{c^3 \text{Subst} \left(\int \frac{x^2(6+2ax^2)}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{2af}$$

$$= -\frac{c^3 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{4a^2 f \sqrt{a + a \sec(e + fx)}} + \frac{c^3 \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx) \tan(e + fx)}{4af(a + a \sec(e + fx))^{3/2}}$$

$$= -\frac{c^3 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{4a^2 f \sqrt{a + a \sec(e + fx)}} + \frac{c^3 \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx) \tan(e + fx)}{4af(a + a \sec(e + fx))^{3/2}}$$

$$= \frac{2c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{a^{5/2} f} - \frac{7c^3 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{2\sqrt{2} a^{5/2} f} - \frac{c^3 \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin^2(e + fx)}{4a^2 f \sqrt{a + a \sec(e + fx)}}$$

Mathematica [A] time = 1.59, size = 136, normalized size = 0.71

$$\frac{c^3 \cot \left(\frac{1}{2}(e + fx) \right) \left((8 \cos(e + fx) - 3 \cos(2(e + fx)) - 5) \sec^4 \left(\frac{1}{2}(e + fx) \right) - 32 \sqrt{\sec(e + fx) - 1} \tan^{-1} \left(\sqrt{\sec(e + fx) - 1} \right) \right)}{16a^2 f \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2), x]

[Out] -1/16*(c^3*Cot[(e + f*x)/2]*((-5 + 8*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4 - 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]])

/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 2.27index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 2.11, size = 553, normalized size = 2.90

$$c^3 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (-1 + \cos(fx + e)) \left(4 \sin(fx + e) (\cos^2(fx + e)) \sqrt{2} \sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\sqrt{\frac{2 \cos(fx+e)}{1+\cos(fx+e)}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x)

[Out] 1/4*c^3/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+8*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+7*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+4*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+14*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+7*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-6*cos(f*x+e)^3+8*cos(f*x+e)^2-2*cos(f*x+e))/(1+cos(f*x+e))/sin(f*x+e)^3/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^3}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)

[Out] int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3 \int \frac{3 \sec(e + fx)}{a^2 \sqrt{a \sec(e + fx) + a} \sec^2(e + fx) + 2a^2 \sqrt{a \sec(e + fx) + a} \sec(e + fx) + a^2 \sqrt{a \sec(e + fx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2), x)

[Out] -c**3*(Integral(3*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
 *2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x)
) + a)), x) + Integral(-3*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*se
 c(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*
 sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a**2*sqrt(a*sec(e + f*x)
 + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2
 *sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)
 *sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt
 (a*sec(e + f*x) + a)), x))

$$3.82 \quad \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=189

$$\frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} - \frac{11c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{4\sqrt{2} a^{5/2} f} - \frac{3c^2 \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2 f \sqrt{a \sec(e+fx)+a}} - \frac{c^2 \sin(e+fx) \cos\left(\frac{1}{2}(e+fx)\right)}{4a^2 f \sqrt{a \sec(e+fx)+a}}$$

[Out] $2*c^2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f-11/8*c^2*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f*2^{(1/2)}-3/8*c^2*\sec(1/2*e+1/2*f*x)^2*\sin(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)}-1/4*c^2*\cos(f*x+e)*\sec(1/2*e+1/2*f*x)^4*\sin(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3904, 3887, 470, 527, 522, 203}

$$-\frac{3c^2 \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2 f \sqrt{a \sec(e+fx)+a}} + \frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2} f} - \frac{11c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{4\sqrt{2} a^{5/2} f} - \frac{c^2 \sin(e+fx) \cos\left(\frac{1}{2}(e+fx)\right)}{4a^2 f \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $(2*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(a^{(5/2)*f} - (11*c^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]])/(4*Sqrt[2]*a^{(5/2)*f} - (3*c^2*Sec[(e + f*x)/2]^2*Sin[e + f*x])/(8*a^2*f*Sqrt[a + a*Sec[e + f*x]]) - (c^2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Sin[e + f*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]]))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +

$d*x^n)^{(q+1)}/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \&\& \text{LtQ}[p, -1]$

Rule 3887

$\text{Int}[\text{cot}[(c_.) + (d_.)*(x_)]^{(m_.)}*(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^{(n_.)}, x_Symbol] :> \text{Dist}[(-2*a^{(m/2 + n + 1/2)})/d, \text{Subst}[\text{Int}[(x^m*(2 + a*x^2)^{(m/2 + n - 1/2)})/(1 + a*x^2), x], x, \text{Cot}[c + d*x]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[n - 1/2]$

Rule 3904

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^{(n_.)}, x_Symbol] :> \text{Dist}[(-a*c)^m, \text{Int}[\text{Cot}[e + f*x]^{(2*m)}*(c + d*\text{Csc}[e + f*x])^{(n - m)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{RationalQ}[n] \&\& !(\text{IntegerQ}[n] \&\& \text{GtQ}[m - n, 0])$

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^2}{(a + a \sec(e + fx))^{5/2}} dx &= (a^2 c^2) \int \frac{\tan^4(e + fx)}{(a + a \sec(e + fx))^{9/2}} dx \\ &= \frac{(2c^2) \text{Subst}\left(\int \frac{x^4}{(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} \\ &= \frac{c^2 \cos(e + fx) \sec^4\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{4a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c^2 \text{Subst}\left(\int \frac{2-2ax^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{2a^2 f} \\ &= \frac{3c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{8a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c^2 \cos(e + fx) \sec^4\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{4a^2 f \sqrt{a + a \sec(e + fx)}} \\ &= \frac{3c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{8a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c^2 \cos(e + fx) \sec^4\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{4a^2 f \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{5/2} f} - \frac{11c^2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{4\sqrt{2} a^{5/2} f} - \frac{3c^2 \sec^2\left(\frac{1}{2}(e + fx)\right) \sin(e + fx)}{8a^2 f \sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.56, size = 136, normalized size = 0.72

$$\frac{c^2 \cot\left(\frac{1}{2}(e + fx)\right) \left((8 \cos(e + fx) - 7 \cos(2(e + fx)) - 1) \sec^4\left(\frac{1}{2}(e + fx)\right) - 64 \sqrt{\sec(e + fx) - 1} \tan^{-1}\left(\sqrt{\sec(e + fx) - 1}\right) \right)}{32a^2 f \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2), x]

[Out] -1/32*(c^2*Cot[(e + f*x)/2]*((-1 + 8*Cos[e + f*x] - 7*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4 - 64*ArcTan[Sqrt[-1 + Sec[e + f*x]])*Sqrt[-1 + Sec[e + f*x]

)] + 44*sqrt(2)*ArcTan[Sqrt[-1 + Sec[e + f*x]]/sqrt(2)]*sqrt[-1 + Sec[e + f*x]])/(a^2*f*sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 1.53, size = 645, normalized size = 3.41

$$\frac{11\sqrt{2}\left(c^2\cos(fx+e)^3 + 3c^2\cos(fx+e)^2 + 3c^2\cos(fx+e) + c^2\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/16*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 16*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(7*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/8*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 16*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(7*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
 2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
 /2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
 t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
 pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
 le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
 gn by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep
 +exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable

/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.83index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.88, size = 545, normalized size = 2.88

$$c^2 \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(8 \sin(fx+e) (\cos^2(fx+e)) \sqrt{2} \sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh}\left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)}\right) + 16 \cos(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2), x)

[Out]
$$-1/8*c^2/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(8*\sin(f*x+e)*\cos(f*x+e)^{2*(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})+16*\cos(f*x+e)*\sin(f*x+e)*2^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})+11*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)+8*2^{(1/2)}*\sin(f*x+e)*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}+22*\cos(f*x+e)*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))+11*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))-14*\cos(f*x+e)^3+8*\cos(f*x+e)^2+6*\cos(f*x+e))/(1+\cos(f*x+e))^2/\sin(f*x+e)/a^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c \sec(fx+e) - c)^2}{(a \sec(fx+e) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2), x, algorithm="maxima")

[Out] integrate((c*sec(f*x+e) - c)^2/(a*sec(f*x+e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^2}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \left(\frac{2 \sec(e + fx)}{a^2 \sqrt{a \sec(e + fx) + a} \sec^2(e + fx) + 2a^2 \sqrt{a \sec(e + fx) + a} \sec(e + fx) + a^2 \sqrt{a \sec(e + fx) + a}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)

[Out] c**2*(Integral(-2*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
 *2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x)
) + a)), x) + Integral(sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec
 (e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
 *x) + a)), x))

$$3.83 \quad \int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=148

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{23c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{8\sqrt{2} a^{5/2}f} - \frac{7c \tan(e+fx)}{8af(a \sec(e+fx) + a)^{3/2}} - \frac{c \tan(e+fx)}{2f(a \sec(e+fx) + a)^{5/2}}$$

[Out] $2*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f-23/16*c*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/a^{(5/2)}/f*2^{(1/2)}-1/2*c*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(5/2)}-7/8*c*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(3/2)}$

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3904, 3887, 471, 527, 522, 203}

$$\frac{7c \sin(e+fx) \sec^2\left(\frac{1}{2}(e+fx)\right)}{16a^2 f \sqrt{a \sec(e+fx) + a}} + \frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}f} - \frac{23c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{8\sqrt{2} a^{5/2}f} - \frac{c \sin(e+fx) \cos(e+fx)}{8a^2 f \sqrt{a \sec(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $(2*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(a^{(5/2)}*f) - (23*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(8*\text{Sqrt}[2]*a^{(5/2)}*f) - (7*c*\text{Sec}[(e + f*x)/2]^2*\text{Sin}[e + f*x])/(16*a^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (c*\text{Cos}[e + f*x]*\text{Sec}[(e + f*x)/2]^4*\text{Sin}[e + f*x])/(8*a^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 471

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(n*(b*c - a*d)*(p+1)), x] - Dist[e^n/(n*(b*c - a*d)*(p+1)), Int[(e*x)^(m-n)*(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[c*(m-n+1) + d*(m+n*(p+q+1)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(b*c - a*d)*(p+1)), x] + Dist[1/(a*n*(b*c - a*d)*(p

+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_.)]^(m_.)*(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])

Rubi steps

$$\begin{aligned} \int \frac{c - c \sec(e + fx)}{(a + a \sec(e + fx))^{5/2}} dx &= - \left((ac) \int \frac{\tan^2(e + fx)}{(a + a \sec(e + fx))^{7/2}} dx \right) \\ &= \frac{(2c) \operatorname{Subst} \left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{af} \\ &= -\frac{c \cos(e + fx) \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{8a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c \operatorname{Subst} \left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{2a^2 f} \\ &= -\frac{7c \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{16a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c \cos(e + fx) \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{8a^2 f \sqrt{a + a \sec(e + fx)}} \\ &= -\frac{7c \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{16a^2 f \sqrt{a + a \sec(e + fx)}} - \frac{c \cos(e + fx) \sec^4 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{8a^2 f \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{a^{5/2} f} - \frac{23c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{8\sqrt{2} a^{5/2} f} - \frac{7c \sec^2 \left(\frac{1}{2}(e + fx) \right) \sin(e + fx)}{16a^2 f \sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.49, size = 134, normalized size = 0.91

$$\frac{c \cot \left(\frac{1}{2}(e + fx) \right) \left((8 \cos(e + fx) - 11 \cos(2(e + fx)) + 3) \sec^4 \left(\frac{1}{2}(e + fx) \right) - 128 \sqrt{\sec(e + fx) - 1} \tan^{-1} \left(\sqrt{\sec(e + fx) - 1} \right) \right)}{64a^2 f \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2), x]

[Out] -1/64*(c*Cot[(e + f*x)/2]*((3 + 8*Cos[e + f*x] - 11*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4 - 128*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*x]] + 92*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e + f*x]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])

maple [B] time = 1.62, size = 543, normalized size = 3.67

$$c \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(16 \sin(fx+e) (\cos^2(fx+e)) \sqrt{2} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) + 32 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x)

[Out]
$$-1/16*c/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(16*\sin(f*x+e)*\cos(f*x+e)^{2*2} \\ ^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})+32*\cos(f*x+e)*\sin(f*x+e)*2 \\ ^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})+23*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)+16*2^{(1/2)}*\sin(f*x+e)*\operatorname{arctanh}(1/2* \\ (-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)/\cos(f*x+e)*2^{(1/2)})*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}+46*\cos(f*x+e)*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))+23*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\ln(((-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*\sin(f*x+e)-\cos(f*x+e)+1)/\sin(f*x+e))-2 \\ 2*\cos(f*x+e)^3+8*\cos(f*x+e)^2+14*\cos(f*x+e))/\cos(f*x+e)^2/\sin(f*x+e)/a^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c \sec(fx+e) - c}{(a \sec(fx+e) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c - \frac{c}{\cos(e+fx)}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \int \frac{\sec(e+fx)}{a^2 \sqrt{a \sec(e+fx) + a} \sec^2(e+fx) + 2a^2 \sqrt{a \sec(e+fx) + a} \sec(e+fx) + a^2 \sqrt{a \sec(e+fx) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)

```
[Out] -c*(Integral(sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 +
2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a
)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**
2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x
))
```

$$3.84 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))} dx$$

Optimal. Leaf size=230

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}cf} - \frac{71 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{32\sqrt{2} a^{5/2}cf} - \frac{7 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{32a^3cf} + \frac{\cos^2(e+fx) \cot(e+fx)}{32a^3cf}$$

[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f-71/64*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f*2^(1/2)-7/32*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+13/32*cos(f*x+e)*cot(f*x+e)*sec(1/2*e+1/2*f*x)^2*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+1/16*cos(f*x+e)^2*cot(f*x+e)*sec(1/2*e+1/2*f*x)^4*(a+a*sec(f*x+e))^(1/2)/a^3/c/f

Rubi [A] time = 0.31, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3904, 3887, 472, 579, 583, 522, 203}

$$-\frac{7 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{32a^3cf} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}cf} - \frac{71 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{32\sqrt{2} a^{5/2}cf} + \frac{\cos^2(e+fx) \cot(e+fx)}{32a^3cf}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]

[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(a^(5/2)*c*f) - (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(32*Sqrt[2]*a^(5/2)*c*f) - (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(32*a^3*c*f) + (13*Cos[e + f*x]*Cot[e + f*x]*Sec[(e + f*x)/2]^2*Sqrt[a + a*Sec[e + f*x]])/(32*a^3*c*f) + (Cos[e + f*x]^2*Cot[e + f*x]*Sec[(e + f*x)/2]^4*Sqrt[a + a*Sec[e + f*x]])/(16*a^3*c*f)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 472

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^(m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 579

Int[((g_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[(b*e - a*f)*(g*x)^(m

```
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^(n*(
m + 1))), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_) + (d_)*(x_)]^(m_)*(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n
_), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(
d_) + (c_))^(n_), x_Symbol] := Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c
+ d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))} dx &= -\frac{\int \frac{\cot^2(e+fx)}{(a+a \sec(e+fx))^{3/2}} dx}{ac} \\
&= \frac{2 \operatorname{Subst} \left(\int \frac{1}{x^2(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{a^3 cf} \\
&= \frac{\cos^2(e + fx) \cot(e + fx) \sec^4 \left(\frac{1}{2}(e + fx) \right) \sqrt{a + a \sec(e + fx)}}{16a^3 cf} \\
&= \frac{13 \cos(e + fx) \cot(e + fx) \sec^2 \left(\frac{1}{2}(e + fx) \right) \sqrt{a + a \sec(e + fx)}}{32a^3 cf} \\
&= -\frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{32a^3 cf} + \frac{13 \cos(e + fx) \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{32a^3 cf} \\
&= -\frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{32a^3 cf} + \frac{13 \cos(e + fx) \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{32a^3 cf} \\
&= \frac{2 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{a^{5/2} cf} - \frac{71 \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{32\sqrt{2} a^{5/2} cf} - \frac{7 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{32a^3 cf}
\end{aligned}$$

Mathematica [A] time = 1.52, size = 158, normalized size = 0.69

$$\frac{\tan^3 \left(\frac{1}{2}(e + fx) \right) \left(24 \cos(e + fx) + 27 \cos(2(e + fx)) + 512 \cos^4 \left(\frac{1}{2}(e + fx) \right) \sqrt{\sec(e + fx) - 1} \tan^{-1} \left(\sqrt{\sec(e + fx) - 1} \right) \right)}{64a^2 cf (\cos(e + fx) - 1)^2 \sqrt{a(\sec(e + fx) - 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]

[Out] ((13 + 24*Cos[e + f*x] + 27*Cos[2*(e + f*x)] + 512*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[(e + f*x)/2]^4*Sqrt[-1 + Sec[e + f*x]] - 284*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x)/2]^4*Sqrt[-1 + Sec[e + f*x]])*Tan[(e + f*x)/2]^3)/(64*a^2*c*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 0.66, size = 608, normalized size = 2.64

$$\frac{71 \sqrt{2} \left(\cos(fx + e)^2 + 2 \cos(fx + e) + 1 \right) \sqrt{-a} \log \left(\frac{2 \sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - 3a \cos(fx+e)^2 - 2a}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)}{64 a^2 c f (\cos(e + fx) - 1)^2 \sqrt{a(\sec(e + fx) - 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [-1/128*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-a)*log(-2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f

```

*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos
s(f*x + e) + 1))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sq
rt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(
-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e)
+ a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(27*cos(f*x + e)^3 + 12*cos(f*x
+ e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c*f
*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/64*(71
*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin
(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*c
os(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(27*cos(f*x + e)^3 +
12*cos(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)
)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by interval
s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
```


+e)*cos(f*x+e)^2*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+128*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))+71*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+64*2^(1/2)*sin(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+142*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))+71*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(((2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/sin(f*x+e))-54*cos(f*x+e)^3-24*cos(f*x+e)^2+14*cos(f*x+e))/sin(f*x+e)^5/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a \sec(fx + e) + a\right)^{\frac{5}{2}} (c \sec(fx + e) - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] -integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{\frac{5}{2}} \left(c - \frac{c}{\cos(e+fx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))),x)

[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2 \sqrt{a \sec(e+fx)+a} \sec^3(e+fx)+a^2 \sqrt{a \sec(e+fx)+a} \sec^2(e+fx)-a^2 \sqrt{a \sec(e+fx)+a} \sec(e+fx)-a^2 \sqrt{a \sec(e+fx)+a}}{c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)

[Out] -Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - a**2*sqrt(a*sec(e + f*x) + a)), x)/c

$$3.85 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^2} dx$$

Optimal. Leaf size=269

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}c^2f} - \frac{107 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{64\sqrt{2} a^{5/2}c^2f} + \frac{43 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{96a^4c^2f} - \frac{\cos^2(e+fx) \cot^3(e+fx)}{96a^4c^2f}$$

[Out] 2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f+43/96*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-15/32*cos(f*x+e)*cot(f*x+e)^3*sec(1/2*e+1/2*f*x)^2*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-1/16*cos(f*x+e)^2*cot(f*x+e)^3*sec(1/2*e+1/2*f*x)^4*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-107/128*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f*2^(1/2)+21/64*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c^2/f

Rubi [A] time = 0.34, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 28, number of rules / integrand size = 0.250, Rules used = {3904, 3887, 472, 579, 583, 522, 203}

$$\frac{43 \cot^3(e+fx)(a \sec(e+fx)+a)^{3/2}}{96a^4c^2f} + \frac{21 \cot(e+fx)\sqrt{a \sec(e+fx)+a}}{64a^3c^2f} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{5/2}c^2f} - \frac{107 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{64\sqrt{2} a^{5/2}c^2f}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2), x]

[Out] (2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/(a^(5/2)*c^2*f) - (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(64*Sqrt[2]*a^(5/2)*c^2*f) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]/(64*a^3*c^2*f) + (43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(96*a^4*c^2*f) - (15*Cos[e + f*x]*Cot[e + f*x]^3*Sec[(e + f*x)/2]^2*(a + a*Sec[e + f*x])^(3/2))/(32*a^4*c^2*f) - (Cos[e + f*x]^2*Cot[e + f*x]^3*Sec[(e + f*x)/2]^4*(a + a*Sec[e + f*x])^(3/2))/(16*a^4*c^2*f)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(-a*c)^m, Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^2} dx &= \frac{\int \frac{\cot^4(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx}{a^2 c^2} \\
&= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{x^4(1+ax^2)(2+ax^2)^3} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^4 c^2 f} \\
&= -\frac{\cos^2(e + fx) \cot^3(e + fx) \sec^4\left(\frac{1}{2}(e + fx)\right) (a + a \sec(e + fx))}{16 a^4 c^2 f} \\
&= -\frac{15 \cos(e + fx) \cot^3(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right) (a + a \sec(e + fx))}{32 a^4 c^2 f} \\
&= \frac{43 \cot^3(e + fx) (a + a \sec(e + fx))^{3/2}}{96 a^4 c^2 f} - \frac{15 \cos(e + fx) \cot^3(e + fx)}{96 a^4 c^2 f} \\
&= \frac{21 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{64 a^3 c^2 f} + \frac{43 \cot^3(e + fx) (a + a \sec(e + fx))^{3/2}}{96 a^4 c^2 f} \\
&= \frac{21 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{64 a^3 c^2 f} + \frac{43 \cot^3(e + fx) (a + a \sec(e + fx))^{3/2}}{96 a^4 c^2 f} \\
&= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{5/2} c^2 f} - \frac{107 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{64 \sqrt{2} a^{5/2} c^2 f} + \frac{21 \cot(e + fx) \sqrt{a + a \sec(e + fx)}}{64 a^3 c^2 f}
\end{aligned}$$

Mathematica [C] time = 24.17, size = 5650, normalized size = 21.00

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2), x]

[Out] Result too large to show

fricas [A] time = 0.67, size = 706, normalized size = 2.62

$$\left[\frac{321 \sqrt{2} \left(\cos^3(fx + e) + \cos^2(fx + e) - \cos(fx + e) - 1 \right) \sqrt{-a} \log \left(-\frac{2 \sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - 3 a \cos(fx+e)^2 - 2 a \cos(fx+e) + a}{\cos(fx+e)^2 + 2 \cos(fx+e) + a} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [-1/768*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f


```
*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 384*(cos(f*x + e)^3 + cos(f
*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*
(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*x + e)^2 - 63*cos(f*x +
e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/384*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*
sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 384*(cos(f*x + e)^3 + cos(f*x + e)^
2 - cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e)
- a))*sin(f*x + e) + 2*(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*
x + e)^2 - 63*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*
c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) -
a^3*c^2*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableUnabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2

$1/2) * \operatorname{arctanh}(1/2 * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e))))^{1/2} * \sin(f * x + e) / \cos(f * x + e) * 2^{1/2}) + 384 * \sin(f * x + e) * \cos(f * x + e)^2 * 2^{1/2} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \operatorname{arctanh}(1/2 * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e))))^{1/2} * \sin(f * x + e) / \cos(f * x + e) * 2^{1/2}) + 321 * \sin(f * x + e) * \cos(f * x + e)^3 * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \ln(((-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \sin(f * x + e) - \cos(f * x + e) + 1) / \sin(f * x + e)) - 384 * \cos(f * x + e) * \sin(f * x + e) * 2^{1/2} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \operatorname{arctanh}(1/2 * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e))))^{1/2} * \sin(f * x + e) / \cos(f * x + e) * 2^{1/2}) + 321 * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \ln(((-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \sin(f * x + e) - \cos(f * x + e) + 1) / \sin(f * x + e)) * \cos(f * x + e)^2 * \sin(f * x + e) - 384 * 2^{1/2} * \sin(f * x + e) * \operatorname{arctanh}(1/2 * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e))))^{1/2} * \sin(f * x + e) / \cos(f * x + e) * 2^{1/2}) * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} - 321 * \cos(f * x + e) * \sin(f * x + e) * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \ln(((-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \sin(f * x + e) - \cos(f * x + e) + 1) / \sin(f * x + e)) - 410 * \cos(f * x + e)^4 - 321 * \sin(f * x + e) * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \ln(((-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{1/2} * \sin(f * x + e) - \cos(f * x + e) + 1) / \sin(f * x + e)) - 142 * \cos(f * x + e)^3 + 298 * \cos(f * x + e)^2 + 126 * \cos(f * x + e) / \sin(f * x + e)^7 / a^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(fx + e) + a)^{\frac{5}{2}} (c \sec(fx + e) - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")

[Out] integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2} \left(c - \frac{c}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2), x)

[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2 \sqrt{a \sec(e+fx)+a} \sec^4(e+fx) - 2a^2 \sqrt{a \sec(e+fx)+a} \sec^2(e+fx) + a^2 \sqrt{a \sec(e+fx)+a}}{c^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)

[Out] Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + a**2*sqrt(a*sec(e + f*x) + a))), x)/c**2

3.86 $\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2} dx$

Optimal. Leaf size=185

$$\frac{ac^4 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac^3 \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}} - \frac{ac^2 \tan(e + fx)(c - c \sec(e + fx))^{3/2}}{2f \sqrt{a \sec(e + fx) + a}}$$

[Out] $-1/2*a*c^2*(c-c*\sec(f*x+e))^{(3/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-1/3*a*c*(c-c*\sec(f*x+e))^{(5/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+a*c^4*\ln(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-a*c^3*(c-c*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3906, 3905, 3475}

$$\frac{ac^3 \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}} - \frac{ac^2 \tan(e + fx)(c - c \sec(e + fx))^{3/2}}{2f \sqrt{a \sec(e + fx) + a}} + \frac{ac^4 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2), x]

[Out] $(a*c^4*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (a*c^3*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (a*c^2*(c - c*\text{Sec}[e + f*x])^{(3/2)}*\text{Tan}[e + f*x])/(2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (a*c*(c - c*\text{Sec}[e + f*x])^{(5/2)}*\text{Tan}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Dist[((-a*c)^(m + 1/2)*Cot[e + f*x]]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rule 3906

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2} dx &= -\frac{ac(c - c \sec(e + fx))^{5/2} \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}} + c \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2} dx \\
&= -\frac{ac^2(c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{a + a \sec(e + fx)}} - \frac{ac(c - c \sec(e + fx))^{5/2}}{3f\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{ac^3\sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{ac^2(c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{ac^3\sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{ac^2(c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{a + a \sec(e + fx)}} \\
&= \frac{ac^4 \log(\cos(e + fx)) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{ac^3\sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 6.14, size = 149, normalized size = 0.81

$$\frac{c^3 \csc\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c - c \sec(e + fx)} (-18 \cos(2(e + fx)) + 3if)}{24f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2), x]

[Out] (c^3*Csc[(e + f*x)/2]*(-22 - 18*Cos[2*(e + f*x)] + (3*I)*f*x*Cos[3*(e + f*x)] + 9*Cos[e + f*x]*(2 + I*f*x - Log[1 + E^((2*I)*(e + f*x))]) - 3*Cos[3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(24*f)

fricas [A] time = 0.59, size = 459, normalized size = 2.48

$$\frac{\left(11c^3 \cos^2(fx + e) - 7c^3 \cos(fx + e) + 2c^3\right) \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \sqrt{\frac{c \cos(fx + e) - c}{\cos(fx + e)}} \sin(fx + e) - 3\left(c^3 \cos^3(fx + e) + f c^3 \cos^2(fx + e)\right)}{6\left(f \cos^3(fx + e) + f c^3 \cos^2(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2), x, algorithm="fricas")

[Out] [-1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 3*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*c^3*cos(f*x + e)^2), -1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 6*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^3 + f*c^3*cos(f*x + e)^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
```



```

*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 44*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*
e) + 3*c^3*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 18*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*
x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 18*((f*x +
e)*c^3*sin(4*f*x + 4*e) + (f*x + e)*c^3*sin(2*f*x + 2*e) - c^3*cos(4*f*x +
4*e) - c^3*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 18*(3*(f*x + e)*c^3*sin(2*
f*x + 2*e) + c^3)*sin(4*f*x + 4*e) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4
*f*x + 4*e) + 3*c^3*cos(2*f*x + 2*e) + c^3)*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 44*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) +
3*c^3*cos(2*f*x + 2*e) + c^3)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 18*(c^3*cos(6*f*x + 6*e) + 3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*
f*x + 2*e) + c^3)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sq
rt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9
*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e) + 1)*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \left(c - \frac{c}{\cos(e + fx)} \right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2), x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(7/2)*(a+a*sec(f*x+e))**(1/2), x)

[Out] Timed out

$$3.87 \quad \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2} dx$$

Optimal. Leaf size=139

$$\frac{ac^3 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac^2 \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}} - \frac{ac \tan(e + fx) (c - c \sec(e + fx))}{2f \sqrt{a \sec(e + fx) + a}}$$

[Out] $-1/2*a*c*(c-c*\sec(f*x+e))^{(3/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+a*c^3*\log(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-a*c^2*(c-c*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3906, 3905, 3475}

$$-\frac{ac^2 \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}} + \frac{ac^3 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx) (c - c \sec(e + fx))}{2f \sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2), x]`

[Out] $(a*c^3*\log(\cos[e + f*x])*Tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]}) - (a*c^2*\sqrt{c - c*\sec[e + f*x]}*Tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}) - (a*c*(c - c*\sec[e + f*x])^{(3/2)}*Tan[e + f*x])/(2*f*\sqrt{a + a*\sec[e + f*x]})$

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3905

`Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[((-a*c)^(m + 1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]`

Rule 3906

`Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]`

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2} dx &= -\frac{ac(c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{a + a \sec(e + fx)}} + c \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2} dx \\
&= -\frac{ac^2\sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{ac(c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{ac^2\sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{ac(c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{a + a \sec(e + fx)}} \\
&= \frac{ac^3 \log(\cos(e + fx)) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{ac^2\sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 2.31, size = 162, normalized size = 1.17

$$\frac{c^2 e^{-3i(e+fx)} (1 + e^{2i(e+fx)})^3 \left(\cot\left(\frac{1}{2}(e+fx)\right) + i \right) \sec^4(e+fx) \sqrt{a(\sec(e+fx)+1)} \sqrt{c - c \sec(e+fx)} \left(\log(1 + e^{2i(e+fx)}) \right)}{16f(1 + e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2), x]

[Out] -1/16*(c^2*(1 + E^((2*I)*(e + f*x)))^3*(I + Cot[(e + f*x)/2])*(-1 - I*f*x + 4*Cos[e + f*x] + Log[1 + E^((2*I)*(e + f*x))] + Cos[2*(e + f*x)]*(-I)*f*x + Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])] *Sqrt[c - c*Sec[e + f*x]]/(E^((3*I)*(e + f*x))*(1 + E^(I*(e + f*x))))*f)

fricas [A] time = 0.57, size = 425, normalized size = 3.06

$$\frac{\left(3c^2 \cos(fx + e) - c^2\right) \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \sqrt{\frac{c \cos(fx + e) - c}{\cos(fx + e)}} \sin(fx + e) - \left(c^2 \cos(fx + e)^2 + c^2 \cos(fx + e)\right) \sqrt{-ac} \log\left(\frac{1}{2} \left(\frac{c \cos(fx + e) - c}{\cos(fx + e)} \right) + \frac{a \cos(fx + e) + a}{\cos(fx + e)}\right)}{2 \left(f \cos(fx + e)^2 + f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2), x, algorithm="fricas")

[Out] [-1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 2*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
```


sign: (4*pi/x/2)>(-4*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{2*c,0%%}+%%{1,2%%})Evaluation time: 3.88Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.35, size = 185, normalized size = 1.33

$$\frac{\left(2 \ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^2(fx+e)\right) - 2 \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right) - 2 \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\right)}{2f \sin(fx+e)}(-1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x)

[Out] 1/2/f*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-5*cos(f*x+e)^2-4*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))^2

maxima [B] time = 1.04, size = 710, normalized size = 5.11

$$\frac{\left((fx+e)c^2 \cos(4fx+4e)^2 + 4(fx+e)c^2 \cos(2fx+2e)^2 + (fx+e)c^2 \sin(4fx+4e)^2 + 4(fx+e)c^2 \sin(2fx+2e)^2\right)}{2f \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -((f*x + e)*c^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e)^2 + (f*x + e)*c^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*sin(2*f*x + 2*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + 2*c^2*sin(2*f*x + 2*e) - (c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e)) *arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + c^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*c^2*sin(2*f*x + 2*e) - c^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e+fx)}} \left(c - \frac{c}{\cos(e+fx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

$$3.88 \quad \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2} dx$$

Optimal. Leaf size=93

$$\frac{ac^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}}$$

[Out] $a*c^2*\ln(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-a*c*(c-c*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3906, 3905, 3475}

$$\frac{ac^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^{(3/2)}, x]$

[Out] $(a*c^2*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (a*c*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3905

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[((-a*c))^{(m + 1/2)}*\text{Cot}[e + f*x]/(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Int}[\text{Cot}[e + f*x]^{(2*m)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m + 1/2]$

Rule 3906

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(2*a*c*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^{(n - 1)})/(f*(2*n - 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[n, 1/2]$

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2} dx &= -\frac{ac \sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} + c \int \sqrt{a + a \sec(e + fx)} \\ &= -\frac{ac \sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} - \frac{(ac^2 \tan(e + fx)) \int \tan(e + fx)}{\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{ac^2 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{ac \sqrt{c - c \sec(e + fx)}}{f \sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.24, size = 99, normalized size = 1.06

$$\frac{ic \left(\cot \left(\frac{1}{2}(e + fx) \right) + i \right) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c - c \sec(e + fx)} \left((fx + i \log(1 + e^{2i(e+fx)})) \cos(e + fx) + i \right)}{f(1 + e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]

[Out] (I*c*(I + Cot[(e + f*x)/2])*(I + Cos[e + f*x]*(f*x + I*Log[1 + E^((2*I)*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x]])*Sqrt[c - c*Sec[e + f*x]]/((1 + E^(I*(e + f*x)))*f)

fricas [A] time = 0.57, size = 350, normalized size = 3.76

$$\frac{2c \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx+e) - \sqrt{-ac} (c \cos(fx+e) + c) \log \left(\frac{ac \cos(fx+e)^4 - (\cos(fx+e)^3 + \cos(fx+e)) \sqrt{-ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{2c \cos(fx+e)} \right)}{2(f \cos(fx+e) + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - sqrt(-a*c)*(c*cos(f*x + e) + c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), -(c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - sqrt(a*c)*(c*cos(f*x + e) + c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):C heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a

maxima [B] time = 0.77, size = 243, normalized size = 2.61

$$\frac{\left((fx + e)c \cos(2fx + 2e)^2 + (fx + e)c \sin(2fx + 2e)^2 + 2(fx + e)c \cos(2fx + 2e) + 2c \cos\left(\frac{1}{2} \arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) \right) \sqrt{a} \sqrt{c}}{\left(\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1 \right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -((f*x + e)*c*cos(2*f*x + 2*e)^2 + (f*x + e)*c*sin(2*f*x + 2*e)^2 + 2*(f*x + e)*c*cos(2*f*x + 2*e) + 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(2*f*x + 2*e) + (f*x + e)*c - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*(c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \left(c - \frac{c}{\cos(e + fx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2),x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e + fx) + 1)} (-c(\sec(e + fx) - 1))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2), x)

$$3.89 \quad \int \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)} dx$$

Optimal. Leaf size=48

$$\frac{ac \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $a*c*\ln(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3905, 3475}

$$\frac{ac \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]

[Out] $(a*c*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(m_.), x_Symbol] := Dist[(-a*c)^(m + 1/2)*Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)} dx &= -\frac{(ac \tan(e + fx)) \int \tan(e + fx) dx}{\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{ac \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 0.58, size = 102, normalized size = 2.12

$$\frac{i e^{\frac{1}{2}i(e+fx)} (fx + i \log(1 + e^{2i(e+fx)})) \cos(e + fx) \csc\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c - c \sec(e + fx)}}{f(1 + e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]

[Out] $(I*E^{((I/2)*(e + f*x))*\text{Cos}[e + f*x]*\text{Csc}[(e + f*x)/2]*(f*x + I*\text{Log}[1 + E^{((2*I)*(e + f*x))}]*)*\text{Sqrt}[a*(1 + \text{Sec}[e + f*x])]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])/(1 + E^{(I*(e + f*x))})*f$

fricas [B] time = 0.57, size = 200, normalized size = 4.17

$$\frac{\sqrt{-ac} \log \left(\frac{ac \cos(fx+e)^4 - (\cos(fx+e)^3 + \cos(fx+e)) \sqrt{-ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx+e) + ac}{2 \cos(fx+e)^2} \right) \sqrt{ac} \arctan \left(\frac{\sqrt{ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx+e)} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))
)*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/f, sqrt(a*c)*arctan(
sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
```


*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{2*c,0%%}+%%{1,2%%})Evaluation time: 3.05Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 2.40, size = 128, normalized size = 2.67

$$\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right) - \ln\left(\frac{2}{1+\cos(fx+e)}\right)\right) \cos(fx+e) \sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}}{f \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x)

[Out] -1/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-ln(2/(1+cos(f*x+e))))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)

maxima [A] time = 0.81, size = 39, normalized size = 0.81

$$\frac{(fx+e - \arctan(\sin(2fx+2e), \cos(2fx+2e)+1))\sqrt{a}\sqrt{c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sqrt(a)*sqrt(c)/f

mapad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + \frac{a}{\cos(e+fx)}} \sqrt{c - \frac{c}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2),x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e+fx)+1)} \sqrt{-c(\sec(e+fx)-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1)), x)

$$3.90 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c-c \sec(e+fx)}} dx$$

Optimal. Leaf size=51

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

[Out] a*ln(1-cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3911, 31}

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]

[Out] (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] :> -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c-c \sec(e+fx)}} dx &= \frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{1}{-c+cx} dx, x, \cos(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= \frac{a \log(1-\cos(e+fx)) \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 0.98, size = 86, normalized size = 1.69

$$\frac{(-1 + e^{i(e+fx)}) (fx + 2i \log(1 - e^{i(e+fx)})) \sqrt{a(\sec(e+fx) + 1)}}{f (1 + e^{i(e+fx)}) \sqrt{c - c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]

```
[Out] -((( -1 + E^(I*(e + f*x))) * (f*x + (2*I)*Log[1 - E^(I*(e + f*x))]) * Sqrt[a*(1 + Sec[e + f*x])]) / ((1 + E^(I*(e + f*x))) * f * Sqrt[c - c*Sec[e + f*x]]))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c \sec(fx + e) - c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assumi
ng -2*a+a is positive. Hint: run assume to make assumptions on a variableWa
rning, assuming -2*a+a is positive. Hint: run assume to make assumptions on
a variableWarning, assuming -2*a+a is positive. Hint: run assume to make a
ssumptions on a variableWarning, assuming -2*a+a is positive. Hint: run ass
ume to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2)
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
```


maple [B] time = 2.12, size = 100, normalized size = 1.96

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(2 \ln \left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)} \right) - \ln \left(\frac{2}{1+\cos(fx+e)} \right) \right) \sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}} \cos(fx+e)}{f \sin(fx+e) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2), x)

[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-ln(2/(1+cos(f*x+e))))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)/c

maxima [A] time = 0.59, size = 65, normalized size = 1.27

$$\frac{2 \sqrt{-a} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}} - \frac{\sqrt{-a} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2+1}\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2), x, algorithm="maxima")

[Out] (2*sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1))/sqrt(c) - sqrt(-a)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\sqrt{c - \frac{c}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2), x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e+fx)+1)}}{\sqrt{-c(\sec(e+fx)-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2), x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(-c*(sec(e + f*x) - 1)), x)

$$3.91 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=96

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{a \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}$$

[Out] $-a*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3907, 3911, 31}

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{a \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2), x]

[Out] $-((a*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(3/2)})) + (a*\log[1 - \cos[e + f*x]]*\tan[e + f*x])/(c*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{3/2}} dx &= -\frac{a \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{\int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c-c \sec(e+fx)}} dx}{c} \\ &= -\frac{a \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{(a \tan(e+fx)) \text{Subst}\left(\int \frac{1}{-c+cx} dx, \frac{a \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}}\sqrt{c-c \sec(e+fx)}\right)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= -\frac{a \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{a \log(1-\cos(e+fx)) \tan(e+fx)}{cf\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.04, size = 107, normalized size = 1.11

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(-2\log(1-e^{i(e+fx)})+(2\log(1-e^{i(e+fx)})-ifx)\cos(e+fx)+\dots\right)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2), x]

[Out] ((-1 + I*f*x - 2*Log[1 - E^(I*(e + f*x))]) + Cos[e + f*x]*((-I)*f*x + 2*Log[1 - E^(I*(e + f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]/(f*(c - c*Sec[e + f*x])^(3/2))

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec(fx+e)+a}\sqrt{-c\sec(fx+e)+c}}{c^2\sec(fx+e)^2-2c^2\sec(fx+e)+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [A] time = 2.55, size = 164, normalized size = 1.71

$$\frac{(-1 + \cos(fx + e))\left(4\ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right)\cos(fx + e) - 2\cos(fx + e)\ln\left(\frac{2}{1 + \cos(fx + e)}\right) - \cos(fx + e) - 4\ln\left(-\dots\right)\right)}{2f\cos(fx + e)\left(\frac{c(-1 + \cos(fx + e))}{\cos(fx + e)}\right)^{\frac{3}{2}}\sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2), x)

[Out] -1/2/f*(-1+cos(f*x+e))*(4*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-2*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-cos(f*x+e)-4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*ln(2/(1+cos(f*x+e)))-1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)

maxima [B] time = 1.03, size = 399, normalized size = 4.16

$$\frac{\left((fx+e)\cos(2fx+2e)^2+4(fx+e)\cos(fx+e)^2+(fx+e)\sin(2fx+2e)^2+4(fx+e)\sin(fx+e)^2+\dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 2*(f*x + e)*cos(f*x + e) + e + sin(f*x + e))*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x + e) - 2*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e + 2*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 - 4*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x + e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(f*x + e) - c^2)*cos(2*f*x + 2*e))*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2),x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e+fx)+1)}}{(-c(\sec(e+fx)-1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(3/2), x)

$$3.92 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=142

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{c^2 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a \tan(e+fx)}{c f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a \tan(e+fx)}{2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}}$$

[Out] $-1/2*a*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a*\tan(f*x+e)/c/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3907, 3911, 31}

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{c^2 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a \tan(e+fx)}{c f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a \tan(e+fx)}{2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2), x]

[Out] $-(a*\tan[e + f*x])/(2*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(5/2)}) - (a*\tan[e + f*x])/(c*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(3/2)}) + (a*\log[1 - \cos[e + f*x]]*\tan[e + f*x])/(c^2*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{5/2}} dx &= -\frac{a \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} + \frac{\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{3/2}} dx}{c} \\
&= -\frac{a \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} - \frac{a \tan(e + fx)}{cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} \\
&= -\frac{a \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} - \frac{a \tan(e + fx)}{cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} \\
&= -\frac{a \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} - \frac{a \tan(e + fx)}{cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 1.25, size = 152, normalized size = 1.07

$$\frac{\tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(6 \log(1 - e^{i(e+fx)}) + (-8 \log(1 - e^{i(e+fx)}) + 4ifx - 4) \cos(e + fx) + (21 - 4ifx) \sin(e + fx)\right)}{2c^2 f (\cos(e + fx) - 1)^2 \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]

[Out] ((3 - (3*I)*f*x + Cos[e + f*x]*(-4 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x))]) + 6*Log[1 - E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])*Tan[(e + f*x)/2]]/(2*c^2*f*(-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c^3 \sec(fx + e)^3 - 3c^3 \sec(fx + e)^2 + 3c^3 \sec(fx + e) - c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out]
$$-\left((f*x + e)*\cos(4*f*x + 4*e)^2 + 16*(f*x + e)*\cos(3*f*x + 3*e)^2 + 36*(f*x + e)*\cos(2*f*x + 2*e)^2 + 16*(f*x + e)*\cos(f*x + e)^2 + (f*x + e)*\sin(4*f*x + 4*e)^2 + 16*(f*x + e)*\sin(3*f*x + 3*e)^2 + 36*(f*x + e)*\sin(2*f*x + 2*e)^2 + 16*(f*x + e)*\sin(f*x + e)^2 + f*x + 2*(2*(4*\cos(3*f*x + 3*e) - 6*\cos(2*f*x + 2*e) + 4*\cos(f*x + e) - 1)*\cos(4*f*x + 4*e) - \cos(4*f*x + 4*e)^2 + 8*(6*\cos(2*f*x + 2*e) - 4*\cos(f*x + e) + 1)*\cos(3*f*x + 3*e) - 16*\cos(3*f*x + 3*e)^2 + 12*(4*\cos(f*x + e) - 1)*\cos(2*f*x + 2*e) - 36*\cos(2*f*x + 2*e)^2 - 16*\cos(f*x + e)^2 + 4*(2*\sin(3*f*x + 3*e) - 3*\sin(2*f*x + 2*e) + 2*\sin(f*x + e))*\sin(4*f*x + 4*e) - \sin(4*f*x + 4*e)^2 + 16*(3*\sin(2*f*x + 2*e) - 2*\sin(f*x + e))*\sin(3*f*x + 3*e) - 16*\sin(3*f*x + 3*e)^2 - 36*\sin(2*f*x + 2*e)^2 + 48*\sin(2*f*x + 2*e)*\sin(f*x + e) - 16*\sin(f*x + e)^2 + 8*\cos(f*x + e) - 1)*\arctan2(\sin(f*x + e), \cos(f*x + e) - 1) + 2*(f*x - 4*(f*x + e)*\cos(3*f*x + 3*e) + 6*(f*x + e)*\cos(2*f*x + 2*e) - 4*(f*x + e)*\cos(f*x + e) + e + 2*\sin(3*f*x + 3*e) - 3*\sin(2*f*x + 2*e) + 2*\sin(f*x + e))*\cos(4*f*x + 4*e) - 8*(f*x + 6*(f*x + e)*\cos(2*f*x + 2*e) - 4*(f*x + e)*\cos(f*x + e) + e)*\cos(3*f*x + 3*e) + 12*(f*x - 4*(f*x + e)*\cos(f*x + e) + e)*\cos(2*f*x + 2*e) - 8*(f*x + e)*\cos(f*x + e) - 2*(4*(f*x + e)*\sin(3*f*x + 3*e) - 6*(f*x + e)*\sin(2*f*x + 2*e) + 4*(f*x + e)*\sin(f*x + e) + 2*\cos(3*f*x + 3*e) - 3*\cos(2*f*x + 2*e) + 2*\cos(f*x + e))*\sin(4*f*x + 4*e) - 4*(12*(f*x + e)*\sin(2*f*x + 2*e) - 8*(f*x + e)*\sin(f*x + e) - 1)*\sin(3*f*x + 3*e) - 6*(8*(f*x + e)*\sin(f*x + e) + 1)*\sin(2*f*x + 2*e) + e + 4*\sin(f*x + e))*\sqrt{a}*\sqrt{c}/\left(\left(c^3*\cos(4*f*x + 4*e)^2 + 16*c^3*\cos(3*f*x + 3*e)^2 + 36*c^3*\cos(2*f*x + 2*e)^2 + 16*c^3*\cos(f*x + e)^2 + c^3*\sin(4*f*x + 4*e)^2 + 16*c^3*\sin(3*f*x + 3*e)^2 + 36*c^3*\sin(2*f*x + 2*e)^2 - 48*c^3*\sin(2*f*x + 2*e)*\sin(f*x + e) + 16*c^3*\sin(f*x + e)^2 - 8*c^3*\cos(f*x + e) + c^3 - 2*(4*c^3*\cos(3*f*x + 3*e) - 6*c^3*\cos(2*f*x + 2*e) + 4*c^3*\cos(f*x + e) - c^3)*\cos(4*f*x + 4*e) - 8*(6*c^3*\cos(2*f*x + 2*e) - 4*c^3*\cos(f*x + e) + c^3)*\cos(3*f*x + 3*e) - 12*(4*c^3*\cos(f*x + e) - c^3)*\cos(2*f*x + 2*e) - 4*(2*c^3*\sin(3*f*x + 3*e) - 3*c^3*\sin(2*f*x + 2*e) + 2*c^3*\sin(f*x + e))*\sin(4*f*x + 4*e) - 16*(3*c^3*\sin(2*f*x + 2*e) - 2*c^3*\sin(f*x + e))*\sin(3*f*x + 3*e)\right)*f)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2),x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e+fx)+1)}}{\left(-c(\sec(e+fx)-1)\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(5/2), x)

$$3.93 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c-c \sec(e+fx))^{7/2}} dx$$

Optimal. Leaf size=188

$$\frac{a \tan(e+fx) \log(1-\cos(e+fx))}{c^3 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a \tan(e+fx)}{c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a \tan(e+fx)}{2cf \sqrt{a \sec(e+fx)+a}}$$

[Out] $-1/3*a*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(7/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/2*a*\tan(f*x+e)/c/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a*\tan(f*x+e)/c^2/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^3/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3907, 3911, 31}

$$-\frac{a \tan(e+fx)}{c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} + \frac{a \tan(e+fx) \log(1-\cos(e+fx))}{c^3 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a \tan(e+fx)}{2cf \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2), x]

[Out] $-(a*\tan[e + f*x])/(3*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(7/2)}) - (a*\tan[e + f*x])/(2*c*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(5/2)}) - (a*\tan[e + f*x])/(c^2*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(3/2)}) + (a*\log[1 - \cos[e + f*x]]*\tan[e + f*x])/(c^3*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{7/2}} dx &= -\frac{a \tan(e + fx)}{3f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} + \frac{\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{5/2}} dx}{c} \\
&= -\frac{a \tan(e + fx)}{3f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} - \frac{a \tan(e + fx)}{2cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} \\
&= -\frac{a \tan(e + fx)}{3f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} - \frac{a \tan(e + fx)}{2cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} \\
&= -\frac{a \tan(e + fx)}{3f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} - \frac{a \tan(e + fx)}{2cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} \\
&= -\frac{a \tan(e + fx)}{3f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}} - \frac{a \tan(e + fx)}{2cf \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{7/2}}
\end{aligned}$$

Mathematica [C] time = 1.94, size = 198, normalized size = 1.05

$$\frac{\tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(-60 \log(1 - e^{i(e+fx)}) - 3ifx \cos(3(e + fx)) + 18i(2i \log(1 - e^{i(e+fx)}) + \dots)\right)}{12c^3 f(\cos(e + fx) + \dots)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
[Out] ((-40 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (18*I)*Cos[2*(e + f*x)]*(I + f*x + (2*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + 9*Cos[e + f*x]*(6 - (5*I)*f*x + 10*Log[1 - E^(I*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/((12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c^4 \sec(fx + e)^4 - 4c^4 \sec(fx + e)^3 + 6c^4 \sec(fx + e)^2 - 4c^4 \sec(fx + e) + c^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 2.18, size = 288, normalized size = 1.53

$$(-1 + \cos(fx + e)) \left(12 \ln \left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)} \right) (\cos^3(fx + e)) - 6 (\cos^3(fx + e)) \ln \left(\frac{2}{1 + \cos(fx + e)} \right) - 36 (\cos^2(fx + e)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x)

[Out] -1/6/f*(-1+cos(f*x+e))*(12*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3-6*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-36*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-7*cos(f*x+e)^3+18*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+36*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+3*cos(f*x+e)^2-18*cos(f*x+e)*ln(2/(1+cos(f*x+e))))-12*ln(-(-1+cos(f*x+e))/sin(f*x+e))+6*cos(f*x+e)+6*ln(2/(1+cos(f*x+e)))-4)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)

maxima [B] time = 4.63, size = 2444, normalized size = 13.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxima")

[Out] -1/3*(3*(f*x + e)*cos(6*f*x + 6*e)^2 + 108*(f*x + e)*cos(5*f*x + 5*e)^2 + 675*(f*x + e)*cos(4*f*x + 4*e)^2 + 1200*(f*x + e)*cos(3*f*x + 3*e)^2 + 675*(f*x + e)*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*cos(f*x + e)^2 + 3*(f*x + e)*sin(6*f*x + 6*e)^2 + 108*(f*x + e)*sin(5*f*x + 5*e)^2 + 675*(f*x + e)*sin(4*f*x + 4*e)^2 + 1200*(f*x + e)*sin(3*f*x + 3*e)^2 + 675*(f*x + e)*sin(2*f*x + 2*e)^2 + 108*(f*x + e)*sin(f*x + e)^2 + 3*f*x + 6*(2*(6*cos(5*f*x + 5*e) - 15*cos(4*f*x + 4*e) + 20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6*cos(f*x + e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 + 12*(15*cos(4*f*x + 4*e) - 20*cos(3*f*x + 3*e) + 15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(5*f*x + 5*e) - 36*cos(5*f*x + 5*e)^2 + 30*(20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - 225*cos(4*f*x + 4*e)^2 + 40*(15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 400*cos(3*f*x + 3*e)^2 + 30*(6*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 225*cos(2*f*x + 2*e)^2 - 36*cos(f*x + e)^2 + 2*(6*sin(5*f*x + 5*e) - 15*sin(4*f*x + 4*e) + 20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(6*f*x + 6*e) - sin(6*f*x + 6*e)^2 + 12*(15*sin(4*f*x + 4*e) - 20*sin(3*f*x + 3*e) + 15*sin(2*f*x + 2*e) - 6*sin(f*x + e))*sin(5*f*x + 5*e) - 36*sin(5*f*x + 5*e)^2 + 30*(20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(4*f*x + 4*e) - 225*sin(4*f*x + 4*e)^2 + 120*(5*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 400*sin(3*f*x + 3*e)^2 - 225*sin(2*f*x + 2*e)^2 + 180*sin(2*f*x + 2*e)*sin(f*x + e) - 36*sin(f*x + e)^2 + 12*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(3*f*x - 18*(f*x + e)*cos(5*f*x + 5*e) + 45*(f*x + e)*cos(4*f*x + 4*e) - 60*(f*x + e)*cos(3*f*x + 3*e) + 45*(f*x + e)*cos(2*f*x + 2*e) - 18*(f*x + e)*cos(f*x + e) + 3*e + 9*sin(5*f*x + 5*e) - 27*sin(4*f*x + 4*e) + 40*sin(3*f*x + 3*e) - 27*sin(2*f*x + 2*e) + 9*sin(f*x + e))*cos(6*f*x + 6*e) - 6*(6*f*x + 90*(f*x + e)*cos(4*f*x + 4*e) - 120*(f*x + e)*cos(3*f*x + 3*e) + 90*(f*x + e)*cos(2*f*x + 2*e) - 36*(f*x + e)*cos(f*x + e) + 6*e - 9*sin(4*f*x + 4*e) + 20*sin(3*f*x + 3*e) - 9*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(15*f*x - 300*(f*x + e)*cos(3*f*x + 3*e) + 225*(f*x + e)*cos(2*f*x + 2*e) - 90*(f*x + e)*cos(f*x + e) + 15*e + 20*sin(3*f*x + 3*e) - 9*sin(f*x + e))*cos(4*f*x + 4*e) - 120*(f*x + 15*(f*x + e)

```

*cos(2*f*x + 2*e) - 6*(f*x + e)*cos(f*x + e) + e + sin(2*f*x + 2*e) - sin(f
*x + e))*cos(3*f*x + 3*e) + 18*(5*f*x - 30*(f*x + e)*cos(f*x + e) + 5*e - 3
*sin(f*x + e))*cos(2*f*x + 2*e) - 36*(f*x + e)*cos(f*x + e) - 2*(18*(f*x +
e)*sin(5*f*x + 5*e) - 45*(f*x + e)*sin(4*f*x + 4*e) + 60*(f*x + e)*sin(3*f*
x + 3*e) - 45*(f*x + e)*sin(2*f*x + 2*e) + 18*(f*x + e)*sin(f*x + e) + 9*cos
(5*f*x + 5*e) - 27*cos(4*f*x + 4*e) + 40*cos(3*f*x + 3*e) - 27*cos(2*f*x +
2*e) + 9*cos(f*x + e))*sin(6*f*x + 6*e) - 6*(90*(f*x + e)*sin(4*f*x + 4*e)
- 120*(f*x + e)*sin(3*f*x + 3*e) + 90*(f*x + e)*sin(2*f*x + 2*e) - 36*(f*x
+ e)*sin(f*x + e) + 9*cos(4*f*x + 4*e) - 20*cos(3*f*x + 3*e) + 9*cos(2*f*x
+ 2*e) - 3)*sin(5*f*x + 5*e) - 6*(300*(f*x + e)*sin(3*f*x + 3*e) - 225*(f*
x + e)*sin(2*f*x + 2*e) + 90*(f*x + e)*sin(f*x + e) + 20*cos(3*f*x + 3*e) -
9*cos(f*x + e) + 9)*sin(4*f*x + 4*e) - 40*(45*(f*x + e)*sin(2*f*x + 2*e) -
18*(f*x + e)*sin(f*x + e) - 3*cos(2*f*x + 2*e) + 3*cos(f*x + e) - 2)*sin(3
*f*x + 3*e) - 54*(10*(f*x + e)*sin(f*x + e) - cos(f*x + e) + 1)*sin(2*f*x +
2*e) + 3*e + 18*sin(f*x + e))*sqrt(a)*sqrt(c)/((c^4*cos(6*f*x + 6*e)^2 + 3
6*c^4*cos(5*f*x + 5*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 400*c^4*cos(3*f*x +
3*e)^2 + 225*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x + e)^2 + c^4*sin(6*f*
x + 6*e)^2 + 36*c^4*sin(5*f*x + 5*e)^2 + 225*c^4*sin(4*f*x + 4*e)^2 + 400*c
^4*sin(3*f*x + 3*e)^2 + 225*c^4*sin(2*f*x + 2*e)^2 - 180*c^4*sin(2*f*x + 2*
e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*(6*
c^4*cos(5*f*x + 5*e) - 15*c^4*cos(4*f*x + 4*e) + 20*c^4*cos(3*f*x + 3*e) -
15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x + 6*e) - 12*(
15*c^4*cos(4*f*x + 4*e) - 20*c^4*cos(3*f*x + 3*e) + 15*c^4*cos(2*f*x + 2*e)
- 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 30*(20*c^4*cos(3*f*x + 3*e)
- 15*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*x + 4*e) - 4
0*(15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*x + 3*e) - 3
0*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 2*(6*c^4*sin(5*f*x + 5*e) -
15*c^4*sin(4*f*x + 4*e) + 20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e
) + 6*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 12*(15*c^4*sin(4*f*x + 4*e) - 20
*c^4*sin(3*f*x + 3*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(f*x + e))*sin(5
*f*x + 5*e) - 30*(20*c^4*sin(3*f*x + 3*e) - 15*c^4*sin(2*f*x + 2*e) + 6*c^4
*sin(f*x + e))*sin(4*f*x + 4*e) - 120*(5*c^4*sin(2*f*x + 2*e) - 2*c^4*sin(f
*x + e))*sin(3*f*x + 3*e))*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2), x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(7/2), x)

[Out] Timed out

3.94 $\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{5/2} dx$

Optimal. Leaf size=190

$$\frac{a^2 c^3 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{a^2 c^2 \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}} - \frac{a^2 c \tan(e + fx) (c - c \sec(e + fx))^{5/2}}{2f \sqrt{a \sec(e + fx) + a}}$$

[Out] $-1/2*a^2*c*(c-c*\sec(f*x+e))^{(3/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+1/3*a^2*(c-c*\sec(f*x+e))^{(5/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+a^2*c^3*\ln(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-a^2*c^2*(c-c*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3909, 3906, 3905, 3475}

$$-\frac{a^2 c^2 \tan(e + fx) \sqrt{c - c \sec(e + fx)}}{f \sqrt{a \sec(e + fx) + a}} + \frac{a^2 c^3 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{a^2 c \tan(e + fx) (c - c \sec(e + fx))^{5/2}}{2f \sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2), x]

[Out] $(a^2*c^3*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (a^2*c^2*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (a^2*c*(c - c*\text{Sec}[e + f*x])^{(3/2)}*\text{Tan}[e + f*x])/(2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (a^2*(c - c*\text{Sec}[e + f*x])^{(5/2)}*\text{Tan}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^(m + 1/2)*Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rule 3906

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]

Rule 3909

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(3/2)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Simp[(-2*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LeQ[n, -2^(-1)]

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{5/2} dx &= \frac{a^2 (c - c \sec(e + fx))^{5/2} \tan(e + fx)}{3f \sqrt{a + a \sec(e + fx)}} + a \int \sqrt{a + a \sec(e + fx)} \\
&= -\frac{a^2 c (c - c \sec(e + fx))^{3/2} \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)}} + \frac{a^2 (c - c \sec(e + fx))^{5/2}}{3f \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{a^2 c^2 \sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} - \frac{a^2 c (c - c \sec(e + fx))^{5/2}}{2f \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{a^2 c^2 \sqrt{c - c \sec(e + fx)} \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)}} - \frac{a^2 c (c - c \sec(e + fx))^{5/2}}{2f \sqrt{a + a \sec(e + fx)}} \\
&= \frac{a^2 c^3 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{a^2 c^2 \sqrt{c - c \sec(e + fx)}}{f \sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 1.28, size = 157, normalized size = 0.83

$$iac^2 \csc\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c - c \sec(e + fx)} (6i \cos(2(e + fx)) + 3$$

24

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]

[Out] ((I/24)*a*c^2*Csc[(e + f*x)/2]*(2*I + (6*I)*Cos[2*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] + Cos[e + f*x]*(6*I + 9*f*x + (9*I)*Log[1 + E^((2*I)*(e + f*x))]) + (3*I)*Cos[3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/f

fricas [A] time = 0.57, size = 467, normalized size = 2.46

$$\left[\frac{(7ac^2 \cos(fx + e)^2 + ac^2 \cos(fx + e) - 2ac^2) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx + e) - 3(ac^2 \cos(fx + e) + a) \sqrt{a + a \sec(e + fx)}}{6(f \cos(fx + e))^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 3*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - 6*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4p
```



```

+ e)*a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x
+ 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 4*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a
*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x
+ 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
e)*a*c^2*sin(4*f*x + 4*e) + 3*(f*x + e)*a*c^2*sin(2*f*x + 2*e) + a*c^2*cos
(4*f*x + 4*e) + a*c^2*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
*c^2*sin(2*f*x + 2*e) - a*c^2)*sin(4*f*x + 4*e) - 6*(a*c^2*cos(6*f*x + 6*e)
+ 3*a*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) + 3*a
*c^2*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(a*c^2*cos(6*f*x + 6*e) + 3*a*c^2*c
os(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + f x)} \right)^{3/2} \left(c - \frac{c}{\cos(e + f x)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.95 $\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{3/2} dx$

Optimal. Leaf size=103

$$\frac{a^2 c^2 \tan^3(e + fx)}{2f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{a^2 c^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $a^2 c^2 \ln(\cos(fx+e)) \tan(fx+e) / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} + 1/2 a^2 c^2 \tan(fx+e)^3 / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2}$

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3905, 3473, 3475}

$$\frac{a^2 c^2 \tan^3(e + fx)}{2f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{a^2 c^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \sec[e + fx])^{3/2} (c - c \sec[e + fx])^{3/2}, x]$

[Out] $(a^2 c^2 \text{Log}[\text{Cos}[e + fx]] \text{Tan}[e + fx]) / (f \sqrt{a + a \sec[e + fx]} \sqrt{c - c \sec[e + fx]}) + (a^2 c^2 \text{Tan}[e + fx]^3) / (2 f \sqrt{a + a \sec[e + fx]} \sqrt{c - c \sec[e + fx]})$

Rule 3473

$\text{Int}[(b \tan[(c \cdot) + (d \cdot)(x \cdot)])^{(n \cdot)}, x_Symbol] \rightarrow \text{Simp}[(b \tan[c + d \cdot x])^{(n - 1)} / (d \cdot (n - 1)), x] - \text{Dist}[b^2, \text{Int}[(b \tan[c + d \cdot x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

$\text{Int}[\tan[(c \cdot) + (d \cdot)(x \cdot)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d \cdot x], x]] / d, x] /;$ FreeQ[{c, d}, x]

Rule 3905

$\text{Int}[(\text{csc}[(e \cdot) + (f \cdot)(x \cdot)] \cdot (b \cdot) + (a \cdot))^{(m \cdot)} (\text{csc}[(e \cdot) + (f \cdot)(x \cdot)] \cdot (d \cdot) + (c \cdot))^{(m \cdot)}, x_Symbol] \rightarrow \text{Dist}[(\text{Cot}[e + f \cdot x]) / (\sqrt{a + b \text{Csc}[e + f \cdot x]} \sqrt{c + d \text{Csc}[e + f \cdot x]}), \text{Int}[\text{Cot}[e + f \cdot x]^{(2 \cdot m)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b \cdot c + a \cdot d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{3/2} dx &= \frac{(a^2 c^2 \tan(e + fx)) \int \tan^3(e + fx) dx}{\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{a^2 c^2 \tan^3(e + fx)}{2f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{(a^2 c^2 \tan(e + fx))}{\sqrt{a + a \sec(e + fx)}} \\ &= \frac{a^2 c^2 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{a^2 c^2 \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.39, size = 159, normalized size = 1.54

$$\frac{iace^{-2i(e+fx)}(1+e^{2i(e+fx)})^2\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(i\log\left(1+\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)\right)}{8f(1+e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]

[Out] ((I/8)*a*c*(1 + E^((2*I)*(e + f*x)))^2*(I + Cot[(e + f*x)/2])*(I + f*x + Cos[2*(e + f*x)]*(f*x + I*Log[1 + E^((2*I)*(e + f*x))]) + I*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x))))*f

fricas [A] time = 0.59, size = 346, normalized size = 3.36

$$\frac{\sqrt{-ac} ac \cos(fx + e) \log\left(\frac{ac \cos(fx+e)^4 - (\cos(fx+e)^3 + \cos(fx+e))\sqrt{-ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx+e)+ac}{2 \cos(fx+e)^2}\right) - ac \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{2 f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(-a*c)*a*c*cos(f*x + e)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2) - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)), 1/2*(2*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e) - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make


```

4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3+t_nostep)]Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign e
rror (%%{2*c,0%%}+%%{1,2%%})Evaluation time: 3.7Limit: Max order reache
d or unable to make series expansion Error: Bad Argument Value

```

maple [A] time = 2.08, size = 172, normalized size = 1.67

$$\left(2 \ln \left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)} \right) (\cos^2(fx+e)) + 2 \ln \left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)} \right) (\cos^2(fx+e)) - 2 \ln \left(\frac{2}{1+\cos(fx+e)} \right) \right) 2f \sin(fx+e) (-1 + \cos(fx+e))$$

3.96 $\int (a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)} dx$

Optimal. Leaf size=93

$$\frac{a^2 c \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx) \sqrt{a \sec(e + fx) + a}}{f \sqrt{c - c \sec(e + fx)}}$$

[Out] $a^2 c \ln(\cos(fx+e)) \tan(fx+e) / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} - a^2 c \tan(fx+e) \sqrt{a \sec(fx+e) + a} / (f \sqrt{c - c \sec(fx+e)})$

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3906, 3905, 3475}

$$\frac{a^2 c \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx) \sqrt{a \sec(e + fx) + a}}{f \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \text{Sec}[e + f*x])^{3/2} \text{Sqrt}[c - c \text{Sec}[e + f*x]], x]$

[Out] $(a^2 c \text{Log}[\text{Cos}[e + f*x]] \text{Tan}[e + f*x]) / (f \text{Sqrt}[a + a \text{Sec}[e + f*x]] \text{Sqrt}[c - c \text{Sec}[e + f*x]]) - (a^2 c \text{Sqrt}[a + a \text{Sec}[e + f*x]] \text{Tan}[e + f*x]) / (f \text{Sqrt}[c - c \text{Sec}[e + f*x]])$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d *x], x]] / d, x] /;$ FreeQ[{c, d}, x]

Rule 3905

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)] * (b_.) + (a_.))^{(m_.)} * (\text{csc}[(e_.) + (f_.)(x_.)] * (d_.) + (c_.))^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[((-a*c))^{(m + 1/2)} * \text{Cot}[e + f*x] / (\text{Sqrt}[a + b*\text{Csc}[e + f*x]] * \text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Int}[\text{Cot}[e + f*x]^{(2*m)}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rule 3906

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_.)] * (b_.) + (a_.)] * (\text{csc}[(e_.) + (f_.)(x_.)] * (d_.) + (c_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(2*a*c*\text{Cot}[e + f*x] * (c + d*\text{Csc}[e + f*x])^{(n - 1)}) / (f*(2*n - 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]] * (c + d*\text{Csc}[e + f*x])^{(n - 1)}, x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)} dx &= -\frac{ac \sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f \sqrt{c - c \sec(e + fx)}} + a \int \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)} dx \\ &= -\frac{ac \sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f \sqrt{c - c \sec(e + fx)}} - \frac{(a^2 c \tan(e + fx)) \int \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)} dx}{\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{a^2 c \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{ac \sqrt{a + a \sec(e + fx)}}{f \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 0.78, size = 128, normalized size = 1.38

$$\frac{ae^{-i(e+fx)}(1+e^{2i(e+fx)})\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(1+(ifx-\log)\right)}{2f(1+e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]

[Out] (a*(1 + E^((2*I)*(e + f*x)))*(I + Cot[(e + f*x)/2])*(1 + Cos[e + f*x]*(I*f*x - Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(2*E^(I*(e + f*x))*(1 + E^(I*(e + f*x))))*f)

fricas [A] time = 0.57, size = 347, normalized size = 3.73

$$\frac{2a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+\sqrt{-ac}(a\cos(fx+e)+a)\log\left(\frac{ac\cos(fx+e)^4-(\cos(fx+e)^3+\cos(fx+e))\sqrt{-ac}}{2\cos(fx+e)}\right)}{2(f\cos(fx+e)+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + sqrt(-a*c)*(a*cos(f*x + e) + a)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/(f*cos(f*x + e) + f), (a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + sqrt(a*c)*(a*cos(f*x + e) + a)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e) + f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a

maxima [B] time = 0.99, size = 243, normalized size = 2.61

$$\frac{\left((fx + e)a \cos(2fx + 2e)^2 + (fx + e)a \sin(2fx + 2e)^2 + 2(fx + e)a \cos(2fx + 2e) - 2a \cos\left(\frac{1}{2} \arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) \right) \sqrt{a} \sqrt{c}}{\left(\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1 \right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -((f*x + e)*a*cos(2*f*x + 2*e)^2 + (f*x + e)*a*sin(2*f*x + 2*e)^2 + 2*(f*x + e)*a*cos(2*f*x + 2*e) - 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(2*f*x + 2*e) + (f*x + e)*a - (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{3/2} \sqrt{c - \frac{c}{\cos(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2),x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a (\sec(e + fx) + 1) \right)^{3/2} \sqrt{-c (\sec(e + fx) - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1)), x)

$$3.97 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{\sqrt{c-c \sec(e+fx)}} dx$$

Optimal. Leaf size=104

$$\frac{2a^2 \tan(e+fx) \log(1-\sec(e+fx))}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{a^2 \tan(e+fx) \log(\cos(e+fx))}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

[Out] $a^2 \ln(\cos(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 2*a^2 \ln(1-\sec(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{2a^2 \tan(e+fx) \log(1-\sec(e+fx))}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{a^2 \tan(e+fx) \log(\cos(e+fx))}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]

[Out] $(a^2 * \text{Log}[\text{Cos}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (2 * a^2 * \text{Log}[1 - \text{Sec}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]])$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sec(e+fx))^{3/2}}{\sqrt{c-c \sec(e+fx)}} dx &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{a+ax}{x(c-cx)} dx, x, \sec(e+fx)\right)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \left(-\frac{2a}{c(-1+x)} + \frac{a}{cx}\right) dx, x, \sec(e+fx)\right)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= \frac{a^2 \log(\cos(e+fx)) \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} + \frac{2a^2 \log(1-\sec(e+fx)) \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.20, size = 105, normalized size = 1.01

$$\frac{a(-1 + e^{i(e+fx)}) \left(4i \log(1 - e^{i(e+fx)}) - i \log(1 + e^{2i(e+fx)}) + fx\right) \sqrt{a(\sec(e+fx) + 1)}}{f(1 + e^{i(e+fx)}) \sqrt{c - c \sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((a*(-1 + E^(I*(e + f*x)))*(f*x + (4*I)*Log[1 - E^(I*(e + f*x))] - I*Log[1 + E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])])/((1 + E^(I*(e + f*x)))*f*Sqrt[c - c*Sec[e + f*x]]))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(a \sec(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sec(fx + e) + c}}{c \sec(fx + e) - c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assumi
ng -2*a+a is positive. Hint: run assume to make assumptions on a variableWa
rning, assuming -2*a+a is positive. Hint: run assume to make assumptions on
a variableWarning, assuming -2*a+a is positive. Hint: run assume to make a
ssumptions on a variableWarning, assuming -2*a+a is positive. Hint: run ass
ume to make assumptions on a variableUnable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
```


*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{-a,0%%}+%%{1,2%%})Sign error (%%{-2*a,0%%}+%%{1,2%%})Evaluation time: 2.71Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 1.95, size = 157, normalized size = 1.51

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(4 \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) - \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right) - \ln\left(\frac{2}{1+\cos(fx+e)}\right) \right)}{f \sin(fx+e) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2), x)

[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-ln(2/(1+cos(f*x+e))))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)/c*a

maxima [A] time = 0.56, size = 60, normalized size = 0.58

$$\frac{\left((fx+e)a + a \arctan(\sin(2fx+2e), \cos(2fx+2e)+1) - 4a \arctan(\sin(fx+e), \cos(fx+e)-1)\right)\sqrt{a}}{\sqrt{c} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2), x, algorithm="maxima")

[Out] -((f*x + e)*a + a*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*a*arctan2(sin(f*x + e), cos(f*x + e) - 1))*sqrt(a)/(sqrt(c)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\sqrt{c - \frac{c}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2), x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sec(e+fx)+1)\right)^{\frac{3}{2}}}{\sqrt{-c(\sec(e+fx)-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2), x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/sqrt(-c*(sec(e + f*x) - 1)), x)

$$3.98 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=100

$$\frac{a^2 \tan(e+fx) \log(1-\cos(e+fx))}{cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{2a^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}$$

[Out] $-2*a^2*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^2*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3908, 3911, 31}

$$\frac{a^2 \tan(e+fx) \log(1-\cos(e+fx))}{cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{2a^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2), x]

[Out] $(-2*a^2*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(3/2)}) + (a^2*\log[1 - \cos[e + f*x]]*\tan[e + f*x])/(c*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3908

Int[(csc[(e_) + (f_.)*(x_)]*(b_.) + (a_))^(n_)*(csc[(e_) + (f_.)*(x_)]*(d_) + (c_))^(m_), x_Symbol] := Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_) + (f_.)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{3/2}} dx &= -\frac{2a^2 \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{a \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c-c \sec(e+fx)}} dx}{c} \\ &= -\frac{2a^2 \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{(a^2 \tan(e+fx)) \text{Subst}\left(\int \frac{1}{-c+cx} dx\right)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= -\frac{2a^2 \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{a^2 \log(1-\cos(e+fx)) \tan(e+fx)}{cf\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 0.68, size = 115, normalized size = 1.15

$$\frac{a \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(-2 \log(1 - e^{i(e+fx)}) + (2 \log(1 - e^{i(e+fx)}) - ifx) \cos(e + fx) + ifx - 2\right)}{cf(\cos(e + fx) - 1) \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2), x]

[Out] (a*(-2 + I*f*x - 2*Log[1 - E^(I*(e + f*x))]) + Cos[e + f*x]*((-I)*f*x + 2*Log[1 - E^(I*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(a \sec(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sec(fx + e) + c}}{c^2 \sec(fx + e)^2 - 2c^2 \sec(fx + e) + c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.97, size = 161, normalized size = 1.61

$$\frac{(-1 + \cos(fx + e)) \left(\cos(fx + e) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 2 \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) \cos(fx + e) + \cos(fx + e) - \ln\left(\frac{2}{1 + \cos(fx + e)}\right) \right)}{f \cos(fx + e) \left(\frac{c(-1 + \cos(fx + e))}{\cos(fx + e)} \right)^{\frac{3}{2}} \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2), x)

[Out] 1/f*(-1+cos(f*x+e))*(cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+cos(f*x+e)-ln(2/(1+cos(f*x+e)))+2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)*a

maxima [A] time = 0.72, size = 95, normalized size = 0.95

$$\frac{\frac{2\sqrt{-a}a \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{-a}a \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2+1}\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}} \sin(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] (2*sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(3/2) - sqrt(-a)*a*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + sqrt(-a)*a*(cos(f*x + e) + 1)^2/(c^(3/2)*sin(f*x + e)^2))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2),x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sec(e+fx)+1)\right)^{3/2}}{\left(-c(\sec(e+fx)-1)\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(3/2), x)

$$3.99 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=146

$$\frac{a^2 \tan(e+fx) \log(1-\cos(e+fx))}{c^2 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^2 \tan(e+fx)}{c f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^2 \tan(e+fx)}{f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}}$$

[Out] $-a^2 \tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a^2 \tan(f*x+e)/c/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^2 \ln(1-\cos(f*x+e))*\tan(f*x+e)/c^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3908, 3907, 3911, 31}

$$\frac{a^2 \tan(e+fx) \log(1-\cos(e+fx))}{c^2 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^2 \tan(e+fx)}{c f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^2 \tan(e+fx)}{f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2), x]

[Out] $-((a^2 \tan[e + f*x])/(f \sqrt{a + a \sec[e + f*x]} * (c - c \sec[e + f*x])^{(5/2)})) - (a^2 \tan[e + f*x])/(c f \sqrt{a + a \sec[e + f*x]} * (c - c \sec[e + f*x])^{(3/2)}) + (a^2 \log[1 - \cos[e + f*x]] * \tan[e + f*x])/(c^2 f \sqrt{a + a \sec[e + f*x]} * \sqrt{c - c \sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3908

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^{5/2}} dx &= -\frac{a^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} + \frac{a \int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{3/2}} dx}{c} \\
&= -\frac{a^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} - \frac{a^2 \tan(e + fx)}{c f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} \\
&= -\frac{a^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} - \frac{a^2 \tan(e + fx)}{c f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} \\
&= -\frac{a^2 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} - \frac{a^2 \tan(e + fx)}{c f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 1.27, size = 153, normalized size = 1.05

$$\frac{a \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(6 \log\left(1 - e^{i(e+fx)}\right) + (-8 \log\left(1 - e^{i(e+fx)}\right) + 4ifx - 6) \cos(e + fx) + (2\right)}{2c^2 f (\cos(e + fx) - 1)^2 \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]

[Out] (a*(4 - (3*I)*f*x + Cos[e + f*x]*(-6 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x))]) + 6*Log[1 - E^(I*(e + f*x))]) + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E^(I*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])*Tan[(e + f*x)/2]]/(2*c^2*f*(-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(a \sec(fx + e) + a)^{3/2} \sqrt{-c \sec(fx + e) + c}}{c^3 \sec(fx + e)^3 - 3c^3 \sec(fx + e)^2 + 3c^3 \sec(fx + e) - c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 2.10, size = 227, normalized size = 1.55

$$(-1 + \cos(fx + e)) \left(8 (\cos^2(fx + e)) \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) - 4 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) (\cos^2(fx + e)) - 5 (\cos^2(fx + e)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sec(f*x+e))^{3/2}/(c-c*\sec(f*x+e))^{5/2},x)$

[Out] $-1/4/f*(-1+\cos(f*x+e))*(8*\cos(f*x+e)^2*\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))-4*\ln(2/(1+\cos(f*x+e)))*\cos(f*x+e)^2-5*\cos(f*x+e)^2-16*\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))*\cos(f*x+e)+8*\cos(f*x+e)*\ln(2/(1+\cos(f*x+e)))-2*\cos(f*x+e)+8*\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))-4*\ln(2/(1+\cos(f*x+e))))+3*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}/\sin(f*x+e)/\cos(f*x+e)^2/(c*(-1+\cos(f*x+e))/\cos(f*x+e))^{5/2}*a$

maxima [B] time = 1.21, size = 1786, normalized size = 12.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+a*\sec(f*x+e))^{3/2}/(c-c*\sec(f*x+e))^{5/2},x, \text{algorithm}="maxima")$

[Out] $-((f*x + e)*a*\cos(4*f*x + 4*e)^2 + 36*(f*x + e)*a*\cos(2*f*x + 2*e)^2 + 16*(f*x + e)*a*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + (f*x + e)*a*\sin(4*f*x + 4*e)^2 + 36*(f*x + e)*a*\sin(2*f*x + 2*e)^2 + 16*(f*x + e)*a*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*a*\cos(2*f*x + 2*e) + (f*x + e)*a - 2*(a*\cos(4*f*x + 4*e)^2 + 36*a*\cos(2*f*x + 2*e)^2 + 16*a*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + a*\sin(4*f*x + 4*e)^2 + 12*a*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 36*a*\sin(2*f*x + 2*e)^2 + 16*a*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*(6*a*\cos(2*f*x + 2*e) + a)*\cos(4*f*x + 4*e) + 12*a*\cos(2*f*x + 2*e) - 8*(a*\cos(4*f*x + 4*e) + 6*a*\cos(2*f*x + 2*e) - 4*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + a)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(a*\cos(4*f*x + 4*e) + 6*a*\cos(2*f*x + 2*e) + a)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(a*\sin(4*f*x + 4*e) + 6*a*\sin(2*f*x + 2*e) - 4*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(a*\sin(4*f*x + 4*e) + 6*a*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a)*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 2*(6*(f*x + e)*a*\cos(2*f*x + 2*e) + (f*x + e)*a - 4*a*\sin(2*f*x + 2*e))*\cos(4*f*x + 4*e) - 2*(4*(f*x + e)*a*\cos(4*f*x + 4*e) + 24*(f*x + e)*a*\cos(2*f*x + 2*e) - 16*(f*x + e)*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 4*(f*x + e)*a + 3*a*\sin(4*f*x + 4*e) + 2*a*\sin(2*f*x + 2*e))*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(4*(f*x + e)*a*\cos(4*f*x + 4*e) + 24*(f*x + e)*a*\cos(2*f*x + 2*e) + 4*(f*x + e)*a + 3*a*\sin(4*f*x + 4*e) + 2*a*\sin(2*f*x + 2*e))*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*a*\sin(2*f*x + 2*e) + 2*a*\cos(2*f*x + 2*e))*\sin(4*f*x + 4*e) - 8*a*\sin(2*f*x + 2*e) - 2*(4*(f*x + e)*a*\sin(4*f*x + 4*e) + 24*(f*x + e)*a*\sin(2*f*x + 2*e) - 16*(f*x + e)*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 3*a*\cos(4*f*x + 4*e) - 2*a*\cos(2*f*x + 2*e) - 3*a)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(4*(f*x + e)*a*\sin(4*f*x + 4*e) + 24*(f*x + e)*a*\sin(2*f*x + 2*e) - 3*a*\cos(4*f*x + 4*e) - 2*a*\cos(2*f*x + 2*e) - 3*a)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sqrt{a}*\sqrt{c}/((c^3*\cos(4*f*x + 4*e)^2 + 36*c^3*\cos(2*f*x + 2*e)^2 + 16*c^3*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*c^3*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + c^3*\sin(4*f*x + 4*e)^2 + 12*c^3*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 36*c^3*\sin(2*f*x + 2*e)^2 + 16*c^3*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*c^3*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 12*c^3*\cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*\cos(2*f*x + 2*e) + c^3)*\cos(4*f*x + 4*e)$

) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) + c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2), x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sec(e+fx)+1)\right)^{\frac{3}{2}}}{\left(-c(\sec(e+fx)-1)\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2), x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(5/2), x)

$$3.100 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c-c \sec(e+fx))^{7/2}} dx$$

Optimal. Leaf size=196

$$\frac{a^2 \tan(e+fx) \log(1-\cos(e+fx))}{c^3 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^2 \tan(e+fx)}{c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^2 \tan(e+fx)}{2cf \sqrt{a \sec(e+fx)+a}}$$

[Out] $-2/3*a^2*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(7/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/2*a^2*\tan(f*x+e)/c/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a^2*\tan(f*x+e)/c^2/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^2*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^3/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.38, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3908, 3907, 3911, 31}

$$-\frac{a^2 \tan(e+fx)}{c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} + \frac{a^2 \tan(e+fx) \log(1-\cos(e+fx))}{c^3 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^2 \tan(e+fx)}{2cf \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2), x]

[Out] $(-2*a^2*\tan[e+f*x])/(3*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(7/2)}) - (a^2*\tan[e+f*x])/(2*c*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(5/2)}) - (a^2*\tan[e+f*x])/(c^2*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(3/2)}) + (a^2*\log[1-\cos[e+f*x]]*\tan[e+f*x])/(c^3*f*\sqrt{a+a*\sec[e+f*x]}*\sqrt{c-c*\sec[e+f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3908

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^{7/2}} dx &= -\frac{2a^2 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} + \frac{a \int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{5/2}} dx}{c} \\
&= -\frac{2a^2 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^2 \tan(e + fx)}{2cf\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} \\
&= -\frac{2a^2 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^2 \tan(e + fx)}{2cf\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} \\
&= -\frac{2a^2 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^2 \tan(e + fx)}{2cf\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} \\
&= -\frac{2a^2 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^2 \tan(e + fx)}{2cf\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}}
\end{aligned}$$

Mathematica [C] time = 2.20, size = 199, normalized size = 1.02

$$\frac{a \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(-60 \log(1 - e^{i(e+fx)}) - 3ifx \cos(3(e + fx)) + 6i(6i \log(1 - e^{i(e+fx)}) + \dots)\right)}{12c^3 f(\cos(e + fx) + \dots)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]

[Out] (a*(-50 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (6*I)*Cos[2*(e + f*x)]*(4*I + 3*f*x + (6*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(66 - (45*I)*f*x + 90*Log[1 - E^(I*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(a \sec(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sec(fx + e) + c}}{c^4 \sec(fx + e)^4 - 4c^4 \sec(fx + e)^3 + 6c^4 \sec(fx + e)^2 - 4c^4 \sec(fx + e) + c^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fricas")

[Out] integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 2.30, size = 289, normalized size = 1.47

$$(-1 + \cos(fx + e)) \left(24 (\cos^3(fx + e)) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 48 \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) (\cos^3(fx + e)) + 35 (\cos^3(fx + e)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x)

[Out] 1/24/f*(-1+cos(f*x+e))*(24*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-48*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+35*cos(f*x+e)^3-72*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+144*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-9*cos(f*x+e)^2+72*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-144*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-27*cos(f*x+e)-24*ln(2/(1+cos(f*x+e)))+48*ln(-(-1+cos(f*x+e))/sin(f*x+e))+17)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)/cos(f*x+e)^3*a

maxima [B] time = 3.66, size = 3480, normalized size = 17.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxima")

[Out] -1/3*(3*(f*x + e)*a*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a*cos(4*f*x + 4*e)^2 + 675*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a*sin(6*f*x + 6*e)^2 + 675*(f*x + e)*a*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 108*(f*x + e)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*(f*x + e)*a*cos(2*f*x + 2*e) + 3*(f*x + e)*a - 6*(a*cos(6*f*x + 6*e)^2 + 225*a*cos(4*f*x + 4*e)^2 + 225*a*cos(2*f*x + 2*e)^2 + 36*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*sin(6*f*x + 6*e)^2 + 225*a*sin(4*f*x + 4*e)^2 + 450*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*a*sin(2*f*x + 2*e)^2 + 36*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*cos(6*f*x + 6*e) + 30*(15*a*cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e) + 30*a*cos(2*f*x + 2*e) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 6*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) - 6*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x + 2*e) - 20*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 6*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))

$$\begin{aligned}
& + 2*e))))*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 40*(a*\sin(6*f*x + 6*e) + 15*a*\sin(4*f*x + 4*e) + 15*a*\sin(2*f*x + 2*e) - 6*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*(a*\sin(6*f*x + 6*e) + 15*a*\sin(4*f*x + 4*e) + 15*a*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 6*(15*(f*x + e)*a*\cos(4*f*x + 4*e) + 15*(f*x + e)*a*\cos(2*f*x + 2*e) + (f*x + e)*a - 11*a*\sin(4*f*x + 4*e) - 11*a*\sin(2*f*x + 2*e))*\cos(6*f*x + 6*e) + 90*(15*(f*x + e)*a*\cos(2*f*x + 2*e) + (f*x + e)*a)*\cos(4*f*x + 4*e) - 12*(3*(f*x + e)*a*\cos(6*f*x + 6*e) + 45*(f*x + e)*a*\cos(4*f*x + 4*e) + 45*(f*x + e)*a*\cos(2*f*x + 2*e) - 60*(f*x + e)*a*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 18*(f*x + e)*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 3*(f*x + e)*a + 2*a*\sin(6*f*x + 6*e) - 3*a*\sin(4*f*x + 4*e) - 3*a*\sin(2*f*x + 2*e) + 10*a*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*\cos(6*f*x + 6*e) + 90*(f*x + e)*a*\cos(4*f*x + 4*e) + 90*(f*x + e)*a*\cos(2*f*x + 2*e) - 36*(f*x + e)*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 6*(f*x + e)*a + 5*a*\sin(6*f*x + 6*e) + 9*a*\sin(4*f*x + 4*e) + 9*a*\sin(2*f*x + 2*e) - 6*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*\cos(6*f*x + 6*e) + 45*(f*x + e)*a*\cos(4*f*x + 4*e) + 45*(f*x + e)*a*\cos(2*f*x + 2*e) + 3*(f*x + e)*a + 2*a*\sin(6*f*x + 6*e) - 3*a*\sin(4*f*x + 4*e) - 3*a*\sin(2*f*x + 2*e))*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 6*(15*(f*x + e)*a*\sin(4*f*x + 4*e) + 15*(f*x + e)*a*\sin(2*f*x + 2*e) + 11*a*\cos(4*f*x + 4*e) + 11*a*\cos(2*f*x + 2*e))*\sin(6*f*x + 6*e) + 6*(225*(f*x + e)*a*\sin(2*f*x + 2*e) - 11*a)*\sin(4*f*x + 4*e) - 66*a*\sin(2*f*x + 2*e) - 12*(3*(f*x + e)*a*\sin(6*f*x + 6*e) + 45*(f*x + e)*a*\sin(4*f*x + 4*e) + 45*(f*x + e)*a*\sin(2*f*x + 2*e) - 60*(f*x + e)*a*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 18*(f*x + e)*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*a*\cos(6*f*x + 6*e) + 3*a*\cos(4*f*x + 4*e) + 3*a*\cos(2*f*x + 2*e) - 10*a*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*a)*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 20*(6*(f*x + e)*a*\sin(6*f*x + 6*e) + 90*(f*x + e)*a*\sin(4*f*x + 4*e) + 90*(f*x + e)*a*\sin(2*f*x + 2*e) - 36*(f*x + e)*a*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 5*a*\cos(6*f*x + 6*e) - 9*a*\cos(4*f*x + 4*e) - 9*a*\cos(2*f*x + 2*e) + 6*a*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 5*a)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*(3*(f*x + e)*a*\sin(6*f*x + 6*e) + 45*(f*x + e)*a*\sin(4*f*x + 4*e) + 45*(f*x + e)*a*\sin(2*f*x + 2*e) - 2*a*\cos(6*f*x + 6*e) + 3*a*\cos(4*f*x + 4*e) + 3*a*\cos(2*f*x + 2*e) - 2*a)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sqrt{a}*\sqrt{c}/((c^4*\cos(6*f*x + 6*e)^2 + 225*c^4*\cos(4*f*x + 4*e)^2 + 225*c^4*\cos(2*f*x + 2*e)^2 + 36*c^4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 400*c^4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 36*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + c^4*\sin(6*f*x + 6*e)^2 + 225*c^4*\sin(4*f*x + 4*e)^2 + 450*c^4*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 225*c^4*\sin(2*f*x + 2*e)^2 + 36*c^4*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 400*c^4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 36*c^4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 30*c^4*\cos(2*f*x + 2*e) + c^4 + 2*(15*c^4*\cos(4*f*x + 4*e) + 15*c^4*\cos(2*f*x + 2*e) + c^4)*\cos(6*f*x + 6*e) + 30*(15*c^4*\cos(2*f*x + 2*e) + c^4)*\cos(4*f*x + 4*e) - 12*(c^4*\cos(6*f*x + 6*e) + 15*c^4*\cos(4*f*x + 4*e) + 15*c^4*\cos(2*f*x + 2*e) - 20*c^4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 6*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + c^4)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 40*(c^4*\cos(6*f*x + 6*e) + 15*c^4*\cos(4*f*x + 4*e) + 15*c^4*\cos(2*f*x + 2*e) - 6*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + c^4)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*(c^4*\cos(6*f*x + 6*e) + 15*c^4*\cos(4*f*x + 4*e) + 15*c^4*\cos(2*f*x + 2*e) + c^4)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 30*(c^
\end{aligned}$$

$4*\sin(4*f*x + 4*e) + c^4*\sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) - 12*(c^4*\sin(6*f*x + 6*e) + 15*c^4*\sin(4*f*x + 4*e) + 15*c^4*\sin(2*f*x + 2*e) - 20*c^4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 6*c^4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 40*(c^4*\sin(6*f*x + 6*e) + 15*c^4*\sin(4*f*x + 4*e) + 15*c^4*\sin(2*f*x + 2*e) - 6*c^4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 12*(c^4*\sin(6*f*x + 6*e) + 15*c^4*\sin(4*f*x + 4*e) + 15*c^4*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*f$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2), x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2), x)

[Out] Timed out

3.101 $\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{5/2} dx$

Optimal. Leaf size=153

$$\frac{a^3 c^3 \tan^5(e + fx)}{4f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c^3 \tan^3(e + fx)}{2f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c^3 \tan(e + fx) \log\left(\frac{a \sec(e + fx) + a}{c - c \sec(e + fx)}\right)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $a^3 c^3 \ln(\cos(fx+e)) \tan(fx+e) / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} + 1/2 a^3 c^3 \tan(fx+e)^3 / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} - 1/4 a^3 c^3 \tan(fx+e)^5 / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2}$

Rubi [A] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3905, 3473, 3475}

$$\frac{a^3 c^3 \tan^5(e + fx)}{4f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c^3 \tan^3(e + fx)}{2f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c^3 \tan(e + fx) \log\left(\frac{a \sec(e + fx) + a}{c - c \sec(e + fx)}\right)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \text{Sec}[e + f*x])^{5/2} * (c - c \text{Sec}[e + f*x])^{5/2}, x]$

[Out] $(a^3 c^3 \text{Log}[\text{Cos}[e + f*x]] * \text{Tan}[e + f*x]) / (f \text{Sqrt}[a + a \text{Sec}[e + f*x]] * \text{Sqrt}[c - c \text{Sec}[e + f*x]]) + (a^3 c^3 \text{Tan}[e + f*x]^3) / (2 * f \text{Sqrt}[a + a \text{Sec}[e + f*x]] * \text{Sqrt}[c - c \text{Sec}[e + f*x]]) - (a^3 c^3 \text{Tan}[e + f*x]^5) / (4 * f \text{Sqrt}[a + a \text{Sec}[e + f*x]] * \text{Sqrt}[c - c \text{Sec}[e + f*x]])$

Rule 3473

$\text{Int}[(b \cdot \tan[(c \cdot) + (d \cdot)(x \cdot)])^{(n \cdot)}, x_Symbol] \rightarrow \text{Simp}[(b * (b \cdot \text{Tan}[c + d * x])^{(n - 1)}) / (d * (n - 1)), x] - \text{Dist}[b^2, \text{Int}[(b \cdot \text{Tan}[c + d * x])^{(n - 2)}, x], x] /;$ $\text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1]$

Rule 3475

$\text{Int}[\tan[(c \cdot) + (d \cdot)(x \cdot)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d * x], x]] / d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rule 3905

$\text{Int}[(\text{csc}[(e \cdot) + (f \cdot)(x \cdot)] * (b \cdot) + (a \cdot))^{(m \cdot)} * (\text{csc}[(e \cdot) + (f \cdot)(x \cdot)] * (d \cdot) + (c \cdot))^{(m \cdot)}, x_Symbol] \rightarrow \text{Dist}[((-a * c))^{(m + 1/2)} * \text{Cot}[e + f * x] / (\text{Sqrt}[a + b * \text{Csc}[e + f * x]] * \text{Sqrt}[c + d * \text{Csc}[e + f * x]]), \text{Int}[\text{Cot}[e + f * x]^{(2 * m)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, m, x\} \ \&\& \ \text{EqQ}[b * c + a * d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m + 1/2]$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{5/2} dx &= -\frac{(a^3 c^3 \tan(e + fx)) \int \tan^5(e + fx) dx}{\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{a^3 c^3 \tan^5(e + fx)}{4f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{(a^3 c^3 \tan(e + fx))}{\sqrt{a + a \sec(e + fx)}} \\
&= \frac{a^3 c^3 \tan^3(e + fx)}{2f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{a^3 c^3 \tan(e + fx)}{4f \sqrt{a + a \sec(e + fx)}} \\
&= \frac{a^3 c^3 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c^3 \tan(e + fx)}{2f \sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 1.54, size = 164, normalized size = 1.07

$$\frac{ia^2 c^2 \csc\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) \sec^3(e + fx) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c - c \sec(e + fx)} (3i \log(1 + e^{2i(e+fx)})) + \dots}{16f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]

[Out] ((I/16)*a^2*c^2*Csc[(e + f*x)/2]*(2*I + 3*f*x + Cos[4*(e + f*x)]*(f*x + I*Log[1 + E^((2*I)*(e + f*x))]) + 4*Cos[2*(e + f*x)]*(I + f*x + I*Log[1 + E^((2*I)*(e + f*x))]) + (3*I)*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/f

fricas [A] time = 0.60, size = 405, normalized size = 2.65

$$\left[\frac{2 \sqrt{-ac} a^2 c^2 \cos(fx + e)^3 \log\left(\frac{ac \cos(fx+e)^4 - (\cos(fx+e)^3 + \cos(fx+e)) \sqrt{-ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx+e)+ac}{2 \cos(fx+e)^2} \right)}{4 f \cos(fx + e)^3} \right] - (3 a^2 c^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(-a*c)*a^2*c^2*cos(f*x + e)^3*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2) - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^3), 1/4*(4*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c))*cos(f*x + e)^3 - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
```


$4*e) + 4*a^2*c^2*\cos(2*f*x + 2*e) + a^2*c^2*\cos(6*f*x + 6*e) + 12*(4*a^2*c^2*\cos(2*f*x + 2*e) + a^2*c^2*\cos(4*f*x + 4*e) + 4*(2*a^2*c^2*\sin(6*f*x + 6*e) + 3*a^2*c^2*\sin(4*f*x + 4*e) + 2*a^2*c^2*\sin(2*f*x + 2*e))*\sin(8*f*x + 8*e) + 16*(3*a^2*c^2*\sin(4*f*x + 4*e) + 2*a^2*c^2*\sin(2*f*x + 2*e))*\sin(6*f*x + 6*e))*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) + 2*(4*(f*x + e)*a^2*c^2*\cos(6*f*x + 6*e) + 6*(f*x + e)*a^2*c^2*\cos(4*f*x + 4*e) + 4*(f*x + e)*a^2*c^2*\cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 - 2*a^2*c^2*\sin(6*f*x + 6*e) - 2*a^2*c^2*\sin(4*f*x + 4*e) - 2*a^2*c^2*\sin(2*f*x + 2*e))*\cos(8*f*x + 8*e) + 8*(6*(f*x + e)*a^2*c^2*\cos(4*f*x + 4*e) + 4*(f*x + e)*a^2*c^2*\cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 + a^2*c^2*\sin(4*f*x + 4*e))*\cos(6*f*x + 6*e) + 4*(12*(f*x + e)*a^2*c^2*\cos(2*f*x + 2*e) + 3*(f*x + e)*a^2*c^2 - 2*a^2*c^2*\sin(2*f*x + 2*e))*\cos(4*f*x + 4*e) + 4*(2*(f*x + e)*a^2*c^2*\sin(6*f*x + 6*e) + 3*(f*x + e)*a^2*c^2*\sin(4*f*x + 4*e) + 2*(f*x + e)*a^2*c^2*\sin(2*f*x + 2*e) + a^2*c^2*\cos(6*f*x + 6*e) + a^2*c^2*\cos(4*f*x + 4*e) + a^2*c^2*\cos(2*f*x + 2*e))*\sin(8*f*x + 8*e) + 4*(12*(f*x + e)*a^2*c^2*\sin(4*f*x + 4*e) + 8*(f*x + e)*a^2*c^2*\sin(2*f*x + 2*e) - 2*a^2*c^2*\cos(4*f*x + 4*e) - a^2*c^2*\sin(6*f*x + 6*e) + 4*(12*(f*x + e)*a^2*c^2*\sin(2*f*x + 2*e) + 2*a^2*c^2*\cos(2*f*x + 2*e) - a^2*c^2*\sin(4*f*x + 4*e))*\sqrt{a}*\sqrt{c}/((2*(4*\cos(6*f*x + 6*e) + 6*\cos(4*f*x + 4*e) + 4*\cos(2*f*x + 2*e) + 1)*\cos(8*f*x + 8*e) + \cos(8*f*x + 8*e)^2 + 8*(6*\cos(4*f*x + 4*e) + 4*\cos(2*f*x + 2*e) + 1)*\cos(6*f*x + 6*e) + 16*\cos(6*f*x + 6*e)^2 + 12*(4*\cos(2*f*x + 2*e) + 1)*\cos(4*f*x + 4*e) + 36*\cos(4*f*x + 4*e)^2 + 16*\cos(2*f*x + 2*e)^2 + 4*(2*\sin(6*f*x + 6*e) + 3*\sin(4*f*x + 4*e) + 2*\sin(2*f*x + 2*e))*\sin(8*f*x + 8*e) + \sin(8*f*x + 8*e)^2 + 16*(3*\sin(4*f*x + 4*e) + 2*\sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) + 16*\sin(6*f*x + 6*e)^2 + 36*\sin(4*f*x + 4*e)^2 + 48*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 16*\sin(2*f*x + 2*e)^2 + 8*\cos(2*f*x + 2*e) + 1)*f)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c - \frac{c}{\cos(e + fx)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2), x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2), x)

[Out] Timed out

3.102 $\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{3/2} dx$

Optimal. Leaf size=190

$$\frac{a^3 c^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{a^2 c^2 \tan(e + fx) \sqrt{a \sec(e + fx) + a}}{f \sqrt{c - c \sec(e + fx)}} - \frac{a c^2 \tan(e + fx) (a \sec(e + fx))}{2 f \sqrt{c - c \sec(e + fx)}}$$

[Out] $-1/2*a*c^2*(a+a*\sec(f*x+e))^{(3/2)}*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(1/2)}+1/3*c^2*(a+a*\sec(f*x+e))^{(5/2)}*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(1/2)}+a^3*c^2*\ln(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-a^2*c^2*(a+a*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3909, 3906, 3905, 3475}

$$-\frac{a^2 c^2 \tan(e + fx) \sqrt{a \sec(e + fx) + a}}{f \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{a c^2 \tan(e + fx) (a \sec(e + fx))}{2 f \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2), x]

[Out] $(a^3*c^2*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (a^2*c^2*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (a*c^2*(a + a*\text{Sec}[e + f*x])^{(3/2)}*\text{Tan}[e + f*x])/(2*f*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (c^2*(a + a*\text{Sec}[e + f*x])^{(5/2)}*\text{Tan}[e + f*x])/(3*f*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[((-a*c)^(m + 1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rule 3906

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]

Rule 3909

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(3/2)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Simp[(-2*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LeQ[n, -2^(-1)]

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{3/2} dx &= \frac{c^2 (a + a \sec(e + fx))^{5/2} \tan(e + fx)}{3f \sqrt{c - c \sec(e + fx)}} + c \int (a + a \sec(e + fx))^{5/2} \\
&= -\frac{ac^2 (a + a \sec(e + fx))^{3/2} \tan(e + fx)}{2f \sqrt{c - c \sec(e + fx)}} + \frac{c^2 (a + a \sec(e + fx))^{5/2}}{3f \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{a^2 c^2 \sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f \sqrt{c - c \sec(e + fx)}} - \frac{ac^2 (a + a \sec(e + fx))^{5/2}}{2f \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{a^2 c^2 \sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f \sqrt{c - c \sec(e + fx)}} - \frac{ac^2 (a + a \sec(e + fx))^{5/2}}{2f \sqrt{c - c \sec(e + fx)}} \\
&= \frac{a^3 c^2 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{a^2 c^2 \sqrt{a + a \sec(e + fx)}}{f \sqrt{c - c \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 1.20, size = 149, normalized size = 0.78

$$\frac{a^2 c \csc\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c - c \sec(e + fx)} (6 \cos(2(e + fx)) + 3ifx)}{24f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2), x]

[Out] (a^2*c*Csc[(e + f*x)/2]*(2 + 6*Cos[2*(e + f*x)] + (3*I)*f*x*Cos[3*(e + f*x)] + Cos[e + f*x]*(-6 + (9*I)*f*x - 9*Log[1 + E^((2*I)*(e + f*x))]) - 3*Cos[3*(e + f*x)]*Log[1 + E^((2*I)*(e + f*x))])*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(24*f)

fricas [A] time = 0.58, size = 467, normalized size = 2.46

$$\frac{\left(a^2 c \cos^2(fx + e) - 5 a^2 c \cos(fx + e) - 2 a^2 c \right) \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \sqrt{\frac{c \cos(fx + e) - c}{\cos(fx + e)}} \sin(fx + e) + 3 \left(a^2 c \cos^3(fx + e) - 6 \left(f \cos(fx + e) \right)^3 + f c \right)}{6 \left(f \cos(fx + e) \right)^3 + f c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + 3*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), 1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + 6*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
```



```

+ e)*a^2*c)*cos(4*f*x + 4*e) - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*
x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 4*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a
^2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x
+ 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(3*(f*x +
e)*a^2*c*sin(4*f*x + 4*e) + 3*(f*x + e)*a^2*c*sin(2*f*x + 2*e) + a^2*c*cos
(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(9*(f*x + e)*a
^2*c*sin(2*f*x + 2*e) - a^2*c)*sin(4*f*x + 4*e) + 6*(a^2*c*cos(6*f*x + 6*e)
+ 3*a^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) + 3*a
^2*c*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(a^2*c*cos(6*f*x + 6*e) + 3*a^2*c*c
os(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^
2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c - \frac{c}{\cos(e + fx)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.103 $\int (a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)} dx$

Optimal. Leaf size=139

$$\frac{a^3 c \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{a^2 c \tan(e + fx) \sqrt{a \sec(e + fx) + a}}{f \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx)(a \sec(e + fx) + a)}{2f \sqrt{c - c \sec(e + fx)}}$$

[Out] $-1/2*a*c*(a+a*\sec(f*x+e))^(3/2)*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^(1/2)+a^3*c*1$
 $n(\cos(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^(1/2)/(c-c*\sec(f*x+e))^(1/2)-a^$
 $2*c*(a+a*\sec(f*x+e))^(1/2)*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^(1/2)$

Rubi [A] time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3906, 3905, 3475}

$$-\frac{a^2 c \tan(e + fx) \sqrt{a \sec(e + fx) + a}}{f \sqrt{c - c \sec(e + fx)}} + \frac{a^3 c \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{ac \tan(e + fx)(a \sec(e + fx) + a)}{2f \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^(5/2)*\text{Sqrt}[c - c*\text{Sec}[e + f*x]],x]$

[Out] $(a^3*c*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c -$
 $c*\text{Sec}[e + f*x]]) - (a^2*c*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[c$
 $- c*\text{Sec}[e + f*x]]) - (a*c*(a + a*\text{Sec}[e + f*x])^(3/2)*\text{Tan}[e + f*x])/(2*f*\text{S}$
 $\text{qrt}[c - c*\text{Sec}[e + f*x]])$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d$
 $*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3905

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d$
 $_.) + (c_.))^(m_.), x_Symbol] \rightarrow \text{Dist}[((-a*c))^(m + 1/2)*\text{Cot}[e + f*x]/(\text{Sqrt}$
 $[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Int}[\text{Cot}[e + f*x]^(2*m), x],$
 $x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^$
 $2, 0] \&\& \text{IntegerQ}[m + 1/2]$

Rule 3906

$\text{Int}[\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d$
 $_.) + (c_.))^(n_.), x_Symbol] \rightarrow \text{Simp}[(2*a*c*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x]$
 $)^(n - 1))/(f*(2*n - 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[c, \text{Int}[\text{Sqrt}[a$
 $+ b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^(n - 1), x], x] /; \text{FreeQ}\{a, b, c, d$
 $, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[n, 1/2]$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)} dx &= -\frac{ac(a + a \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{c - c \sec(e + fx)}} + a \int (a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)} dx \\
&= -\frac{a^2c\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f\sqrt{c - c \sec(e + fx)}} - \frac{ac(a + a \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{c - c \sec(e + fx)}} \\
&= -\frac{a^2c\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f\sqrt{c - c \sec(e + fx)}} - \frac{ac(a + a \sec(e + fx))^{3/2} \tan(e + fx)}{2f\sqrt{c - c \sec(e + fx)}} \\
&= \frac{a^3c \log(\cos(e + fx)) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{a^2c\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{f\sqrt{c - c \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 1.40, size = 164, normalized size = 1.18

$$\frac{a^2 e^{-i(e+fx)} (1 + e^{2i(e+fx)}) \left(\cot\left(\frac{1}{2}(e+fx)\right) + i \right) \sec^2(e+fx) \sqrt{a(\sec(e+fx)+1)} \sqrt{c - c \sec(e+fx)} (-\log(1 + e^{2i(e+fx)}))}{4f(1 + e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]], x]

[Out] (a^2*(1 + E^((2*I)*(e + f*x)))*(I + Cot[(e + f*x)/2])*(1 + I*f*x + 4*Cos[e + f*x] + Cos[2*(e + f*x)]*(I*f*x - Log[1 + E^((2*I)*(e + f*x))]) - Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*E^(I*(e + f*x))*(1 + E^(I*(e + f*x))))*f

fricas [A] time = 0.56, size = 420, normalized size = 3.02

$$\frac{\left(5a^2 \cos(fx + e) + a^2\right) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx + e) + \left(a^2 \cos(fx + e)^2 + a^2 \cos(fx + e)\right) \sqrt{-ac} \log\left(\frac{2\left(f \cos(fx + e)^2 + f \cos(fx + e)\right)}{\dots}\right)}{2\left(f \cos(fx + e)^2 + f \cos(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2), x, algorithm="fricas")

[Out] [1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + (a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^2 + f*cos(f*x + e)), 1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*c*cos(f*x + e)^2 + a*c)))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real):C
heck [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions
on a variableWarning, assuming -2*a+a is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -2*a+a is positive. Hint: run a
ssume to make assumptions on a variableWarning, assuming -2*a+a is positive
. Hint: run assume to make assumptions on a variableUnable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/

```


sign: (4*pi/x/2)>(-4*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{2*c,0%%}+%%{1,2%%})Evaluation time: 3.78Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.32, size = 180, normalized size = 1.29

$$\frac{\left(2 \ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^2(fx+e)\right) - 2 \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right) - 2 \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\right)}{2f \cos(fx+e) \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x)

[Out] 1/2/f*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-2*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+3*cos(f*x+e)^2+4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x+e)/sin(f*x+e)*a^2

maxima [B] time = 1.05, size = 710, normalized size = 5.11

$$\frac{\left((fx+e)a^2 \cos(4fx+4e)^2 + 4(fx+e)a^2 \cos(2fx+2e)^2 + (fx+e)a^2 \sin(4fx+4e)^2 + 4(fx+e)a^2 \sin(2fx+2e)^2\right)}{2f \cos(fx+e) \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -((f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + (f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + 2*a^2*sin(2*f*x + 2*e) - (a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e)) *arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*x + e)*a^2*sin(2*f*x + 2*e) - a^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e+fx)}\right)^{5/2} \sqrt{c - \frac{c}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

$$3.104 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{\sqrt{c-c \sec(e+fx)}} dx$$

Optimal. Leaf size=152

$$\frac{a^3 \tan(e+fx) \sec(e+fx)}{f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}} + \frac{4a^3 \tan(e+fx) \log(1-\sec(e+fx))}{f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}} + \frac{a^3 \tan(e+fx) \log(\cos(e+fx))}{f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}}$$

[Out] $a^3 \ln(\cos(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 4*a^3 \ln(1-\sec(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + a^3 \ln(\cos(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{a^3 \tan(e+fx) \sec(e+fx)}{f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}} + \frac{4a^3 \tan(e+fx) \log(1-\sec(e+fx))}{f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}} + \frac{a^3 \tan(e+fx) \log(\cos(e+fx))}{f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]

[Out] $(a^3 * \text{Log}[\text{Cos}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (4 * a^3 * \text{Log}[1 - \text{Sec}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (a^3 * \text{Sec}[e + f*x] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]])$

Rule 72

Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sec(e+fx))^{5/2}}{\sqrt{c-c \sec(e+fx)}} dx &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{(a+ax)^2}{x(c-cx)} dx, x, \sec(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \left(-\frac{a^2}{c} - \frac{4a^2}{c(-1+x)} + \frac{a^2}{cx}\right) dx, x, \sec(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= \frac{a^3 \log(\cos(e+fx)) \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} + \frac{4a^3 \log(1-\sec(e+fx)) \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 6.76, size = 292, normalized size = 1.92

$$\frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right) \sec(e + fx) (a(\sec(e + fx) + 1))^{5/2} \sqrt{(\cos(e + fx) + 1) \sec(e + fx)}}{f(\sec(e + fx) + 1)^{5/2} \sqrt{c - c \sec(e + fx)}} + \frac{\sqrt{2} e^{\frac{1}{2}i(e+fx)} \sqrt{\frac{(1+e^{i(e+fx)})^2}{1+e^{2i(e+fx)}}}}{\sqrt{2} e^{\frac{1}{2}i(e+fx)}} (8 \log(1 + e^{i(e+fx)}))$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]], x]

[Out] (Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*((-I)*f*x + 8*Log[1 - E^(I*(e + f*x))] - 3*Log[1 + E^((2*I)*(e + f*x))])*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2])/((1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*(1 + Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (Sec[e + f*x]*Sqrt[(1 + Cos[e + f*x])*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Tan[e/2 + (f*x)/2])/((f*(1 + Sec[e + f*x]))^(5/2)*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2\right) \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c \sec(fx + e) - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assumptions on a variableWa

check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{-a,0%%}+%%{1,2%%})Sign error (%%{-2*a,0%%}+%%{1,2%%})Evaluation time: 2.92Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.06, size = 184, normalized size = 1.21

$$\frac{\left(8 \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) \cos(fx+e) - 3 \cos(fx+e) \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 3 \cos(fx+e) \ln\left(-\frac{-\sin(fx+e)}{\sin(fx+e)}\right)\right)}{f \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)

[Out] -1/f*(8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-3*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*cos(f*x+e)*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(1+cos(f*x+e)))+cos(f*x+e)+1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c*a^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\sqrt{c - \frac{c}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2), x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2), x)
```

```
[Out] Timed out
```

$$3.105 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=96

$$\frac{a^3 \tan(e+fx) \log(\cos(e+fx))}{cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{4a^3 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}$$

[Out] $-4*a^3*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^3*\ln(\cos(f*x+e))*\tan(f*x+e)/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3910, 3905, 3475}

$$\frac{a^3 \tan(e+fx) \log(\cos(e+fx))}{cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{4a^3 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}(c-c \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2), x]

[Out] $(-4*a^3*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(3/2)}) + (a^3*\log[\cos[e + f*x]]*\tan[e + f*x])/(c*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[((-a*c)^(m + 1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rule 3910

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(5/2)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{3/2}} dx &= -\frac{4a^3 \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{a^2 \int \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}}{c^2} \\ &= -\frac{4a^3 \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} - \frac{(a^3 \tan(e+fx)) \int \tan(e+fx)}{c\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= -\frac{4a^3 \tan(e+fx)}{f\sqrt{a+a \sec(e+fx)}(c-c \sec(e+fx))^{3/2}} + \frac{a^3 \log(\cos(e+fx)) \tan(e+fx)}{cf\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.29, size = 111, normalized size = 1.16

$$\frac{a^2 \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(-\log\left(1 + e^{2i(e+fx)}\right) + \left(\log\left(1 + e^{2i(e+fx)}\right) - ifx\right) \cos(e + fx) + ifx - 4\right)}{cf(\cos(e + fx) - 1)\sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2), x]

[Out] (a^2*(-4 + I*f*x - Log[1 + E^((2*I)*(e + f*x))]) + Cos[e + f*x]*((-I)*f*x + Log[1 + E^((2*I)*(e + f*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2] / (c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])

fricas [B] time = 0.58, size = 442, normalized size = 4.60

$$\frac{\left((a^2 c \cos(fx + e) - a^2 c) \sqrt{-\frac{a}{c}} \log \left(\frac{a \cos(fx+e)^4 - (\cos(fx+e)^3 + \cos(fx+e)) \sqrt{-\frac{a}{c}} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \sin(fx+e)+a}{2 \cos(fx+e)^2} \right) \sin(fx+e) \right)}{2(c^2 f \cos(fx + e) - c^2 f) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [1/2*((a^2*c*cos(f*x + e) - a^2*c)*sqrt(-a/c)*log(1/2*(a*cos(f*x + e))^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a)/cos(f*x + e)^2)*sin(f*x + e) + 4*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), ((a^2*c*cos(f*x + e) - a^2*c)*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*cos(f*x + e)^2 + a))*sin(f*x + e) + 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.26, size = 239, normalized size = 2.49

$$\frac{(-1 + \cos(fx + e)) \left(\cos(fx + e) \ln \left(-\frac{-1 + \cos(fx+e) + \sin(fx+e)}{\sin(fx+e)} \right) + \cos(fx + e) \ln \left(-\frac{-\sin(fx+e) - 1 + \cos(fx+e)}{\sin(fx+e)} \right) - \cos(fx + e) \right)}{f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)

[Out] $-1/f*(-1+\cos(f*x+e))*(\cos(f*x+e)*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))+\cos(f*x+e)*\ln(-(-\sin(f*x+e)-1+\cos(f*x+e))/\sin(f*x+e))-\cos(f*x+e)*\ln(2/(1+\cos(f*x+e)))-\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-\ln(-(-\sin(f*x+e)-1+\cos(f*x+e))/\sin(f*x+e))-2*\cos(f*x+e)+\ln(2/(1+\cos(f*x+e))))-2*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}/\cos(f*x+e)/(c*(-1+\cos(f*x+e))/\cos(f*x+e))^{3/2}/\sin(f*x+e)*a^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2),x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)

[Out] Timed out

$$3.106 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=100

$$\frac{a^3 \tan(e+fx) \log(1-\cos(e+fx))}{c^2 f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}} - \frac{2a^3 \tan(e+fx)}{f \sqrt{a \sec(e+fx) + a} (c-c \sec(e+fx))^{5/2}}$$

[Out] $-2*a^3*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^3*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3910, 3911, 31}

$$\frac{a^3 \tan(e+fx) \log(1-\cos(e+fx))}{c^2 f \sqrt{a \sec(e+fx) + a} \sqrt{c-c \sec(e+fx)}} - \frac{2a^3 \tan(e+fx)}{f \sqrt{a \sec(e+fx) + a} (c-c \sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2), x]

[Out] $(-2*a^3*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(5/2)}) + (a^3*\log[1 - \cos[e + f*x]]*\tan[e + f*x])/(c^2*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3910

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(m_), x_Symbol] := Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{5/2}} dx &= -\frac{2a^3 \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2}} + \frac{a^2 \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c-c \sec(e+fx)}} dx}{c^2} \\ &= -\frac{2a^3 \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2}} + \frac{(a^3 \tan(e+fx)) \text{Subst}\left(\int \frac{1}{-c+cx} dx, \frac{a+a \sec(e+fx)}{c-c \sec(e+fx)}\right)}{cf \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= -\frac{2a^3 \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2}} + \frac{a^3 \log(1-\cos(e+fx)) \tan(e+fx)}{c^2 f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.39, size = 155, normalized size = 1.55

$$\frac{a^2 \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(6 \log(1 - e^{i(e+fx)}) + (-8 \log(1 - e^{i(e+fx)}) + 4ifx - 8) \cos(e + fx) + \dots\right)}{2c^2 f (\cos(e + fx) - 1)^2 \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2), x]

[Out] (a^2*(4 - (3*I)*f*x + Cos[e + f*x]*(-8 + (4*I)*f*x - 8*Log[1 - E^(I*(e + f*x))]) + 6*Log[1 - E^(I*(e + f*x))]) + Cos[2*(e + f*x)]*((-I)*f*x + 2*Log[1 - E^(I*(e + f*x))]) * Sqrt[a*(1 + Sec[e + f*x])] * Tan[(e + f*x)/2]) / (2*c^2*f*(-1 + Cos[e + f*x])^2 * Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\left(a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2\right) \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c^3 \sec(fx + e)^3 - 3c^3 \sec(fx + e)^2 + 3c^3 \sec(fx + e) - c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2), x, algorithm="fricas")

[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.25, size = 229, normalized size = 2.29

$$\frac{(-1 + \cos(fx + e)) \left(4 (\cos^2(fx + e)) \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) - 2 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) (\cos^2(fx + e)) - 3 (\cos^2(fx + e) + \dots)\right)}{c^3 \sin(fx + e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2), x)

[Out] -1/2/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-3*cos(f*x+e)^2-8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+4*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*ln(2/(1+cos(f*x+e)))+1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)/cos(f*x+e)^2*a^2

maxima [A] time = 0.57, size = 139, normalized size = 1.39

$$\frac{4 \sqrt{-a} a^2 \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}} - \frac{2 \sqrt{-a} a^2 \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2+1}\right)}{c^{\frac{5}{2}}} - \frac{\left(\sqrt{-a} a^2 \sqrt{c} - \frac{2 \sqrt{-a} a^2 \sqrt{c} \sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right) (\cos(fx+e)+1)^4}{c^3 \sin(fx+e)^4}$$

$2f$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] 1/2*(4*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) - 2*sqrt(-a)*a^2*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) - (sqrt(-a)*a^2*sqrt(c) - 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^4/(c^3*sin(f*x + e)^4))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2),x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)

[Out] Timed out

$$3.107 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{7/2}} dx$$

Optimal. Leaf size=148

$$\frac{a^3 \tan(e+fx) \log(1-\cos(e+fx))}{c^3 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^3 \tan(e+fx)}{c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{4a^3 \tan(e+fx)}{3f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}}$$

[Out] $-4/3*a^3*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(7/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a^3*\tan(f*x+e)/c^2/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^3*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^3/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3910, 3907, 3911, 31}

$$-\frac{a^3 \tan(e+fx)}{c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} + \frac{a^3 \tan(e+fx) \log(1-\cos(e+fx))}{c^3 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{4a^3 \tan(e+fx)}{3f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(5/2)}/(c - c*\text{Sec}[e + f*x])^{(7/2)}, x]$

[Out] $(-4*a^3*\text{Tan}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^{(7/2)}) - (a^3*\text{Tan}[e + f*x])/(c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^{(3/2)}) + (a^3*\text{Log}[1 - \text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(c^3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 31

$\text{Int}[(a + b*x)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 3907

$\text{Int}[\text{Sqrt}[\text{csc}[e + f*x] + (f*x)]*(b + a)]*(\text{csc}[e + f*x] + (f*x))^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-2*a*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[1/c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -2^{(-1)}]$

Rule 3910

$\text{Int}[(\text{csc}[e + f*x] + (f*x))^{(5/2)}*(\text{csc}[e + f*x] + (f*x))^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-8*a^3*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[a^2/c^2, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^{(n + 2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -2^{(-1)}]$

Rule 3911

$\text{Int}[(\text{csc}[e + f*x] + (f*x))^{(m)}*(\text{csc}[e + f*x] + (f*x))^{(n)}, x_Symbol] \rightarrow -\text{Dist}[(a*c*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(b + a*x)^{(m - 1/2)}*(d + c*x)^{(n - 1/2)}/x^{(m + n)}, x], x, \text{Sin}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m - 1/2] \&\& \text{EqQ}[m + n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{7/2}} dx &= -\frac{4a^3 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} + \frac{a^2 \int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{3/2}} dx}{c^2} \\
&= -\frac{4a^3 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^3 \tan(e + fx)}{c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} \\
&= -\frac{4a^3 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^3 \tan(e + fx)}{c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} \\
&= -\frac{4a^3 \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}} - \frac{a^3 \tan(e + fx)}{c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{7/2}}
\end{aligned}$$

Mathematica [C] time = 2.67, size = 202, normalized size = 1.36

$$\frac{a^2 \tan\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(-60 \log(1 - e^{i(e+fx)}) - 3ifx \cos(3(e + fx)) + 6i(6i \log(1 - e^{i(e+fx)}) + 3)\right)}{12c^3 f(\cos(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2), x]

[Out] (a^2*(-58 + (30*I)*f*x - (3*I)*f*x*Cos[3*(e + f*x)] + (6*I)*Cos[2*(e + f*x)]*(5*I + 3*f*x + (6*I)*Log[1 - E^(I*(e + f*x))]) - 60*Log[1 - E^(I*(e + f*x))] + 6*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + 9*Cos[e + f*x]*(8 - (5*I)*f*x + 10*Log[1 - E^(I*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(12*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2 \right) \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c^4 \sec(fx + e)^4 - 4c^4 \sec(fx + e)^3 + 6c^4 \sec(fx + e)^2 - 4c^4 \sec(fx + e) + c^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fricas")

[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.37, size = 281, normalized size = 1.90

$$(-1 + \cos(fx + e)) \left(3 (\cos^3(fx + e)) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 6 \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) (\cos^3(fx + e)) - 9 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x)

[Out] 1/3/f*(-1+cos(f*x+e))*(3*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-6*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3-9*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+5*cos(f*x+e)^3+18*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+9*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-18*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-3*ln(2/(1+cos(f*x+e)))-3*cos(f*x+e)+6*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)/cos(f*x+e)^3*a^2

maxima [B] time = 4.40, size = 3738, normalized size = 25.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxima")

[Out] -1/3*(3*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2 - 72*a^2*sin(2*f*x + 2*e) - 6*(a^2*cos(6*f*x + 6*e)^2 + 225*a^2*cos(4*f*x + 4*e)^2 + 225*a^2*cos(2*f*x + 2*e)^2 + 36*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(6*f*x + 6*e)^2 + 225*a^2*sin(4*f*x + 4*e)^2 + 450*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*a^2*sin(2*f*x + 2*e)^2 + 36*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*a^2*cos(2*f*x + 2*e) + a^2 + 2*(15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) + 30*(15*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 12*(a^2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 20*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(a^2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) - 6*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(a^2*cos(6*f*x + 6*e) + 15*a^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 12*(a^2*sin(6*f*x + 6*e) + 15*a^2*sin(4*f*x + 4*e) + 15*a^2*sin(2*f*x + 2*e) - 20*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + a^2

$$\begin{aligned}
& 2*f*x + 2*e))) - 6*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) \\
&)*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 40*(a^2*\sin(6*f*x \\
& + 6*e) + 15*a^2*\sin(4*f*x + 4*e) + 15*a^2*\sin(2*f*x + 2*e) - 6*a^2*\sin(1/2* \\
& \arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2 \\
& *e), \cos(2*f*x + 2*e))) - 12*(a^2*\sin(6*f*x + 6*e) + 15*a^2*\sin(4*f*x + 4*e \\
&) + 15*a^2*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + \\
& 2*e))))*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1 \\
& /2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 6*(15*(f*x + e)*a^2* \\
& \cos(4*f*x + 4*e) + 15*(f*x + e)*a^2*\cos(2*f*x + 2*e) + (f*x + e)*a^2 - 12*a \\
& ^2*\sin(4*f*x + 4*e) - 12*a^2*\sin(2*f*x + 2*e))*\cos(6*f*x + 6*e) + 90*(15*(f \\
& *x + e)*a^2*\cos(2*f*x + 2*e) + (f*x + e)*a^2)*\cos(4*f*x + 4*e) - 6*(6*(f*x \\
& + e)*a^2*\cos(6*f*x + 6*e) + 90*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 90*(f*x + e \\
&)*a^2*\cos(2*f*x + 2*e) - 120*(f*x + e)*a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e) \\
& , \cos(2*f*x + 2*e))) - 36*(f*x + e)*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), c \\
& os(2*f*x + 2*e))) + 6*(f*x + e)*a^2 + 5*a^2*\sin(6*f*x + 6*e) + 3*a^2*\sin(4* \\
& f*x + 4*e) + 3*a^2*\sin(2*f*x + 2*e) + 16*a^2*\sin(3/2*\arctan2(\sin(2*f*x + 2* \\
& e), \cos(2*f*x + 2*e))))*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)) \\
&) - 4*(30*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 450*(f*x + e)*a^2*\cos(4*f*x + 4* \\
& e) + 450*(f*x + e)*a^2*\cos(2*f*x + 2*e) - 180*(f*x + e)*a^2*\cos(1/2*\arctan2 \\
& (\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 30*(f*x + e)*a^2 + 29*a^2*\sin(6*f*x \\
& + 6*e) + 75*a^2*\sin(4*f*x + 4*e) + 75*a^2*\sin(2*f*x + 2*e) - 24*a^2*\sin(1/ \\
& 2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\cos(3/2*\arctan2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e))) - 6*(6*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 90*(f*x + \\
& e)*a^2*\cos(4*f*x + 4*e) + 90*(f*x + e)*a^2*\cos(2*f*x + 2*e) + 6*(f*x + e)* \\
& a^2 + 5*a^2*\sin(6*f*x + 6*e) + 3*a^2*\sin(4*f*x + 4*e) + 3*a^2*\sin(2*f*x + 2 \\
& *e))*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 18*(5*(f*x + e) \\
& *a^2*\sin(4*f*x + 4*e) + 5*(f*x + e)*a^2*\sin(2*f*x + 2*e) + 4*a^2*\cos(4*f*x \\
& + 4*e) + 4*a^2*\cos(2*f*x + 2*e))*\sin(6*f*x + 6*e) + 18*(75*(f*x + e)*a^2*si \\
& n(2*f*x + 2*e) - 4*a^2)*\sin(4*f*x + 4*e) - 6*(6*(f*x + e)*a^2*\sin(6*f*x + 6 \\
& *e) + 90*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 90*(f*x + e)*a^2*\sin(2*f*x + 2*e) \\
& - 120*(f*x + e)*a^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - \\
& 36*(f*x + e)*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 5* \\
& a^2*\cos(6*f*x + 6*e) - 3*a^2*\cos(4*f*x + 4*e) - 3*a^2*\cos(2*f*x + 2*e) - 16 \\
& *a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 5*a^2)*\sin(5/2* \\
& \arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(30*(f*x + e)*a^2*\sin(6*f* \\
& x + 6*e) + 450*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 450*(f*x + e)*a^2*\sin(2*f*x \\
& + 2*e) - 180*(f*x + e)*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2 \\
& *e))) - 29*a^2*\cos(6*f*x + 6*e) - 75*a^2*\cos(4*f*x + 4*e) - 75*a^2*\cos(2*f* \\
& x + 2*e) + 24*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 29 \\
& *a^2)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 6*(6*(f*x + e) \\
& *a^2*\sin(6*f*x + 6*e) + 90*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 90*(f*x + e)*a^ \\
& 2*\sin(2*f*x + 2*e) - 5*a^2*\cos(6*f*x + 6*e) - 3*a^2*\cos(4*f*x + 4*e) - 3*a^ \\
& 2*\cos(2*f*x + 2*e) - 5*a^2)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2 \\
& *e))))*\sqrt{a}*\sqrt{c}/((c^4*\cos(6*f*x + 6*e))^2 + 225*c^4*\cos(4*f*x + 4*e))^ \\
& 2 + 225*c^4*\cos(2*f*x + 2*e))^2 + 36*c^4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), c \\
& os(2*f*x + 2*e)))^2 + 400*c^4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + \\
& 2*e)))^2 + 36*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \\
& c^4*\sin(6*f*x + 6*e))^2 + 225*c^4*\sin(4*f*x + 4*e))^2 + 450*c^4*\sin(4*f*x + \\
& 4*e)*\sin(2*f*x + 2*e) + 225*c^4*\sin(2*f*x + 2*e))^2 + 36*c^4*\sin(5/2*\arctan2 \\
& (\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 400*c^4*\sin(3/2*\arctan2(\sin(2*f*x \\
& + 2*e), \cos(2*f*x + 2*e)))^2 + 36*c^4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), co \\
& s(2*f*x + 2*e)))^2 + 30*c^4*\cos(2*f*x + 2*e) + c^4 + 2*(15*c^4*\cos(4*f*x + \\
& 4*e) + 15*c^4*\cos(2*f*x + 2*e) + c^4)*\cos(6*f*x + 6*e) + 30*(15*c^4*\cos(2*f \\
& *x + 2*e) + c^4)*\cos(4*f*x + 4*e) - 12*(c^4*\cos(6*f*x + 6*e) + 15*c^4*\cos(4 \\
& *f*x + 4*e) + 15*c^4*\cos(2*f*x + 2*e) - 20*c^4*\cos(3/2*\arctan2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e))) - 6*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f* \\
& x + 2*e))) + c^4)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 40 \\
& *(c^4*\cos(6*f*x + 6*e) + 15*c^4*\cos(4*f*x + 4*e) + 15*c^4*\cos(2*f*x + 2*e) \\
& - 6*c^4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + c^4)*\cos(3/2
\end{aligned}$$


```

*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) +
15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) - 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x +
4*e) + 15*c^4*sin(2*f*x + 2*e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*
f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x +
4*e) + 15*c^4*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

$$3.108 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{9/2}} dx$$

Optimal. Leaf size=194

$$\frac{a^3 \tan(e+fx) \log(1-\cos(e+fx))}{c^4 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^3 \tan(e+fx)}{c^3 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^3 \tan(e+fx)}{2c^2 f \sqrt{a \sec(e+fx)+a}}$$

[Out] $-a^3 \tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(9/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/2*a^3*\tan(f*x+e)/c^2/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a^3*\tan(f*x+e)/c^3/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^3*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^4/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.38, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3910, 3907, 3911, 31}

$$-\frac{a^3 \tan(e+fx)}{c^3 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^3 \tan(e+fx)}{2c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{5/2}} + \frac{a^3 \tan(e+fx)}{c^4 f \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2), x]

[Out] $-((a^3*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(9/2)})) - (a^3*\tan[e + f*x])/(2*c^2*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(5/2)}) - (a^3*\tan[e + f*x])/(c^3*f*\sqrt{a + a*\sec[e + f*x]}*(c - c*\sec[e + f*x])^{(3/2)}) + (a^3*\log[1 - \cos[e + f*x]]*\tan[e + f*x])/(c^4*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3910

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{9/2}} dx &= -\frac{a^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} + \frac{a^2 \int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{5/2}} dx}{c^2} \\
&= -\frac{a^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} - \frac{a^3 \tan(e + fx)}{2c^2 f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} \\
&= -\frac{a^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} - \frac{a^3 \tan(e + fx)}{2c^2 f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} \\
&= -\frac{a^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} - \frac{a^3 \tan(e + fx)}{2c^2 f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} \\
&= -\frac{a^3 \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}} - \frac{a^3 \tan(e + fx)}{2c^2 f \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{9/2}}
\end{aligned}$$

Mathematica [C] time = 5.50, size = 285, normalized size = 1.47

$$\frac{\sin^9\left(\frac{1}{2}(e + fx)\right) \sec^{\frac{9}{2}}(e + fx) (a(\sec(e + fx) + 1))^{5/2} \left(\frac{(89 \cos(e + fx) - 60 \cos(2(e + fx)) + 23 \cos(3(e + fx)) - 6 \cos(4(e + fx)) - 54) \csc^8\left(\frac{e + fx}{2}\right)}{8f} \right)}{(\sec(e + fx) + 1)^{5/2} (c - c \sec(e + fx))^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2), x]

[Out] (Sec[e + f*x]^(9/2)*(a*(1 + Sec[e + f*x]))^(5/2)*((16*Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*((-I)*f*x + 2*Log[1 - E^(I*(e + f*x))]))/(1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))])*f) + ((-54 + 89*Cos[e + f*x] - 60*Cos[2*(e + f*x)] + 23*Cos[3*(e + f*x)] - 6*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]^8*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/(8*f)*Sin[(e + f*x)/2]^9)/((1 + Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(9/2))

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(a^2 \sec^2(fx + e) + 2a^2 \sec(fx + e) + a^2) \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{c^5 \sec^5(fx + e) - 5c^5 \sec^4(fx + e) + 10c^5 \sec^3(fx + e) - 10c^5 \sec^2(fx + e) + 5c^5 \sec(fx + e) - c^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="fricas")

[Out] integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^5*sec(f*x + e)^5 - 5*c^5*sec(f*x + e)^4 + 10*c^5*sec(f*x + e)^3 - 10*c^5*sec(f*x + e)^2 + 5*c^5*sec(f*x + e) - c^5), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 2.35, size = 353, normalized size = 1.82

$$\frac{(-1 + \cos(fx + e)) \left(32 (\cos^4(fx + e)) \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) - 16 (\cos^4(fx + e)) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 29 (\cos^4(fx + e)) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/16/f*(-1+cos(f*x+e))*(32*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-16*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))-29*cos(f*x+e)^4-128*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+64*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))+20*cos(f*x+e)^3+192*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-96*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+10*cos(f*x+e)^2-128*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+64*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-28*cos(f*x+e)+32*ln(-(-1+cos(f*x+e))/sin(f*x+e))-16*ln(2/(1+cos(f*x+e)))+11)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(9/2)*a^2
```

maxima [B] time = 24.80, size = 6134, normalized size = 31.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 4900*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 64*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 4900*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 46*a^2*sin(2*f*x + 2*e) - 2*(a^2*cos(8*f*x + 8*e)^2 + 784*a^2*cos(6*f*x + 6*e)^2 + 4900*a^2*cos(4*f*x + 4*e)^2 + 784*a^2*cos(2*f*x + 2*e)^2 + 64*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(8*f*x + 8*e)^2 + 784*a^2*sin(6*f*x + 6*e)^2 + 4900*a^2*sin(4*f*x + 4*e)^2 + 3920*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*a^2*sin(2*f*x + 2*e)^2 + 64*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*a^2*cos(2*f*x + 2
```

$$\begin{aligned}
& *e) + a^2 + 2*(28*a^2*\cos(6*f*x + 6*e) + 70*a^2*\cos(4*f*x + 4*e) + 28*a^2*\cos(2*f*x + 2*e) + a^2)*\cos(8*f*x + 8*e) + 56*(70*a^2*\cos(4*f*x + 4*e) + 28*a^2*\cos(2*f*x + 2*e) + a^2)*\cos(6*f*x + 6*e) + 140*(28*a^2*\cos(2*f*x + 2*e) + a^2)*\cos(4*f*x + 4*e) - 16*(a^2*\cos(8*f*x + 8*e) + 28*a^2*\cos(6*f*x + 6*e) + 70*a^2*\cos(4*f*x + 4*e) + 28*a^2*\cos(2*f*x + 2*e) - 56*a^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 56*a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a^2)*\cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 112*(a^2*\cos(8*f*x + 8*e) + 28*a^2*\cos(6*f*x + 6*e) + 70*a^2*\cos(4*f*x + 4*e) + 28*a^2*\cos(2*f*x + 2*e) - 56*a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 8*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a^2)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 112*(a^2*\cos(8*f*x + 8*e) + 28*a^2*\cos(6*f*x + 6*e) + 70*a^2*\cos(4*f*x + 4*e) + 28*a^2*\cos(2*f*x + 2*e) - 8*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a^2)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*(a^2*\cos(8*f*x + 8*e) + 28*a^2*\cos(6*f*x + 6*e) + 70*a^2*\cos(4*f*x + 4*e) + 28*a^2*\cos(2*f*x + 2*e) + a^2)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 28*(2*a^2*\sin(6*f*x + 6*e) + 5*a^2*\sin(4*f*x + 4*e) + 2*a^2*\sin(2*f*x + 2*e))*\sin(8*f*x + 8*e) + 784*(5*a^2*\sin(4*f*x + 4*e) + 2*a^2*\sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) - 16*(a^2*\sin(8*f*x + 8*e) + 28*a^2*\sin(6*f*x + 6*e) + 70*a^2*\sin(4*f*x + 4*e) + 28*a^2*\sin(2*f*x + 2*e) - 56*a^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 56*a^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) * \sin(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 112*(a^2*\sin(8*f*x + 8*e) + 28*a^2*\sin(6*f*x + 6*e) + 70*a^2*\sin(4*f*x + 4*e) + 28*a^2*\sin(2*f*x + 2*e) - 56*a^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 8*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 112*(a^2*\sin(8*f*x + 8*e) + 28*a^2*\sin(6*f*x + 6*e) + 70*a^2*\sin(4*f*x + 4*e) + 28*a^2*\sin(2*f*x + 2*e) - 8*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*(a^2*\sin(8*f*x + 8*e) + 28*a^2*\sin(6*f*x + 6*e) + 70*a^2*\sin(4*f*x + 4*e) + 28*a^2*\sin(2*f*x + 2*e)) * \arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 2*(28*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 70*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 28*(f*x + e)*a^2*\cos(2*f*x + 2*e) + (f*x + e)*a^2 - 23*a^2*\sin(6*f*x + 6*e) - 66*a^2*\sin(4*f*x + 4*e) - 23*a^2*\sin(2*f*x + 2*e))*\cos(8*f*x + 8*e) + 28*(140*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 56*(f*x + e)*a^2*\cos(2*f*x + 2*e) + 2*(f*x + e)*a^2 - 17*a^2*\sin(4*f*x + 4*e))*\cos(6*f*x + 6*e) + 28*(140*(f*x + e)*a^2*\cos(2*f*x + 2*e) + 5*(f*x + e)*a^2 + 17*a^2*\sin(2*f*x + 2*e))*\cos(4*f*x + 4*e) - 4*(4*(f*x + e)*a^2*\cos(8*f*x + 8*e) + 112*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 280*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 112*(f*x + e)*a^2*\cos(2*f*x + 2*e) - 224*(f*x + e)*a^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 224*(f*x + e)*a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 32*(f*x + e)*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(f*x + e)*a^2 + 3*a^2*\sin(8*f*x + 8*e) - 8*a^2*\sin(6*f*x + 6*e) - 54*a^2*\sin(4*f*x + 4*e) - 8*a^2*\sin(2*f*x + 2*e) + 48*a^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 48*a^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) * \cos(7/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*\cos(8*f*x + 8*e) + 784*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 1960*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 784*(f*x + e)*a^2*\cos(2*f*x + 2*e) - 1568*(f*x + e)*a^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 224*(f*x + e)*a^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 28*(f*x + e)*a^2 + 27*a^2*\sin(8*f*x + 8*e) + 112*a^2*\sin(6*f*x + 6*e) + 42*a^2*\sin(4*f*x + 4*e) + 112*a^2*\sin(2*f*x + 2*e) - 48*a^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*\cos(8*f*x + 8*e) + 784*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 1960*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 784*(f*x + e)*a^2*\cos(2*f*x + 2*e) - 224*(f*x + e)*a^2*\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& * \arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 28*(f*x + e)*a^2 + 27*a^2*s \\
& \sin(8*f*x + 8*e) + 112*a^2*\sin(6*f*x + 6*e) + 42*a^2*\sin(4*f*x + 4*e) + 112* \\
& a^2*\sin(2*f*x + 2*e) - 48*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + \\
& 2*e))))*\cos(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(4*(f*x + \\
& e)*a^2*\cos(8*f*x + 8*e) + 112*(f*x + e)*a^2*\cos(6*f*x + 6*e) + 280*(f*x + \\
& e)*a^2*\cos(4*f*x + 4*e) + 112*(f*x + e)*a^2*\cos(2*f*x + 2*e) + 4*(f*x + e)* \\
& a^2 + 3*a^2*\sin(8*f*x + 8*e) - 8*a^2*\sin(6*f*x + 6*e) - 54*a^2*\sin(4*f*x + \\
& 4*e) - 8*a^2*\sin(2*f*x + 2*e))*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x \\
& + 2*e))) + 2*(28*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 70*(f*x + e)*a^2*\sin(4*f* \\
& x + 4*e) + 28*(f*x + e)*a^2*\sin(2*f*x + 2*e) + 23*a^2*\cos(6*f*x + 6*e) + 66 \\
& *a^2*\cos(4*f*x + 4*e) + 23*a^2*\cos(2*f*x + 2*e))*\sin(8*f*x + 8*e) + 2*(1960 \\
& *(f*x + e)*a^2*\sin(4*f*x + 4*e) + 784*(f*x + e)*a^2*\sin(2*f*x + 2*e) + 238* \\
& a^2*\cos(4*f*x + 4*e) - 23*a^2*\sin(6*f*x + 6*e) + 4*(980*(f*x + e)*a^2*\sin(\\
& 2*f*x + 2*e) - 119*a^2*\cos(2*f*x + 2*e) - 33*a^2)*\sin(4*f*x + 4*e) - 4*(4*(\\
& f*x + e)*a^2*\sin(8*f*x + 8*e) + 112*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 280*(f \\
& *x + e)*a^2*\sin(4*f*x + 4*e) + 112*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 224*(f* \\
& x + e)*a^2*\sin(5/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 224*(f*x \\
& + e)*a^2*\sin(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 32*(f*x + e \\
&)*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 3*a^2*\cos(8*f* \\
& x + 8*e) + 8*a^2*\cos(6*f*x + 6*e) + 54*a^2*\cos(4*f*x + 4*e) + 8*a^2*\cos(2*f \\
& *x + 2*e) - 48*a^2*\cos(5/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4 \\
& 8*a^2*\cos(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 3*a^2*\sin(7/2 \\
& *\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*\sin(8*f \\
& *x + 8*e) + 784*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 1960*(f*x + e)*a^2*\sin(4*f \\
& *x + 4*e) + 784*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 1568*(f*x + e)*a^2*\sin(3/2 \\
& *\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 224*(f*x + e)*a^2*\sin(1/2*a \\
& rctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 27*a^2*\cos(8*f*x + 8*e) - 112 \\
& *a^2*\cos(6*f*x + 6*e) - 42*a^2*\cos(4*f*x + 4*e) - 112*a^2*\cos(2*f*x + 2*e) \\
& + 48*a^2*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 27*a^2*\sin \\
& (5/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(28*(f*x + e)*a^2*\sin \\
& (8*f*x + 8*e) + 784*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 1960*(f*x + e)*a^2*\sin \\
& (4*f*x + 4*e) + 784*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 224*(f*x + e)*a^2*\sin(\\
& 1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 27*a^2*\cos(8*f*x + 8*e) \\
& - 112*a^2*\cos(6*f*x + 6*e) - 42*a^2*\cos(4*f*x + 4*e) - 112*a^2*\cos(2*f*x + \\
& 2*e) + 48*a^2*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 27*a^2 \\
&)*\sin(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(4*(f*x + e)*a^2 \\
& *\sin(8*f*x + 8*e) + 112*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 280*(f*x + e)*a^2* \\
& \sin(4*f*x + 4*e) + 112*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 3*a^2*\cos(8*f*x + 8 \\
& *e) + 8*a^2*\cos(6*f*x + 6*e) + 54*a^2*\cos(4*f*x + 4*e) + 8*a^2*\cos(2*f*x + \\
& 2*e) - 3*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sqrt{a} \\
& *\sqrt{c}/((c^5*\cos(8*f*x + 8*e)^2 + 784*c^5*\cos(6*f*x + 6*e)^2 + 4900*c^5*c \\
& \cos(4*f*x + 4*e)^2 + 784*c^5*\cos(2*f*x + 2*e)^2 + 64*c^5*\cos(7/2*\arctan 2(\sin \\
& (2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 3136*c^5*\cos(5/2*\arctan 2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e)))^2 + 3136*c^5*\cos(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos \\
& (2*f*x + 2*e)))^2 + 64*c^5*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2* \\
& e)))^2 + c^5*\sin(8*f*x + 8*e)^2 + 784*c^5*\sin(6*f*x + 6*e)^2 + 4900*c^5*\sin \\
& (4*f*x + 4*e)^2 + 3920*c^5*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 784*c^5*\sin(\\
& 2*f*x + 2*e)^2 + 64*c^5*\sin(7/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)) \\
&)^2 + 3136*c^5*\sin(5/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 313 \\
& 6*c^5*\sin(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 64*c^5*\sin(1 \\
& /2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 56*c^5*\cos(2*f*x + 2*e) \\
& + c^5 + 2*(28*c^5*\cos(6*f*x + 6*e) + 70*c^5*\cos(4*f*x + 4*e) + 28*c^5*\cos(\\
& 2*f*x + 2*e) + c^5)*\cos(8*f*x + 8*e) + 56*(70*c^5*\cos(4*f*x + 4*e) + 28*c^5 \\
& *\cos(2*f*x + 2*e) + c^5)*\cos(6*f*x + 6*e) + 140*(28*c^5*\cos(2*f*x + 2*e) + \\
& c^5)*\cos(4*f*x + 4*e) - 16*(c^5*\cos(8*f*x + 8*e) + 28*c^5*\cos(6*f*x + 6*e) \\
& + 70*c^5*\cos(4*f*x + 4*e) + 28*c^5*\cos(2*f*x + 2*e) - 56*c^5*\cos(5/2*\arctan \\
& 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 56*c^5*\cos(3/2*\arctan 2(\sin(2*f*x + \\
& 2*e), \cos(2*f*x + 2*e))) - 8*c^5*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f \\
& *x + 2*e))) + c^5)*\cos(7/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 1
\end{aligned}$$

```

12*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e)
) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos
(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*
cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*co
s(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*
*cos(2*f*x + 2*e) + c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x
+ 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x +
2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e)
+ 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c
^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 2
8*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*
e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
2*e) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) +
28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e)
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2), x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2), x)

[Out] Timed out

$$3.109 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c-c \sec(e+fx))^{11/2}} dx$$

Optimal. Leaf size=244

$$\frac{a^3 \tan(e+fx) \log(1-\cos(e+fx))}{c^5 f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{a^3 \tan(e+fx)}{c^4 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^3 \tan(e+fx)}{2c^3 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{5/2}}$$

[Out] $-4/5*a^3*\tan(f*x+e)/f/(c-c*\sec(f*x+e))^{(11/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/3*a^3*\tan(f*x+e)/c^2/f/(c-c*\sec(f*x+e))^{(7/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/2*a^3*\tan(f*x+e)/c^3/f/(c-c*\sec(f*x+e))^{(5/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a^3*\tan(f*x+e)/c^4/f/(c-c*\sec(f*x+e))^{(3/2)}/(a+a*\sec(f*x+e))^{(1/2)}+a^3*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c^5/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.47, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3910, 3907, 3911, 31}

$$\frac{a^3 \tan(e+fx)}{c^4 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{3/2}} - \frac{a^3 \tan(e+fx)}{2c^3 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{5/2}} - \frac{a^3 \tan(e+fx)}{3c^2 f \sqrt{a \sec(e+fx)+a} (c-c \sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2), x]

[Out] $(-4*a^3*\tan[e+f*x])/(5*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(11/2)}) - (a^3*\tan[e+f*x])/(3*c^2*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(7/2)}) - (a^3*\tan[e+f*x])/(2*c^3*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(5/2)}) - (a^3*\tan[e+f*x])/(c^4*f*\sqrt{a+a*\sec[e+f*x]}*(c-c*\sec[e+f*x])^{(3/2)}) + (a^3*\log[1-\cos[e+f*x]]*\tan[e+f*x])/(c^5*f*\sqrt{a+a*\sec[e+f*x]}*\sqrt{c-c*\sec[e+f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3910

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-8*a^3*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a^2/c^2, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ

[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{11/2}} dx &= -\frac{4a^3 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} + \frac{a^2 \int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{7/2}} dx}{c^2} \\
&= -\frac{4a^3 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} - \frac{a^3 \tan(e + fx)}{3c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} \\
&= -\frac{4a^3 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} - \frac{a^3 \tan(e + fx)}{3c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} \\
&= -\frac{4a^3 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} - \frac{a^3 \tan(e + fx)}{3c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} \\
&= -\frac{4a^3 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} - \frac{a^3 \tan(e + fx)}{3c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} \\
&= -\frac{4a^3 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}} - \frac{a^3 \tan(e + fx)}{3c^2 f \sqrt{a + a \sec(e + fx)}(c - c \sec(e + fx))^{11/2}}
\end{aligned}$$

Mathematica [C] time = 5.99, size = 299, normalized size = 1.23

$$\sin^{11}\left(\frac{1}{2}(e + fx)\right) \sec^{\frac{11}{2}}(e + fx) (a(\sec(e + fx) + 1))^{5/2} \left(-\frac{(5612 \cos(e + fx) - 5(736 \cos(2(e + fx)) - 367 \cos(3(e + fx)) + 111 \cos(4(e + fx))))}{(\sec(e + fx) + 1)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2), x]

```
[Out] (Sec[e + f*x]^(11/2)*(a*(1 + Sec[e + f*x]))^(5/2)*(((32*I)*Sqrt[2]*E^((I/2)
*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x)))]*(f*x +
(2*I)*Log[1 - E^(I*(e + f*x))]))/(1 + E^(I*(e + f*x))*Sqrt[E^(I*(e + f*x))
]/(1 + E^((2*I)*(e + f*x)))]*f) - ((5612*Cos[e + f*x] - 5*(625 + 736*Cos[2*
(e + f*x)] - 367*Cos[3*(e + f*x)] + 111*Cos[4*(e + f*x)] - 21*Cos[5*(e + f*
x)]))*Csc[(e + f*x)/2]^10*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[
e + f*x]]/(240*f))*Sin[(e + f*x)/2]^11)/((1 + Sec[e + f*x])^(5/2)*(c - c*S
ec[e + f*x])^(11/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(a^2 \sec(fx + e))^2 + 2a^2 \sec(fx + e) + a^2}{c^6 \sec(fx + e)^6 - 6c^6 \sec(fx + e)^5 + 15c^6 \sec(fx + e)^4 - 20c^6 \sec(fx + e)^3 + 15c^6 \sec(fx + e)^2} \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2), x, algorithm="fricas")

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^6*sec(f*x + e)^6 - 6*c^6*sec(f*x + e)^5 + 15*c^6*sec(f*x + e)^4 - 20*c^6*sec(f*x + e)^3 + 15*c^6*sec(f*x + e)^2 - 6*c^6*sec(f*x + e) + c^6), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 2.19, size = 415, normalized size = 1.70

$$(-1 + \cos(fx + e)) \left(240 (\cos^5(fx + e)) \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) - 120 (\cos^5(fx + e)) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 1200 (\cos^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x)
```

```
[Out] -1/120/f*(-1+cos(f*x+e))*(240*cos(f*x+e)^5*ln(-(-1+cos(f*x+e))/sin(f*x+e))-120*cos(f*x+e)^5*ln(2/(1+cos(f*x+e)))-1200*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-233*cos(f*x+e)^5+600*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))+2400*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+325*cos(f*x+e)^4-1200*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-2400*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-10*cos(f*x+e)^3+1200*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+1200*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-290*cos(f*x+e)^2-600*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-240*ln(-(-1+cos(f*x+e))/sin(f*x+e))+295*cos(f*x+e)+120*ln(2/(1+cos(f*x+e)))-83)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(11/2)/sin(f*x+e)/cos(f*x+e)^5*a^2
```

maxima [B] time = 146.31, size = 9150, normalized size = 37.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a^2*cos(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 661500*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 661500*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 1500*(f*x + e)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952560*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1500*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 15*(f*x + e)*a^2*sin(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 661500*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 661500*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 1500*(f*x + e)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952560*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*(f*x +
```

$$\begin{aligned}
& e) * a^2 * \sin(3/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 1500 * (f * x \\
& + e) * a^2 * \sin(1/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 1350 * (f * x \\
& + e) * a^2 * \cos(2 * f * x + 2 * e) + 15 * (f * x + e) * a^2 - 1110 * a^2 * \sin(2 * f * x + 2 * e) - \\
& 30 * (a^2 * \cos(10 * f * x + 10 * e)^2 + 2025 * a^2 * \cos(8 * f * x + 8 * e)^2 + 44100 * a^2 * \cos \\
& (6 * f * x + 6 * e)^2 + 44100 * a^2 * \cos(4 * f * x + 4 * e)^2 + 2025 * a^2 * \cos(2 * f * x + 2 * e)^2 \\
& + 100 * a^2 * \cos(9/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 14400 * \\
& a^2 * \cos(7/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 63504 * a^2 * \cos(\\
& 5/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 14400 * a^2 * \cos(3/2 * \arct \\
& an2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 100 * a^2 * \cos(1/2 * \arctan2(\sin(2 * \\
& f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + a^2 * \sin(10 * f * x + 10 * e)^2 + 2025 * a^2 * \sin(\\
& 8 * f * x + 8 * e)^2 + 44100 * a^2 * \sin(6 * f * x + 6 * e)^2 + 44100 * a^2 * \sin(4 * f * x + 4 * e)^2 \\
& + 18900 * a^2 * \sin(4 * f * x + 4 * e) * \sin(2 * f * x + 2 * e) + 2025 * a^2 * \sin(2 * f * x + 2 * e) \\
& ^2 + 100 * a^2 * \sin(9/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 14400 \\
& * a^2 * \sin(7/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 63504 * a^2 * \sin \\
& (5/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 14400 * a^2 * \sin(3/2 * \arct \\
& an2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 100 * a^2 * \sin(1/2 * \arctan2(\sin(2 \\
& * f * x + 2 * e), \cos(2 * f * x + 2 * e)))^2 + 90 * a^2 * \cos(2 * f * x + 2 * e) + a^2 + 2 * (45 * a \\
& ^2 * \cos(8 * f * x + 8 * e) + 210 * a^2 * \cos(6 * f * x + 6 * e) + 210 * a^2 * \cos(4 * f * x + 4 * e) + \\
& 45 * a^2 * \cos(2 * f * x + 2 * e) + a^2) * \cos(10 * f * x + 10 * e) + 90 * (210 * a^2 * \cos(6 * f * x \\
& + 6 * e) + 210 * a^2 * \cos(4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x + 2 * e) + a^2) * \cos(8 * f * \\
& x + 8 * e) + 420 * (210 * a^2 * \cos(4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x + 2 * e) + a^2) * \c \\
& os(6 * f * x + 6 * e) + 420 * (45 * a^2 * \cos(2 * f * x + 2 * e) + a^2) * \cos(4 * f * x + 4 * e) - 20 \\
& * (a^2 * \cos(10 * f * x + 10 * e) + 45 * a^2 * \cos(8 * f * x + 8 * e) + 210 * a^2 * \cos(6 * f * x + 6 * \\
& e) + 210 * a^2 * \cos(4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x + 2 * e) - 120 * a^2 * \cos(7/2 * a \\
& rctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 252 * a^2 * \cos(5/2 * \arctan2(\sin(2 \\
& * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 120 * a^2 * \cos(3/2 * \arctan2(\sin(2 * f * x + 2 * e), \\
& \cos(2 * f * x + 2 * e))) - 10 * a^2 * \cos(1/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + \\
& 2 * e))) + a^2) * \cos(9/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 240 * (a \\
& ^2 * \cos(10 * f * x + 10 * e) + 45 * a^2 * \cos(8 * f * x + 8 * e) + 210 * a^2 * \cos(6 * f * x + 6 * e) \\
& + 210 * a^2 * \cos(4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x + 2 * e) - 252 * a^2 * \cos(5/2 * \arct \\
& an2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 120 * a^2 * \cos(3/2 * \arctan2(\sin(2 * f * \\
& x + 2 * e), \cos(2 * f * x + 2 * e))) - 10 * a^2 * \cos(1/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos \\
& (2 * f * x + 2 * e))) + a^2) * \cos(7/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) \\
& - 504 * (a^2 * \cos(10 * f * x + 10 * e) + 45 * a^2 * \cos(8 * f * x + 8 * e) + 210 * a^2 * \cos(6 * f * \\
& x + 6 * e) + 210 * a^2 * \cos(4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x + 2 * e) - 120 * a^2 * \cos \\
& (3/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 10 * a^2 * \cos(1/2 * \arctan2(\\
& \sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) + a^2) * \cos(5/2 * \arctan2(\sin(2 * f * x + 2 * e) \\
&), \cos(2 * f * x + 2 * e))) - 240 * (a^2 * \cos(10 * f * x + 10 * e) + 45 * a^2 * \cos(8 * f * x + 8 * \\
& e) + 210 * a^2 * \cos(6 * f * x + 6 * e) + 210 * a^2 * \cos(4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x \\
& + 2 * e) - 10 * a^2 * \cos(1/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) + a^2 \\
&) * \cos(3/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 20 * (a^2 * \cos(10 * f * x \\
& + 10 * e) + 45 * a^2 * \cos(8 * f * x + 8 * e) + 210 * a^2 * \cos(6 * f * x + 6 * e) + 210 * a^2 * \cos \\
& (4 * f * x + 4 * e) + 45 * a^2 * \cos(2 * f * x + 2 * e) + a^2) * \cos(1/2 * \arctan2(\sin(2 * f * x + \\
& 2 * e), \cos(2 * f * x + 2 * e))) + 30 * (3 * a^2 * \sin(8 * f * x + 8 * e) + 14 * a^2 * \sin(6 * f * x + \\
& 6 * e) + 14 * a^2 * \sin(4 * f * x + 4 * e) + 3 * a^2 * \sin(2 * f * x + 2 * e)) * \sin(10 * f * x + 10 * e) \\
& + 1350 * (14 * a^2 * \sin(6 * f * x + 6 * e) + 14 * a^2 * \sin(4 * f * x + 4 * e) + 3 * a^2 * \sin(2 * f * \\
& x + 2 * e)) * \sin(8 * f * x + 8 * e) + 6300 * (14 * a^2 * \sin(4 * f * x + 4 * e) + 3 * a^2 * \sin(2 * f * \\
& x + 2 * e)) * \sin(6 * f * x + 6 * e) - 20 * (a^2 * \sin(10 * f * x + 10 * e) + 45 * a^2 * \sin(8 * f * x \\
& + 8 * e) + 210 * a^2 * \sin(6 * f * x + 6 * e) + 210 * a^2 * \sin(4 * f * x + 4 * e) + 45 * a^2 * \sin(2 \\
& * f * x + 2 * e) - 120 * a^2 * \sin(7/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) \\
& - 252 * a^2 * \sin(5/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 120 * a^2 * \si \\
& n(3/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 10 * a^2 * \sin(1/2 * \arctan2 \\
& (\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) * \sin(9/2 * \arctan2(\sin(2 * f * x + 2 * e), co \\
& s(2 * f * x + 2 * e))) - 240 * (a^2 * \sin(10 * f * x + 10 * e) + 45 * a^2 * \sin(8 * f * x + 8 * e) + \\
& 210 * a^2 * \sin(6 * f * x + 6 * e) + 210 * a^2 * \sin(4 * f * x + 4 * e) + 45 * a^2 * \sin(2 * f * x + 2 * \\
& e) - 252 * a^2 * \sin(5/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 120 * a^2 \\
& * \sin(3/2 * \arctan2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e))) - 10 * a^2 * \sin(1/2 * \arct \\
& an2(\sin(2 * f * x + 2 * e), \cos(2 * f * x + 2 * e)))) * \sin(7/2 * \arctan2(\sin(2 * f * x + 2 * e), \\
& \cos(2 * f * x + 2 * e))) - 504 * (a^2 * \sin(10 * f * x + 10 * e) + 45 * a^2 * \sin(8 * f * x + 8 * e)
\end{aligned}$$

$$\begin{aligned}
& + 210a^2\sin(6fx + 6e) + 210a^2\sin(4fx + 4e) + 45a^2\sin(2fx + 2e) - 120a^2\sin\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 10a^2\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)\sin\left(\frac{5}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 240(a^2\sin(10fx + 10e) + 45a^2\sin(8fx + 8e) + 210a^2\sin(6fx + 6e) + 210a^2\sin(4fx + 4e) + 45a^2\sin(2fx + 2e) - 10a^2\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right))\sin\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 20(a^2\sin(10fx + 10e) + 45a^2\sin(8fx + 8e) + 210a^2\sin(6fx + 6e) + 210a^2\sin(4fx + 4e) + 45a^2\sin(2fx + 2e))\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)\arctan\left(\frac{\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)}{\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)}\right) - 1) + 10(135(fx + e)a^2\cos(8fx + 8e) + 630(fx + e)a^2\cos(6fx + 6e) + 630(fx + e)a^2\cos(4fx + 4e) + 135(fx + e)a^2\cos(2fx + 2e) + 3(fx + e)a^2 - 111a^2\sin(8fx + 8e) - 625a^2\sin(6fx + 6e) - 625a^2\sin(4fx + 4e) - 111a^2\sin(2fx + 2e))\cos(10fx + 10e) + 450(630(fx + e)a^2\cos(6fx + 6e) + 630(fx + e)a^2\cos(4fx + 4e) + 135(fx + e)a^2\cos(2fx + 2e) + 3(fx + e)a^2 - 107a^2\sin(6fx + 6e) - 107a^2\sin(4fx + 4e))\cos(8fx + 8e) + 450(2940(fx + e)a^2\cos(4fx + 4e) + 630(fx + e)a^2\cos(2fx + 2e) + 14(fx + e)a^2 + 107a^2\sin(2fx + 2e))\cos(6fx + 6e) + 450(630(fx + e)a^2\cos(2fx + 2e) + 14(fx + e)a^2 + 107a^2\sin(2fx + 2e))\cos(4fx + 4e) - 10(30(fx + e)a^2\cos(10fx + 10e) + 1350(fx + e)a^2\cos(8fx + 8e) + 6300(fx + e)a^2\cos(6fx + 6e) + 6300(fx + e)a^2\cos(4fx + 4e) + 1350(fx + e)a^2\cos(2fx + 2e) - 3600(fx + e)a^2\cos\left(\frac{7}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 7560(fx + e)a^2\cos\left(\frac{5}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 3600(fx + e)a^2\cos\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 300(fx + e)a^2\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) + 30(fx + e)a^2 + 21a^2\sin(10fx + 10e) - 165a^2\sin(8fx + 8e) - 1840a^2\sin(6fx + 6e) - 1840a^2\sin(4fx + 4e) - 165a^2\sin(2fx + 2e) + 940a^2\sin\left(\frac{7}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) + 2472a^2\sin\left(\frac{5}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) + 940a^2\sin\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right))\cos\left(\frac{9}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 20(180(fx + e)a^2\cos(10fx + 10e) + 8100(fx + e)a^2\cos(8fx + 8e) + 37800(fx + e)a^2\cos(6fx + 6e) + 37800(fx + e)a^2\cos(4fx + 4e) + 8100(fx + e)a^2\cos(2fx + 2e) - 45360(fx + e)a^2\cos\left(\frac{5}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 21600(fx + e)a^2\cos\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 1800(fx + e)a^2\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) + 180(fx + e)a^2 + 173a^2\sin(10fx + 10e) + 1125a^2\sin(8fx + 8e) - 1170a^2\sin(6fx + 6e) - 1170a^2\sin(4fx + 4e) + 1125a^2\sin(2fx + 2e) + 2988a^2\sin\left(\frac{5}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 470a^2\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right))\cos\left(\frac{7}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 12(630(fx + e)a^2\cos(10fx + 10e) + 28350(fx + e)a^2\cos(8fx + 8e) + 132300(fx + e)a^2\cos(6fx + 6e) + 132300(fx + e)a^2\cos(4fx + 4e) + 28350(fx + e)a^2\cos(2fx + 2e) - 75600(fx + e)a^2\cos\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 6300(fx + e)a^2\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) + 630(fx + e)a^2 + 647a^2\sin(10fx + 10e) + 5805a^2\sin(8fx + 8e) + 4620a^2\sin(6fx + 6e) + 4620a^2\sin(4fx + 4e) + 5805a^2\sin(2fx + 2e) - 4980a^2\sin\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 2060a^2\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right))\cos\left(\frac{5}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) - 20(180(fx + e)a^2\cos(10fx + 10e) + 8100(fx + e)a^2\cos(8fx + 8e) + 37800(fx + e)a^2\cos(6fx + 6e) + 37800(fx + e)a^2\cos(4fx + 4e) + 8100(fx + e)a^2\cos(2fx + 2e) - 1800(fx + e)a^2\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right) + 180(fx + e)a^2 + 173a^2\sin(10fx + 10e) + 1125a^2\sin(8fx + 8e) - 1170a^2\sin(6fx + 6e) - 1170a^2\sin(4fx + 4e) + 1125a^2\sin(2fx + 2e) - 470a^2\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right))\cos\left(\frac{3}{2}\arctan\left(\frac{\sin(2fx + 2e)}{\cos(2fx + 2e)}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& * \arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 10*(30*(f*x + e)*a^2*\cos(10 \\
& *f*x + 10*e) + 1350*(f*x + e)*a^2*\cos(8*f*x + 8*e) + 6300*(f*x + e)*a^2*\cos \\
& (6*f*x + 6*e) + 6300*(f*x + e)*a^2*\cos(4*f*x + 4*e) + 1350*(f*x + e)*a^2*\cos \\
& s(2*f*x + 2*e) + 30*(f*x + e)*a^2 + 21*a^2*\sin(10*f*x + 10*e) - 165*a^2*\sin \\
& (8*f*x + 8*e) - 1840*a^2*\sin(6*f*x + 6*e) - 1840*a^2*\sin(4*f*x + 4*e) - 165 \\
& *a^2*\sin(2*f*x + 2*e))*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) \\
& + 10*(135*(f*x + e)*a^2*\sin(8*f*x + 8*e) + 630*(f*x + e)*a^2*\sin(6*f*x + 6 \\
& *e) + 630*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 135*(f*x + e)*a^2*\sin(2*f*x + 2 \\
& *e) + 111*a^2*\cos(8*f*x + 8*e) + 625*a^2*\cos(6*f*x + 6*e) + 625*a^2*\cos(4*f* \\
& x + 4*e) + 111*a^2*\cos(2*f*x + 2*e))*\sin(10*f*x + 10*e) + 30*(9450*(f*x + e \\
&)*a^2*\sin(6*f*x + 6*e) + 9450*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 2025*(f*x + \\
& e)*a^2*\sin(2*f*x + 2*e) + 1605*a^2*\cos(6*f*x + 6*e) + 1605*a^2*\cos(4*f*x + \\
& 4*e) - 37*a^2)*\sin(8*f*x + 8*e) + 50*(26460*(f*x + e)*a^2*\sin(4*f*x + 4*e) \\
& + 5670*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 963*a^2*\cos(2*f*x + 2*e) - 125*a^2) \\
& *\sin(6*f*x + 6*e) + 50*(5670*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 963*a^2*\cos(2 \\
& *f*x + 2*e) - 125*a^2)*\sin(4*f*x + 4*e) - 10*(30*(f*x + e)*a^2*\sin(10*f*x + \\
& 10*e) + 1350*(f*x + e)*a^2*\sin(8*f*x + 8*e) + 6300*(f*x + e)*a^2*\sin(6*f*x \\
& + 6*e) + 6300*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 1350*(f*x + e)*a^2*\sin(2*f* \\
& x + 2*e) - 3600*(f*x + e)*a^2*\sin(7/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + \\
& 2*e))) - 7560*(f*x + e)*a^2*\sin(5/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + \\
& 2*e))) - 3600*(f*x + e)*a^2*\sin(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2 \\
& *e))) - 300*(f*x + e)*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e \\
&))) - 21*a^2*\cos(10*f*x + 10*e) + 165*a^2*\cos(8*f*x + 8*e) + 1840*a^2*\cos(6 \\
& *f*x + 6*e) + 1840*a^2*\cos(4*f*x + 4*e) + 165*a^2*\cos(2*f*x + 2*e) - 940*a^ \\
& 2*\cos(7/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2472*a^2*\cos(5/2*a \\
& rctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 940*a^2*\cos(3/2*\arctan 2(\sin(2 \\
& *f*x + 2*e), \cos(2*f*x + 2*e))) - 21*a^2)*\sin(9/2*\arctan 2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) - 20*(180*(f*x + e)*a^2*\sin(10*f*x + 10*e) + 8100*(f*x \\
& + e)*a^2*\sin(8*f*x + 8*e) + 37800*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 37800*(f \\
& *x + e)*a^2*\sin(4*f*x + 4*e) + 8100*(f*x + e)*a^2*\sin(2*f*x + 2*e) - 45360* \\
& (f*x + e)*a^2*\sin(5/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 21600* \\
& (f*x + e)*a^2*\sin(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 1800*(\\
& f*x + e)*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 173*a^2 \\
& *\cos(10*f*x + 10*e) - 1125*a^2*\cos(8*f*x + 8*e) + 1170*a^2*\cos(6*f*x + 6*e) \\
& + 1170*a^2*\cos(4*f*x + 4*e) - 1125*a^2*\cos(2*f*x + 2*e) - 2988*a^2*\cos(5/2 \\
& *\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 470*a^2*\cos(1/2*\arctan 2(\sin \\
& (2*f*x + 2*e), \cos(2*f*x + 2*e))) - 173*a^2)*\sin(7/2*\arctan 2(\sin(2*f*x + 2* \\
& e), \cos(2*f*x + 2*e))) - 12*(630*(f*x + e)*a^2*\sin(10*f*x + 10*e) + 28350*(\\
& f*x + e)*a^2*\sin(8*f*x + 8*e) + 132300*(f*x + e)*a^2*\sin(6*f*x + 6*e) + 132 \\
& 300*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 28350*(f*x + e)*a^2*\sin(2*f*x + 2*e) - \\
& 75600*(f*x + e)*a^2*\sin(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - \\
& 6300*(f*x + e)*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - \\
& 647*a^2*\cos(10*f*x + 10*e) - 5805*a^2*\cos(8*f*x + 8*e) - 4620*a^2*\cos(6*f*x \\
& + 6*e) - 4620*a^2*\cos(4*f*x + 4*e) - 5805*a^2*\cos(2*f*x + 2*e) + 4980*a^2* \\
& \cos(3/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2060*a^2*\cos(1/2*\arctan \\
& 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 647*a^2)*\sin(5/2*\arctan 2(\sin(2* \\
& f*x + 2*e), \cos(2*f*x + 2*e))) - 20*(180*(f*x + e)*a^2*\sin(10*f*x + 10*e) + \\
& 8100*(f*x + e)*a^2*\sin(8*f*x + 8*e) + 37800*(f*x + e)*a^2*\sin(6*f*x + 6*e) \\
& + 37800*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 8100*(f*x + e)*a^2*\sin(2*f*x + 2 \\
& *e) - 1800*(f*x + e)*a^2*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)) \\
&) - 173*a^2*\cos(10*f*x + 10*e) - 1125*a^2*\cos(8*f*x + 8*e) + 1170*a^2*\cos(6 \\
& *f*x + 6*e) + 1170*a^2*\cos(4*f*x + 4*e) - 1125*a^2*\cos(2*f*x + 2*e) + 470*a \\
& ^2*\cos(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 173*a^2)*\sin(3/2* \\
& arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 10*(30*(f*x + e)*a^2*\sin(10* \\
& f*x + 10*e) + 1350*(f*x + e)*a^2*\sin(8*f*x + 8*e) + 6300*(f*x + e)*a^2*\sin(\\
& 6*f*x + 6*e) + 6300*(f*x + e)*a^2*\sin(4*f*x + 4*e) + 1350*(f*x + e)*a^2*\sin \\
& (2*f*x + 2*e) - 21*a^2*\cos(10*f*x + 10*e) + 165*a^2*\cos(8*f*x + 8*e) + 1840 \\
& *a^2*\cos(6*f*x + 6*e) + 1840*a^2*\cos(4*f*x + 4*e) + 165*a^2*\cos(2*f*x + 2*e \\
&) - 21*a^2)*\sin(1/2*\arctan 2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sqrt{a}*s
\end{aligned}$$

$$\begin{aligned} & \text{qrt}(c)/((c^6 \cos(10fx + 10e)^2 + 2025c^6 \cos(8fx + 8e)^2 + 44100c^6 \\ & * \cos(6fx + 6e)^2 + 44100c^6 \cos(4fx + 4e)^2 + 2025c^6 \cos(2fx + 2 \\ & * e)^2 + 100c^6 \cos(9/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 14 \\ & 400c^6 \cos(7/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 63504c^6 * \\ & \cos(5/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 14400c^6 \cos(3/2 * \\ & \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 100c^6 \cos(1/2 \arctan2(\sin \\ & (2fx + 2e), \cos(2fx + 2e)))^2 + c^6 \sin(10fx + 10e)^2 + 2025c^6 * \\ & \sin(8fx + 8e)^2 + 44100c^6 \sin(6fx + 6e)^2 + 44100c^6 \sin(4fx + 4 \\ & * e)^2 + 18900c^6 \sin(4fx + 4e) \sin(2fx + 2e) + 2025c^6 \sin(2fx + \\ & 2e)^2 + 100c^6 \sin(9/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 1 \\ & 4400c^6 \sin(7/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 63504c^6 \\ & * \sin(5/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 14400c^6 \sin(3/2 \\ & * \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 100c^6 \sin(1/2 \arctan2(\sin \\ & (2fx + 2e), \cos(2fx + 2e)))^2 + 90c^6 \cos(2fx + 2e) + c^6 + 2 * (\\ & 45c^6 \cos(8fx + 8e) + 210c^6 \cos(6fx + 6e) + 210c^6 \cos(4fx + 4 \\ & * e) + 45c^6 \cos(2fx + 2e) + c^6) \cos(10fx + 10e) + 90 * (210c^6 \cos(6 * \\ & fx + 6e) + 210c^6 \cos(4fx + 4e) + 45c^6 \cos(2fx + 2e) + c^6) \cos(\\ & 8fx + 8e) + 420 * (210c^6 \cos(4fx + 4e) + 45c^6 \cos(2fx + 2e) + c^ \\ & 6) \cos(6fx + 6e) + 420 * (45c^6 \cos(2fx + 2e) + c^6) \cos(4fx + 4e) \\ & - 20 * (c^6 \cos(10fx + 10e) + 45c^6 \cos(8fx + 8e) + 210c^6 \cos(6fx \\ & + 6e) + 210c^6 \cos(4fx + 4e) + 45c^6 \cos(2fx + 2e) - 120c^6 \cos(7 \\ & /2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 252c^6 \cos(5/2 \arctan2(\sin \\ & (2fx + 2e), \cos(2fx + 2e))) - 120c^6 \cos(3/2 \arctan2(\sin(2fx + 2 \\ & * e), \cos(2fx + 2e))) - 10c^6 \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx * \\ & x + 2e))) + c^6) \cos(9/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 24 \\ & 0 * (c^6 \cos(10fx + 10e) + 45c^6 \cos(8fx + 8e) + 210c^6 \cos(6fx + 6 \\ & * e) + 210c^6 \cos(4fx + 4e) + 45c^6 \cos(2fx + 2e) - 252c^6 \cos(5/2 * \\ & \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 120c^6 \cos(3/2 \arctan2(\sin(\\ & 2fx + 2e), \cos(2fx + 2e))) - 10c^6 \cos(1/2 \arctan2(\sin(2fx + 2e), \\ & \cos(2fx + 2e))) + c^6) \cos(7/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2 * \\ & e))) - 504 * (c^6 \cos(10fx + 10e) + 45c^6 \cos(8fx + 8e) + 210c^6 \cos(\\ & 6fx + 6e) + 210c^6 \cos(4fx + 4e) + 45c^6 \cos(2fx + 2e) - 120c^6 \\ & * \cos(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 10c^6 \cos(1/2 \arct \\ & an2(\sin(2fx + 2e), \cos(2fx + 2e))) + c^6) \cos(5/2 \arctan2(\sin(2fx + \\ & 2e), \cos(2fx + 2e))) - 240 * (c^6 \cos(10fx + 10e) + 45c^6 \cos(8fx \\ & + 8e) + 210c^6 \cos(6fx + 6e) + 210c^6 \cos(4fx + 4e) + 45c^6 \cos(2 \\ & * fx + 2e) - 10c^6 \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + \\ & c^6) \cos(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 20 * (c^6 \cos(10 \\ & * fx + 10e) + 45c^6 \cos(8fx + 8e) + 210c^6 \cos(6fx + 6e) + 210c^6 \\ & * \cos(4fx + 4e) + 45c^6 \cos(2fx + 2e) + c^6) \cos(1/2 \arctan2(\sin(2fx * \\ & x + 2e), \cos(2fx + 2e))) + 30 * (3c^6 \sin(8fx + 8e) + 14c^6 \sin(6fx \\ & + 6e) + 14c^6 \sin(4fx + 4e) + 3c^6 \sin(2fx + 2e)) * \sin(10fx + 1 \\ & 0e) + 1350 * (14c^6 \sin(6fx + 6e) + 14c^6 \sin(4fx + 4e) + 3c^6 \sin(\\ & 2fx + 2e)) * \sin(8fx + 8e) + 6300 * (14c^6 \sin(4fx + 4e) + 3c^6 \sin(\\ & 2fx + 2e)) * \sin(6fx + 6e) - 20 * (c^6 \sin(10fx + 10e) + 45c^6 \sin(8 * \\ & fx + 8e) + 210c^6 \sin(6fx + 6e) + 210c^6 \sin(4fx + 4e) + 45c^6 \sin \\ & (2fx + 2e) - 120c^6 \sin(7/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e \\ & * e))) - 252c^6 \sin(5/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 120c^ \\ & 6 \sin(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 10c^6 \sin(1/2 \arct \\ & an2(\sin(2fx + 2e), \cos(2fx + 2e))) * \sin(9/2 \arctan2(\sin(2fx + 2e) \\ & , \cos(2fx + 2e))) - 240 * (c^6 \sin(10fx + 10e) + 45c^6 \sin(8fx + 8e \\ & * e) + 210c^6 \sin(6fx + 6e) + 210c^6 \sin(4fx + 4e) + 45c^6 \sin(2fx \\ & + 2e) - 252c^6 \sin(5/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 120 \\ & * c^6 \sin(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - 10c^6 \sin(1/2 * \\ & \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) * \sin(7/2 \arctan2(\sin(2fx + 2 \\ & * e), \cos(2fx + 2e))) - 504 * (c^6 \sin(10fx + 10e) + 45c^6 \sin(8fx + \\ & 8e) + 210c^6 \sin(6fx + 6e) + 210c^6 \sin(4fx + 4e) + 45c^6 \sin(2fx \\ & * x + 2e) - 120c^6 \sin(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) - \\ & 10c^6 \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) * \sin(5/2 \arctan \end{aligned}$$

```
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c
^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) +
45*c^6*sin(2*f*x + 2*e) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6
*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) +
210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+fx)}\right)^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2), x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2), x)
```

```
[Out] Timed out
```

$$3.110 \quad \int \frac{(c - c \sec(e + fx))^{7/2}}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=204

$$\frac{c^4 \tan(e + fx) \sec^2(e + fx)}{2f\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \tan(e + fx) \sec(e + fx)}{f\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{8c^4 \tan(e + fx) \log(\sec(e + fx))}{f\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $c^4 \ln(\cos(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 8*c^4 \ln(1+\sec(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} - 4*c^4 * \sec(f*x+e) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 1/2*c^4 * \sec(f*x+e)^2 * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{c^4 \tan(e + fx) \sec^2(e + fx)}{2f\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \tan(e + fx) \sec(e + fx)}{f\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{8c^4 \tan(e + fx) \log(\sec(e + fx))}{f\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]

[Out] $(c^4 * \text{Log}[\text{Cos}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (8 * c^4 * \text{Log}[1 + \text{Sec}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) - (4 * c^4 * \text{Sec}[e + f*x] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (c^4 * \text{Sec}[e + f*x]^2 * \text{Tan}[e + f*x]) / (2 * f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]])$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{7/2}}{\sqrt{a + a \sec(e + fx)}} dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \left(\frac{4c^3}{a} + \frac{c^3}{ax} - \frac{c^3x}{a} - \frac{8c^3}{a(1+x)}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{c^4 \log(\cos(e + fx)) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{8c^4 \log(1 + \sec(e + fx)) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 16.35, size = 153, normalized size = 0.75

$$\frac{c^3 \cot\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{c - c \sec(e + fx)} \left(-16 \log(1 + e^{i(e+fx)}) + 7 \log(1 + e^{2i(e+fx)}) + 8 \cos(e + fx)\right)}{2f \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (c^3*Cot[(e + f*x)/2]*(-1 + I*f*x + 8*Cos[e + f*x] - 16*Log[1 + E^(I*(e + f*x))] + 7*Log[1 + E^((2*I)*(e + f*x))] + Cos[2*(e + f*x)]*(I*f*x - 16*Log[1 + E^(I*(e + f*x))] + 7*Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]]/(2*f*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(c^3 \sec(fx + e)^3 - 3c^3 \sec(fx + e)^2 + 3c^3 \sec(fx + e) - c^3 \right) \sqrt{-c \sec(fx + e) + c}}{\sqrt{a \sec(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
 on of abs or sign assumes constant sign by intervals (correct if the argume
 nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
 2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
 x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
 /x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
 k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
 nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
 >(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
 >(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
 2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integr
 ation of abs or sign assumes constant sign by intervals (correct if the arg
 ument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of
 t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
 /2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec

2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
 x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs
 or sign assumes constant sign by intervals (correct if the argument is rea
 l):Check [abs(t_nostep^2-1),abs(t_nostep^3+t_nostep)]Warning, integratio
 n of abs or sign assumes constant sign by intervals (correct if the argument i
 s real):Check [abs(t_nostep)]Sign error (%%{2*c,0%%}+%%{1,2%%})Evaluati
 on time: 4.5Limit: Max order reached or unable to make series expansion Err
 or: Bad Argument Value

maple [A] time = 2.22, size = 190, normalized size = 0.93

$$\frac{\left(2 \ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^2(fx+e)\right) + 14 \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right) + 14 \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\right)}{2f \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x)

[Out] 1/2/f*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+14*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+14*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+9*cos(f*x+e)^2+8*cos(f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2/sin(f*x+e)/(-1+cos(f*x+e))^3/a

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{7/2}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(1/2),x)

[Out] Timed out

$$3.111 \quad \int \frac{(c - c \sec(e + fx))^{5/2}}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=151

$$-\frac{c^3 \tan(e + fx) \sec(e + fx)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{4c^3 \tan(e + fx) \log(\sec(e + fx) + 1)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{c^3 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $c^3 \ln(\cos(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 4*c^3 \ln(1+\sec(f*x+e)) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} - c^3 \sec(f*x+e) * \tan(f*x+e) / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$-\frac{c^3 \tan(e + fx) \sec(e + fx)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{4c^3 \tan(e + fx) \log(\sec(e + fx) + 1)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{c^3 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]

[Out] $(c^3 * \text{Log}[\text{Cos}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (4 * c^3 * \text{Log}[1 + \text{Sec}[e + f*x]] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) - (c^3 * \text{Sec}[e + f*x] * \text{Tan}[e + f*x]) / (f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]])$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{5/2}}{\sqrt{a + a \sec(e + fx)}} dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^2}{x(a+ax)} dx, x, \sec(e + fx)\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \left(\frac{c^2}{a} + \frac{c^2}{ax} - \frac{4c^2}{a(1+x)}\right) dx, x, \sec(e + fx)\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{c^3 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{4c^3 \log(1 + \sec(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 4.02, size = 181, normalized size = 1.20

$$\frac{c^2 e^{-3i(e+fx)} (1 + e^{2i(e+fx)})^3 \cos\left(\frac{1}{2}(e + fx)\right) \cot\left(\frac{1}{2}(e + fx)\right) \sec^4(e + fx) \sqrt{c - c \sec(e + fx)} (1 + (-8 \log(1 + e^{i(e+fx)}))}{4f (1 + e^{i(e+fx)}) \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (c^2*(1 + E^((2*I)*(e + f*x)))^3*cos[(e + f*x)/2]*cot[(e + f*x)/2]*(1 + Cos[e + f*x]*(I*f*x - 8*Log[1 + E^(I*(e + f*x))] + 3*Log[1 + E^((2*I)*(e + f*x))]))*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2]))/(4*E^((3*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(c^2 \sec(fx + e)^2 - 2c^2 \sec(fx + e) + c^2 \right) \sqrt{-c \sec(fx + e) + c}}{\sqrt{a \sec(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integr
ation of abs or sign assumes constant sign by intervals (correct if the argu
ment is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of
t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
```


maple [A] time = 2.24, size = 170, normalized size = 1.13

$$\frac{\left(3 \cos (f x+e) \ln \left(-\frac{-1+\cos (f x+e)+\sin (f x+e)}{\sin (f x+e)}\right)+3 \cos (f x+e) \ln \left(-\frac{-\sin (f x+e)-1+\cos (f x+e)}{\sin (f x+e)}\right)+\cos (f x+e) \ln \left(\frac{1+\cos (f x+e)}{1-\cos (f x+e)}\right)\right)}{f \sin (f x+e)(-1+\cos (f x+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)

[Out] 1/f*(3*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*cos(f*x+e)*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(2/(1+cos(f*x+e))))+cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2/sin(f*x+e)/(-1+cos(f*x+e))^2/a

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)

[Out] Timed out

$$3.112 \quad \int \frac{(c - c \sec(e + fx))^{3/2}}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=102

$$\frac{2c^2 \tan(e + fx) \log(\sec(e + fx) + 1)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{c^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $c^2 \ln(\cos(fx + e)) \tan(fx + e) / f / (a + a \sec(fx + e))^{1/2} / (c - c \sec(fx + e))^{1/2} + 2c^2 \ln(1 + \sec(fx + e)) \tan(fx + e) / f / (a + a \sec(fx + e))^{1/2} / (c - c \sec(fx + e))^{1/2}$

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{2c^2 \tan(e + fx) \log(\sec(e + fx) + 1)}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} + \frac{c^2 \tan(e + fx) \log(\cos(e + fx))}{f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]

[Out] $(c^2 \text{Log}[\text{Cos}[e + f*x]] \text{Tan}[e + f*x]) / (f \text{Sqrt}[a + a \text{Sec}[e + f*x]] \text{Sqrt}[c - c \text{Sec}[e + f*x]]) + (2c^2 \text{Log}[1 + \text{Sec}[e + f*x]] \text{Tan}[e + f*x]) / (f \text{Sqrt}[a + a \text{Sec}[e + f*x]] \text{Sqrt}[c - c \text{Sec}[e + f*x]])$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{3/2}}{\sqrt{a + a \sec(e + fx)}} dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{c - cx}{x(a + ax)} dx, x, \sec(e + fx)\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \left(\frac{c}{ax} - \frac{2c}{a(1+x)}\right) dx, x, \sec(e + fx)\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{c^2 \log(\cos(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{2c^2 \log(1 + \sec(e + fx)) \tan(e + fx)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 9.72, size = 103, normalized size = 1.01

$$\frac{c(1 + e^{i(e+fx)}) \left(4i \log(1 + e^{i(e+fx)}) - i \log(1 + e^{2i(e+fx)}) + fx\right) \sqrt{c - c \sec(e + fx)}}{f(-1 + e^{i(e+fx)}) \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*(1 + E^(I*(e + f*x)))*(f*x + (4*I)*Log[1 + E^(I*(e + f*x))] - I*Log[1 + E^((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]]/((-1 + E^(I*(e + f*x)))*f *Sqrt[a*(1 + Sec[e + f*x]))))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(-c \sec(fx + e) + c)^{\frac{3}{2}}}{\sqrt{a \sec(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-c*sec(f*x + e) + c)^(3/2)/sqrt(a*sec(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integrati
on of abs or sign assumes constant sign by intervals (correct if the argume
nt is real):Check [abs(cos(f*x+exp(1)))]Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integr
ation of abs or sign assumes constant sign by intervals (correct if the arg
ument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of
t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
```


maxima [A] time = 0.84, size = 60, normalized size = 0.59

$$\frac{\left(\left(fx + e\right)c + c \arctan\left(\sin\left(2fx + 2e\right), \cos\left(2fx + 2e\right) + 1\right) - 4c \arctan\left(\sin\left(fx + e\right), \cos\left(fx + e\right) + 1\right)\right)\sqrt{a}}{\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -((f*x + e)*c + c*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*c*arctan2(sin(f*x + e), cos(f*x + e) + 1))*sqrt(c)/(sqrt(a)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sec(e+fx) - 1\right)\right)^{3/2}}{\sqrt{a\left(\sec(e+fx) + 1\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.113 \quad \int \frac{\sqrt{c-c \sec(e+fx)}}{\sqrt{a+a \sec(e+fx)}} dx$$

Optimal. Leaf size=49

$$\frac{c \tan(e+fx) \log(\cos(e+fx)+1)}{f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

[Out] c*ln(1+cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3911, 31}

$$\frac{c \tan(e+fx) \log(\cos(e+fx)+1)}{f \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{c-c \sec(e+fx)}}{\sqrt{a+a \sec(e+fx)}} dx &= \frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{1}{a+ax} dx, x, \cos(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= \frac{c \log(1+\cos(e+fx)) \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 0.98, size = 127, normalized size = 2.59

$$\frac{(1 + e^{i(e+fx)}) \sqrt{\frac{c(-1+e^{i(e+fx)})^2}{1+e^{2i(e+fx)}}} (fx + 2i \log(1 + e^{i(e+fx)}))}{f(-1 + e^{i(e+fx)}) \sqrt{\frac{a(1+e^{i(e+fx)})^2}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]

[Out] -(((1 + E^(I*(e + f*x)))*Sqrt[(c*(-1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*x)))]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]))/((-1 + E^(I*(e + f*x)))*Sqrt[(a*(1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*x)))]*f))

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{-c \sec(fx + e) + c}}{\sqrt{a \sec(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)c*sqrt(-a*c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*sign(cos(f*x+exp(1)))*ln(abs(c*tan(1/2*(f*x+exp(1)))^2+c))/a/f/abs(c)

maple [A] time = 2.41, size = 75, normalized size = 1.53

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}} \cos(fx+e) \ln\left(\frac{2}{1+\cos(fx+e)}\right)}{f \sin(fx+e) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)

[Out] 1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*ln(2/(1+cos(f*x+e)))/sin(f*x+e)/a

maxima [A] time = 0.73, size = 34, normalized size = 0.69

$$\frac{\sqrt{c} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{\sqrt{-a} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c - \frac{c}{\cos(e+fx)}}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)

[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sec(e+fx)-1)}}{\sqrt{a(\sec(e+fx)+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2), x)

[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.114 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} dx$$

Optimal. Leaf size=46

$$\frac{\tan(e+fx) \log(\sin(e+fx))}{f \sqrt{a \sec(e+fx) + a} \sqrt{c - c \sec(e+fx)}}$$

[Out] $\ln(\sin(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3905, 3475}

$$\frac{\tan(e+fx) \log(\sin(e+fx))}{f \sqrt{a \sec(e+fx) + a} \sqrt{c - c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]

[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(m_.), x_Symbol] :> Dist[(((a*c)^(m + 1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} dx &= \frac{\tan(e+fx) \int \cot(e+fx) dx}{\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= \frac{\log(\sin(e+fx)) \tan(e+fx)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.09, size = 104, normalized size = 2.26

$$\frac{2(-1 + e^{i(e+fx)}) (fx + i \log(1 - e^{2i(e+fx)})) \cos^2\left(\frac{1}{2}(e+fx)\right) \sec(e+fx)}{f(1 + e^{i(e+fx)}) \sqrt{a(\sec(e+fx) + 1)} \sqrt{c - c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]

[Out] (-2*(-1 + E^(I*(e + f*x)))*Cos[(e + f*x)/2]^2*(f*x + I*Log[1 - E^((2*I)*(e + f*x))])*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])

fricas [B] time = 0.73, size = 272, normalized size = 5.91

$$\sqrt{-ac} \log \left(\frac{8 \left((256 \cos(fx+e))^5 - 512 \cos(fx+e)^3 + 175 \cos(fx+e) \right) \sqrt{-ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} - (256 ac \cos(fx+e)^4 - 512 ac \cos(fx+e)^2 + 337 a^2 c) \sin(fx+e)}{(\cos(fx+e)^2 - 1) \sin(fx+e)} \right) - \frac{\sqrt{ac} \arctan\left(\frac{(16 \cos(fx+e)^3 - 7 \cos(fx+e)) \sqrt{ac} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}}}{(16 a^2 \cos(fx+e)^2 - 25 a^2 c) \sin(fx+e)}\right)}{2 ac f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*c)*log(-8*((256*cos(f*x + e))^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), -sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e))/(a*c*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
 g, integration of abs or sign assumes constant sign by intervals (correct i
 f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
 tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
 ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
 i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
 *pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
 able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
 n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
 /2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
 ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
 eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
 /t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:

ostep were not checkedDiscontinuities at zeroes of $\cos(f*t_nostep+\exp(1))$ were not checkedUnable to check sign: $(2*\pi/t_nostep/2)>(-2*\pi/t_nostep/2)$ Unable to check sign: $(2*\pi/t_nostep/2)>(-2*\pi/t_nostep/2)$ Unable to check sign: $(2*\pi/t_nostep/2)>(-2*\pi/t_nostep/2)$ Unable to check sign: $(2*\pi/x/2)>(-2*\pi/x/2)$ Unable to check sign: $(2*\pi/x/2)>(-2*\pi/x/2)$ Unable to check sign: $(2*\pi/x/2)>(-2*\pi/x/2)$ Unable to check sign: $(2*\pi/x/2)>(-2*\pi/x/2)$ Unable to check sign: $(2*\pi/x/2)>(-2*\pi/x/2)$ Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error $\{\%,2\%\}$ Sign error ($\{\%,0\%\}+\{\%,2\%\}$)Evaluation time: 2.4Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 2.38, size = 101, normalized size = 2.20

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) - \ln\left(\frac{2}{1+\cos(fx+e)}\right) \right) \sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}} \cos(fx+e)}{f \sin(fx+e) ca}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)

[Out] $-1/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*(\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))-1)$
 $\ln(2/(1+\cos(f*x+e)))*c*(-1+\cos(f*x+e))/\cos(f*x+e)^{1/2}*\cos(f*x+e)/\sin(f*x+e)/c/a$

maxima [A] time = 0.84, size = 39, normalized size = 0.85

$$\frac{fx+e - \arctan(\sin(2fx+2e), \cos(2fx+2e)-1)}{\sqrt{a}\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] $-(f*x + e - \arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) - 1))/(\sqrt{a}*\sqrt{c})*f$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \sqrt{c - \frac{c}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))), x)

$$3.115 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=168

$$\frac{\tan(e+fx)}{2cf(1-\cos(e+fx))\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{3 \tan(e+fx) \log(1-\cos(e+fx))}{4cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx)}{4cf\sqrt{a}}$$

[Out] $1/2*\tan(f*x+e)/c/f/(1-\cos(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}+3/4*\ln(1-\cos(f*x+e))*\tan(f*x+e)/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}+1/4*\ln(1+\cos(f*x+e))*\tan(f*x+e)/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 217, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$-\frac{\tan(e+fx)}{2cf(1-\sec(e+fx))\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{3 \tan(e+fx) \log(1-\sec(e+fx))}{4cf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx)}{4cf\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]

[Out] (Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Tan[e + f*x]/(2*c*f*(1 - Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{3/2}} dx &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^2} dx, x, \sec(e+fx)\right)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \left(\frac{1}{2ac^2(-1+x)^2} - \frac{3}{4ac^2(-1+x)} + \frac{1}{ac^2x} - \frac{1}{4ac^2}\right) dx, x, \sec(e+fx)\right)}{f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= \frac{\log(\cos(e+fx)) \tan(e+fx)}{cf\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} + \frac{3 \log(1-\sec(e+fx))}{4cf\sqrt{a+a \sec(e+fx)}} \end{aligned}$$

[abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{c,0%%}+%%{c,2%%})Sign error %%{-c,2%%}Evaluation time: 2.46Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.59, size = 167, normalized size = 0.99

$$\frac{(-1 + \cos(fx + e)) \left(6 \ln \left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)} \right) \cos(fx + e) - 4 \cos(fx + e) \ln \left(\frac{2}{1 + \cos(fx + e)} \right) - 6 \ln \left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)} \right) \right) - 4f \left(\frac{c(-1 + \cos(fx + e))}{\cos(fx + e)} \right)^{\frac{3}{2}} \cos(fx + e) \sin(fx + e) a}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)

[Out] -1/4/f*(-1+cos(f*x+e))*(6*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)-4*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-6*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)+4*ln(2/(1+cos(f*x+e)))-1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)/sin(f*x+e)/a

maxima [B] time = 0.95, size = 818, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - (cos(2*f*x + 2*e))^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) - 3*(cos(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))

+ 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(4*f*x + 4*(f*x + e)*cos(2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e) - 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*e)/((c*cos(2*f*x + 2*e)^2 + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(2*f*x + 2*e)^2 - 4*c*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*c*cos(2*f*x + 2*e) - 4*(c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*sqrt(a)*sqrt(c)*f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(e+fx)+1)} (-c(\sec(e+fx)-1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)), x)

$$3.116 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=274

$$\frac{3 \tan(e+fx)}{4c^2 f(1-\sec(e+fx))\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{4c^2 f(1-\sec(e+fx))^2 \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

[Out] $\ln(\cos(f*x+e))*\tan(f*x+e)/c^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}+7/8*\ln(1-\sec(f*x+e))*\tan(f*x+e)/c^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}+1/8*\ln(1+\sec(f*x+e))*\tan(f*x+e)/c^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-1/4*\tan(f*x+e)/c^2/f/(1-\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-3/4*\tan(f*x+e)/c^2/f/(1-\sec(f*x+e))/((a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)})$

Rubi [A] time = 0.16, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{3 \tan(e+fx)}{4c^2 f(1-\sec(e+fx))\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{4c^2 f(1-\sec(e+fx))^2 \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x]))^{(5/2)}, x]$

[Out] $(\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (7*\text{Log}[1 - \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(8*c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (\text{Log}[1 + \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(8*c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(4*c^2*f*(1 - \text{Sec}[e + f*x])^2*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (3*\text{Tan}[e + f*x])/(4*c^2*f*(1 - \text{Sec}[e + f*x])*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 72

$\text{Int}[(e + f*x)^p/((a + b*x)*(c + d*x)), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{IntegerQ}[p]$

Rule 3912

$\text{Int}[(\text{csc}[e + f*x])^m*(\text{csc}[e + f*x])^n, x_Symbol] \rightarrow \text{Dist}[(a*c*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{m-1/2}*(c + d*x)^{n-1/2}/x, x], x, \text{Csc}[e + f*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, x\} \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c-c \sec(e+fx))^{5/2}} dx &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{1}{x(a+ax)(c-cx)^3} dx, x, \sec(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \left(-\frac{1}{2ac^3(-1+x)^3} + \frac{3}{4ac^3(-1+x)^2} - \frac{7}{8ac^3(-1+x)}\right) dx, x, \sec(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= \frac{\log(\cos(e+fx)) \tan(e+fx)}{c^2 f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} + \frac{7 \log(1-\sec(e+fx))}{8c^2 f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.93, size = 194, normalized size = 0.71

$$\frac{\tan(e + fx) \left(21 \log(1 - e^{i(e+fx)}) + 3 \log(1 + e^{i(e+fx)}) \right) + (-28 \log(1 - e^{i(e+fx)}) - 4 \log(1 + e^{i(e+fx)}) + 16ifx - 1)}{8c^2 f (\cos(e + fx) - 1)^2 \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]

[Out] ((8 - (12*I)*f*x + 21*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-10 + (16*I)*f*x - 28*Log[1 - E^(I*(e + f*x))] - 4*Log[1 + E^(I*(e + f*x))]) + 3*Log[1 + E^(I*(e + f*x))] + Cos[2*(e + f*x)]*((-4*I)*f*x + 7*Log[1 - E^(I*(e + f*x))]) + Log[1 + E^(I*(e + f*x))])*Tan[e + f*x])/(8*c^2*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{ac^3 \sec(fx + e)^4 - 2ac^3 \sec(fx + e)^3 + 2ac^3 \sec(fx + e) - ac^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^3*sec(f*x + e)^4 - 2*a*c^3*sec(f*x + e)^3 + 2*a*c^3*sec(f*x + e) - a*c^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
 (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
 2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
 /2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
 t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
 pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
 le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
 gn by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep
 +exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
 (-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
 ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no

e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Evaluation time: 2.74index.cc index_m_i_lex_is_greater Error: Bad Argument Value

maple [A] time = 2.59, size = 229, normalized size = 0.84

$$\frac{(-1 + \cos(fx + e)) \left(16 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) (\cos^2(fx + e)) - 28 (\cos^2(fx + e)) \ln\left(\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) - 32 \cos(fx + e) \right)}{16f}$$

16f

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)

[Out] 1/16/f*(-1+cos(f*x+e))*(16*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-28*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*cos(f*x+e)*ln(2/(1+cos(f*x+e))))+56*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+9*cos(f*x+e)^2+16*ln(2/(1+cos(f*x+e))))-28*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*cos(f*x+e)-7)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)/cos(f*x+e)^2/a

maxima [B] time = 1.28, size = 2206, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 64*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3

$$\begin{aligned}
& /2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))^2 + 64*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))^2 + 4*f*x - (2*(6*\cos(2*f*x + 2*e) + 1)*\cos(4*f*x + 4*e) + \cos(4*f*x + 4*e)^2 + 36*\cos(2*f*x + 2*e)^2 - 8*(\cos(4*f*x + 4*e) + 6*\cos(2*f*x + 2*e) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\cos(4*f*x + 4*e) + 6*\cos(2*f*x + 2*e) + 1)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(4*f*x + 4*e)^2 + 12*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 36*\sin(2*f*x + 2*e)^2 - 8*(\sin(4*f*x + 4*e) + 6*\sin(2*f*x + 2*e) - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\sin(4*f*x + 4*e) + 6*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 12*\cos(2*f*x + 2*e) + 1)*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) - 7*(2*(6*\cos(2*f*x + 2*e) + 1)*\cos(4*f*x + 4*e) + \cos(4*f*x + 4*e)^2 + 36*\cos(2*f*x + 2*e)^2 - 8*(\cos(4*f*x + 4*e) + 6*\cos(2*f*x + 2*e) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\cos(4*f*x + 4*e) + 6*\cos(2*f*x + 2*e) + 1)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + \sin(4*f*x + 4*e)^2 + 12*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 36*\sin(2*f*x + 2*e)^2 - 8*(\sin(4*f*x + 4*e) + 6*\sin(2*f*x + 2*e) - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\sin(4*f*x + 4*e) + 6*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 12*\cos(2*f*x + 2*e) + 1)*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 8*(f*x + 6*(f*x + e)*\cos(2*f*x + 2*e) + e - 2*\sin(2*f*x + 2*e))*\cos(4*f*x + 4*e) + 48*(f*x + e)*\cos(2*f*x + 2*e) - 2*(16*f*x + 16*(f*x + e)*\cos(4*f*x + 4*e) + 96*(f*x + e)*\cos(2*f*x + 2*e) - 64*(f*x + e)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 16*e + 5*\sin(4*f*x + 4*e) - 2*\sin(2*f*x + 2*e))*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(16*f*x + 16*(f*x + e)*\cos(4*f*x + 4*e) + 96*(f*x + e)*\cos(2*f*x + 2*e) + 16*e + 5*\sin(4*f*x + 4*e) - 2*\sin(2*f*x + 2*e))*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*(3*(f*x + e)*\sin(2*f*x + 2*e) + \cos(2*f*x + 2*e))*\sin(4*f*x + 4*e) - 2*(16*(f*x + e)*\sin(4*f*x + 4*e) + 96*(f*x + e)*\sin(2*f*x + 2*e) - 64*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 5*\cos(4*f*x + 4*e) + 2*\cos(2*f*x + 2*e) - 5)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*\sin(4*f*x + 4*e) + 96*(f*x + e)*\sin(2*f*x + 2*e) - 5*\cos(4*f*x + 4*e) + 2*\cos(2*f*x + 2*e) - 5)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*e - 16*\sin(2*f*x + 2*e))/((c^2*\cos(4*f*x + 4*e)^2 + 36*c^2*\cos(2*f*x + 2*e)^2 + 16*c^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + c^2*\sin(4*f*x + 4*e)^2 + 12*c^2*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 36*c^2*\sin(2*f*x + 2*e)^2 + 16*c^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*c^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 12*c^2*\cos(2*f*x + 2*e) + c^2 + 2*(6*c^2*\cos(2*f*x + 2*e) + c^2)*\cos(4*f*x + 4*e) - 8*(c^2*\cos(4*f*x + 4*e) + 6*c^2*\cos(2*f*x + 2*e) - 4*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + c^2)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(c^2*\cos(4*f*x + 4*e) + 6*c^2*\cos(2*f*x + 2*e) + c^2)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(c^2*\sin(4*f*x + 4*e) + 6*c^2*\sin(2*f*x + 2*e) - 4*c^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(c^2*\sin(4*f*x + 4*e) + 6*c^2*\sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sqrt{a}*\sqrt{c}*f)
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)), x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(e+fx)+1)} (-c(\sec(e+fx)-1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2), x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(5/2)), x)

$$3.117 \quad \int \frac{(c - c \sec(e + fx))^{7/2}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=215

$$\frac{c^4 \tan(e + fx) \sec(e + fx)}{af\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{8c^4 \tan(e + fx)}{af(\sec(e + fx) + 1)\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \tan(e + fx)}{af\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $c^4 \ln(\cos(fx+e)) \tan(fx+e) / a / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} - 4c^4 \ln(1+\sec(fx+e)) \tan(fx+e) / a / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} + c^4 \sec(fx+e) \tan(fx+e) / a / f / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2} - 8c^4 \tan(fx+e) / a / f / (1+\sec(fx+e)) / (a+a \sec(fx+e))^{1/2} / (c-c \sec(fx+e))^{1/2}$

Rubi [A] time = 0.14, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 88}

$$\frac{c^4 \tan(e + fx) \sec(e + fx)}{af\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{8c^4 \tan(e + fx)}{af(\sec(e + fx) + 1)\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \tan(e + fx)}{af\sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $(c^4 \text{Log}[\text{Cos}[e + f*x]] \text{Tan}[e + f*x]) / (a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (4*c^4*\text{Log}[1 + \text{Sec}[e + f*x]]*\text{Tan}[e + f*x]) / (a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (c^4*\text{Sec}[e + f*x]*\text{Tan}[e + f*x]) / (a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (8*c^4*\text{Tan}[e + f*x]) / (a*f*(1 + \text{Sec}[e + f*x])*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 88

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{7/2}}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \left(-\frac{c^3}{a^2} + \frac{c^3}{a^2x} - \frac{8c^3}{a^2(1+x)^2} + \frac{4c^3}{a^2(1+x)}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{c^4 \log(\cos(e + fx)) \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \log(1 + \sec(e + fx)) \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 2.27, size = 204, normalized size = 0.95

$$\frac{c^3 \cot\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{c - c \sec(e + fx)} \left(8 \log\left(1 + e^{i(e+fx)}\right) - 5 \log\left(1 + e^{2i(e+fx)}\right) + 2\left(8 \log\left(1 + e^{i(e+fx)}\right)\right)\right)}{2af(\cos(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2), x]

[Out] (c^3*Cot[(e + f*x)/2]*(-2 + I*f*x + 8*Log[1 + E^(I*(e + f*x))]) + 2*Cos[e + f*x]*(-9 + I*f*x + 8*Log[1 + E^(I*(e + f*x))]) - 5*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e + f*x)]*(I*f*x + 8*Log[1 + E^(I*(e + f*x))]) - 5*Log[1 + E^((2*I)*(e + f*x))]) - 5*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]]/(2*a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\left(c^3 \sec(fx + e)^3 - 3c^3 \sec(fx + e)^2 + 3c^3 \sec(fx + e) - c^3\right) \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [A] time = 2.27, size = 278, normalized size = 1.29

$$\frac{\left(5 \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right) + 5 \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right) - \ln\left(\frac{2}{1+\cos(fx+e)}\right)\right) \cos(fx+e)^2}{\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2), x)

[Out] 1/f*(5*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+5*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+5*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+5*cos(f*x+e)*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-3*cos(f*x+e)^2-cos(f*x+e)*ln(2/(1+cos(f*x+e)))+6*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*cos(f*x+e)^3*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e))^2/a^2

maxima [B] time = 1.48, size = 2393, normalized size = 11.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out]
$$-\left((f*x + e)*c^3*\cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*\cos(2*f*x + 2*e)^2 + (f*x + e)*c^3*\sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*\sin(2*f*x + 2*e)^2 + 4*(f*x + e)*c^3*\cos(2*f*x + 2*e) + (f*x + e)*c^3 - 4*c^3*\sin(2*f*x + 2*e) + 4*((f*x + e)*c^3 - 5*c^3*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\cos\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*((f*x + e)*c^3 - 5*c^3*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*((f*x + e)*c^3 - 5*c^3*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\sin\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*((f*x + e)*c^3 - 5*c^3*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 - 5*(c^3*\cos(4*f*x + 4*e)^2 + 4*c^3*\cos(2*f*x + 2*e)^2 + c^3*\sin(4*f*x + 4*e)^2 + 4*c^3*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 4*c^3*\sin(2*f*x + 2*e)^2 + 4*c^3*\cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*\cos(2*f*x + 2*e) + c^3)*\cos(4*f*x + 4*e)\right)*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) + 8*(c^3*\cos(4*f*x + 4*e)^2 + 4*c^3*\cos(2*f*x + 2*e)^2 + 4*c^3*\cos\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*c^3*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + c^3*\sin(4*f*x + 4*e)^2 + 4*c^3*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 4*c^3*\sin(2*f*x + 2*e)^2 + 4*c^3*\sin\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*c^3*\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*c^3*\cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*\cos(2*f*x + 2*e) + c^3)*\cos(4*f*x + 4*e) + 4*(c^3*\cos(4*f*x + 4*e) + 2*c^3*\cos(2*f*x + 2*e) + 2*c^3*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + c^3)*\cos\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 4*(c^3*\cos(4*f*x + 4*e) + 2*c^3*\cos(2*f*x + 2*e) + c^3)*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 4*(c^3*\sin(4*f*x + 4*e) + 2*c^3*\sin(2*f*x + 2*e) + 2*c^3*\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right))*\sin\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 4*(c^3*\sin(4*f*x + 4*e) + 2*c^3*\sin(2*f*x + 2*e))*\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right))*\arctan2(\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right), \cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 1) + 2*(2*(f*x + e)*c^3*\cos(2*f*x + 2*e) + (f*x + e)*c^3 - 2*c^3*\sin(2*f*x + 2*e))*\cos(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*\cos(4*f*x + 4*e) + 4*(f*x + e)*c^3*\cos(2*f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*\sin(4*f*x + 4*e) + 14*c^3*\sin(2*f*x + 2*e) - 10*(c^3*\cos(4*f*x + 4*e) + 2*c^3*\cos(2*f*x + 2*e) + c^3)*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 - 5*c^3*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right))*\cos\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 2*(2*(f*x + e)*c^3*\cos(4*f*x + 4*e) + 4*(f*x + e)*c^3*\cos(2*f*x + 2*e) + 2*(f*x + e)*c^3 + 9*c^3*\sin(4*f*x + 4*e) + 14*c^3*\sin(2*f*x + 2*e) - 10*(c^3*\cos(4*f*x + 4*e) + 2*c^3*\cos(2*f*x + 2*e) + c^3)*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 4*((f*x + e)*c^3*\sin(2*f*x + 2*e) + c^3*\cos(2*f*x + 2*e))*\sin(4*f*x + 4*e) + 2*(2*(f*x + e)*c^3*\sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*\sin(2*f*x + 2*e) - 9*c^3*\cos(4*f*x + 4*e) - 14*c^3*\cos(2*f*x + 2*e) - 9*c^3 - 10*(c^3*\sin(4*f*x + 4*e) + 2*c^3*\sin(2*f*x + 2*e))*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1) + 4*((f*x + e)*c^3 - 5*c^3*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right))*\sin\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right) + 2*(2*(f*x + e)*c^3*\sin(4*f*x + 4*e) + 4*(f*x + e)*c^3*\sin(2*f*x + 2*e) - 9*c^3*\cos(4*f*x + 4*e) - 14*c^3*\cos(2*f*x + 2*e) - 9*c^3 - 10*(c^3*\sin(4*f*x + 4*e) + 2*c^3*\sin(2*f*x + 2*e))*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e) + 1))*\sin\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right))*\sqrt{c}/\left((a*\cos(4*f*x + 4*e))^2 + 4*a*\cos(2*f*x + 2*e)^2 + 4*a*\cos\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + 4*a*\cos\left(\frac{1}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 + a*\sin(4*f*x + 4*e)^2 + 4*a*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + 4*a*\sin(2*f*x + 2*e)^2 + 4*a*\sin\left(\frac{3}{2}*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))\right)^2 +$$

```

4*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(2*a*cos(2*
f*x + 2*e) + a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + 4*(a*cos(4*f*x +
4*e) + 2*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(
a*cos(4*f*x + 4*e) + 2*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e) + 2
*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x
+ 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sqrt(a)*f
)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{7/2}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

$$3.118 \quad \int \frac{(c - c \sec(e + fx))^{5/2}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=96

$$\frac{c^3 \tan(e + fx) \log(\cos(e + fx))}{af \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^3 \tan(e + fx)}{f(a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}}$$

[Out] $-4*c^3*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^(3/2)/(c-c*\sec(f*x+e))^(1/2)+c^3*\ln(\cos(f*x+e))*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^(1/2)/(c-c*\sec(f*x+e))^(1/2)$

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3910, 3905, 3475}

$$\frac{c^3 \tan(e + fx) \log(\cos(e + fx))}{af \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^3 \tan(e + fx)}{f(a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c*\text{Sec}[e + f*x])^(5/2)/(a + a*\text{Sec}[e + f*x])^(3/2), x]$

[Out] $(-4*c^3*\text{Tan}[e + f*x])/(f*(a + a*\text{Sec}[e + f*x])^(3/2)*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (c^3*\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 3905

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(m_.), x_Symbol] \rightarrow \text{Dist}[((-a*c)^(m + 1/2)*\text{Cot}[e + f*x])/(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Int}[\text{Cot}[e + f*x]^(2*m), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m + 1/2]$

Rule 3910

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(5/2)*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] \rightarrow \text{Simp}[(-8*a^3*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[a^2/c^2, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^(n + 2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -2^(-1)]$

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{5/2}}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{4c^3 \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{c^2 \int \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)} dx}{a^2} \\ &= -\frac{4c^3 \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} - \frac{(c^3 \tan(e + fx)) \int \tan(e + fx) dx}{a \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{4c^3 \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{c^3 \log(\cos(e + fx)) \tan(e + fx)}{af \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 0.74, size = 116, normalized size = 1.21

$$\frac{ic^2 \cot\left(\frac{1}{2}(e+fx)\right) \sqrt{c-c\sec(e+fx)} \left(i \log\left(1+e^{2i(e+fx)}\right) + (fx+i \log\left(1+e^{2i(e+fx)}\right)) \cos(e+fx) + fx + 4i\right)}{af(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2), x]

[Out] (I*c^2*Cot[(e + f*x)/2]*(4*I + f*x + Cos[e + f*x]*(f*x + I*Log[1 + E^((2*I)*(e + f*x))]) + I*Log[1 + E^((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])

fricas [B] time = 0.58, size = 453, normalized size = 4.72

$$\frac{4c^2 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)-c}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - \left(ac^2 \cos(fx+e)^2 + 2ac^2 \cos(fx+e) + ac^2\right) \sqrt{-\frac{c}{a}}}{2 \left(a^2 f \cos(fx+e)^2 + 2a^2 f \cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(-c/a)*log(1/2*(c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + c)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(c*cos(f*x + e)^2 + c)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.16, size = 238, normalized size = 2.48

$$\frac{\cos(fx+e) \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + \cos(fx+e) \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right) - \cos(fx+e) \ln\left(\frac{2}{1+\cos(fx+e)}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x)`

[Out] `1/f*(cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)*ln(2/(1+cos(f*x+e)))+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)-ln(2/(1+cos(f*x+e)))+2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*cos(f*x+e)^3*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e))/a^2`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2),x)`

[Out] `int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)`

[Out] Timed out

$$3.119 \quad \int \frac{(c - c \sec(e + fx))^{3/2}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=98

$$\frac{c^2 \tan(e + fx) \log(\cos(e + fx) + 1)}{af \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{2c^2 \tan(e + fx)}{f(a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}}$$

[Out] $-2*c^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(3/2)}/(c-c*\sec(f*x+e))^{(1/2)}+c^2*\ln(1+\cos(f*x+e))*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3908, 3911, 31}

$$\frac{c^2 \tan(e + fx) \log(\cos(e + fx) + 1)}{af \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{2c^2 \tan(e + fx)}{f(a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c*\text{Sec}[e + f*x])^{(3/2)}/(a + a*\text{Sec}[e + f*x])^{(3/2)}, x]$

[Out] $(-2*c^2*\text{Tan}[e + f*x])/(f*(a + a*\text{Sec}[e + f*x])^{(3/2)}*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (c^2*\text{Log}[1 + \text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 31

$\text{Int}[(a + (b*x)^{-1}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 3908

$\text{Int}[(\text{csc}[(e + f*x)]*(b + a))^{(3/2)}*(\text{csc}[(e + f*x)]*(d + c))^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-4*a^2*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^{(n)})/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[a/c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -2^{(-1)}]$

Rule 3911

$\text{Int}[(\text{csc}[(e + f*x)]*(b + a))^{(m)}*(\text{csc}[(e + f*x)]*(d + c))^{(n)}, x_Symbol] \rightarrow -\text{Dist}[(a*c*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(b + a*x)^{(m - 1/2)}*(d + c*x)^{(n - 1/2)}/x^{(m + n)}, x], x, \text{Sin}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m - 1/2] \&\& \text{EqQ}[m + n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{3/2}}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{2c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{c \int \frac{\sqrt{c - c \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}} dx}{a} \\ &= -\frac{2c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{(c^2 \tan(e + fx)) \text{Subst}\left(\int \frac{1}{a + ax} dx, x\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{2c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{c^2 \log(1 + \cos(e + fx)) \tan(e + fx)}{af \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.14, size = 114, normalized size = 1.16

$$\frac{ic \cot\left(\frac{1}{2}(e + fx)\right) \sqrt{c - c \sec(e + fx)} \left(2i \log(1 + e^{i(e+fx)}) + (fx + 2i \log(1 + e^{i(e+fx)}))\right) \cos(e + fx) + fx + 2i}{af(\cos(e + fx) + 1)\sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2), x]

[Out] (I*c*Cot[(e + f*x)/2]*(2*I + f*x + Cos[e + f*x]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + (2*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]]/(a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} (-c \sec(fx + e) + c)^{\frac{3}{2}}}{a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2*(-1/2*c*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1)))))/a^2/abs(c)-1/2*c^2*sqrt(-a*c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*ln(2*abs(c))/a^2/abs(c)+1/2*c^2*sqrt(-a*c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*ln(abs(c*tan(1/2*(f*x+exp(1)))^2+c))/a^2/abs(c))/f

maple [A] time = 2.19, size = 106, normalized size = 1.08

$$\frac{\left(\cos(fx + e) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) + \cos(fx + e) + \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 1\right) \left(\frac{c(-1 + \cos(fx + e))}{\cos(fx + e)}\right)^{\frac{3}{2}} (\cos^2(fx + e)) \sqrt{\frac{a(1 + \cos(fx + e))}{a^2}}}{f \sin(fx + e)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2), x)

[Out] -1/f*(cos(f*x+e)*ln(2/(1+cos(f*x+e)))+cos(f*x+e)+ln(2/(1+cos(f*x+e)))-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/a^2

maxima [A] time = 0.73, size = 70, normalized size = 0.71

$$\frac{\frac{c^{\frac{3}{2}} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2+1}\right)}{\sqrt{-a}a} - \frac{c^{\frac{3}{2}} \sin(fx+e)^2}{\sqrt{-a}a(\cos(fx+e)+1)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] (c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a) - c^(3/2)*sin(f*x + e)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)

[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(3/2), x)

$$3.120 \quad \int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=94

$$\frac{c \tan(e + fx) \log(\cos(e + fx) + 1)}{af \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{f(a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}}$$

[Out] $-c*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(3/2)}/(c-c*\sec(f*x+e))^{(1/2)}+c*\ln(1+\cos(f*x+e))*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3907, 3911, 31}

$$\frac{c \tan(e + fx) \log(\cos(e + fx) + 1)}{af \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{f(a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c - c*\text{Sec}[e + f*x]]/(a + a*\text{Sec}[e + f*x])^{(3/2)}, x]$

[Out] $-((c*\text{Tan}[e + f*x])/(f*(a + a*\text{Sec}[e + f*x])^{(3/2)}*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])) + (c*\text{Log}[1 + \text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 31

$\text{Int}[(a + (b*x))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 3907

$\text{Int}[\text{Sqrt}[\text{csc}[e + f*x] + (f*x)]*(b + a)]*(\text{csc}[e + f*x] + (f*x)]*(d + c)^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-2*a*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[1/c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -2^{(-1)}]$

Rule 3911

$\text{Int}[(\text{csc}[e + f*x] + (f*x)]*(b + a)^{(m)}*(\text{csc}[e + f*x] + (f*x)]*(d + c)^{(n)}, x_Symbol] \rightarrow -\text{Dist}[(a*c*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(b + a*x)^{(m - 1/2)}*(d + c*x)^{(n - 1/2)}/x^{(m + n)}, x], x, \text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m - 1/2] \&\& \text{EqQ}[m + n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{c \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{\int \frac{\sqrt{c - c \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}} dx}{a} \\ &= -\frac{c \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{(c \tan(e + fx)) \text{Subst}\left(\int \frac{1}{a + ax} dx, \sqrt{a + a \sec(e + fx)}\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{c \tan(e + fx)}{f(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} + \frac{c \log(1 + \cos(e + fx)) \tan(e + fx)}{af \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 0.63, size = 106, normalized size = 1.13

$$\frac{i \cot\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{c - c \sec(e + fx)} \left(2i \log\left(1 + e^{i(e+fx)}\right) + (fx + 2i \log\left(1 + e^{i(e+fx)}\right)) \cos(e + fx) + f\right)}{f(a(\sec(e + fx) + 1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2), x]

[Out] (I*Cot[(e + f*x)/2]*(I + f*x + Cos[e + f*x]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + (2*I)*Log[1 + E^(I*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(f*(a*(1 + Sec[e + f*x]))^(3/2))

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*sqrt(2)*(-1/4*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/sqrt(2)/a^2/abs(c)+1/2*c*sqrt(-a*c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*ln(abs(2*(c*tan(1/2*(f*x+exp(1)))^2-c)+4*c))/sqrt(2)/a^2/abs(c))*sign(cos(f*x+exp(1)))/f

maple [A] time = 2.30, size = 119, normalized size = 1.27

$$\frac{\left(2 \ln\left(\frac{2}{1+\cos(fx+e)}\right) (\cos^2(fx+e)) + \cos^2(fx+e) - 2 \cos(fx+e) - 2 \ln\left(\frac{2}{1+\cos(fx+e)}\right) + 1\right) \cos(fx+e) \sqrt{\frac{c(-1+\cos(fx+e))}{c}}}{2f \sin(fx+e)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2), x)

[Out] -1/2/f*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)-2*ln(2/(1+cos(f*x+e)))+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/a^2

maxima [B] time = 0.96, size = 395, normalized size = 4.20

$$\frac{\left((fx+e) \cos(2fx+2e)^2 + 4(fx+e) \cos(fx+e)^2 + (fx+e) \sin(2fx+2e)^2 + 4(fx+e) \sin(fx+e)^2 + f\right)}{f(a(\sec(e + fx) + 1))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 2*(f*x + e)*cos(f*x + e) + e - sin(f*x + e))*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + 2*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e - 2*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(f*x + e)^2 + a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*sin(f*x + e)^2 + 4*a^2*cos(f*x + e) + a^2 + 2*(2*a^2*cos(f*x + e) + a^2)*cos(2*f*x + 2*e))*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{\cos(e+fx)}}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sec(e+fx)-1)}}{(a(\sec(e+fx)+1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)

[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(3/2), x)

$$3.121 \quad \int \frac{1}{(a+a \sec(e+fx))^{3/2} \sqrt{c-c \sec(e+fx)}} dx$$

Optimal. Leaf size=215

$$\frac{\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx) \log(1-\sec(e+fx))}{4af\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} + \frac{3}{4af\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

[Out] $\ln(\cos(f*x+e))*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$
 $+1/4*\ln(1-\sec(f*x+e))*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$
 $+3/4*\ln(1+\sec(f*x+e))*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$
 $-1/2*\tan(f*x+e)/a/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{\tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx) \log(1-\sec(e+fx))}{4af\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} + \frac{3}{4af\sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]`

[Out] $(\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (\text{Log}[1 - \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(4*a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (3*\text{Log}[1 + \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(4*a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(2*a*f*(1 + \text{Sec}[e + f*x])*Sqrt[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 72

`Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 3912

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\int \frac{1}{(a+a \sec(e+fx))^{3/2} \sqrt{c-c \sec(e+fx)}} dx = -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)} dx, x, \sec(e+fx)\right)}{f\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}}$$

$$= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \left(-\frac{1}{4a^2c(-1+x)} + \frac{1}{a^2cx} - \frac{1}{2a^2c(1+x)^2} - \frac{1}{4a^2c}\right) dx, x, \sec(e+fx)\right)}{f\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}}$$

$$= \frac{\log(\cos(e+fx)) \tan(e+fx)}{af\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} + \frac{\log(1-\sec(e+fx))}{4af\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} + \frac{3}{4af\sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}}$$

Mathematica [C] time = 1.42, size = 141, normalized size = 0.66

$$\frac{\tan(e + fx) \left(\log(1 - e^{i(e+fx)}) + 3 \log(1 + e^{i(e+fx)}) + (\log(1 - e^{i(e+fx)}) + 3 \log(1 + e^{i(e+fx)}) - 2ifx) \cos(e + fx) \right)}{2af(\cos(e + fx) + 1)\sqrt{a(\sec(e + fx) + 1)}\sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]

[Out] ((1 - (2*I)*f*x + Log[1 - E^(I*(e + f*x))] + 3*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*((-2*I)*f*x + Log[1 - E^(I*(e + f*x))] + 3*Log[1 + E^(I*(e + f*x))]))*Tan[e + f*x]/(2*a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{a^2 c \sec(fx + e)^3 + a^2 c \sec(fx + e)^2 - a^2 c \sec(fx + e) - a^2 c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c*sec(f*x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e) - a^2*c), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
 2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
 -2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
 ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
 step/2)Warning, integration of abs or sign assumes constant sign by interval
 s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
 : (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
 2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
 2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>

[abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error %%{c,2%%}Sign error (%%{c,0%%}+%%{c,2%%})Evaluation time: 2.37Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.43, size = 159, normalized size = 0.74

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (-1 + \cos(fx + e))^2 \left(4 \cos(fx + e) \ln\left(\frac{2}{1+\cos(fx+e)}\right) - 2 \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) \cos(fx + e) + 4 \ln\left(\frac{2}{1+\cos(fx+e)}\right) \right)}{4f \sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}} \sin(fx + e)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)

[Out] -1/4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(4*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+4*ln(2/(1+cos(f*x+e)))+cos(f*x+e)-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/a^2

maxima [B] time = 0.96, size = 818, normalized size = 3.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - 3*(cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) - (cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) + 2*(4*f*x + 4*(f*x + e)*cos(

$2fx + 2e) + 4e + \sin(2fx + 2e)) \cdot \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 2 \cdot (4 \cdot (fx + e) \cdot \sin(2fx + 2e) - \cos(2fx + 2e) - 1) \cdot \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 2e) / ((a \cdot \cos(2fx + 2e))^2 + 4a \cdot \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))^2 + a \cdot \sin(2fx + 2e)^2 + 4a \cdot \sin(2fx + 2e) \cdot \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 4a \cdot \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 2a \cdot \cos(2fx + 2e) + 4 \cdot (a \cdot \cos(2fx + 2e) + a) \cdot \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + a) \cdot \sqrt{a} \cdot \sqrt{c} \cdot f$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2} \sqrt{c - \frac{c}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),x)

[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a(\sec(e+fx)+1)\right)^{3/2} \sqrt{-c(\sec(e+fx)-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)

[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))), x)

$$3.122 \quad \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=101

$$\frac{\cot(e+fx)}{2acf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx) \log(\sin(e+fx))}{acf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

[Out] 1/2*cot(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin(f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3905, 3473, 3475}

$$\frac{\cot(e+fx)}{2acf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx) \log(\sin(e+fx))}{acf\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)), x]

[Out] Cot[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_)^(m_)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_)^(m_), x_Symbol] :> Dist[(-a*c)^(m + 1/2)*Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^{3/2}} dx &= -\frac{\tan(e+fx) \int \cot^3(e+fx) dx}{ac\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= \frac{\cot(e+fx)}{2acf\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx)}{ac\sqrt{a+a \sec(e+fx)}} \\ &= \frac{\cot(e+fx)}{2acf\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} + \frac{\log(\sin(e+fx))}{acf\sqrt{a+a \sec(e+fx)}} \end{aligned}$$

Mathematica [C] time = 1.53, size = 121, normalized size = 1.20

$$\frac{\tan(e+fx) \sec^2(e+fx) \left(\log(1 - e^{2i(e+fx)}) + (ifx - \log(1 - e^{2i(e+fx)})) \cos(2(e+fx)) - ifx + 1 \right)}{2cf(\sec(e+fx) - 1)(a(\sec(e+fx) + 1))^{3/2}\sqrt{c - c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]

[Out] ((1 - I*f*x + Cos[2*(e + f*x)]*(I*f*x - Log[1 - E^((2*I)*(e + f*x))]) + Log[1 - E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Tan[e + f*x])/(2*c*f*(-1 + Sec[e + f*x])*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])

fricas [B] time = 0.86, size = 492, normalized size = 4.87

$$\frac{9\sqrt{-ac}\left(\cos(fx+e)^2-1\right)\log\left(\frac{8\left(\left(256\cos(fx+e)^5-512\cos(fx+e)^3+175\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}-\left(256a^2c^2f\cos(fx+e)^4-512a^2c^2f\cos(fx+e)^2+337a^2c^2f\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)}{18\left(a^2c^2f\cos(fx+e)^4-512a^2c^2f\cos(fx+e)^2+337a^2c^2f\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/18*(9*sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-8*((256*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), -1/18*(18*sqrt(a*c)*(cos(f*x + e)^2 - 1)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Una

sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error (%%{c,0%%}+%%{c,2%%})Sign error %%{-c,2%%}Evaluation time: 2.4Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.39, size = 173, normalized size = 1.71

$$\frac{(-1 + \cos(fx + e))^2 \left(4 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) (\cos^2(fx + e)) - 4 (\cos^2(fx + e)) \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) + \cos^2(fx + e) - 4f \sin(fx + e)^3 \left(\frac{c(-1 + \cos(fx + e))}{\cos(fx + e)}\right)^{\frac{3}{2}} \cos(fx + e) a^2 \right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)

[Out] -1/4/f*(-1+cos(f*x+e))^2*(4*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+cos(f*x+e)^2-4*ln(2/(1+cos(f*x+e)))+4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+1)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)/a^2

maxima [B] time = 0.88, size = 486, normalized size = 4.81

$$\left((fx + e) \cos(4fx + 4e)^2 + 4(fx + e) \cos(2fx + 2e)^2 + (fx + e) \sin(4fx + 4e)^2 + 4(fx + e) \sin(2fx + 2e)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)^2

```
- sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
- 1) + 2*(f*x - 2*(f*x + e)*cos(2*f*x + 2*e) + e + sin(2*f*x + 2*e))*cos(4
*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)*sin(2*f*x + 2*e
) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + e + 2*sin(2*f*x + 2*e))*sqrt(a)*sq
rt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2
*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*c
^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2 - 2*(2*a^2*c^2
*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2} \left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a(\sec(e+fx)+1)\right)^{3/2} \left(-c(\sec(e+fx)-1)\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2)),
x)
```

$$3.123 \quad \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c-c \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=347

$$\frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

[Out] $\ln(\cos(f*x+e))*\tan(f*x+e)/a/c^2/f/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}+11/16*\ln(1-\sec(f*x+e))*\tan(f*x+e)/a/c^2/f/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}+5/16*\ln(1+\sec(f*x+e))*\tan(f*x+e)/a/c^2/f/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}-1/8*\tan(f*x+e)/a/c^2/f/(1-\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}-1/2*\tan(f*x+e)/a/c^2/f/(1-\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}-1/8*\tan(f*x+e)/a/c^2/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.18, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 88}

$$\frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(\sec(e+fx)+1)\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((a + a*\text{Sec}[e + f*x])^{3/2}*(c - c*\text{Sec}[e + f*x])^{5/2}), x]$

[Out] $(\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a*c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (11*\text{Log}[1 - \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(16*a*c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (5*\text{Log}[1 + \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(16*a*c^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(8*a*c^2*f*(1 - \text{Sec}[e + f*x])^2*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(2*a*c^2*f*(1 - \text{Sec}[e + f*x])*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(8*a*c^2*f*(1 + \text{Sec}[e + f*x])*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 88

$\text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] /; \text{FreeQ}\{a, b, c, d, e, f, p, x\} \ \&\& \ \text{IntegersQ}[m, n] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -1]))$

Rule 3912

$\text{Int}[(\text{csc}[e + f*x] + (f*x)*\text{Csc}[e + f*x])*(b + a*x)^m*(\text{csc}[e + f*x] + (f*x)*\text{Csc}[e + f*x])^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, x\} \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{5/2}} dx = -\frac{(ac \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x(a+ax)^2(c-cx)^3} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}}$$

$$= -\frac{(ac \tan(e + fx)) \operatorname{Subst}\left(\int \left(-\frac{1}{4a^2c^3(-1+x)^3} + \frac{1}{2a^2c^3(-1+x)^2} - \frac{1}{10a^2c^3(-1+x)}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}}$$

$$= \frac{\log(\cos(e + fx)) \tan(e + fx)}{ac^2 f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{11 \log(\cos(e + fx))}{16ac^2 f \sqrt{a + a \sec(e + fx)}}$$

Mathematica [C] time = 2.62, size = 275, normalized size = 0.79

$$\tan(e + fx) \left(22 \log(1 - e^{i(e+fx)}) + 10 \log(1 + e^{i(e+fx)}) - 8ifx \cos(3(e + fx)) + 11 \log(1 - e^{i(e+fx)}) \cos(3(e + fx)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
[Out] ((14 - (16*I)*f*x - (8*I)*f*x*Cos[3*(e + f*x)] + 22*Log[1 - E^(I*(e + f*x))] + 11*Cos[3*(e + f*x)]*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-12 + (8*I)*f*x - 11*Log[1 - E^(I*(e + f*x))] - 5*Log[1 + E^(I*(e + f*x))]) + 2*Cos[2*(e + f*x)]*(-5 + (8*I)*f*x - 11*Log[1 - E^(I*(e + f*x))] - 5*Log[1 + E^(I*(e + f*x))]) + 10*Log[1 + E^(I*(e + f*x))] + 5*Cos[3*(e + f*x)]*Log[1 + E^(I*(e + f*x))])*Tan[e + f*x])/(32*a*c^2*f*(-1 + Cos[e + f*x])^2*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{a^2c^3 \sec(fx + e)^5 - a^2c^3 \sec(fx + e)^4 - 2a^2c^3 \sec(fx + e)^3 + 2a^2c^3 \sec(fx + e)^2 + a^2c^3 \sec(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c^3*sec(f*x + e)^5 - a^2*c^3*sec(f*x + e)^4 - 2*a^2*c^3*sec(f*x + e)^3 + 2*a^2*c^3*sec(f*x + e)^2 + a^2*c^3*sec(f*x + e) - a^2*c^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
```



```

step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning,
integration of abs or sign assumes constant sign by intervals (correct if t
he argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zero
es of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Disc
ontinuitities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuitie
s at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(
-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]W
arning, integration of abs or sign assumes constant sign by intervals (corr
ect if the argument is real):Check [abs(t_nostep)]Evaluation time: 2.99inde
x.cc index_m i_lex_is_greater Error: Bad Argument Value

```

maple [A] time = 2.95, size = 293, normalized size = 0.84

$$\frac{(-1 + \cos(fx + e))^2 \left(32 (\cos^3(fx + e)) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 44 \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) (\cos^3(fx + e)) - 32 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)

[Out] -1/32/f*(-1+cos(f*x+e))^2*(32*cos(f*x+e)^3*ln(2/(1+cos(f*x+e)))-44*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3-32*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+4

$$4\cos(f*x+e)^2\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))+13\cos(f*x+e)^3-32\cos(f*x+e)*\ln(2/(1+\cos(f*x+e)))+44\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))*\cos(f*x+e)+7\cos(f*x+e)^2+32\ln(2/(1+\cos(f*x+e)))-44\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))-\cos(f*x+e)-11*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{(1/2)}/(c*(-1+\cos(f*x+e))/\cos(f*x+e))^{(5/2)}/\sin(f*x+e)^3/\cos(f*x+e)^2/a^2$$

maxima [B] time = 4.18, size = 4272, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out]
$$-1/8*(8*(f*x + e)*\cos(6*f*x + 6*e)^2 + 8*(f*x + e)*\cos(4*f*x + 4*e)^2 + 8*(f*x + e)*\cos(2*f*x + 2*e)^2 + 32*(f*x + e)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*(f*x + e)*\sin(6*f*x + 6*e)^2 + 8*(f*x + e)*\sin(4*f*x + 4*e)^2 + 8*(f*x + e)*\sin(2*f*x + 2*e)^2 + 32*(f*x + e)*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*f*x + 5*(2*(\cos(4*f*x + 4*e) + \cos(2*f*x + 2*e) - 1)*\cos(6*f*x + 6*e) - \cos(6*f*x + 6*e)^2 - 2*(\cos(2*f*x + 2*e) - 1)*\cos(4*f*x + 4*e) - \cos(4*f*x + 4*e)^2 - \cos(2*f*x + 2*e)^2 + 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) - 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 1)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*(\sin(4*f*x + 4*e) + \sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) - \sin(6*f*x + 6*e)^2 - \sin(4*f*x + 4*e)^2 - 2*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) - \sin(2*f*x + 2*e)^2 + 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e) + 4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) * \sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e) - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e)) * \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\cos(2*f*x + 2*e) - 1)*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 11*(2*(\cos(4*f*x + 4*e) + \cos(2*f*x + 2*e) - 1)*\cos(6*f*x + 6*e) - \cos(6*f*x + 6*e)^2 - 2*(\cos(2*f*x + 2*e) - 1)*\cos(4*f*x + 4*e) - \cos(4*f*x + 4*e)^2 - \cos(2*f*x + 2*e)^2 + 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 8*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) - 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 1)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*(\sin(4*f*x + 4*e) + \sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) - \sin(6*f$$

$$\begin{aligned}
& *x + 6*e)^2 - \sin(4*f*x + 4*e)^2 - 2*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) - \sin(2*f*x + 2*e)^2 + 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e)) \\
& + 4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) \\
& *\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 \\
& - 8*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e) - 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) \\
& *\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 \\
& + 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) \\
& - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\cos(2*f*x + 2*e) - 1)*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \\
& \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 4*(4*f*x - 4*(f*x + e)*\cos(4*f*x + 4*e) - 4*(f*x + e)*\cos(2*f*x + 2*e) + 4*e + 3*\sin(4*f*x + 4*e) + 3*\sin(2*f*x + 2*e)) \\
& *\cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*\cos(2*f*x + 2*e) + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) - 2*(16*f*x + 16*(f*x + e)*\cos(6*f*x + 6*e) \\
& - 16*(f*x + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) + 64*(f*x + e)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 32*(f*x + e) \\
& *\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 16*e + 5*\sin(6*f*x + 6*e) + 7*\sin(4*f*x + 4*e) + 7*\sin(2*f*x + 2*e) - 8*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))))*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 4*(16*f*x + 16*(f*x + e)*\cos(6*f*x + 6*e) - 16*(f*x + e)*\cos(4*f*x + 4*e) \\
& - 16*(f*x + e)*\cos(2*f*x + 2*e) - 32*(f*x + e)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*e + 7*\sin(6*f*x + 6*e) + 5*\sin(4*f*x + 4*e) \\
& + 5*\sin(2*f*x + 2*e) - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 2*(16*f*x + 16*(f*x + e)*\cos(6*f*x + 6*e) \\
& - 16*(f*x + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) + 64*(f*x + e)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 32*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) - 5*\cos(6*f*x + 6*e) - 7*\cos(4*f*x + 4*e) - 7*\cos(2*f*x + 2*e) + 8*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 5)*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) + 4*(16*(f*x + e)*\sin(6*f*x + 6*e) - 16*(f*x + e)*\sin(4*f*x + 4*e) - 16*(f*x + e)*\sin(2*f*x + 2*e) - 32*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) - 7*\cos(6*f*x + 6*e) - 5*\cos(4*f*x + 4*e) - 5*\cos(2*f*x + 2*e) + 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 7)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) - 2*(16*(f*x + e)*\sin(6*f*x + 6*e) - 16*(f*x + e)*\sin(4*f*x + 4*e) - 16*(f*x + e)*\sin(2*f*x + 2*e) - 5*\cos(6*f*x + 6*e) - 7*\cos(4*f*x + 4*e) - 7*\cos(2*f*x + 2*e) - 5)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \\
& \cos(2*f*x + 2*e))) + 8*e + 12*\sin(2*f*x + 2*e))/((a*c^2*\cos(6*f*x + 6*e)^2 + a*c^2*\cos(4*f*x + 4*e)^2 + a*c^2*\cos(2*f*x + 2*e)^2 + 4*a*c^2*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*a*c^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*a*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + a*c^2*\sin(6*f*x + 6*e)^2 + a*c^2*\sin(4*f*x + 4*e)^2 + 2*a*c^2*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + a*c^2*\sin(2*f*x + 2*e)^2 + 4*a*c^2*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*a*c^2*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*a*c^2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 2*a*c^2*\cos(2*f*x + 2*e) + a*c^2 - 2*(a*c^2*\cos(4*f*x + 4*e) + a*c^2*\cos(2*f*x + 2*e) - a*c^2)*\cos(6*f*x + 6*e) + 2*(a*c^2*\cos(2*f*x + 2*e) - a*c^2)*\cos(4*f*x + 4*e) - 4*(a*c^2*\cos(6*f*x + 6*e) - a*c^2*\cos(4*f*x + 4*e) - a*c^2*\cos(2*f*x + 2*e) + 4*a*c^2*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 2*a*c^2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a*c^2)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 8*(a*c^2*\cos(6*f*x + 6*e) -
\end{aligned}$$


```

a*c^2*cos(4*f*x + 4*e) - a*c^2*cos(2*f*x + 2*e) - 2*a*c^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + a*c^2*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) - 4*(a*c^2*cos(6*f*x + 6*e) - a*c^2*cos(4*f*x + 4*e)
- a*c^2*cos(2*f*x + 2*e) + a*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(a*c^2*sin(4*f*x + 4*e) + a*c^2*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) - 4*(a*c^2*sin(6*f*x + 6*e) - a*c^2*sin(4*f*x + 4*e) - a*c^2*sin(2
*f*x + 2*e) + 4*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
- 2*a*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(a*c^2*sin(6*f*x + 6*e) - a*c
^2*sin(4*f*x + 4*e) - a*c^2*sin(2*f*x + 2*e) - 2*a*c^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 4*(a*c^2*sin(6*f*x + 6*e) - a*c^2*sin(4*f*x + 4*e) - a*c^2*si
n(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(
a)*sqrt(c)*f

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2} \left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

$$3.124 \quad \int \frac{(c - c \sec(e + fx))^{7/2}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=220

$$\frac{4c^4 \tan(e + fx)}{a^2 f (\sec(e + fx) + 1) \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \tan(e + fx)}{a^2 f (\sec(e + fx) + 1)^2 \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

[Out] $c^4 \ln(\cos(f*x+e)) * \tan(f*x+e) / a^2 / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 2*c^4 \ln(1+\sec(f*x+e)) * \tan(f*x+e) / a^2 / f / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} - 4*c^4 * \tan(f*x+e) / a^2 / f / (1+\sec(f*x+e))^2 / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)} + 4*c^4 * \tan(f*x+e) / a^2 / f / (1+\sec(f*x+e)) / (a+a*\sec(f*x+e))^{(1/2)} / (c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 88}

$$\frac{4c^4 \tan(e + fx)}{a^2 f (\sec(e + fx) + 1) \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{4c^4 \tan(e + fx)}{a^2 f (\sec(e + fx) + 1)^2 \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]

[Out] $(c^4 * \text{Log}[\text{Cos}[e + f*x]] * \text{Tan}[e + f*x]) / (a^2 * f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (2 * c^4 * \text{Log}[1 + \text{Sec}[e + f*x]] * \text{Tan}[e + f*x]) / (a^2 * f * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) - (4 * c^4 * \text{Tan}[e + f*x]) / (a^2 * f * (1 + \text{Sec}[e + f*x])^2 * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]]) + (4 * c^4 * \text{Tan}[e + f*x]) / (a^2 * f * (1 + \text{Sec}[e + f*x]) * \text{Sqrt}[a + a * \text{Sec}[e + f*x]] * \text{Sqrt}[c - c * \text{Sec}[e + f*x]])$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{7/2}}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^3}{x(a+ax)^3} dx, x, \sec(e + fx)\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \left(\frac{c^3}{a^3 x} - \frac{8c^3}{a^3(1+x)^3} + \frac{4c^3}{a^3(1+x)^2} - \frac{2c^3}{a^3(1+x)}\right) dx, x, \sec(e + fx)\right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= \frac{c^4 \log(\cos(e + fx)) \tan(e + fx)}{a^2 f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} + \frac{2c^4 \log(1 + \sec(e + fx)) \tan(e + fx)}{a^2 f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 2.54, size = 157, normalized size = 0.71

$$\frac{c^3 \cot\left(\frac{1}{2}(e + fx)\right) \sqrt{c - c \sec(e + fx)} \left(4\left(-4 \log\left(1 + e^{i(e+fx)}\right) + \log\left(1 + e^{2i(e+fx)}\right) + ifx - 2\right) \cos(e + fx) + (-2 + \cos(2(e + fx)))\right)}{2a^2 f (\cos(e + fx) + 1)^2 \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]

[Out] (c^3*Cot[(e + f*x)/2]*(4*Cos[e + f*x]*(-2 + I*f*x - 4*Log[1 + E^(I*(e + f*x))]) + Log[1 + E^((2*I)*(e + f*x))]) + (3 + Cos[2*(e + f*x)])*(I*f*x - 4*Log[1 + E^(I*(e + f*x))]) + Log[1 + E^((2*I)*(e + f*x))]) * Sqrt[c - c*Sec[e + f*x]])/(2*a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(c^3 \sec(fx + e)^3 - 3c^3 \sec(fx + e)^2 + 3c^3 \sec(fx + e) - c^3\right) \sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{a^3 \sec(fx + e)^3 + 3a^3 \sec(fx + e)^2 + 3a^3 \sec(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)*c*(a^2*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)^2*abs(c)+2*a^2*c*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)*abs(c))*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/a^5/c/f

maple [A] time = 2.60, size = 338, normalized size = 1.54

$$\frac{\left(\ln\left(\frac{2}{1+\cos(fx+e)}\right)\cos^2(fx+e) + \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\cos^2(fx+e) + \ln\left(-\frac{-\sin(fx+e)-1+\cos(fx+e)}{\sin(fx+e)}\right)\cos^2(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x)

[Out] 1/f*(ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*cos(f*x+e)*ln(2/(1+cos(f*x+e)))+cos(f*x+e)^2+2*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-sin(f*x+e)-1+cos(f*x+e))/sin(f*x+e))+1)*(c*(-1+cos(f*x+e))

)/cos(f*x+e))^(7/2)*cos(f*x+e)^4*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/(-1+cos(f*x+e))/a^3

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{7/2}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(5/2),x)

[Out] Timed out

$$3.125 \quad \int \frac{(c - c \sec(e + fx))^{5/2}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=98

$$\frac{c^3 \tan(e + fx) \log(\cos(e + fx) + 1)}{a^2 f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{2c^3 \tan(e + fx)}{f(a \sec(e + fx) + a)^{5/2} \sqrt{c - c \sec(e + fx)}}$$

[Out] $-2*c^3*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(5/2)}/(c-c*\sec(f*x+e))^{(1/2)}+c^3*\ln(1+\cos(f*x+e))*\tan(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3910, 3911, 31}

$$\frac{c^3 \tan(e + fx) \log(\cos(e + fx) + 1)}{a^2 f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{2c^3 \tan(e + fx)}{f(a \sec(e + fx) + a)^{5/2} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c*\text{Sec}[e + f*x])^{(5/2)}/(a + a*\text{Sec}[e + f*x])^{(5/2)}, x]$

[Out] $(-2*c^3*\text{Tan}[e + f*x])/(f*(a + a*\text{Sec}[e + f*x])^{(5/2)}*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (c^3*\text{Log}[1 + \text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]$

Rule 31

$\text{Int}[(a + b*x)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 3910

$\text{Int}[(csc[(e + f*x)]*(b + a*x))^{(5/2)}*(csc[(e + f*x)]*(d + c*x))^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-8*a^3*\text{Cot}[e + f*x]*(c + d*\text{Csc}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] + \text{Dist}[a^2/c^2, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -2^{(-1)}]$

Rule 3911

$\text{Int}[(csc[(e + f*x)]*(b + a*x))^{(m)}*(csc[(e + f*x)]*(d + c*x))^{(n)}, x_Symbol] \rightarrow -\text{Dist}[(a*c*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(b + a*x)^{(m - 1/2)}*(d + c*x)^{(n - 1/2)}/x^{(m + n)}, x], x, \text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m - 1/2] \&\& \text{EqQ}[m + n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^{5/2}}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{2c^3 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} + \frac{c^2 \int \frac{\sqrt{c - c \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}} dx}{a^2} \\ &= -\frac{2c^3 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} + \frac{(c^3 \tan(e + fx)) \text{Subst}\left(\int \frac{1}{a + ax} dx\right)}{af \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\ &= -\frac{2c^3 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} + \frac{c^3 \log(1 + \cos(e + fx)) \tan(e + fx)}{a^2 f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.57, size = 154, normalized size = 1.57

$$\frac{ic^2 \cot\left(\frac{1}{2}(e+fx)\right) \sqrt{c-c\sec(e+fx)} \left(6i \log\left(1+e^{i(e+fx)}\right) + (fx+2i \log\left(1+e^{i(e+fx)}\right)) \cos(2(e+fx))\right) + 4\left(2i \log\left(1+e^{i(e+fx)}\right)\right)}{2a^2 f(\cos(e+fx)+1)^2 \sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2), x]

[Out] ((I/2)*c^2*Cot[(e + f*x)/2]*(4*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + 4*Cos[e + f*x]*(2*I + f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(c^2 \sec^2(fx+e) - 2c^2 \sec(fx+e) + c^2 \right) \sqrt{a \sec(fx+e) + a} \sqrt{-c \sec(fx+e) + c}}{a^3 \sec^3(fx+e) + 3a^3 \sec^2(fx+e) + 3a^3 \sec(fx+e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2*(-1/2*c^3*sqrt(-a*c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*ln(2*abs(c))/a^3/abs(c)+1/2*c^3*sqrt(-a*c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*ln(abs(c*tan(1/2*(f*x+exp(1)))^2+c))/a^3/abs(c)+1/4*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)^2*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))/a^3/c)/f

maple [A] time = 2.75, size = 144, normalized size = 1.47

$$\frac{\left(2 \ln\left(\frac{2}{1+\cos(fx+e)}\right)\right) \left(\cos^2(fx+e)\right) + 3 \left(\cos^2(fx+e)\right) + 4 \cos(fx+e) \ln\left(\frac{2}{1+\cos(fx+e)}\right) - 2 \cos(fx+e) + 2 \ln\left(\frac{2}{1+\cos(fx+e)}\right)}{2f \sin(fx+e)^5 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2), x)

[Out] 1/2/f*(2*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+3*cos(f*x+e)^2+4*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+2*ln(2/(1+cos(f*x+e)))-1)*(c*(-1+cos(f*x+e)))/

$\cos(f*x+e))^{(5/2)}*\cos(f*x+e)^3*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{(1/2)}/\sin(f*x+e)^5/a^3$

maxima [A] time = 0.71, size = 102, normalized size = 1.04

$$\frac{2c^{\frac{5}{2}} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2+1}\right)}{\sqrt{-a}a^2} + \frac{\frac{2\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}$$

$$2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] 1/2*(2*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a^2) + (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(-a)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)

[Out] Timed out

$$3.126 \quad \int \frac{(c - c \sec(e + fx))^{3/2}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=144

$$\frac{c^2 \tan(e + fx) \log(\cos(e + fx) + 1)}{a^2 f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{a f (a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{f (a \sec(e + fx) + a)^{5/2} \sqrt{c - c \sec(e + fx)}}$$

[Out] $-c^2 \tan(f*x+e)/f/(a+a*\sec(f*x+e))^{5/2}/(c-c*\sec(f*x+e))^{1/2}-c^2 \tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{3/2}/(c-c*\sec(f*x+e))^{1/2}+c^2 \ln(1+\cos(f*x+e))*\tan(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{1/2}/(c-c*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.29, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3908, 3907, 3911, 31}

$$\frac{c^2 \tan(e + fx) \log(\cos(e + fx) + 1)}{a^2 f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{a f (a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{f (a \sec(e + fx) + a)^{5/2} \sqrt{c - c \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $-((c^2 \tan[e + f*x])/(f*(a + a*\sec[e + f*x])^{5/2} \sqrt{c - c*\sec[e + f*x]}) - (c^2 \tan[e + f*x])/(a*f*(a + a*\sec[e + f*x])^{3/2} \sqrt{c - c*\sec[e + f*x]}) + (c^2 \log[1 + \cos[e + f*x]]*\tan[e + f*x])/(a^2*f*\sqrt{a + a*\sec[e + f*x]}*\sqrt{c - c*\sec[e + f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3908

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-4*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^{3/2}}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} + \frac{c \int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{3/2}} dx}{a} \\
&= -\frac{c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{af(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{af(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{c^2 \tan(e + fx)}{f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} - \frac{c^2 \tan(e + fx)}{af(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 0.87, size = 152, normalized size = 1.06

$$\frac{ic \cot\left(\frac{1}{2}(e + fx)\right) \sqrt{c - c \sec(e + fx)} \left(6i \log\left(1 + e^{i(e+fx)}\right) + \left(fx + 2i \log\left(1 + e^{i(e+fx)}\right)\right) \cos(2(e + fx)) + \left(8i \log\left(1 + e^{i(e+fx)}\right)\right) \cos(2(e + fx))\right)}{2a^2 f(\cos(e + fx) + 1)^2 \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2), x]

[Out] ((I/2)*c*Cot[(e + f*x)/2]*(4*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + Cos[e + f*x]*(6*I + 4*f*x + (8*I)*Log[1 + E^(I*(e + f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]]/(a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} (-c \sec(fx + e) + c)^{3/2}}{a^3 \sec(fx + e)^3 + 3a^3 \sec(fx + e)^2 + 3a^3 \sec(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2*(1/16*(2*a^3*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1))))^2-c)^2*abs(c)*sign(tan(1/2*(f*x+exp(1))))^3+tan(1/2*(f*x+exp(1))))-4*a^3*c*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1))))^2-c)*abs(c)*sign(tan(1/2*(f*x+exp(1))))^3+ta

$$\frac{n(1/2*(f*x+\exp(1))))}{a^6/c^2-1/2*c^2*\sqrt{-a*c}*sign(\tan(1/2*(f*x+\exp(1))))^3+\tan(1/2*(f*x+\exp(1)))}*\ln(2*\text{abs}(c))/a^3/\text{abs}(c)+1/2*c^2*\sqrt{-a*c}*sign(\tan(1/2*(f*x+\exp(1))))^3+\tan(1/2*(f*x+\exp(1)))}*\ln(\text{abs}(c*\tan(1/2*(f*x+\exp(1))))^2+c))/a^3/\text{abs}(c))/f$$

maple [A] time = 2.65, size = 152, normalized size = 1.06

$$\frac{(-1 + \cos(fx + e)) \left(4 \ln \left(\frac{2}{1 + \cos(fx + e)} \right) (\cos^2(fx + e)) + 5 (\cos^2(fx + e)) + 8 \cos(fx + e) \ln \left(\frac{2}{1 + \cos(fx + e)} \right) - 2 \cos(fx + e) \right)}{4f \sin(fx + e)^5 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x)

[Out] 1/4/f*(-1+cos(f*x+e))*(4*ln(2/(1+cos(f*x+e))))*cos(f*x+e)^2+5*cos(f*x+e)^2+8*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+4*ln(2/(1+cos(f*x+e)))-3)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3

maxima [B] time = 1.15, size = 1786, normalized size = 12.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -((f*x + e)*c*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*c*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x + e)*c*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*c*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*c*cos(2*f*x + 2*e) + (f*x + e)*c - 2*(c*cos(4*f*x + 4*e)^2 + 36*c*cos(2*f*x + 2*e)^2 + 16*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(4*f*x + 4*e)^2 + 12*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c*sin(2*f*x + 2*e)^2 + 16*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*c*cos(2*f*x + 2*e) + c)*cos(4*f*x + 4*e) + 12*c*cos(2*f*x + 2*e) + 8*(c*cos(4*f*x + 4*e) + 6*c*cos(2*f*x + 2*e) + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + c)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*cos(4*f*x + 4*e) + 6*c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e) + 4*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 1) + 2*(6*(f*x + e)*c*cos(2*f*x + 2*e) + (f*x + e)*c - 4*c*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(4*(f*x + e)*c*cos(4*f*x + 4*e) + 24*(f*x + e)*c*cos(2*f*x + 2*e) + 16*(f*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) + 4*(f*x + e)*c + 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(4*(f*x + e)*c*cos(4*f*x + 4*e) + 24*(f*x + e)*c*cos(2*f*x + 2*e) + 4*(f*x + e)*c + 3*c*sin(4*f*x + 4*e) + 2*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(3*(f*x + e)*c*sin(2*f*x + 2*e) + 2*c*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 8*c*sin(2*f*x + 2*e) + 2*(4*(f*x + e)*c*sin(4*f*x + 4*e) + 24*(f*x + e)*c*sin(2*f*x + 2*e) + 16*(f*x + e)*c*sin(1/2*arctan

$2(\sin(2fx + 2e), \cos(2fx + 2e))) - 3c \cos(4fx + 4e) - 2c \cos(2fx + 2e) - 3c \sin(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 2(4(fx + e)c \sin(4fx + 4e) + 24(fx + e)c \sin(2fx + 2e) - 3c \cos(4fx + 4e) - 2c \cos(2fx + 2e) - 3c \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \sqrt{a} \sqrt{c} / ((a^3 \cos(4fx + 4e)^2 + 36a^3 \cos(2fx + 2e)^2 + 16a^3 \cos(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 16a^3 \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + a^3 \sin(4fx + 4e)^2 + 12a^3 \sin(4fx + 4e) \sin(2fx + 2e) + 36a^3 \sin(2fx + 2e)^2 + 16a^3 \sin(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 16a^3 \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 12a^3 \cos(2fx + 2e) + a^3 + 2(6a^3 \cos(2fx + 2e) + a^3) \cos(4fx + 4e) + 8(a^3 \cos(4fx + 4e) + 6a^3 \cos(2fx + 2e) + 4a^3 \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + a^3) \cos(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 8(a^3 \cos(4fx + 4e) + 6a^3 \cos(2fx + 2e) + a^3) \cos(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 8(a^3 \sin(4fx + 4e) + 6a^3 \sin(2fx + 2e) + 4a^3 \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \sin(3/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 8(a^3 \sin(4fx + 4e) + 6a^3 \sin(2fx + 2e)) \sin(1/2 \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \cdot f$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2), x)

[Out] int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e+fx) - 1))^{\frac{3}{2}}}{(a(\sec(e+fx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2), x)

[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(5/2), x)

$$3.127 \quad \int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal. Leaf size=140

$$\frac{c \tan(e + fx) \log(\cos(e + fx) + 1)}{a^2 f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{a f (a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{2 f (a \sec(e + fx) + a)^{5/2}}$$

[Out] $-1/2 * c * \tan(f * x + e) / f / (a + a * \sec(f * x + e))^{5/2} / (c - c * \sec(f * x + e))^{1/2} - c * \tan(f * x + e) / a / f / (a + a * \sec(f * x + e))^{3/2} / (c - c * \sec(f * x + e))^{1/2} + c * \ln(1 + \cos(f * x + e)) * \tan(f * x + e) / a^2 / f / (a + a * \sec(f * x + e))^{1/2} / (c - c * \sec(f * x + e))^{1/2}$

Rubi [A] time = 0.28, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3907, 3911, 31}

$$\frac{c \tan(e + fx) \log(\cos(e + fx) + 1)}{a^2 f \sqrt{a \sec(e + fx) + a} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{a f (a \sec(e + fx) + a)^{3/2} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{2 f (a \sec(e + fx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]

[Out] $-(c * \tan[e + f * x]) / (2 * f * (a + a * \sec[e + f * x])^{5/2} * \sqrt{c - c * \sec[e + f * x]}) - (c * \tan[e + f * x]) / (a * f * (a + a * \sec[e + f * x])^{3/2} * \sqrt{c - c * \sec[e + f * x]}) + (c * \log[1 + \cos[e + f * x]] * \tan[e + f * x]) / (a^2 * f * \sqrt{a + a * \sec[e + f * x]} * \sqrt{c - c * \sec[e + f * x]})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3907

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Simp[(-2*a*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]

Rule 3911

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := -Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((b + a*x)^(m - 1/2)*(d + c*x)^(n - 1/2))/x^(m + n), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && EqQ[m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{c \tan(e + fx)}{2f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} + \frac{\int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{3/2}} dx}{a} \\
&= -\frac{c \tan(e + fx)}{2f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{af(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{c \tan(e + fx)}{2f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{af(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} \\
&= -\frac{c \tan(e + fx)}{2f(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} - \frac{c \tan(e + fx)}{af(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 0.67, size = 151, normalized size = 1.08

$$\frac{i \cot\left(\frac{1}{2}(e + fx)\right) \sqrt{c - c \sec(e + fx)} \left(6i \log\left(1 + e^{i(e+fx)}\right) + \left(fx + 2i \log\left(1 + e^{i(e+fx)}\right)\right) \cos(2(e + fx)) + 4\left(2i \log\left(1 + e^{i(e+fx)}\right)\right) \cos(2(e + fx))\right)}{2a^2 f(\cos(e + fx) + 1)^2 \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2), x]

[Out] ((I/2)*Cot[(e + f*x)/2]*(3*I + 3*f*x + Cos[2*(e + f*x)]*(f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + 4*Cos[e + f*x]*(I + f*x + (2*I)*Log[1 + E^(I*(e + f*x))]) + (6*I)*Log[1 + E^(I*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]]/(a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{a \sec(fx + e) + a} \sqrt{-c \sec(fx + e) + c}}{a^3 \sec(fx + e)^3 + 3a^3 \sec(fx + e)^2 + 3a^3 \sec(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*sqrt(2)*(1/128*(4*sqrt(2)*a^3*c*sqrt(-a*c))*(c*tan(1/2*(f*x+exp(1)))^2-c)^2*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))-16*sqrt(2)*a^3*c^2*sqrt(-a*c)*(c*tan(1/2*(f*x+exp(1)))^2-c)*abs(c)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1)))))/a^6/c^4+1/2*c*sqrt(-a*c

)*sign(tan(1/2*(f*x+exp(1)))^3+tan(1/2*(f*x+exp(1))))*ln(abs(c*tan(1/2*(f*x+exp(1)))^2+c))/sqrt(2)/a^3/abs(c))*sign(cos(f*x+exp(1)))/f

maple [A] time = 2.79, size = 152, normalized size = 1.09

$$\frac{(-1 + \cos(fx + e))^2 \left(8 \ln\left(\frac{2}{1 + \cos(fx + e)}\right) (\cos^2(fx + e)) + 7 (\cos^2(fx + e)) + 16 \cos(fx + e) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 2 \right)}{8f \sin(fx + e)^5 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x)

[Out] 1/8/f*(-1+cos(f*x+e))^2*(8*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+7*cos(f*x+e)^2+16*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-2*cos(f*x+e)+8*ln(2/(1+cos(f*x+e)))-5*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3

maxima [B] time = 1.40, size = 1165, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x + e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f*x + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2*e)^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 4*(f*x + e)*cos(3*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e - 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*cos(4*f*x + 4*e) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x + e) + e)*cos(3*f*x + 3*e) + 12*(f*x + 4*(f*x + e)*cos(f*x + e) + e)*cos(2*f*x + 2*e) + 8*(f*x + e)*cos(f*x + e) + 2*(4*(f*x + e)*sin(3*f*x + 3*e) + 6*(f*x + e)*sin(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x + 3*e) + 3*cos(2*f*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) + 4*(12*(f*x + e)*sin(2*f*x + 2*e) + 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e) + 6*(8*(f*x + e)*sin(f*x + e) - 1)*sin(2*f*x + 2*e) + e - 4*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*cos(4*f*x + 4*e)^2 + 16*a^3*cos(3*f*x + 3*e)^2 + 36*a^3*cos(2*f*x + 2*e)^2 + 16*a^3*cos(f*x + e)^2 + a^3*sin(4*f*x + 4*e)^2 + 16*a^3*sin(3*f*x + 3*e)^2 + 36*a^3*sin(2*f*x + 2*e)^2 + 48*a^3*sin(2*f*x + 2*e)*sin(f*x + e) + 16*a^3*sin(f*x + e)^2 + 8*a^3*cos(f*x + e) + a^3 + 2*(4*a^3*cos(3*f*x + 3*e) + 6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(4*f*x + 4*e) + 8*(6*a^3*cos(2*f*x + 2*e) + 4*a^3*cos(f*x + e) + a^3)*cos(3*f*x + 3*e) + 12*(4*a^3*cos(f*x + e) + a^3)*cos(2*f*x + 2*e) + 4*(2*a^3*sin(3*f*x + 3*e) + 3*a^3*sin(2*f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*sin(2*f*x + 2*e) + 2*a^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - \frac{c}{\cos(e+fx)}}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2), x)

[Out] int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sec(e+fx) - 1)}}{(a(\sec(e+fx) + 1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2), x)

[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(5/2), x)

$$3.128 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2} \sqrt{c-c \sec(e+fx)}} dx$$

Optimal. Leaf size=270

$$\frac{3 \tan(e+fx)}{4a^2 f(\sec(e+fx)+1) \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{4a^2 f(\sec(e+fx)+1)^2 \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

[Out] $\ln(\cos(f*x+e))*\tan(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)+1/8*\ln(1-\sec(f*x+e))*\tan(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)+7/8*\ln(1+\sec(f*x+e))*\tan(f*x+e)/a^2/f/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)-1/4*\tan(f*x+e)/a^2/f/(1+\sec(f*x+e))^{(1/2)/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)-3/4*\tan(f*x+e)/a^2/f/(1+\sec(f*x+e))^{(1/2)/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 72}

$$\frac{3 \tan(e+fx)}{4a^2 f(\sec(e+fx)+1) \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{4a^2 f(\sec(e+fx)+1)^2 \sqrt{a \sec(e+fx)+a} \sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]`

[Out] $(\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (\text{Log}[1 - \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(8*a^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (7*\text{Log}[1 + \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(8*a^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(4*a^2*f*(1 + \text{Sec}[e + f*x])^2*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - (3*\text{Tan}[e + f*x])/(4*a^2*f*(1 + \text{Sec}[e + f*x])*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 72

`Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 3912

`Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^{5/2} \sqrt{c-c \sec(e+fx)}} dx &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \frac{1}{x(a+ax)^3(c-cx)} dx, x, \sec(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= -\frac{(ac \tan(e+fx)) \text{Subst}\left(\int \left(-\frac{1}{8a^3c(-1+x)} + \frac{1}{a^3cx} - \frac{1}{2a^3c(1+x)^3} - \frac{1}{4a^3c}\right) dx, x, \sec(e+fx)\right)}{f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \\ &= \frac{\log(\cos(e+fx)) \tan(e+fx)}{a^2 f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} + \frac{\log(1-\sec(e+fx))}{8a^2 f \sqrt{a+a \sec(e+fx)} \sqrt{c-c \sec(e+fx)}} \end{aligned}$$

e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error %%%{c,2%%}%Sign error (%%%{c,0%%}%+%%){c,2%%}%Evaluation time: 2.71Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.71, size = 223, normalized size = 0.83

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} (-1 + \cos(fx + e))^3 \left(16 \ln\left(\frac{2}{1+\cos(fx+e)}\right) (\cos^2(fx + e)) - 4 (\cos^2(fx + e)) \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)

[Out] 1/16/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^3*(16*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+32*cos(f*x+e)*ln(2/(1+cos(f*x+e)))-8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)+9*cos(f*x+e)^2+16*ln(2/(1+cos(f*x+e)))-4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)-7)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3

maxima [B] time = 1.14, size = 2206, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] -1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 + 64*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x + e)*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1/2*ar

$$\begin{aligned}
& \text{ctan2}(\sin(2fx + 2e), \cos(2fx + 2e))^2 + 4fx - 7*(2*(6*\cos(2fx + 2e) + 1)*\cos(4fx + 4e) + \cos(4fx + 4e)^2 + 36*\cos(2fx + 2e)^2 + 8 \\
& *(\cos(4fx + 4e) + 6*\cos(2fx + 2e) + 4*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1)*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) \\
& + 16*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 8*(\cos(4fx + 4e) + 6*\cos(2fx + 2e) + 1)*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) \\
& + 16*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + \sin(4fx + 4e)^2 + 12*\sin(4fx + 4e)*\sin(2fx + 2e) + 36*\sin(2fx + 2e)^2 + 8*(\sin(4fx + 4e) + 6*\sin(2fx + 2e) + 4*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 8*(\sin(4fx + 4e) + 6*\sin(2fx + 2e))*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 12*\cos(2fx + 2e) + 1)*\arctan2(\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))), \cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1) - (2*(6*\cos(2fx + 2e) + 1)*\cos(4fx + 4e) + \cos(4fx + 4e)^2 + 36*\cos(2fx + 2e)^2 + 8*(\cos(4fx + 4e) + 6*\cos(2fx + 2e) + 4*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1)*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 8*(\cos(4fx + 4e) + 6*\cos(2fx + 2e) + 1)*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + \sin(4fx + 4e)^2 + 12*\sin(4fx + 4e)*\sin(2fx + 2e) + 36*\sin(2fx + 2e)^2 + 8*(\sin(4fx + 4e) + 6*\sin(2fx + 2e) + 4*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 8*(\sin(4fx + 4e) + 6*\sin(2fx + 2e))*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 12*\cos(2fx + 2e) + 1)*\arctan2(\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))), \cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) - 1) + 8*(fx + 6*(fx + e)*\cos(2fx + 2e) + e - 2*\sin(2fx + 2e))*\cos(4fx + 4e) + 48*(fx + e)*\cos(2fx + 2e) + 2*(16*fx + 16*(fx + e)*\cos(4fx + 4e) + 96*(fx + e)*\cos(2fx + 2e) + 64*(fx + e)*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 16*e + 5*\sin(4fx + 4e) - 2*\sin(2fx + 2e))*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 2*(16*fx + 16*(fx + e)*\cos(4fx + 4e) + 96*(fx + e)*\cos(2fx + 2e) + 16*e + 5*\sin(4fx + 4e) - 2*\sin(2fx + 2e))*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 16*(3*(fx + e)*\sin(2fx + 2e) + \cos(2fx + 2e))*\sin(4fx + 4e) + 2*(16*(fx + e)*\sin(4fx + 4e) + 96*(fx + e)*\sin(2fx + 2e) + 64*(fx + e)*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) - 5*\cos(4fx + 4e) + 2*\cos(2fx + 2e) - 5)*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 2*(16*(fx + e)*\sin(4fx + 4e) + 96*(fx + e)*\sin(2fx + 2e) - 5*\cos(4fx + 4e) + 2*\cos(2fx + 2e) - 5)*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 4e - 16*\sin(2fx + 2e))/((a^2*\cos(4fx + 4e)^2 + 36*a^2*\cos(2fx + 2e)^2 + 16*a^2*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 16*a^2*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + a^2*\sin(4fx + 4e)^2 + 12*a^2*\sin(4fx + 4e)*\sin(2fx + 2e) + 36*a^2*\sin(2fx + 2e)^2 + 16*a^2*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 16*a^2*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))^2 + 12*a^2*\cos(2fx + 2e) + a^2 + 2*(6*a^2*\cos(2fx + 2e) + a^2)*\cos(4fx + 4e) + 8*(a^2*\cos(4fx + 4e) + 6*a^2*\cos(2fx + 2e) + 4*a^2*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + a^2)*\cos(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 8*(a^2*\cos(4fx + 4e) + 6*a^2*\cos(2fx + 2e) + a^2)*\cos(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 8*(a^2*\sin(4fx + 4e) + 6*a^2*\sin(2fx + 2e) + 4*a^2*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))*\sin(3/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))) + 8*(a^2*\sin(4fx + 4e) + 6*a^2*\sin(2fx + 2e))*\sin(1/2*\arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))*\sqrt{a}*\sqrt{c}*f)
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2} \sqrt{c - \frac{c}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),x)

[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a(\sec(e+fx)+1)\right)^{5/2} \sqrt{-c(\sec(e+fx)-1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)

[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x) - 1))), x)

$$3.129 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=345

$$\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{2a^2cf(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a}\sqrt{c}}$$

[Out] $\ln(\cos(f*x+e))*\tan(f*x+e)/a^2/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}+5/16*\ln(1-\sec(f*x+e))*\tan(f*x+e)/a^2/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}+11/16*\ln(1+\sec(f*x+e))*\tan(f*x+e)/a^2/c/f/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-1/8*\tan(f*x+e)/a^2/c/f/(1-\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-1/8*\tan(f*x+e)/a^2/c/f/(1+\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}-1/2*\tan(f*x+e)/a^2/c/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}/(c-c*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3912, 88}

$$\frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{2a^2cf(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]

[Out] $(\text{Log}[\text{Cos}[e + f*x]]*\text{Tan}[e + f*x])/(a^2*c*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (5*\text{Log}[1 - \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(16*a^2*c*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) + (11*\text{Log}[1 + \text{Sec}[e + f*x]]*\text{Tan}[e + f*x])/(16*a^2*c*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(8*a^2*c*f*(1 - \text{Sec}[e + f*x])* \text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(8*a^2*c*f*(1 + \text{Sec}[e + f*x])^2*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]]) - \text{Tan}[e + f*x]/(2*a^2*c*f*(1 + \text{Sec}[e + f*x])* \text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c - c*\text{Sec}[e + f*x]])$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)]/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or
sign assumes constant sign by intervals (correct if the argument is real)
:Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_
nostep were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, in
tegration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroe
s of t_nostep^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f
*t_nostep+exp(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep),abs(t_nostep^2-1)]W
arning, integration of abs or sign assumes constant sign by intervals (corr
ect if the argument is real):Check [abs(t_nostep)]Sign error (%%{c,0%%}%+
%%{c,2%%})Sign error %%{-c,2%%}Evaluation time: 3.17Limit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value

maple [A] time = 2.60, size = 284, normalized size = 0.82

$$\left(20 \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right) (\cos^3(fx+e)) - 32 (\cos^3(fx+e)) \ln\left(\frac{2}{1+\cos(fx+e)}\right) + 20 (\cos^2(fx+e)) \ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2), x)

[Out] -1/32/f*(20*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^3-32*cos(f*x+e)^3*ln
(2/(1+cos(f*x+e)))+20*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*ln(2/

$$(1+\cos(f*x+e)))*\cos(f*x+e)^2-13*\cos(f*x+e)^3-20*\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))*\cos(f*x+e)+32*\cos(f*x+e)*\ln(2/(1+\cos(f*x+e)))+7*\cos(f*x+e)^2-20*\ln(-(-1+\cos(f*x+e))/\sin(f*x+e))+32*\ln(2/(1+\cos(f*x+e)))+\cos(f*x+e)-11)*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*\cos(f*x+e)^2*(c*(-1+\cos(f*x+e))/\cos(f*x+e))^{3/2}/c^3/\sin(f*x+e)^5/a^3$$

maxima [B] time = 4.63, size = 4272, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/8*(8*(f*x + e)*\cos(6*f*x + 6*e)^2 + 8*(f*x + e)*\cos(4*f*x + 4*e)^2 + 8*(f*x + e)*\cos(2*f*x + 2*e)^2 + 32*(f*x + e)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*(f*x + e)*\sin(6*f*x + 6*e)^2 + 8*(f*x + e)*\sin(4*f*x + 4*e)^2 + 8*(f*x + e)*\sin(2*f*x + 2*e)^2 + 32*(f*x + e)*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*f*x + 11*(2*(\cos(4*f*x + 4*e) + \cos(2*f*x + 2*e) - 1)*\cos(6*f*x + 6*e) - \cos(6*f*x + 6*e)^2 - 2*(\cos(2*f*x + 2*e) - 1)*\cos(4*f*x + 4*e) - \cos(4*f*x + 4*e)^2 - \cos(2*f*x + 2*e)^2 - 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) - 4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 1)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*(\sin(4*f*x + 4*e) + \sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) - \sin(6*f*x + 6*e)^2 - \sin(4*f*x + 4*e)^2 - 2*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) - \sin(2*f*x + 2*e)^2 - 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e) - 4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e) + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))))*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e))*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\cos(2*f*x + 2*e) - 1)*\arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1) + 5*(2*(\cos(4*f*x + 4*e) + \cos(2*f*x + 2*e) - 1)*\cos(6*f*x + 6*e) - \cos(6*f*x + 6*e)^2 - 2*(\cos(2*f*x + 2*e) - 1)*\cos(4*f*x + 4*e) - \cos(4*f*x + 4*e)^2 - \cos(2*f*x + 2*e)^2 - 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) - 4*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 1)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 2*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 1)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 4*(\cos(6*f*x + 6*e) - \cos(4*f*x + 4*e) - \cos(2*f*x + 2*e) + 1)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*(\sin(4*f*x + 4*e) + \sin(2*f*x + 2*e))*\sin(6*f*x + 6*e) - \sin(6*f$$

$$\begin{aligned}
& *x + 6*e)^2 - \sin(4*f*x + 4*e)^2 - 2*\sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) - \sin(2*f*x + 2*e)^2 - 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e)) - 4*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) * \sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 8*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e) + 2*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 16*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 4*(\sin(6*f*x + 6*e) - \sin(4*f*x + 4*e) - \sin(2*f*x + 2*e)) * \sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 2*\cos(2*f*x + 2*e) - 1) * \arctan2(\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))), \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 1) + 4*(4*f*x - 4*(f*x + e)*\cos(4*f*x + 4*e) - 4*(f*x + e)*\cos(2*f*x + 2*e) + 4*e + 3*\sin(4*f*x + 4*e) + 3*\sin(2*f*x + 2*e)) * \cos(6*f*x + 6*e) - 16*(f*x - (f*x + e)*\cos(2*f*x + 2*e) + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) + 2*(16*f*x + 16*(f*x + e)*\cos(6*f*x + 6*e) - 16*(f*x + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) - 64*(f*x + e)*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 32*(f*x + e) * \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 16*e + 5*\sin(6*f*x + 6*e) + 7*\sin(4*f*x + 4*e) + 7*\sin(2*f*x + 2*e) + 8*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(16*f*x + 16*(f*x + e)*\cos(6*f*x + 6*e) - 16*(f*x + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) + 32*(f*x + e)*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 16*e + 7*\sin(6*f*x + 6*e) + 5*\sin(4*f*x + 4*e) + 5*\sin(2*f*x + 2*e) + 4*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) * \cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(16*f*x + 16*(f*x + e)*\cos(6*f*x + 6*e) - 16*(f*x + e)*\cos(4*f*x + 4*e) - 16*(f*x + e)*\cos(2*f*x + 2*e) + 16*e + 5*\sin(6*f*x + 6*e) + 7*\sin(4*f*x + 4*e) + 7*\sin(2*f*x + 2*e)) * \cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(4*(f*x + e)*\sin(4*f*x + 4*e) + 4*(f*x + e)*\sin(2*f*x + 2*e) + 3*\cos(4*f*x + 4*e) + 3*\cos(2*f*x + 2*e)) * \sin(6*f*x + 6*e) + 4*(4*(f*x + e)*\sin(2*f*x + 2*e) + 3)*\sin(4*f*x + 4*e) + 2*(16*(f*x + e)*\sin(6*f*x + 6*e) - 16*(f*x + e)*\sin(4*f*x + 4*e) - 16*(f*x + e)*\sin(2*f*x + 2*e) - 64*(f*x + e)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 32*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 5*\cos(6*f*x + 6*e) - 7*\cos(4*f*x + 4*e) - 7*\cos(2*f*x + 2*e) - 8*\cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 5)*\sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 4*(16*(f*x + e)*\sin(6*f*x + 6*e) - 16*(f*x + e)*\sin(4*f*x + 4*e) - 16*(f*x + e)*\sin(2*f*x + 2*e) + 32*(f*x + e)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) - 7*\cos(6*f*x + 6*e) - 5*\cos(4*f*x + 4*e) - 5*\cos(2*f*x + 2*e) - 4*\cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 7)*\sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 2*(16*(f*x + e)*\sin(6*f*x + 6*e) - 16*(f*x + e)*\sin(4*f*x + 4*e) - 16*(f*x + e)*\sin(2*f*x + 2*e) - 5*\cos(6*f*x + 6*e) - 7*\cos(4*f*x + 4*e) - 7*\cos(2*f*x + 2*e) - 5)*\sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + 8*e + 12*\sin(2*f*x + 2*e)) / ((a^2*c*cos(6*f*x + 6*e)^2 + a^2*c*cos(4*f*x + 4*e)^2 + a^2*c*cos(2*f*x + 2*e)^2 + 4*a^2*c*cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*a^2*c*cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*a^2*c*cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + a^2*c*sin(6*f*x + 6*e)^2 + a^2*c*sin(4*f*x + 4*e)^2 + 2*a^2*c*sin(4*f*x + 4*e)*\sin(2*f*x + 2*e) + a^2*c*sin(2*f*x + 2*e)^2 + 4*a^2*c*sin(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 16*a^2*c*sin(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 + 4*a^2*c*sin(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))^2 - 2*a^2*c*cos(2*f*x + 2*e) + a^2*c - 2*(a^2*c*cos(4*f*x + 4*e) + a^2*c*cos(2*f*x + 2*e) - a^2*c)*\cos(6*f*x + 6*e) + 2*(a^2*c*cos(2*f*x + 2*e) - a^2*c)*\cos(4*f*x + 4*e) + 4*(a^2*c*cos(6*f*x + 6*e) - a^2*c*cos(4*f*x + 4*e) - a^2*c*cos(2*f*x + 2*e) - 4*a^2*c*cos(3/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e)))) + 2*a^2*c*cos(1/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) + a^2*c)*\cos(5/2*\arctan2(\sin(2*f*x + 2*e), \cos(2*f*x + 2*e))) - 8*(a^2*c*cos(6*f*x + 6*e) -
\end{aligned}$$

```

a^2*c*cos(4*f*x + 4*e) - a^2*c*cos(2*f*x + 2*e) + 2*a^2*c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + a^2*c*cos(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 4*(a^2*c*cos(6*f*x + 6*e) - a^2*c*cos(4*f*x + 4*e)
- a^2*c*cos(2*f*x + 2*e) + a^2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 2*(a^2*c*sin(4*f*x + 4*e) + a^2*c*sin(2*f*x + 2*e))*sin(6*f*
x + 6*e) + 4*(a^2*c*sin(6*f*x + 6*e) - a^2*c*sin(4*f*x + 4*e) - a^2*c*sin(2
*f*x + 2*e) - 4*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 2*a^2*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a^2*c*sin(6*f*x + 6*e) - a^2
*c*sin(4*f*x + 4*e) - a^2*c*sin(2*f*x + 2*e) + 2*a^2*c*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 4*(a^2*c*sin(6*f*x + 6*e) - a^2*c*sin(4*f*x + 4*e) - a^2*c*si
n(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(
a)*sqrt(c)*f

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2} \left(c - \frac{c}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

$$3.130 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=151

$$-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx)}{a^2c^2f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

[Out] $1/2*\cot(f*x+e)/a^2/c^2/f/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)}-1/4*\cot(f*x+e)^3/a^2/c^2/f/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)}+\ln(\sin(f*x+e))*\tan(f*x+e)/a^2/c^2/f/(a+a*\sec(f*x+e))^{(1/2)/(c-c*\sec(f*x+e))^{(1/2)}}$

Rubi [A] time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3905, 3473, 3475}

$$-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}} + \frac{\tan(e+fx)}{a^2c^2f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]

[Out] Cot[e + f*x]/(2*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - Cot[e + f*x]^3/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3905

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(m_), x_Symbol] := Dist[(-a*c)^(m + 1/2)*Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c-c \sec(e+fx))^{5/2}} dx &= \frac{\tan(e+fx) \int \cot^5(e+fx) dx}{a^2c^2\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= -\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} - \frac{\tan(e+fx)}{a^2c^2\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} - \frac{4a^2c^2f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}}{4a^2c^2f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \\ &= \frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} - \frac{1}{4a^2c^2f\sqrt{a+a \sec(e+fx)}\sqrt{c-c \sec(e+fx)}} \end{aligned}$$


```

to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
s constant sign by intervals (correct if the argument is real):Check [abs(t
_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep^3+t_nostep were n
ot checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep^3+t_nostep)]Discontinuities at zeroes of t_nostep
^3+t_nostep were not checkedDiscontinuities at zeroes of cos(f*t_nostep+exp
(1)) were not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs
or sign assumes constant sign by intervals (correct if the argument is real
):Check [abs(t_nostep),abs(t_nostep^2-1)]Warning, integration of abs or sig
n assumes constant sign by intervals (correct if the argument is real):Chec
k [abs(t_nostep)]Evaluation time: 3.11index.cc index_m i_lex_is_greater Err
or: Bad Argument Value

```

maple [A] time = 2.48, size = 237, normalized size = 1.57

$$\left(-1 + \cos(fx + e)\right)^3 \left(32 \left(\cos^4(fx + e)\right) \ln\left(-\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) - 32 \left(\cos^4(fx + e)\right) \ln\left(\frac{2}{1 + \cos(fx + e)}\right) - 13 \left(\cos^4(fx + e)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2), x)

[Out] -1/32/f*(-1+cos(f*x+e))^3*(32*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*cos(f*x+e)^4*ln(2/(1+cos(f*x+e)))-13*cos(f*x+e)^4-64*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+64*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-6*cos(f*x+e)^2+32*ln(-(-1+cos(f*x+e))/sin(f*x+e))-32*ln(2/(1+cos(f*x+e)))+11)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/a^3

maxima [B] time = 1.34, size = 1386, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2), x, algorithm="maxima")

[Out] -((f*x + e)*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*cos(6*f*x + 6*e)^2 + 36*(f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin(8*f*x + 8*e)^2 + 16*(f*x + e)*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*sin(4*f*x + 4*e)^2 + 16*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(4*cos(6*f*x + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) - 1) + 2*(f*x - 4*(f*x + e)*cos(6*f*x + 6*e) + 6*(f*x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + 2*sin(6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) - 8*(f*x + 6*(f*x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + sin(4*f*x + 4*e))*cos(6*f*x + 6*e) + 4*(3*f*x - 12*(f*x + e)*cos(2*f*x + 2*e) + 3*e + 2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 8*(f*x + e)*cos(2*f*x + 2*e) - 4*(2*(f*x + e)*sin(6*f*x + 6*e) - 3*(f*x + e)*sin(4*f*x + 4*e) + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(6*f*x + 6*e) - cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - 4*(12*(f*x + e)*sin(4*f*x + 4*e) - 8*(f*x + e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e) - 1)*sin(6*f*x + 6*e) - 4*(12*(f*x + e)*sin(2*f*x + 2*e) + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + e + 4*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e)^2 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c^3*sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*x + 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(2*f*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*f*x + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*c^3)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x +

$4*e) + 2*a^3*c^3*\sin(2*f*x + 2*e))*\sin(8*f*x + 8*e) - 16*(3*a^3*c^3*\sin(4*f*x + 4*e) - 2*a^3*c^3*\sin(2*f*x + 2*e))*\sin(6*f*x + 6*e))*f)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2} \left(c - \frac{c}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),x)

[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)

[Out] Timed out

3.131 $\int (1 + \sec(e + fx))^m (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=92

$$\frac{2^{m+\frac{1}{2}} \tan(e + fx) (c - c \sec(e + fx))^n F_1\left(n + \frac{1}{2}; \frac{1}{2} - m, 1; n + \frac{3}{2}; \frac{1}{2}(1 - \sec(e + fx)), 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{\sec(e + fx) + 1}}$$

[Out] $2^{(1/2+m)} \text{AppellF1}(1/2+n, 1, 1/2-m, 3/2+n, 1-\sec(f*x+e), 1/2-1/2*\sec(f*x+e)) * (c - c*\sec(f*x+e))^n * \tan(f*x+e) / f / (1+2*n) / (1+\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3912, 136}

$$\frac{2^{m+\frac{1}{2}} \tan(e + fx) (c - c \sec(e + fx))^n F_1\left(n + \frac{1}{2}; \frac{1}{2} - m, 1; n + \frac{3}{2}; \frac{1}{2}(1 - \sec(e + fx)), 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{\sec(e + fx) + 1}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Sec}[e + f*x])^m * (c - c*\text{Sec}[e + f*x])^n, x]$

[Out] $(2^{(1/2 + m)} \text{AppellF1}[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - \text{Sec}[e + f*x])/2, 1 - \text{Sec}[e + f*x]] * (c - c*\text{Sec}[e + f*x])^n * \text{Tan}[e + f*x]) / (f*(1 + 2*n)*\text{Sqrt}[1 + \text{Sec}[e + f*x]])$

Rule 136

$\text{Int}[(a + (b*x)^m * (c + (d*x)^n * (e + (f*x)^p))] / x, x] \rightarrow \text{Simp}[(b*e - a*f)^p * (a + b*x)^{m+1} * \text{AppellF1}[m+1, -n, -p, m+2, -(d*(a+b*x))/(b*c - a*d), -(f*(a+b*x))/(b*e - a*f)]] / (b^{p+1} * (m+1) * (b/(b*c - a*d))^n, x) /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0]) && SimplerQ[c + d*x, a + b*x]

Rule 3912

$\text{Int}[(\text{csc}[e + (f*x)] * (b + (a*x)^m) * (\text{csc}[e + (f*x)] * (d + (c*x)^n))] / x, x] \rightarrow \text{Dist}[(a*c*\text{Cot}[e + f*x]) / (f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]] * \text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{m-1/2} * (c + d*x)^{n-1/2} / x, x], x, \text{Csc}[e + f*x]], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (1 + \sec(e + fx))^m (c - c \sec(e + fx))^n dx = -\frac{(c \tan(e + fx)) \text{Subst}\left(\int \frac{(1+x)^{\frac{1}{2}+m} (c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx)\right)}{f\sqrt{1 + \sec(e + fx)}\sqrt{c - c \sec(e + fx)}} \\ = \frac{2^{\frac{1}{2}+m} F_1\left(\frac{1}{2} + n; \frac{1}{2} - m, 1; \frac{3}{2} + n; \frac{1}{2}(1 - \sec(e + fx)), 1 - \sec(e + fx)\right)}{f(1 + 2n)\sqrt{1 + \sec(e + fx)}}$$

Mathematica [F] time = 1.10, size = 0, normalized size = 0.00

$$\int (1 + \sec(e + fx))^m (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-c \sec(fx + e) + c\right)^n \left(\sec(fx + e) + 1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c \sec(fx + e) + c\right)^n \left(\sec(fx + e) + 1\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)

maple [F] time = 2.14, size = 0, normalized size = 0.00

$$\int \left(1 + \sec(fx + e)\right)^m \left(c - c \sec(fx + e)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)

[Out] int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c \sec(fx + e) + c\right)^n \left(\sec(fx + e) + 1\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(e + fx)} + 1\right)^m \left(c - \frac{c}{\cos(e + fx)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n,x)

[Out] int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c \left(\sec(e + fx) - 1\right)\right)^n \left(\sec(e + fx) + 1\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)

[Out] Integral((-c*(sec(e + f*x) - 1))**n*(sec(e + f*x) + 1)**m, x)

3.132 $\int (a + a \sec(e + fx))^m (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=109

$$\frac{c^{2^{n+\frac{1}{2}}} \tan(e + fx) (1 - \sec(e + fx))^{\frac{1}{2}-n} (a \sec(e + fx) + a)^m (c - c \sec(e + fx))^{n-1} F_1\left(m + \frac{1}{2}; \frac{1}{2} - n, 1; m + \frac{3}{2}; \frac{1}{2}(\sec(e + fx) + 1)\right)}{f(2m + 1)}$$

[Out] $2^{(1/2+n)} * c * \text{AppellF1}(1/2+m, 1, 1/2-n, 3/2+m, 1+\sec(f*x+e), 1/2+1/2*\sec(f*x+e)) * (1-\sec(f*x+e))^{(1/2-n)} * (a+a*\sec(f*x+e))^m * (c-c*\sec(f*x+e))^{(-1+n)} * \tan(f*x+e) / f / (1+2*m)$

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3912, 137, 136}

$$\frac{c^{2^{n+\frac{1}{2}}} \tan(e + fx) (1 - \sec(e + fx))^{\frac{1}{2}-n} (a \sec(e + fx) + a)^m (c - c \sec(e + fx))^{n-1} F_1\left(m + \frac{1}{2}; \frac{1}{2} - n, 1; m + \frac{3}{2}; \frac{1}{2}(\sec(e + fx) + 1)\right)}{f(2m + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^m * (c - c*\text{Sec}[e + f*x])^n, x]$

[Out] $(2^{(1/2 + n)} * c * \text{AppellF1}[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + \text{Sec}[e + f*x])/2, 1 + \text{Sec}[e + f*x]] * (1 - \text{Sec}[e + f*x])^{(1/2 - n)} * (a + a*\text{Sec}[e + f*x])^m * (c - c*\text{Sec}[e + f*x])^{(-1 + n)} * \text{Tan}[e + f*x]) / (f*(1 + 2*m))$

Rule 136

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] :> \text{Simp}[(b*e - a*f)^p * (a + b*x)^{m+1} * \text{AppellF1}[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^{(p+1)} * (m+1) * (b/(b*c - a*d))^n), x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplrQ[c + d*x, a + b*x])

Rule 137

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] :> \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * ((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n * (e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 3912

$\text{Int}[(\text{csc}[e + f*x] * (b + a*x))^m * (\text{csc}[e + f*x] * (d + c*x))^n, x_Symbol] :> \text{Dist}[(a*c*\text{Cot}[e + f*x]) / (f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]] * \text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{m-1/2} * (c + d*x)^{n-1/2} / x, x], x, \text{Csc}[e + f*x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (a + a \sec(e + fx))^m (c - c \sec(e + fx))^n dx = - \frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \frac{(a+ax)^{-\frac{1}{2}+m} (c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= - \frac{\left(2^{-\frac{1}{2}+n} ac (c - c \sec(e + fx))^{-1+n} \left(\frac{c - c \sec(e + fx)}{c} \right)^{\frac{1}{2}-n} \tan(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= \frac{2^{\frac{1}{2}+n} c F_1 \left(\frac{1}{2} + m; \frac{1}{2} - n, 1; \frac{3}{2} + m; \frac{1}{2} (1 + \sec(e + fx)), 1 + \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (a + a \sec(e + fx))^m (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral} \left((a \sec(fx + e) + a)^m (-c \sec(fx + e) + c)^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^m (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 2.39, size = 0, normalized size = 0.00

$$\int (a + a \sec(fx + e))^m (c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^m (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^m \left(c - \frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec(e + fx) + 1 \right) \right)^m \left(-c \left(\sec(e + fx) - 1 \right) \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)

[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n, x)

3.133 $\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=101

$$\frac{c^{2^{n+\frac{1}{2}}} \tan(e + fx) (a \sec(e + fx) + a)^3 (1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(\frac{7}{2}; \frac{1}{2} - n, 1; \frac{9}{2}; \frac{1}{2} (\sec(e + fx) + 1), \sec(e + fx) + 1\right)}{7f}$$

[Out] $1/7 * 2^{(1/2+n)} * c * \text{AppellF1}(7/2, 1, 1/2-n, 9/2, 1+\sec(f*x+e), 1/2+1/2*\sec(f*x+e)) * (1-\sec(f*x+e))^{(1/2-n)} * (a+a*\sec(f*x+e))^3 * (c-c*\sec(f*x+e))^{(-1+n)} * \tan(f*x+e) / f$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3912, 137, 136}

$$\frac{c^{2^{n+\frac{1}{2}}} \tan(e + fx) (a \sec(e + fx) + a)^3 (1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(\frac{7}{2}; \frac{1}{2} - n, 1; \frac{9}{2}; \frac{1}{2} (\sec(e + fx) + 1), \sec(e + fx) + 1\right)}{7f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^3 * (c - c*\text{Sec}[e + f*x])^n, x]$

[Out] $(2^{(1/2 + n)} * c * \text{AppellF1}[7/2, 1/2 - n, 1, 9/2, (1 + \text{Sec}[e + f*x])/2, 1 + \text{Sec}[e + f*x]]) * (1 - \text{Sec}[e + f*x])^{(1/2 - n)} * (a + a*\text{Sec}[e + f*x])^3 * (c - c*\text{Sec}[e + f*x])^{(-1 + n)} * \text{Tan}[e + f*x] / (7*f)$

Rule 136

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x] := \text{Simp}[(b*e - a*f)^p * (a + b*x)^{m+1} * \text{AppellF1}[m+1, -n, -p, m+2, -(d*(a+b*x))/(b*c - a*d), -(f*(a+b*x))/(b*e - a*f)]] / (b^{p+1} * (m+1) * (b/(b*c - a*d))^n, x) /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 137

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x] := \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * ((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n * (e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 3912

$\text{Int}[(\text{csc}[e + f*x] + (b + a*x)^m * (\text{csc}[e + f*x] + (d + c*x)^n), x] := \text{Dist}[(a*c*\text{Cot}[e + f*x]) / (f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{m-1/2} * (c + d*x)^{n-1/2} / x, x], x, \text{Csc}[e + f*x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^n dx = - \frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \frac{(a+ax)^{5/2} (c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= - \frac{\left(2^{-\frac{1}{2}+n} ac (c - c \sec(e + fx))^{-1+n} \left(\frac{c - c \sec(e + fx)}{c} \right)^{\frac{1}{2}-n} \tan(e + fx) \right) S}{f \sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2^{\frac{1}{2}+n} c F_1 \left(\frac{7}{2}; \frac{1}{2} - n, 1; \frac{9}{2}; \frac{1}{2} (1 + \sec(e + fx)), 1 + \sec(e + fx) \right) (1 - s)}{f \sqrt{a + a \sec(e + fx)}}$$

Mathematica [F] time = 3.24, size = 0, normalized size = 0.00

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(\left(a^3 \sec(fx + e)^3 + 3a^3 \sec(fx + e)^2 + 3a^3 \sec(fx + e) + a^3 \right) (-c \sec(fx + e) + c)^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3)*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^3 (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 2.86, size = 0, normalized size = 0.00

$$\int (a + a \sec(fx + e))^3 (c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^3 (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^3 \left(c - \frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int 3(-c \sec(e + fx) + c)^n \sec(e + fx) dx + \int 3(-c \sec(e + fx) + c)^n \sec^2(e + fx) dx + \int (-c \sec(e + fx) + c)^n dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**n,x)

[Out] a**3*(Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x)**3, x) + Integral((-c*sec(e + f*x) + c)**n, x))

3.134 $\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=101

$$\frac{c^{2n+\frac{1}{2}} \tan(e + fx) (a \sec(e + fx) + a)^2 (1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(\frac{5}{2}; \frac{1}{2} - n, 1; \frac{7}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right)}{5f}$$

[Out] $\frac{1}{5} 2^{(1/2+n)} c \text{AppellF1}(5/2, 1, 1/2-n, 7/2, 1+\sec(f*x+e), 1/2+1/2*\sec(f*x+e)) * (1-\sec(f*x+e))^{(1/2-n)} * (a+a*\sec(f*x+e))^2 * (c-c*\sec(f*x+e))^{(-1+n)} * \tan(f*x+e) / f$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3912, 137, 136}

$$\frac{c^{2n+\frac{1}{2}} \tan(e + fx) (a \sec(e + fx) + a)^2 (1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(\frac{5}{2}; \frac{1}{2} - n, 1; \frac{7}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right)}{5f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^2 * (c - c*\text{Sec}[e + f*x])^n, x]$

[Out] $(2^{(1/2 + n)} c \text{AppellF1}[5/2, 1/2 - n, 1, 7/2, (1 + \text{Sec}[e + f*x])/2, 1 + \text{Sec}[e + f*x]]) * (1 - \text{Sec}[e + f*x])^{(1/2 - n)} * (a + a*\text{Sec}[e + f*x])^2 * (c - c*\text{Sec}[e + f*x])^{(-1 + n)} * \text{Tan}[e + f*x] / (5*f)$

Rule 136

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] \rightarrow \text{Simp}[(b*e - a*f)^p * (a + b*x)^{m+1} * \text{AppellF1}[m+1, -n, -p, m+2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^{(p+1)} * (m+1) * (b/(b*c - a*d))^n), x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplifierQ[c + d*x, a + b*x])

Rule 137

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] \rightarrow \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * ((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n * (e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplifierQ[c + d*x, a + b*x]

Rule 3912

$\text{Int}[(\text{csc}[e + f*x] * (b + a*x))^m * (\text{csc}[e + f*x] * (d + c*x))^n, x_Symbol] \rightarrow \text{Dist}[(a*c*\text{Cot}[e + f*x]) / (f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]) * \text{Sqrt}[c + d*\text{Csc}[e + f*x]], \text{Subst}[\text{Int}[(a + b*x)^{(m-1/2)} * (c + d*x)^{(n-1/2)} / x, x], x, \text{Csc}[e + f*x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^n dx = - \frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \frac{(a+ax)^{3/2} (c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= - \frac{\left(2^{-\frac{1}{2}+n} ac (c - c \sec(e + fx))^{-1+n} \left(\frac{c - c \sec(e + fx)}{c} \right)^{\frac{1}{2}-n} \tan(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2^{\frac{1}{2}+n} c F_1 \left(\frac{5}{2}; \frac{1}{2} - n, 1; \frac{7}{2}; \frac{1}{2} (1 + \sec(e + fx)), 1 + \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)}}$$

Mathematica [F] time = 1.53, size = 0, normalized size = 0.00

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(\left(a^2 \sec^2(fx + e) + 2a^2 \sec(fx + e) + a^2 \right) (-c \sec(fx + e) + c)^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^2 (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 2.29, size = 0, normalized size = 0.00

$$\int (a + a \sec(fx + e))^2 (c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^2 (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^2 \left(c - \frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2(-c \sec(e + fx) + c)^n \sec(e + fx) dx + \int (-c \sec(e + fx) + c)^n \sec^2(e + fx) dx + \int (-c \sec(e + fx) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**n,x)

[Out] a**2*(Integral(2*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)**n, x))

3.135 $\int (a + a \sec(e + fx))(c - c \sec(e + fx))^n dx$

Optimal. Leaf size=99

$$\frac{c2^{n+\frac{1}{2}} \tan(e + fx)(a \sec(e + fx) + a)(1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(\frac{3}{2}; \frac{1}{2} - n, 1; \frac{5}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right)}{3f}$$

[Out] $1/3*2^{(1/2+n)}*c*AppellF1(3/2, 1, 1/2-n, 5/2, 1+\sec(f*x+e), 1/2+1/2*\sec(f*x+e))*(1-\sec(f*x+e))^{(1/2-n)}*(a+a*\sec(f*x+e))*(c-c*\sec(f*x+e))^{(-1+n)}*\tan(f*x+e)/f$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}} \tan(e + fx)(a \sec(e + fx) + a)(1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(\frac{3}{2}; \frac{1}{2} - n, 1; \frac{5}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right)}{3f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]

[Out] $(2^{(1/2 + n)}*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Sec[e + f*x]])*(1 - Sec[e + f*x])^{(1/2 - n)}*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^{(-1 + n)}*Tan[e + f*x])/(3*f)$

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 137

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)]/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (a + a \sec(e + fx))(c - c \sec(e + fx))^n dx = -\frac{(ac \tan(e + fx)) \operatorname{Subst}\left(\int \frac{\sqrt{a+ax}(c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c - c \sec(e + fx))^{-1+n}\left(\frac{c-c \sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e + fx)\right) \operatorname{Su}}{f\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2^{\frac{1}{2}+n}cF_1\left(\frac{3}{2}; \frac{1}{2} - n, 1; \frac{5}{2}; \frac{1}{2}(1 + \sec(e + fx)), 1 + \sec(e + fx)\right)(1 - \sec(e + fx))}{3}$$

Mathematica [F] time = 1.60, size = 0, normalized size = 0.00

$$\int (a + a \sec(e + fx))(c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(a \sec(fx + e) + a\right)\left(-c \sec(fx + e) + c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)(-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int (a + a \sec(fx + e))(c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)(-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right) \left(c - \frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int (-c \sec(e + fx) + c)^n \sec(e + fx) dx + \int (-c \sec(e + fx) + c)^n dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)

[Out] a*(Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e + f*x) + c)**n, x))

$$3.136 \quad \int \frac{(c - c \sec(e + fx))^n}{a + a \sec(e + fx)} dx$$

Optimal. Leaf size=99

$$\frac{c2^{n+\frac{1}{2}} \tan(e + fx)(1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(-\frac{1}{2}; \frac{1}{2} - n, 1; \frac{1}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right) (c - c \sec(e + fx))}{f(a \sec(e + fx) + a)}$$

[Out] $-2^{(1/2+n)} * c * \text{AppellF1}(-1/2, 1, 1/2-n, 1/2, 1+\sec(f*x+e), 1/2+1/2*\sec(f*x+e)) * (1-\sec(f*x+e))^{(1/2-n)} * (c-c*\sec(f*x+e))^{(-1+n)} * \tan(f*x+e) / f / (a+a*\sec(f*x+e))$

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}} \tan(e + fx)(1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(-\frac{1}{2}; \frac{1}{2} - n, 1; \frac{1}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right) (c - c \sec(e + fx))}{f(a \sec(e + fx) + a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c - c*\text{Sec}[e + f*x])^n / (a + a*\text{Sec}[e + f*x]), x]$

[Out] $-((2^{(1/2 + n)} * c * \text{AppellF1}[-1/2, 1/2 - n, 1, 1/2, (1 + \text{Sec}[e + f*x])/2, 1 + \text{Sec}[e + f*x]] * (1 - \text{Sec}[e + f*x])^{(1/2 - n)} * (c - c*\text{Sec}[e + f*x])^{(-1 + n)} * \text{Tan}[e + f*x]) / (f * (a + a*\text{Sec}[e + f*x])))$

Rule 136

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] :> \text{Simp}[(b*e - a*f)^p * (a + b*x)^{m+1} * \text{AppellF1}[m+1, -n, -p, m+2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^{p+1} * (m+1) * (b/(b*c - a*d))^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& !(\text{GtQ}[d/(d*a - c*b), 0]) \&\& \text{SimplerQ}[c + d*x, a + b*x]$

Rule 137

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] :> \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * (b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)^n * (e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& !\text{GtQ}[b/(b*c - a*d), 0] \&\& !\text{SimplerQ}[c + d*x, a + b*x]$

Rule 3912

$\text{Int}[(\text{csc}[e + f*x] + (f*x)) * (b + a*x)^m * (\text{csc}[e + f*x] + (f*x)) * (d + c*x)^n, x_Symbol] :> \text{Dist}[(a*c*\text{Cot}[e + f*x]) / (f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]] * \text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{m-1/2} * (c + d*x)^{n-1/2} / x, x], x, \text{Csc}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\int \frac{(c - c \sec(e + fx))^n}{a + a \sec(e + fx)} dx = - \frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{3/2}} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= - \frac{\left(2^{-\frac{1}{2}+n} ac (c - c \sec(e + fx))^{-1+n} \left(\frac{c - c \sec(e + fx)}{c} \right)^{\frac{1}{2}-n} \tan(e + fx) \right) \operatorname{Subst} \left(\int \frac{\left(\frac{1-x}{2} \right)^{-\frac{1}{2}}}{x(a+ax)^3} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)}}$$

$$= - \frac{2^{\frac{1}{2}+n} c F_1 \left(-\frac{1}{2}; \frac{1}{2} - n, 1; \frac{1}{2}; \frac{1}{2} (1 + \sec(e + fx)), 1 + \sec(e + fx) \right) (1 - \sec(e + fx))^{\frac{1}{2}}}{f (a + a \sec(e + fx))}$$

Mathematica [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(e + fx))^n}{a + a \sec(e + fx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]

[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]), x]

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(\frac{(-c \sec(fx + e) + c)^n}{a \sec(fx + e) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)

maple [F] time = 2.27, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(fx + e))^n}{a + a \sec(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)

[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^n}{a + \frac{a}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)

[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(-c \sec(e+fx)+c)^n}{\sec(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))*n/(a+a*sec(f*x+e)),x)

[Out] Integral((-c*sec(e + f*x) + c)*n/(sec(e + f*x) + 1), x)/a

$$3.137 \quad \int \frac{(c - c \sec(e + fx))^n}{(a + a \sec(e + fx))^2} dx$$

Optimal. Leaf size=101

$$\frac{c2^{n+\frac{1}{2}} \tan(e + fx)(1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(-\frac{3}{2}; \frac{1}{2} - n, 1; -\frac{1}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right) (c - c \sec(e + fx))}{3f(a \sec(e + fx) + a)^2}$$

[Out] $-1/3*2^{(1/2+n)*c}*AppellF1(-3/2, 1, 1/2-n, -1/2, 1+\sec(f*x+e), 1/2+1/2*\sec(f*x+e))*(1-\sec(f*x+e))^{(1/2-n)}*(c-c*\sec(f*x+e))^{(-1+n)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^2$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3912, 137, 136}

$$\frac{c2^{n+\frac{1}{2}} \tan(e + fx)(1 - \sec(e + fx))^{\frac{1}{2}-n} F_1\left(-\frac{3}{2}; \frac{1}{2} - n, 1; -\frac{1}{2}; \frac{1}{2}(\sec(e + fx) + 1), \sec(e + fx) + 1\right) (c - c \sec(e + fx))}{3f(a \sec(e + fx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]

[Out] $-(2^{(1/2 + n)*c}*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2, 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^{(1/2 - n)}*(c - c*Sec[e + f*x])^{(-1 + n)}*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)$

Rule 136

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 137

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 3912

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(c - c \sec(e + fx))^n}{(a + a \sec(e + fx))^2} dx &= - \frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} \\
&= - \frac{\left(2^{-\frac{1}{2}+n} ac (c - c \sec(e + fx))^{-1+n} \left(\frac{c - c \sec(e + fx)}{c} \right)^{\frac{1}{2}-n} \tan(e + fx) \right) \operatorname{Subst} \left(\int \frac{\left(\frac{1}{2} - \frac{x}{2} \right)^{-\frac{1}{2}+n}}{x(a+ax)^{5/2}} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)}} \\
&= - \frac{2^{\frac{1}{2}+n} c F_1 \left(-\frac{3}{2}; \frac{1}{2} - n, 1; -\frac{1}{2}; \frac{1}{2} (1 + \sec(e + fx)), 1 + \sec(e + fx) \right) (1 - \sec(e + fx))^{\frac{1}{2}-n}}{3f(a + a \sec(e + fx))^2}
\end{aligned}$$

Mathematica [F] time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(e + fx))^n}{(a + a \sec(e + fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]

[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(\frac{(-c \sec(fx + e) + c)^n}{a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral((-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{(a \sec(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)

maple [F] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(fx + e))^n}{(a + a \sec(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)

[Out] `int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{(a \sec(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")`

[Out] `integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^n}{\left(a + \frac{a}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)`

[Out] `int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(-c \sec(e+fx)+c)^n}{\sec^2(e+fx)+2 \sec(e+fx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**2,x)`

[Out] `Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2`

3.138 $\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=172

$$\frac{2a^3 \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} + \frac{6a^3 \tan(e + fx)(c - c \sec(e + fx))^n}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} - \frac{2a^3 \tan(e + fx)(c - c \sec(e + fx))^n}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

[Out] $6a^3(c - c \sec(fx + e))^n \tan(fx + e) / f / (1 + 2n) / (a + a \sec(fx + e))^{1/2} + 2a^3 \text{hypergeom}([1, 1/2 + n], [3/2 + n], 1 - \sec(fx + e)) * (c - c \sec(fx + e))^n \tan(fx + e) / f / (1 + 2n) / (a + a \sec(fx + e))^{1/2} - 2a^3(c - c \sec(fx + e))^{1+n} \tan(fx + e) / c / f / (3 + 2n) / (a + a \sec(fx + e))^{1/2}$

Rubi [A] time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3912, 88, 65}

$$\frac{2a^3 \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} + \frac{6a^3 \tan(e + fx)(c - c \sec(e + fx))^n}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} - \frac{2a^3 \tan(e + fx)(c - c \sec(e + fx))^n}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \text{Sec}[e + f*x])^{5/2} * (c - c \text{Sec}[e + f*x])^n, x]$

[Out] $(6a^3(c - c \text{Sec}[e + f*x])^n \text{Tan}[e + f*x]) / (f * (1 + 2n) * \text{Sqrt}[a + a \text{Sec}[e + f*x]]) + (2a^3 \text{Hypergeometric2F1}[1, 1/2 + n, 3/2 + n, 1 - \text{Sec}[e + f*x]] * (c - c \text{Sec}[e + f*x])^n \text{Tan}[e + f*x]) / (f * (1 + 2n) * \text{Sqrt}[a + a \text{Sec}[e + f*x]]) - (2a^3(c - c \text{Sec}[e + f*x])^{1+n} \text{Tan}[e + f*x]) / (c * f * (3 + 2n) * \text{Sqrt}[a + a \text{Sec}[e + f*x]])$

Rule 65

$\text{Int}[(b \cdot x)^m * ((c) + (d \cdot x)^n), x_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^{n+1} * \text{Hypergeometric2F1}[-m, n+1, n+2, 1 + (d \cdot x)/c] / (d * (n+1) * (-d/(b \cdot c))^m), x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-(d/(b \cdot c)), 0])$

Rule 88

$\text{Int}[(a \cdot x)^m * ((c \cdot x)^n * ((e \cdot x)^p + (f \cdot x)^p)), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x)^m * (c + d \cdot x)^n * (e + f \cdot x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x \ \&\& \ \text{IntegersQ}[m, n] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{GeQ}[n, -1]))$

Rule 3912

$\text{Int}[(\text{csc}[(e \cdot x) + (f \cdot x) * x] * (b \cdot x) + (a \cdot x)^m) * (\text{csc}[(e \cdot x) + (f \cdot x) * x] * (d \cdot x) + (c \cdot x)^n), x_Symbol] \rightarrow \text{Dist}[(a * \text{Cot}[e + f * x]) / (f * \text{Sqrt}[a + b * \text{Csc}[e + f * x]] * \text{Sqrt}[c + d * \text{Csc}[e + f * x]]), \text{Subst}[\text{Int}[(a + b * x)^{m-1/2} * (c + d * x)^{n-1/2} / x, x], x, \text{Csc}[e + f * x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \ \&\& \ \text{EqQ}[b * c + a * d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^n dx = -\frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \frac{(a+ax)^2 (c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= -\frac{(ac \tan(e + fx)) \operatorname{Subst} \left(\int \left(3a^2 (c - cx)^{-\frac{1}{2}+n} + \frac{a^2 (c-cx)^{-\frac{1}{2}+n}}{x} \right) dx, x, \sec(e + fx) \right)}{f \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}}$$

$$= \frac{6a^3 (c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n) \sqrt{a + a \sec(e + fx)}} - \frac{2a^3 (c - c \sec(e + fx))^{n+1}}{cf(3 + 2n) \sqrt{a + a \sec(e + fx)}}$$

$$= \frac{6a^3 (c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n) \sqrt{a + a \sec(e + fx)}} + \frac{2a^3 {}_2F_1 \left(1, \frac{1}{2} + n; \frac{3}{2} + n; \frac{c - c \sec(e + fx)}{a} \right)}{f(1 + 2n) \sqrt{a + a \sec(e + fx)}}$$

Mathematica [F] time = 8.54, size = 0, normalized size = 0.00

$$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(\left(a^2 \sec^2(fx + e) + 2a^2 \sec(fx + e) + a^2 \right) \sqrt{a \sec(fx + e) + a} (-c \sec(fx + e) + c)^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^{\frac{5}{2}} (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 1.74, size = 0, normalized size = 0.00

$$\int (a + a \sec(fx + e))^{\frac{5}{2}} (c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^{\frac{5}{2}} (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{\frac{5}{2}} \left(c - \frac{c}{\cos(e + fx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**n,x)

[Out] Timed out

3.139 $\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=119

$$\frac{2a^2 \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} + \frac{2a^2 \tan(e + fx)(c - c \sec(e + fx))^n}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

[Out] $2*a^2*(c-c*\sec(f*x+e))^n*\tan(f*x+e)/f/(1+2*n)/(a+a*\sec(f*x+e))^{(1/2)}+2*a^2*$
 hypergeom([1, 1/2+n], [3/2+n], 1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/
 (1+2*n)/(a+a*sec(f*x+e))^(1/2)

Rubi [A] time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3909, 3912, 65}

$$\frac{2a^2 \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} + \frac{2a^2 \tan(e + fx)(c - c \sec(e + fx))^n}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]

[Out] $(2*a^2*(c - c*\text{Sec}[e + f*x])^n*\text{Tan}[e + f*x])/(f*(1 + 2*n)*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^2*\text{Hypergeometric2F1}[1, 1/2 + n, 3/2 + n, 1 - \text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^n*\text{Tan}[e + f*x])/(f*(1 + 2*n)*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 65

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 3909

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Simp[(-2*a^2*Cot[e + f*x]*(c + d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LeQ[n, -2^(-1)]

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^n dx &= \frac{2a^2(c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} + a \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^n dx \\ &= \frac{2a^2(c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} - \frac{(a^2 c \tan(e + fx)) \operatorname{Subst}\left(\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^n dx\right)}{f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^2(c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} + \frac{2a^2 {}_2F_1\left(1, \frac{1}{2} + n; \frac{3}{2} + n; 1 - \frac{c \sec(e + fx)}{a}\right)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [F] time = 11.83, size = 0, normalized size = 0.00

$$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(a \sec(fx + e) + a\right)^{\frac{3}{2}} \left(-c \sec(fx + e) + c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^{\frac{3}{2}} (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 1.73, size = 0, normalized size = 0.00

$$\int (a + a \sec(fx + e))^{\frac{3}{2}} (c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^{\frac{3}{2}} (-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + f x)} \right)^{3/2} \left(c - \frac{c}{\cos(e + f x)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**n,x)

[Out] Timed out

3.140 $\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^n dx$

Optimal. Leaf size=68

$$\frac{2a \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

[Out] 2*a*hypergeom([1, 1/2+n], [3/2+n], 1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3912, 65}

$$\frac{2a \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]

[Out] (2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])

Rule 65

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 3912

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^n dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}} \\ &= \frac{2a {}_2F_1\left(1, \frac{1}{2} + n; \frac{3}{2} + n; 1 - \sec(e + fx)\right) (c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]

[Out] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n, x]

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a \sec(fx + e) + a}(-c \sec(fx + e) + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(fx + e) + a}(-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int \sqrt{a + a \sec(fx + e)}(c - c \sec(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)

[Out] int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(fx + e) + a}(-c \sec(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}}\left(c - \frac{c}{\cos(e + fx)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n,x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e + fx) + 1)}(-c(\sec(e + fx) - 1))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**n,x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**n, x)

$$3.141 \quad \int \frac{(c - c \sec(e + fx))^n}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal. Leaf size=139

$$\frac{2 \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} - \frac{\tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

[Out] -hypergeom([1, 1/2+n], [3/2+n], 1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n], [3/2+n], 1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3912, 86, 65, 68}

$$\frac{2 \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}} - \frac{\tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; 1 - \sec(e + fx)\right)}{f(2n + 1)\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]], x]

[Out] -((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])) + (2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])

Rule 65

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 68

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x)/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 86

Int[((e_) + (f_)*(x_))^(p_)/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 3912

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(a*c*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2))/x, x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{(c - c \sec(e + fx))^n}{\sqrt{a + a \sec(e + fx)}} dx = -\frac{(ac \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}}$$

$$= -\frac{(c \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}} + \frac{(ac \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}}$$

$$= -\frac{{}_2F_1\left(1, \frac{1}{2} + n; \frac{3}{2} + n; \frac{1}{2}(1 - \sec(e + fx))\right)(c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} + \frac{{}_2F_1\left(1, \frac{1}{2} + n; \frac{3}{2} + n; \frac{1}{2}(1 - \sec(e + fx))\right)(c - c \sec(e + fx))^n \tan(e + fx)}{f(1 + 2n)\sqrt{a + a \sec(e + fx)}} + \dots$$

Mathematica [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(e + fx))^n}{\sqrt{a + a \sec(e + fx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]], x]

[Out] Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]], x]

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(-c \sec(fx + e) + c)^n}{\sqrt{a \sec(fx + e) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2), x, algorithm="fricas")

[Out] integral((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{\sqrt{a \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2), x, algorithm="giac")

[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)

maple [F] time = 1.93, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(fx + e))^n}{\sqrt{a + a \sec(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2), x)

[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{\sqrt{a \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^n}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e + fx) - 1))^n}{\sqrt{a(\sec(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral((-c*(sec(e + f*x) - 1))**n/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.142 \quad \int \frac{(c - c \sec(e + fx))^n}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal. Leaf size=205

$$\frac{(5 - 2n) \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; \frac{1}{2}(1 - \sec(e + fx))\right)}{4af(2n + 1)\sqrt{a \sec(e + fx) + a}} + \frac{2 \tan(e + fx)(c - c \sec(e + fx))^n}{af(2n + 1)}$$

[Out] $-1/4*(5-2*n)*\text{hypergeom}([1, 1/2+n], [3/2+n], 1/2-1/2*\text{sec}(f*x+e))*(c-c*\text{sec}(f*x+e))^n*\text{tan}(f*x+e)/a/f/(1+2*n)/(a+a*\text{sec}(f*x+e))^{(1/2)+2}*\text{hypergeom}([1, 1/2+n], [3/2+n], 1-\text{sec}(f*x+e))*(c-c*\text{sec}(f*x+e))^n*\text{tan}(f*x+e)/a/f/(1+2*n)/(a+a*\text{sec}(f*x+e))^{(1/2)-1/2}*(c-c*\text{sec}(f*x+e))^n*\text{tan}(f*x+e)/a/f/(1+\text{sec}(f*x+e))/(a+a*\text{sec}(f*x+e))^{(1/2)}$

Rubi [A] time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3912, 103, 156, 65, 68}

$$\frac{(5 - 2n) \tan(e + fx)(c - c \sec(e + fx))^n {}_2F_1\left(1, n + \frac{1}{2}; n + \frac{3}{2}; \frac{1}{2}(1 - \sec(e + fx))\right)}{4af(2n + 1)\sqrt{a \sec(e + fx) + a}} + \frac{2 \tan(e + fx)(c - c \sec(e + fx))^n}{af(2n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $-((5 - 2*n)*\text{Hypergeometric2F1}[1, 1/2 + n, 3/2 + n, (1 - \text{Sec}[e + f*x])/2]*(c - c*\text{Sec}[e + f*x])^n*\text{Tan}[e + f*x])/(4*a*f*(1 + 2*n)*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*\text{Hypergeometric2F1}[1, 1/2 + n, 3/2 + n, 1 - \text{Sec}[e + f*x]]*(c - c*\text{Sec}[e + f*x])^n*\text{Tan}[e + f*x])/(a*f*(1 + 2*n)*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - ((c - c*\text{Sec}[e + f*x])^n*\text{Tan}[e + f*x])/(2*a*f*(1 + \text{Sec}[e + f*x])*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 65

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 68

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x]$

Rule 3912

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_.)]*(b_.) + (a_.)^{(m_.)}*(\text{csc}[e_.] + (f_.)*(x_.)]*(d_.) + (c_.)^{(n_.)}), x_Symbol] :> \text{Dist}[(a*c*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]])*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]], \text{Subst}[\text{Int}[(a + b*x)^{(m - 1/2)}*(c + d*x)^{(n - 1/2)}/x, x], x, \text{Csc}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(c - c \sec(e + fx))^n}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{(ac \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x(a+ax)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(c - c \sec(e + fx))^n \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx) \text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}\left(2ac - \frac{1}{2}a\right)}{x(a+ax)} dx, x, \sec(e + fx)\right)}{2af\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(c - c \sec(e + fx))^n \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} - \frac{(c \tan(e + fx)) \text{Subst}\left(\int \frac{(c-cx)^{-\frac{1}{2}+n}}{x} dx, x, \sec(e + fx)\right)}{af\sqrt{a + a \sec(e + fx)}\sqrt{c - c \sec(e + fx)}} \\ &= -\frac{(5 - 2n) {}_2F_1\left(1, \frac{1}{2} + n; \frac{3}{2} + n; \frac{1}{2}(1 - \sec(e + fx))\right) (c - c \sec(e + fx))^n \tan(e + fx)}{4af(1 + 2n)\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [F] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(e + fx))^n}{(a + a \sec(e + fx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2), x]

[Out] Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2), x]

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \sec(fx + e) + a}(-c \sec(fx + e) + c)^n}{a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{(a \sec(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \frac{(c - c \sec(fx + e))^n}{(a + a \sec(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)

[Out] int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c \sec(fx + e) + c)^n}{(a \sec(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c - \frac{c}{\cos(e+fx)}\right)^n}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e + fx) - 1))^n}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)

[Out] Integral((-c*(sec(e + f*x) - 1))**n/(a*(sec(e + f*x) + 1))**(3/2), x)

$$3.143 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{c+c \sec(e+fx)} dx$$

Optimal. Leaf size=91

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} - \frac{\sqrt{2} \sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{cf}$$

[Out] $2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/c/f-\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}*a^{(1/2)}/c/f$

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {21, 3776, 3774, 203, 3795}

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} - \frac{\sqrt{2} \sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c*f) - (\text{Sqrt}[2]*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])])/(c*f)$

Rule 21

Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3776

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \sec(e + fx)}}{c + c \sec(e + fx)} dx &= \frac{a \int \frac{1}{\sqrt{a + a \sec(e + fx)}} dx}{c} \\
&= \frac{\int \sqrt{a + a \sec(e + fx)} dx}{c} - \frac{a \int \frac{\sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx}{c} \\
&= \frac{(2a) \text{Subst} \left(\int \frac{1}{a + x^2} dx, x, -\frac{a \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}} \right)}{cf} + \frac{(2a) \text{Subst} \left(\int \frac{1}{2a + x^2} dx, x, -\frac{a \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}} \right)}{cf} \\
&= \frac{2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}} \right)}{cf} - \frac{\sqrt{2} \sqrt{a} \tan^{-1} \left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{2} \sqrt{a + a \sec(e + fx)}} \right)}{cf}
\end{aligned}$$

Mathematica [C] time = 0.65, size = 133, normalized size = 1.46

$$\frac{i\sqrt{1 + e^{2i(e+fx)}} \sqrt{a(\sec(e + fx) + 1)} \left(\sinh^{-1} \left(e^{i(e+fx)} \right) - \sqrt{2} \tanh^{-1} \left(\frac{-1 + e^{i(e+fx)}}{\sqrt{2} \sqrt{1 + e^{2i(e+fx)}}} \right) - \tanh^{-1} \left(\sqrt{1 + e^{2i(e+fx)}} \right) \right)}{cf(1 + e^{i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]

[Out] ((-I)*Sqrt[1 + E^((2*I)*(e + f*x))])*(ArcSinh[E^(I*(e + f*x))]) - Sqrt[2]*ArcTanh[(-1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] - ArcTanh[Sqrt[1 + E^((2*I)*(e + f*x))]]*Sqrt[a*(1 + Sec[e + f*x])]/(c*(1 + E^(I*(e + f*x))))*f

fricas [A] time = 0.56, size = 293, normalized size = 3.22

$$\frac{\sqrt{2} \sqrt{-a} \log \left(\frac{2 \sqrt{2} \sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + 3a \cos(fx+e)^2 + 2a \cos(fx+e) - a}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right) + 2 \sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 - 2 \sqrt{-a} \cos(fx+e) + a}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)}{2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(sqrt(2)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), (sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)4*sqrt(2)*(-1/8*sqrt(-a)*ln((sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2)/c-1/4*a*sqrt(-a)*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a)/sqrt(2)/c/abs(a))*sign(cos(f*x+exp(1)))/f

maple [A] time = 1.78, size = 141, normalized size = 1.55

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \sqrt{-\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \left(\ln \left(-\frac{-\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) + \cos(fx+e) - 1}{\sin(fx+e)} \right) + \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e)}{2\cos(fx+e)} \right) \right)}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)

[Out] -1/c/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))+2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec(fx + e) + a}}{c \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{c + \frac{c}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a \sec(e+fx)+a}}{\sec(e+fx)+1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)

[Out] Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) + 1), x)/c

$$3.144 \quad \int \frac{(c+d \sec(e+fx))^{3/2}}{a+a \sec(e+fx)} dx$$

Optimal. Leaf size=231

$$\frac{2c \cot(e+fx) \sqrt{\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}} \sqrt{\frac{d(\sec(e+fx)+1)}{c+d \sec(e+fx)}} (c+d \sec(e+fx)) \Pi\left(\frac{c}{c+d}; \sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \middle| \frac{c-d}{c+d}\right) (c-d) \sqrt{\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}}}{af\sqrt{c+d}}$$

[Out] $-2*c*\cot(f*x+e)*\text{EllipticPi}((c+d)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}, c/(c+d), ((c-d)/(c+d))^{(1/2)})*(c+d*\sec(f*x+e))*(-d*(1-\sec(f*x+e))/(c+d*\sec(f*x+e)))^{(1/2)}*(d*(1+\sec(f*x+e))/(c+d*\sec(f*x+e)))^{(1/2)}/a/f/(c+d)^{(1/2)}-(c-d)*\text{EllipticE}(\tan(f*x+e)/(1+\sec(f*x+e)), ((c-d)/(c+d))^{(1/2)})*(1/(1+\sec(f*x+e)))^{(1/2)}*(c+d*\sec(f*x+e))^{(1/2)}/a/f/((c+d*\sec(f*x+e))/(c+d)/(1+\sec(f*x+e)))^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3927, 3780, 3968}

$$\frac{2c \cot(e+fx) \sqrt{\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}} \sqrt{\frac{d(\sec(e+fx)+1)}{c+d \sec(e+fx)}} (c+d \sec(e+fx)) \Pi\left(\frac{c}{c+d}; \sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \middle| \frac{c-d}{c+d}\right) (c-d) \sqrt{\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}}}{af\sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*\text{Sec}[e + f*x])^{(3/2)}/(a + a*\text{Sec}[e + f*x]), x]$

[Out] $(-2*c*\text{Cot}[e + f*x]*\text{EllipticPi}[c/(c + d), \text{ArcSin}[\text{Sqrt}[c + d]/\text{Sqrt}[c + d*\text{Sec}[e + f*x]]], (c - d)/(c + d)*\text{Sqrt}[-((d*(1 - \text{Sec}[e + f*x]))/(c + d*\text{Sec}[e + f*x]))]*\text{Sqrt}[(d*(1 + \text{Sec}[e + f*x]))/(c + d*\text{Sec}[e + f*x])]*(c + d*\text{Sec}[e + f*x])]/(a*\text{Sqrt}[c + d]*f) - ((c - d)*\text{EllipticE}[\text{ArcSin}[\text{Tan}[e + f*x]/(1 + \text{Sec}[e + f*x])], (c - d)/(c + d)*\text{Sqrt}[(1 + \text{Sec}[e + f*x])^{(-1)}]*\text{Sqrt}[c + d*\text{Sec}[e + f*x])]/(a*f*\text{Sqrt}[(c + d*\text{Sec}[e + f*x])]/((c + d)*(1 + \text{Sec}[e + f*x]))))$

Rule 3780

$\text{Int}[\text{Sqrt}[\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(2*(a + b*\text{Csc}[c + d*x])*\text{Sqrt}[(b*(1 + \text{Csc}[c + d*x]))/(a + b*\text{Csc}[c + d*x])]*\text{Sqrt}[-((b*(1 - \text{Csc}[c + d*x]))/(a + b*\text{Csc}[c + d*x]))]*\text{EllipticPi}[a/(a + b), \text{ArcSin}[\text{Rt}[a + b, 2]/\text{Sqrt}[a + b*\text{Csc}[c + d*x]]], (a - b)/(a + b)]/(d*\text{Rt}[a + b, 2]*\text{Cot}[c + d*x]), x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3927

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(3/2)}/(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] \rightarrow \text{Dist}[a/c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[(b*c - a*d)/c, \text{Int}[(\text{Csc}[e + f*x]*\text{Sqrt}[a + b*\text{Csc}[e + f*x]])/(c + d*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ (\text{EqQ}[a^2 - b^2, 0] \ || \ \text{EqQ}[c^2 - d^2, 0])$

Rule 3968

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] \rightarrow -\text{Simp}[(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[c/(c + d*\text{Csc}[e + f*x])]*\text{EllipticE}[\text{ArcSin}[(c*\text{Cot}[e + f*x])/(c + d*\text{Csc}[e + f*x])], -((b*c - a*d)/(b*c + a*d))]/(d*f*\text{Sqrt}[(c*d*(a + b*\text{Csc}[e + f*x]))/(b*c + a*d)*(c + d*\text{Csc}[e + f*x])]), x] /; \text{FreeQ}\{a, b, c, d, e,$

$f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]$

Rubi steps

$$\int \frac{(c + d \sec(e + fx))^{3/2}}{a + a \sec(e + fx)} dx = \frac{c \int \sqrt{c + d \sec(e + fx)} dx}{a} + (-c + d) \int \frac{\sec(e + fx) \sqrt{c + d \sec(e + fx)}}{a + a \sec(e + fx)} dx$$

$$= -\frac{2c \cot(e + fx) \Pi\left(\frac{c}{c+d}; \sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \middle| \frac{c-d}{c+d}\right) \sqrt{-\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}} \sqrt{\frac{d(1+\sec(e+fx))}{c+d \sec(e+fx)}}}{a\sqrt{c+d} f}$$

Mathematica [B] time = 18.23, size = 810, normalized size = 3.51

$$\frac{(c + d \sec(e + fx))^{3/2} \left(2 \sec\left(\frac{1}{2}(e + fx)\right) \left(d \sin\left(\frac{1}{2}(e + fx)\right) - c \sin\left(\frac{1}{2}(e + fx)\right)\right) - 2(d - c) \sin(e + fx)\right) \cos^2\left(\frac{e}{2}\right)}{f(d + c \cos(e + fx))(\sec(e + fx)a + a)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]

[Out] (Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(2*Sec[(e + f*x)/2]*(-c*Sin[(e + f*x)/2]) + d*Sin[(e + f*x)/2]) - 2*(-c + d)*Sin[e + f*x]))/(f*(d + c*Cos[e + f*x])*(a + a*Sec[e + f*x])) + (2*Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(c^2*Tan[(e + f*x)/2] - d^2*Tan[(e + f*x)/2] - 2*c^2*Tan[(e + f*x)/2]^3 + 2*c*d*Tan[(e + f*x)/2]^3 + c^2*Tan[(e + f*x)/2]^5 - 2*c*d*Tan[(e + f*x)/2]^5 + d^2*Tan[(e + f*x)/2]^5 - 4*c^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] - 4*c^2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]]], (c - d)/(c + d)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + (c^2 - d^2)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 2*c*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)))/(f*(d + c*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*x]]*(a + a*Sec[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(d \sec(fx + e) + c)^{\frac{3}{2}}}{a \sec(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec(fx + e) + c)^{\frac{3}{2}}}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)

maple [A] time = 1.92, size = 295, normalized size = 1.28

$$\sqrt{\frac{d+c \cos(fx+e)}{\cos(fx+e)}} \sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}} (1 + \cos(fx + e))^2 \sqrt{\frac{d+c \cos(fx+e)}{(1+\cos(fx+e))(c+d)}} (-1 + \cos(fx + e)) \left(2 \operatorname{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x)

[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*((d+c*cos(f*x+e))/(1+cos(f*x+e)))/(c+d)^(1/2)*(-1+cos(f*x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c^2-2*c*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c^2-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d^2-4*c^2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+d))^(1/2)))/(d+c*cos(f*x+e))/sin(f*x+e)^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec(fx + e) + c)^{\frac{3}{2}}}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^{\frac{3}{2}}}{a + \frac{a}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)),x)

[Out] int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)+1} dx + \int \frac{d\sqrt{c+d\sec(e+fx)} \sec(e+fx)}{\sec(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x) + Integral(d*sqrt(c + d*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

$$3.145 \quad \int \frac{\sqrt{c+d \sec(e+fx)}}{a+a \sec(e+fx)} dx$$

Optimal. Leaf size=225

$$\frac{2 \cot(e+fx) \sqrt{-\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}} \sqrt{\frac{d(\sec(e+fx)+1)}{c+d \sec(e+fx)}} (c+d \sec(e+fx)) \Pi\left(\frac{c}{c+d}; \sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \Big|_{\frac{c-d}{c+d}}\right) \sqrt{\frac{1}{\sec(e+fx)+1}}}{af\sqrt{c+d}}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((c+d)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}, c/(c+d), ((c-d)/(c+d))^{(1/2)})*(c+d*\sec(f*x+e))*(-d*(1-\sec(f*x+e))/(c+d*\sec(f*x+e)))^{(1/2)}*(d*(1+\sec(f*x+e))/(c+d*\sec(f*x+e)))^{(1/2)}/a/f/(c+d)^{(1/2)}-\text{EllipticE}(\tan(f*x+e)/(1+\sec(f*x+e)), ((c-d)/(c+d))^{(1/2)})*(1/(1+\sec(f*x+e)))^{(1/2)}*(c+d*\sec(f*x+e))^{(1/2)}/a/f/((c+d*\sec(f*x+e))/(c+d)/(1+\sec(f*x+e)))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3925, 3780, 3968}

$$\frac{2 \cot(e+fx) \sqrt{-\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}} \sqrt{\frac{d(\sec(e+fx)+1)}{c+d \sec(e+fx)}} (c+d \sec(e+fx)) \Pi\left(\frac{c}{c+d}; \sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \Big|_{\frac{c-d}{c+d}}\right) \sqrt{\frac{1}{\sec(e+fx)+1}}}{af\sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]), x]

[Out] $(-2*\text{Cot}[e + f*x]*\text{EllipticPi}[c/(c + d), \text{ArcSin}[\text{Sqrt}[c + d]/\text{Sqrt}[c + d*\text{Sec}[e + f*x]]], (c - d)/(c + d)*\text{Sqrt}[-((d*(1 - \text{Sec}[e + f*x]))/(c + d*\text{Sec}[e + f*x]))]*\text{Sqrt}[(d*(1 + \text{Sec}[e + f*x]))/(c + d*\text{Sec}[e + f*x])]*(c + d*\text{Sec}[e + f*x])]/(a*\text{Sqrt}[c + d]*f) - (\text{EllipticE}[\text{ArcSin}[\text{Tan}[e + f*x]/(1 + \text{Sec}[e + f*x])], (c - d)/(c + d)*\text{Sqrt}[(1 + \text{Sec}[e + f*x])^{-1}]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]]]/(a*f*\text{Sqrt}[(c + d*\text{Sec}[e + f*x])/(c + d)*(1 + \text{Sec}[e + f*x])]))$

Rule 3780

Int[Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*(a + b*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)]/(d*Rt[a + b, 2]*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3925

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])

Rule 3968

Int[(csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c + d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))]/(d*f*Sqrt[(c*d*(a + b*Csc[e + f*x]))/(b*c + a*d)*(c + d*Csc[e + f*x])]), x] /; FreeQ[{a, b, c, d, e,

$f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]$

Rubi steps

$$\int \frac{\sqrt{c + d \sec(e + fx)}}{a + a \sec(e + fx)} dx = \frac{\int \sqrt{c + d \sec(e + fx)} dx}{a} - \int \frac{\sec(e + fx) \sqrt{c + d \sec(e + fx)}}{a + a \sec(e + fx)} dx$$

$$= \frac{2 \cot(e + fx) \Pi\left(\frac{c}{c+d}; \sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \middle| \frac{c-d}{c+d}\right) \sqrt{\frac{d(1-\sec(e+fx))}{c+d \sec(e+fx)}} \sqrt{\frac{d(1+\sec(e+fx))}{c+d \sec(e+fx)}}}{a \sqrt{c+d} f}$$

Mathematica [A] time = 8.75, size = 178, normalized size = 0.79

$$\frac{4 \cos^4\left(\frac{1}{2}(e + fx)\right) \sqrt{\frac{1}{\sec(e+fx)+1}} \sqrt{c + d \sec(e + fx)} \left(2(c - d) F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right) \middle| \frac{c-d}{c+d}\right) + (c + d) E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right) \middle| \frac{c-d}{c+d}\right)\right)}{af(c + d)(\cos(e + fx) + 1)^2 \sqrt{\frac{c \cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]

[Out] $(-4 \cos^4\left(\frac{1}{2}(e + fx)\right) \sqrt{\frac{1}{\sec(e+fx)+1}} \sqrt{c + d \sec(e + fx)} \left(2(c - d) F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right) \middle| \frac{c-d}{c+d}\right) + (c + d) E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right) \middle| \frac{c-d}{c+d}\right)\right) / (a \sqrt{c+d} f) + \dots$

fricas [F] time = 22.53, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{d \sec(fx + e) + c}}{a \sec(fx + e) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec(fx + e) + c}}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)

maple [A] time = 1.98, size = 285, normalized size = 1.27

$$\frac{\sqrt{\frac{d+c \cos(fx+e)}{\cos(fx+e)}} \sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}} \sqrt{\frac{d+c \cos(fx+e)}{(1+\cos(fx+e))(c+d)}} (1 + \cos(fx + e))^2 \left(2 \text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}, \sqrt{\frac{c-d}{c+d}}\right) c - 2 E\left(\sin^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c+d \sec(e+fx)}}\right) \middle| \frac{c-d}{c+d}\right)\right)}{af(c + d)(\cos(e + fx) + 1)^2 \sqrt{\frac{c \cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x)`

[Out] `-1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))^(1/2)*(1+cos(f*x+e))^2*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c-2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+d))^(1/2)))*(-1+cos(f*x+e))/(d+c*cos(f*x+e))/sin(f*x+e)^2`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec(fx + e) + c}}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c + \frac{d}{\cos(e+fx)}}}{a + \frac{a}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)),x)`

[Out] `int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{c+d \sec(e+fx)}}{\sec(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)`

[Out] `Integral(sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x)/a`

$$3.146 \quad \int \frac{1}{(a+a \sec(e+fx))\sqrt{c+d \sec(e+fx)}} dx$$

Optimal. Leaf size=319

$$\frac{2\sqrt{c+d} \cot(e+fx) \sqrt{\frac{d(1-\sec(e+fx))}{c+d}} \sqrt{\frac{d(\sec(e+fx)+1)}{c-d}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{c+d}}\right) \middle| \frac{c+d}{c-d}\right) - 2\sqrt{c+d} \cot(e+fx) \sqrt{\frac{d(1-\sec(e+fx))}{c+d}}}{af(c-d)}$$

[Out] $2*\cot(f*x+e)*\text{EllipticF}((c+d*\sec(f*x+e))^{1/2}/(c+d)^{1/2},((c+d)/(c-d))^{1/2})*(c+d)^{1/2}*(d*(1-\sec(f*x+e))/(c+d))^{1/2}*(-d*(1+\sec(f*x+e))/(c-d))^{1/2}/a/(c-d)/f-2*\cot(f*x+e)*\text{EllipticPi}((c+d*\sec(f*x+e))^{1/2}/(c+d)^{1/2},(c+d)/c,((c+d)/(c-d))^{1/2})*(c+d)^{1/2}*(d*(1-\sec(f*x+e))/(c+d))^{1/2}*(-d*(1+\sec(f*x+e))/(c-d))^{1/2}/a/c/f-\text{EllipticE}(\tan(f*x+e)/(1+\sec(f*x+e)),((c-d)/(c+d))^{1/2})*(1/(1+\sec(f*x+e)))^{1/2}*(c+d*\sec(f*x+e))^{1/2}/a/(c-d)/f/((c+d*\sec(f*x+e))/(c+d)/(1+\sec(f*x+e)))^{1/2}$

Rubi [A] time = 0.37, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3929, 3921, 3784, 3832, 3968}

$$\frac{2\sqrt{c+d} \cot(e+fx) \sqrt{\frac{d(1-\sec(e+fx))}{c+d}} \sqrt{\frac{d(\sec(e+fx)+1)}{c-d}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{c+d}}\right) \middle| \frac{c+d}{c-d}\right) - 2\sqrt{c+d} \cot(e+fx) \sqrt{\frac{d(1-\sec(e+fx))}{c+d}}}{af(c-d)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] $(2*\text{Sqrt}[c + d]*\text{Cot}[e + f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sec}[e + f*x]]]/\text{Sqrt}[c + d]],(c + d)/(c - d))*\text{Sqrt}[(d*(1 - \text{Sec}[e + f*x]))/(c + d)]*\text{Sqrt}[-((d*(1 + \text{Sec}[e + f*x]))/(c - d))]/(a*(c - d)*f) - (2*\text{Sqrt}[c + d]*\text{Cot}[e + f*x]*\text{EllipticPi}[(c + d)/c, \text{ArcSin}[\text{Sqrt}[c + d*\text{Sec}[e + f*x]]]/\text{Sqrt}[c + d]],(c + d)/(c - d))*\text{Sqrt}[(d*(1 - \text{Sec}[e + f*x]))/(c + d)]*\text{Sqrt}[-((d*(1 + \text{Sec}[e + f*x]))/(c - d))]/(a*c*f) - (\text{EllipticE}[\text{ArcSin}[\text{Tan}[e + f*x]/(1 + \text{Sec}[e + f*x])],(c - d)/(c + d)]*\text{Sqrt}[(1 + \text{Sec}[e + f*x])^{-1}]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])/(a*(c - d)*f*\text{Sqrt}[(c + d*\text{Sec}[e + f*x])/((c + d)*(1 + \text{Sec}[e + f*x]))])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D

ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 3929

Int[1/(Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))), x_Symbol] :> Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])

Rule 3968

Int[(csc[(e_) + (f_)*(x_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)])/(csc[(e_) + (f_)*(x_)]*(d_) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c + d*Csc[e + f*x])], -(b*c - a*d)/(b*c + a*d))]/(d*f*Sqrt[(c*d*(a + b*Csc[e + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))\sqrt{c + d \sec(e + fx)}} dx = -\frac{\int \frac{-ac+ad-ad \sec(e+fx)}{\sqrt{c+d \sec(e+fx)}} dx}{a^2(c-d)} + \frac{a \int \frac{\sec(e+fx)\sqrt{c+d \sec(e+fx)}}{a+a \sec(e+fx)} dx}{-ac+ad}$$

$$= -\frac{E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}\sqrt{c+d \sec(e+fx)}}}{a(c-d)f\sqrt{\frac{c+d \sec(e+fx)}{(c+d)(1+\sec(e+fx))}}} + \dots$$

$$= \frac{2\sqrt{c+d} \cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{c+d}}\right)\middle|\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}}{a(c-d)f}$$

Mathematica [A] time = 12.71, size = 187, normalized size = 0.59

$$\frac{2 \cos^2\left(\frac{1}{2}(e + fx)\right) \sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}} \sec(e + fx) \sqrt{\frac{c \cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}} \left(2(c - 2d)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right)\middle|\frac{c-d}{c+d}\right) + (c + a)\right)}{af(d - c)\sqrt{c + d \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
 [Out] (2*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 2*(c - 2*d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 4*(-c + d)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sec[e + f*x])/(a*(-c + d)*f*Sqrt[c + d*Sec[e + f*x]])

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d \sec(fx + e) + c}}{ad \sec(fx + e)^2 + ac + (ac + ad) \sec(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*sec(f*x + e) + c)/(a*d*sec(f*x + e)^2 + a*c + (a*c + a*d)*sec(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(fx + e) + a) \sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

maple [A] time = 2.10, size = 327, normalized size = 1.03

$$\sqrt{\frac{d+c \cos(fx+e)}{\cos(fx+e)}} \sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}} \sqrt{\frac{d+c \cos(fx+e)}{(1+\cos(fx+e))(c+d)}} (1 + \cos(fx + e))^2 \left(2 \operatorname{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}, \sqrt{\frac{c-d}{c+d}}\right) c - 4 E\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x)

[Out] -1/a/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*(1+cos(f*x+e))^2*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*c-4*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((c-d)/(c+d))^(1/2))-4*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+d))^(1/2))+4*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((c-d)/(c+d))^(1/2))*d)*(-1+cos(f*x+e))/(d+c*cos(f*x+e))/sin(f*x+e)^2/(c-d)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(fx + e) + a) \sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right) \sqrt{c + \frac{d}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)),x)

[Out] int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+d \sec(e+fx)} \sec(e+fx) + \sqrt{c+d \sec(e+fx)}} dx$$

a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(c + d*sec(e + f*x))*sec(e + f*x) + sqrt(c + d*sec(e + f*x))), x)/a

3.147 $\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^4 dx$

Optimal. Leaf size=271

$$\frac{2a^{3/2}c^4 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} - \frac{2d^4 \tan(e + fx)(a - a \sec(e + fx))^3}{7a^2 f \sqrt{a \sec(e + fx) + a}} - \frac{2d^2(6c^2 + 8cd + 3d^2) \tan(e + fx)}{3f\sqrt{a \sec(e + fx) + a}}$$

```
[Out] 2*a*d*(2*c+d)*(2*c^2+2*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(6*c^2+8*c*d+3*d^2)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*d^3*(4*c+3*d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2/7*d^4*(a-a*sec(f*x+e))^3*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^4*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3940, 88, 63, 206}

$$\frac{2a^{3/2}c^4 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} - \frac{2d^4 \tan(e + fx)(a - a \sec(e + fx))^3}{7a^2 f \sqrt{a \sec(e + fx) + a}} - \frac{2d^2(6c^2 + 8cd + 3d^2) \tan(e + fx)}{3f\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (2*a*d*(2*c + d)*(2*c^2 + 2*c*d + d^2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^(3/2)*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*d^2*(6*c^2 + 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]) + (2*d^3*(4*c + 3*d)*(a - a*Sec[e + f*x])^2*Tan[e + f*x])/(5*a*f*Sqrt[a + a*Sec[e + f*x]]) - (2*d^4*(a - a*Sec[e + f*x])^3*Tan[e + f*x])/(7*a^2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x]
```

$e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]$

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^4 dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^4}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{d(2c+d)(2c^2+2cd+d^2)}{\sqrt{a-ax}} + \frac{c^4}{x\sqrt{a-ax}} - \frac{d^2(6c^2+d^2)}{\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}} \\ &= \frac{2ad(2c + d)(2c^2 + 2cd + d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^2(6c^2 + 8cd + 3d^2)}{3f\sqrt{a - a \sec(e + fx)}} \\ &= \frac{2ad(2c + d)(2c^2 + 2cd + d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^2(6c^2 + 8cd + 3d^2)}{3f\sqrt{a - a \sec(e + fx)}} \\ &= \frac{2ad(2c + d)(2c^2 + 2cd + d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^{3/2}c^4 \tanh^{-1}\left(\frac{\sqrt{a-a}}{\sqrt{a-a}}\right)}{f\sqrt{a - a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 14.30, size = 587, normalized size = 2.17

$$\frac{\cos^4(e + fx) \sec\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))^4 \left(\frac{4}{105} \sec(e + fx) \left(105c^2d^2 \sin\left(\frac{1}{2}(e + fx)\right) + \dots\right)\right)}{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]

[Out] (Cos[e + f*x]^4*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^4*((8*d*(105*c^3 + 105*c^2*d + 56*c*d^2 + 12*d^3)*Sin[(e + f*x)/2])/105 + (2*d^4*Sec[e + f*x]^3*Sin[(e + f*x)/2])/7 + (4*Sec[e + f*x]^2*(14*c*d^3*Sin[(e + f*x)/2] + 3*d^4*Sin[(e + f*x)/2]))/35 + (4*Sec[e + f*x]*(105*c^2*d^2*Sin[(e + f*x)/2] + 56*c*d^3*Sin[(e + f*x)/2] + 12*d^4*Sin[(e + f*x)/2]))/105)/(f*(d + c*Cos[e + f*x])^4 - (8*(-3 - 2*Sqrt[2])*c^4*Cos[(e + f*x)/4]^4*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(e + f*x)/2])]/(1 + Cos[(e + f*x)/2]))*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])]/(1 + Cos[(e + f*x)/2]))*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Cos[e + f*x]^3*(EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(e + f*x)/2])*Sec[(e + f*x)/4]^2*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^4*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(f*(d + c*Cos[e + f*x])^4)


```

))))*tan(1/2*(f*x+exp(1)))^2+1/3087000*(-1543500*sqrt(2)*a^4*d^4*sign(cos(
f*x+exp(1)))-10290000*sqrt(2)*a^4*c*d^3*sign(cos(f*x+exp(1)))-21609000*sqrt
(2)*a^4*c^2*d^2*sign(cos(f*x+exp(1)))-18522000*sqrt(2)*a^4*c^3*d*sign(cos(f
*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2+1/14700*(7350*sqrt(2)*a^4*d^4*sign(co
s(f*x+exp(1)))+29400*sqrt(2)*a^4*c*d^3*sign(cos(f*x+exp(1)))+44100*sqrt(2)*
a^4*c^2*d^2*sign(cos(f*x+exp(1)))+29400*sqrt(2)*a^4*c^3*d*sign(cos(f*x+exp(
1)))))/sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)/(-a*tan(1/2*(f*x+exp(1)))^2+a)^3*
tan(1/2*(f*x+exp(1)))-1/2*a*sqrt(-a)*c^4*sign(cos(f*x+exp(1)))*ln(abs(2*(sq
rt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2
)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f
*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a))/abs(a))/f

```

maple [B] time = 2.13, size = 546, normalized size = 2.01

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(105\sqrt{2} (\cos^3(fx+e)) \sin(fx+e) \left(\frac{-2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{7}{2}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{-2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \right) c^4 + 315$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x)

```

[Out] 1/840/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(105*2^(1/2)*cos(f*x+e)^3*sin(f
*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^4+315*2^(1/2)*cos(f*x+e)^
2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^4+315*2^(1/2)*cos(
f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^4+105*2^(1/2)
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^4*sin(f*x+e)-6720*cos(f*x+e)^4
*c^3*d-6720*cos(f*x+e)^4*c^2*d^2-3584*cos(f*x+e)^4*c*d^3-768*cos(f*x+e)^4*d
^4+6720*cos(f*x+e)^3*c^3*d+3360*cos(f*x+e)^3*c^2*d^2+1792*cos(f*x+e)^3*c*d^
3+384*cos(f*x+e)^3*d^4+3360*cos(f*x+e)^2*c^2*d^2+448*cos(f*x+e)^2*c*d^3+96*
cos(f*x+e)^2*d^4+1344*cos(f*x+e)*c*d^3+48*cos(f*x+e)*d^4+240*d^4)/cos(f*x+e
)^3/sin(f*x+e)

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \left(c + \frac{d}{\cos(e + fx)} \right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4,x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**4*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**4, x)
```

3.148 $\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^3 dx$

Optimal. Leaf size=205

$$\frac{2a^{3/2}c^3 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} + \frac{2ad(3c^2 + 3cd + d^2) \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}} - \frac{2d^2(3c + 2d) \tan(e + fx)(a - a \sec(e + fx))}{3f\sqrt{a \sec(e + fx) + a}}$$

[Out] $2*a*d*(3*c^2+3*c*d+d^2)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2/3*d^2*(3*c+2*d)*(a-a*\sec(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/5*d^3*(a-a*\sec(f*x+e))^2*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}+2*a^{(3/2)}*c^3*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3940, 88, 63, 206}

$$\frac{2a^{3/2}c^3 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} + \frac{2ad(3c^2 + 3cd + d^2) \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}} - \frac{2d^2(3c + 2d) \tan(e + fx)(a - a \sec(e + fx))}{3f\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]

[Out] $(2*a*d*(3*c^2 + 3*c*d + d^2)*\operatorname{Tan}[e + f*x])/(f*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) + (2*a^{(3/2)}*c^3*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]/\operatorname{Sqrt}[a]]*\operatorname{Tan}[e + f*x])/(f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - (2*d^2*(3*c + 2*d)*(a - a*\operatorname{Sec}[e + f*x])*\operatorname{Tan}[e + f*x])/(3*f*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) + (2*d^3*(a - a*\operatorname{Sec}[e + f*x])^2*\operatorname{Tan}[e + f*x])/(5*a*f*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]])$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

&& IntegerQ[m - 1/2]

Rubi steps

$$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^3 dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{d(3c^2+3cd+d^2)}{\sqrt{a-ax}} + \frac{c^3}{x\sqrt{a-ax}} - \frac{d^2(3c+2d)}{a}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2ad(3c^2 + 3cd + d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^2(3c + 2d)(a - a \sec(e + fx))}{3f\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2ad(3c^2 + 3cd + d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^2(3c + 2d)(a - a \sec(e + fx))}{3f\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2ad(3c^2 + 3cd + d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^{3/2}c^3 \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a}}$$

Mathematica [C] time = 14.19, size = 517, normalized size = 2.52

$$\frac{\cos^3(e + fx) \sec\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))^3 \left(\frac{2}{15}d(45c^2 + 30cd + 8d^2) \sin\left(\frac{1}{2}(e + fx)\right)\right)}{f(c \cos(e + fx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]

[Out] (Cos[e + f*x]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^3*((2*d*(45*c^2 + 30*c*d + 8*d^2)*Sin[(e + f*x)/2])/15 + (2*d^3*Sec[e + f*x]^2*Sin[(e + f*x)/2])/5 + (2*Sec[e + f*x]*(15*c*d^2*Sin[(e + f*x)/2] + 4*d^3*Sin[(e + f*x)/2]))/15)/(f*(d + c*Cos[e + f*x])^3) - (8*(-3 - 2*Sqrt[2])*c^3*Cos[(e + f*x)/4]^4*Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Cos[e + f*x]^2*(EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*Sqrt[(-1 - Sqrt[2] + (2 + Sqrt[2])*Cos[(e + f*x)/2])*Sec[(e + f*x)/4]^2*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^3*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2])/(f*(d + c*Cos[e + f*x])^3)

fricas [A] time = 0.48, size = 392, normalized size = 1.91

$$\frac{15 \left(c^3 \cos^3(fx + e) + c^3 \cos^2(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right)}{15 \left(f \cos(fx + e) \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] [1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*d^3 + (45*c^
2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*d^2 + 4*d^3)*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*
cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a
)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(
f*x + e))) - (3*d^3 + (45*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*
d^2 + 4*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)2*(2*((1/450*(-300*a^3*d^3*sign(cos(f*x+exp(1))))-1800*a^3*c*d^2*sign(cos(f
*x+exp(1)))-2700*a^3*c^2*d*sign(cos(f*x+exp(1))))/sqrt(2)+1/450*(210*a^3*d^
3*sign(cos(f*x+exp(1)))+450*a^3*c*d^2*sign(cos(f*x+exp(1)))+1350*a^3*c^2*d*
sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2/sqrt(2))*tan(1/2*(f*x+exp(1)
))^2+1/450*(450*a^3*d^3*sign(cos(f*x+exp(1)))+1350*a^3*c*d^2*sign(cos(f*x+e
xp(1)))+1350*a^3*c^2*d*sign(cos(f*x+exp(1))))/sqrt(2))/sqrt(-a*tan(1/2*(f*x
+exp(1)))^2+a)/(-a*tan(1/2*(f*x+exp(1)))^2+a)^2*tan(1/2*(f*x+exp(1)))-1/2*a
*sqrt(-a)*c^3*sign(cos(f*x+exp(1)))*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))
^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-
a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*ab
s(a)-6*a))/abs(a))/f
maple [B] time = 1.96, size = 389, normalized size = 1.90
```

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(15 \left(\cos^2(fx+e) \right) \sin(fx+e) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \left(\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{5}{2}} c^3 + 30 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x)
```



```
[Out] -1/60/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(15*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*c^3+30*cos(f*x+e)*sin(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*c^3+15*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*2^(1/2)*c^3*sin(f*x+e)+360*cos(f*x+e)^3*c^2*d+240*cos(f*x+e)^3*c*d^2+64*cos(f*x+e)^3*d^3-360*cos(f*x+e)^2*c^2*d-120*cos(f*x+e)^2*c*d^2-32*cos(f*x+e)^2*d^3-120*cos(f*x+e)*c*d^2-8*cos(f*x+e)*d^3-24*d^3)/cos(f*x+e)^2/sin(f*x+e)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \left(c + \frac{d}{\cos(e + fx)} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3,x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**3*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3, x)
```

3.149 $\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^2 dx$

Optimal. Leaf size=144

$$\frac{2a^{3/2}c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} + \frac{2ad(2c + d) \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}} - \frac{2d^2 \tan(e + fx)(a - a \sec(e + fx))}{3f\sqrt{a \sec(e + fx) + a}}$$

[Out] $2*a*d*(2*c+d)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2/3*d^2*(a-a*\sec(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*a^{(3/2)}*c^2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3940, 88, 63, 206}

$$\frac{2a^{3/2}c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} + \frac{2ad(2c + d) \tan(e + fx)}{f\sqrt{a \sec(e + fx) + a}} - \frac{2d^2 \tan(e + fx)(a - a \sec(e + fx))}{3f\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]

[Out] $(2*a*d*(2*c + d)*\operatorname{Tan}[e + f*x])/(f*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) + (2*a^{(3/2)}*c^2*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]/\operatorname{Sqrt}[a]]*\operatorname{Tan}[e + f*x])/(f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - (2*d^2*(a - a*\operatorname{Sec}[e + f*x])*\operatorname{Tan}[e + f*x])/(3*f*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]])$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^n/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^2 dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{d(2c+d)}{\sqrt{a-ax}} + \frac{c^2}{x\sqrt{a-ax}} - \frac{d^2\sqrt{a-ax}}{a}\right) dx, x\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= \frac{2ad(2c + d) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^2(a - a \sec(e + fx)) \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}} \\
&= \frac{2ad(2c + d) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^2(a - a \sec(e + fx)) \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}} \\
&= \frac{2ad(2c + d) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^{3/2}c^2 \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 6.59, size = 444, normalized size = 3.08

$$\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}} \sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)} \csc^3\left(\frac{1}{2}(e+fx)\right) \sec\left(\frac{1}{2}(e+fx)\right) \sqrt{a(\sec(e+fx)+1)} (c+d \sec(e+fx))^2$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]

[Out] (Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^2*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]^2*(256*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(c + d - 2*c*Sin[(e + f*x)/2]^2)^2 + 1024*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(d^2 + c*d*(2 - 3*Sin[(e + f*x)/2]^2) + c^2*(1 - 3*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]^4)) - (7*Sqrt[2]*(-3*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]] + Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(3 + 4*Sin[(e + f*x)/2]^2))*(15*d^2 + 10*c*d*(3 - 2*Sin[(e + f*x)/2]^2) + c^2*(15 - 20*Sin[(e + f*x)/2]^2 + 12*Sin[(e + f*x)/2]^4))/Sqrt[Sin[(e + f*x)/2]^2]))/(672*f*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2))

fricas [A] time = 0.48, size = 320, normalized size = 2.22

$$\frac{3\left(c^2 \cos^2(fx + e) + c^2 \cos(fx + e)\right) \sqrt{-a} \log\left(\frac{2a \cos(fx+e)^2 - 2\sqrt{-a} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + a \cos(fx+e) - a}{\cos(fx+e)+1}\right)}{3\left(f \cos^2(fx + e) + f \cos(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \left(c + \frac{d}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**2*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2, x)

3.150 $\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx)) dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2ad \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

[Out] $2*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/f+2*a*d*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3915, 3774, 203, 3792}

$$\frac{2\sqrt{a}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2ad \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]

[Out] $(2*\text{Sqrt}[a]*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/f + (2*a*d*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3915

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps


```
[In] integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)), x)
```

$$3.151 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{c+d \sec(e+fx)} dx$$

Optimal. Leaf size=105

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} - \frac{2\sqrt{a} \sqrt{d} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e+fx)}{\sqrt{c+d} \sqrt{a \sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

[Out] $2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}/c/f-2*\arctan(a^{(1/2)}*d^{(1/2)}*\tan(f*x+e)/(c+d)^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*a^{(1/2)}*d^{(1/2)}/c/f/(c+d)^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3925, 3774, 203, 3967, 205}

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf} - \frac{2\sqrt{a} \sqrt{d} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e+fx)}{\sqrt{c+d} \sqrt{a \sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(c*f) - (2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Tan}[e + f*x])/(\text{Sqrt}[c + d]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(c*\text{Sqrt}[c + d]*f))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3925

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_) / (csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] := Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])

Rule 3967

Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]) / (csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

]

Rubi steps

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{c + d \sec(e + fx)} dx = \frac{\int \sqrt{a + a \sec(e + fx)} dx}{c} - \frac{d \int \frac{\sec(e + fx) \sqrt{a + a \sec(e + fx)}}{c + d \sec(e + fx)} dx}{c}$$

$$= \frac{(2a) \text{Subst}\left(\int \frac{1}{a + x^2} dx, x, -\frac{a \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{cf} + \frac{(2ad) \text{Subst}\left(\int \frac{1}{ac + ad + dx^2} dx, x, -\frac{a}{\sqrt{a + a \sec(e + fx)}}\right)}{cf}$$

$$= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{cf} - \frac{2\sqrt{a} \sqrt{d} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e + fx)}{\sqrt{c + d} \sqrt{a + a \sec(e + fx)}}\right)}{c\sqrt{c + d} f}$$

Mathematica [C] time = 25.07, size = 2650, normalized size = 25.24

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]

```
[Out] (-4*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2]))*Cos[(e + f*x)/2]]/(1 + Cos[(e + f*x)/2]))*(d + c*Cos[e + f*x])*(c*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(EllipticPi[-((( -3 + 2*Sqrt[2]))*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((( -3 + 2*Sqrt[2]))*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2])))*Sec[(e + f*x)/2]*((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(2*(d + c*Cos[e + f*x]))) *Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]/(c*(c + d)*f*(c + d*Sec[e + f*x])*((Sqrt[2]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2]))*Cos[(e + f*x)/2]]/(1 + Cos[(e + f*x)/2]))*(c*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(EllipticPi[-((( -3 + 2*Sqrt[2]))*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((( -3 + 2*Sqrt[2]))*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2])))*Sqrt[Sec[e + f*x]]*Tan[(e + f*x)/4]/(c*(c + d)*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]) + (2*Sqrt[2]*Cos[(e + f*x)/4]*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2]))*Cos[(e + f*x)/2]]/(1 + Cos[(e + f*x)/2]))*(c*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(EllipticPi[-((( -3 + 2*Sqrt[2]))*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((( -3 + 2*Sqrt[2]))*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2])))*Sqrt[Sec[e + f*x]]*Sin[(e + f*x)/4]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]/(c*(c + d)) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*(c*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(EllipticPi[-((( -3 + 2*Sqrt[2]))*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + Elli
```

```
pticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*Sqrt[Sec[e + f*x]]*(((-2 + Sqrt[2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])) + ((-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])*Sin[(e + f*x)/2])/(2*(1 + Cos[(e + f*x)/2])^2))*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]/(c*(c + d)*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]) - (2*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])])*(c*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*(c + d)*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + d*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[e + f*x]^(3/2)*Sin[e + f*x]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]/(c*(c + d)) - (4*Sqrt[2]*Cos[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])/(1 + Cos[(e + f*x)/2])]*Sqrt[Sec[e + f*x]]*Sqrt[3 - 2*Sqrt[2] - Tan[(e + f*x)/4]^2]*((c*Sec[(e + f*x)/4]^2)/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])])*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])]) - ((c + d)*Sec[(e + f*x)/4]^2)/(2*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])])*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])])*(1 - ((-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2]))) + d*(Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])])*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])])*(1 + ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)))) + Sec[(e + f*x)/4]^2/(4*Sqrt[3 - 2*Sqrt[2]]*Sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*Sqrt[2])])*Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2)/(3 - 2*Sqrt[2])])*(1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)/((3 - 2*Sqrt[2])*(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d)))))/(c*(c + d))
```

fricas [A] time = 0.87, size = 669, normalized size = 6.37

$$\left[\frac{\sqrt{-\frac{ad}{c+d}} \log \left(\frac{2(c+d)\sqrt{-\frac{ad}{c+d}} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + (ac+2ad) \cos(fx+e)^2 - ad + (ac+ad) \cos(fx+e)}{c \cos(fx+e)^2 + (c+d) \cos(fx+e) + d} \right) + \sqrt{-a} \log \left(\frac{2a \cos(fx+e)}{c \cos(fx+e)^2 + (c+d) \cos(fx+e) + d} \right)}{cf} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")

```
[Out] [(sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), (2*sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*(sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sq
```


[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{c + \frac{d}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e + fx) + 1)}}{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x)), x)

$$3.152 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=219

$$\frac{a^{3/2} \sqrt{d} (3c + 2d) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a - a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}} \right)}{c^2 f (c + d)^{3/2} \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2a^{3/2} \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a - a \sec(e+fx)}}{\sqrt{a}} \right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} - \frac{cf(c + d)}{c^2 f (c + d)^{3/2} \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}}$$

[Out] $-a*d*\tan(f*x+e)/c/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)+2*a^{(3/2)}* \arctanh((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/c^2/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a^{(3/2)}*(3*c+2*d)*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)/(c+d)^{(1/2)})}*d^{(1/2)}*\tan(f*x+e)/c^2/(c+d)^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 103, 156, 63, 206, 208}

$$\frac{a^{3/2} \sqrt{d} (3c + 2d) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a - a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}} \right)}{c^2 f (c + d)^{3/2} \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2a^{3/2} \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a - a \sec(e+fx)}}{\sqrt{a}} \right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} - \frac{cf(c + d)}{c^2 f (c + d)^{3/2} \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]

[Out] $(2*a^{(3/2)}*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^{(3/2)}*Sqrt[d]*(3*c + 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d]])*Tan[e + f*x]/(c^2*(c + d)^{(3/2)}*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a*d*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c + d \sec(e + fx))^2} dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}(c+dx)^2} dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{ad \tan(e + fx)}{c(c + d)f \sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} - \frac{(a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{a(c+d)}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c(c + d)f \sqrt{a - a \sec(e + fx)}}$$

$$= -\frac{ad \tan(e + fx)}{c(c + d)f \sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} - \frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a - a \sec(e + fx)}}$$

$$= -\frac{ad \tan(e + fx)}{c(c + d)f \sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} + \frac{(2a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1 - \frac{x^2}{a}} dx, x, \sec(e + fx)\right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a - a \sec(e + fx)}}$$

$$= \frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{a^{3/2} \sqrt{d} (3c + 2d) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{c^2 (c + d)^{3/2} f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

Mathematica [C] time = 28.58, size = 2907, normalized size = 13.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e +
f*x]))*(-((d*Sin[(e + f*x)/2])/(c^2*(c + d))) + (d^2*Sin[(e + f*x)/2])/(c^
2*(c + d)*(d + c*Cos[e + f*x])))/(f*(c + d*Sec[e + f*x])^2) - (2*Sqrt[2]*C
os[(e + f*x)/4]^2*Sqrt[(-1 + Sqrt[2] - (-2 + Sqrt[2])*Cos[(e + f*x)/2])]/(1
+ Cos[(e + f*x)/2]))*(d + c*Cos[e + f*x])^2*(c*(2*c + d)*EllipticF[ArcSin[T
an[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*Ellipt
icPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*
Sqrt[2]] + d*(3*c + 2*d)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*
Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]]
, 17 - 12*Sqrt[2]] + EllipticPi[(-(-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2
```


$$\begin{aligned}
&]*\text{Sqrt}[c*(c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - \\
& 12*\text{Sqrt}[2]])*\text{Sec}[(e + f*x)/2]*((\text{Cos}[(e + f*x)/2]*\text{Sqrt}[\text{Sec}[e + f*x]])/(2*(c \\
& + d)*(d + c*\text{Cos}[e + f*x])) + (\text{Cos}[(3*(e + f*x))/2]*\text{Sqrt}[\text{Sec}[e + f*x]])/(2* \\
& (c + d)*(d + c*\text{Cos}[e + f*x])) + (d*\text{Cos}[(3*(e + f*x))/2]*\text{Sqrt}[\text{Sec}[e + f*x]]) \\
& / (2*c*(c + d)*(d + c*\text{Cos}[e + f*x])))*\text{Sec}[e + f*x]^2*\text{Sqrt}[a*(1 + \text{Sec}[e + f*x \\
&])]*\text{Sqrt}[3 - 2*\text{Sqrt}[2] - \text{Tan}[(e + f*x)/4]^2]/(c^2*(c + d)^2*f*(c + d*\text{Sec}[e \\
& + f*x])^2*((\text{Sqrt}[(-1 + \text{Sqrt}[2] - (-2 + \text{Sqrt}[2])*\text{Cos}[(e + f*x)/2])/(1 + \text{Cos} \\
& [(e + f*x)/2]))*(c*(2*c + d)*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{S} \\
& \text{qrt}[2]]], 17 - 12*\text{Sqrt}[2]] - 4*(c + d)^2*\text{EllipticPi}[-3 + 2*\text{Sqrt}[2], \text{ArcSin} \\
& \text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + d*(3*c + 2*d)*(El \\
& lipticPi[-(((-3 + 2*\text{Sqrt}[2])*(c + d))/(3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] - d) \\
&), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{Ellipti \\
& cPi}[((-3 + 2*\text{Sqrt}[2])*(c + d))/(-3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] + d), \text{ArcS} \\
& in[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]])*\text{Sqrt}[\text{Sec}[e + f \\
& *x]]*\text{Tan}[(e + f*x)/4]/(\text{Sqrt}[2]*c^2*(c + d)^2*\text{Sqrt}[3 - 2*\text{Sqrt}[2] - \text{Tan}[(e + \\
& f*x)/4]^2) + (\text{Sqrt}[2]*\text{Cos}[(e + f*x)/4]*\text{Sqrt}[(-1 + \text{Sqrt}[2] - (-2 + \text{Sqrt}[2] \\
&)*\text{Cos}[(e + f*x)/2])/(1 + \text{Cos}[(e + f*x)/2])])*(c*(2*c + d)*\text{EllipticF}[\text{ArcSin}[\text{T} \\
& an[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - 4*(c + d)^2*\text{Ellipt \\
& icPi}[-3 + 2*\text{Sqrt}[2], \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12* \\
& \text{Sqrt}[2]] + d*(3*c + 2*d)*(\text{EllipticPi}[-(((-3 + 2*\text{Sqrt}[2])*(c + d))/(3*c + 2* \\
& \text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] - d)), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]] \\
& , 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 + 2*\text{Sqrt}[2])*(c + d))/(-3*c + 2*\text{Sqrt}[2] \\
&]*\text{Sqrt}[c*(c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - \\
& 12*\text{Sqrt}[2]))*\text{Sqrt}[\text{Sec}[e + f*x]]*\text{Sin}[(e + f*x)/4]*\text{Sqrt}[3 - 2*\text{Sqrt}[2] - \text{Tan} \\
& (e + f*x)/4]^2)/(c^2*(c + d)^2) - (\text{Sqrt}[2]*\text{Cos}[(e + f*x)/4]^2*(c*(2*c + d) \\
& *\text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - \\
& 4*(c + d)^2*\text{EllipticPi}[-3 + 2*\text{Sqrt}[2], \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2* \\
& \text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + d*(3*c + 2*d)*(\text{EllipticPi}[-(((-3 + 2*\text{Sqrt}[2]) \\
&)*(c + d))/(3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] - d)), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{S} \\
& \text{qrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 + 2*\text{Sqrt}[2])*(c + d) \\
&)/(-3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - \\
& 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]))*\text{Sqrt}[\text{Sec}[e + f*x]]*(((-2 + \text{Sqrt}[2])*\text{Sin}[(e \\
& + f*x)/2])/(2*(1 + \text{Cos}[(e + f*x)/2]))) + ((-1 + \text{Sqrt}[2] - (-2 + \text{Sqrt}[2])*\text{Co} \\
& s[(e + f*x)/2])* \text{Sin}[(e + f*x)/2])/(2*(1 + \text{Cos}[(e + f*x)/2])^2)*\text{Sqrt}[3 - 2* \\
& \text{Sqrt}[2] - \text{Tan}[(e + f*x)/4]^2)/(c^2*(c + d)^2*\text{Sqrt}[(-1 + \text{Sqrt}[2] - (-2 + \text{Sqr} \\
& t[2])* \text{Cos}[(e + f*x)/2])/(1 + \text{Cos}[(e + f*x)/2])]) - (\text{Sqrt}[2]*\text{Cos}[(e + f*x)/ \\
& 4]^2*\text{Sqrt}[(-1 + \text{Sqrt}[2] - (-2 + \text{Sqrt}[2])* \text{Cos}[(e + f*x)/2])/(1 + \text{Cos}[(e + f* \\
& x)/2])])*(c*(2*c + d)*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]] \\
& , 17 - 12*\text{Sqrt}[2]] - 4*(c + d)^2*\text{EllipticPi}[-3 + 2*\text{Sqrt}[2], \text{ArcSin}[\text{Tan}[(e + \\
& f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + d*(3*c + 2*d)*(\text{EllipticPi} \\
& [-(((-3 + 2*\text{Sqrt}[2])*(c + d))/(3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] - d)), \text{ArcSi} \\
& n[\text{Tan}[(e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 \\
& + 2*\text{Sqrt}[2])*(c + d))/(-3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] + d), \text{ArcSin}[\text{Tan}[(\\
& e + f*x)/4]/\text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]))*\text{Sec}[e + f*x]^(3/2)*\text{Sin} \\
& [e + f*x]*\text{Sqrt}[3 - 2*\text{Sqrt}[2] - \text{Tan}[(e + f*x)/4]^2)/(c^2*(c + d)^2) - (2*\text{Sq} \\
& rt[2]*\text{Cos}[(e + f*x)/4]^2*\text{Sqrt}[(-1 + \text{Sqrt}[2] - (-2 + \text{Sqrt}[2])* \text{Cos}[(e + f*x)/ \\
& 2])/(1 + \text{Cos}[(e + f*x)/2])])*\text{Sqrt}[\text{Sec}[e + f*x]]*\text{Sqrt}[3 - 2*\text{Sqrt}[2] - \text{Tan}[(e \\
& + f*x)/4]^2]*((c*(2*c + d)*\text{Sec}[(e + f*x)/4]^2)/(4*\text{Sqrt}[3 - 2*\text{Sqrt}[2]])*\text{Sqrt} \\
& [1 - \text{Tan}[(e + f*x)/4]^2/(3 - 2*\text{Sqrt}[2])])*\text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2])* \text{Tan}[(e \\
& + f*x)/4]^2)/(3 - 2*\text{Sqrt}[2])]) - ((c + d)^2*\text{Sec}[(e + f*x)/4]^2)/(\text{Sqrt}[3 - 2 \\
& * \text{Sqrt}[2]]*\text{Sqrt}[1 - \text{Tan}[(e + f*x)/4]^2/(3 - 2*\text{Sqrt}[2])])*\text{Sqrt}[1 - ((17 - 12*\text{S} \\
& \text{qrt}[2])* \text{Tan}[(e + f*x)/4]^2)/(3 - 2*\text{Sqrt}[2])])*(1 - ((-3 + 2*\text{Sqrt}[2])* \text{Tan}[(e \\
& + f*x)/4]^2)/(3 - 2*\text{Sqrt}[2])) + d*(3*c + 2*d)*(\text{Sec}[(e + f*x)/4]^2/(4*\text{Sqrt} \\
& [3 - 2*\text{Sqrt}[2]]*\text{Sqrt}[1 - \text{Tan}[(e + f*x)/4]^2/(3 - 2*\text{Sqrt}[2])])*\text{Sqrt}[1 - ((17 - \\
& 12*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4]^2)/(3 - 2*\text{Sqrt}[2])])*(1 + ((-3 + 2*\text{Sqrt}[2])*(c \\
& + d)* \text{Tan}[(e + f*x)/4]^2)/((3 - 2*\text{Sqrt}[2])*(3*c + 2*\text{Sqrt}[2]*\text{Sqrt}[c*(c - d)] \\
& - d))) + \text{Sec}[(e + f*x)/4]^2/(4*\text{Sqrt}[3 - 2*\text{Sqrt}[2]]*\text{Sqrt}[1 - \text{Tan}[(e + f*x) \\
& /4]^2/(3 - 2*\text{Sqrt}[2])])*\text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4]^2)/(3 - \\
& 2*\text{Sqrt}[2])])*(1 - ((-3 + 2*\text{Sqrt}[2])*(c + d)* \text{Tan}[(e + f*x)/4]^2)/((3 - 2*\text{Sqr}
\end{aligned}$$

able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)4*sqrt(2)*((-3*a*sqrt(-a)*d^2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-a*sqrt(-a)*c*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a^2*sqrt(-a)*d^2-a^2*sqrt(-a)*c*d)/(-2*c^3+2*c*d^2)/(c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4-d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4+2*a*c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+6*a*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a^2*c-a^2*d)+1/4*(-2*a*sqrt(-a)*d^2-3*a*sqrt(-a)*c*d)*atan(1/2*(c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a*c+3*a*d)/sqrt(2)/sqrt(-d^2-c*d)/a)/sqrt(2)/sqrt(-d^2-c*d)/a/(c^3+c^2*d)-1/4*a*sqrt(-a)*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a))/sqrt(2)/c^2/abs(a))*sign(cos(f*x+exp(1)))/f

maple [B] time = 3.00, size = 97265, normalized size = 444.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec(fx + e) + a}}{(d \sec(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c + \frac{d}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e + fx) + 1)}}{(c + d \sec(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**2, x)

3.153 $\int \frac{\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^3} dx$

Optimal. Leaf size=287

$$\frac{2a^{3/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{c^3 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{a^{3/2} \sqrt{d} (15c^2 + 20cd + 8d^2) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{4c^3 f (c+d)^{5/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

[Out] $-1/2*a*d*\tan(f*x+e)/c/(c+d)/f/(c+d*\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{1/2}-1/4*a*d*(7*c+4*d)*\tan(f*x+e)/c^2/(c+d)^2/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}+2*a^{3/2}*arctanh((a-a*\sec(f*x+e))^{1/2}/a^{1/2})*\tan(f*x+e)/c^3/f/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}-1/4*a^{3/2}*(15*c^2+20*c*d+8*d^2)*arctanh(d^{1/2}*(a-a*\sec(f*x+e))^{1/2}/a^{1/2}/(c+d)^{1/2})*d^{1/2}*\tan(f*x+e)/c^3/(c+d)^{5/2}/f/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.31, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3940, 103, 151, 156, 63, 206, 208}

$$\frac{a^{3/2} \sqrt{d} (15c^2 + 20cd + 8d^2) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{4c^3 f (c+d)^{5/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2a^{3/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{c^3 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]`

[Out] $(2*a^{3/2}*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^{3/2}*Sqrt[d]*(15*c^2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(4*c^3*(c + d)^{5/2}*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a*d*Tan[e + f*x])/(2*c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]])*(c + d*Sec[e + f*x])^2 - (a*d*(7*c + 4*d)*Tan[e + f*x])/(4*c^2*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]])*(c + d*Sec[e + f*x])$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 103

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])`

Rule 151

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d`

$x)^n(e + fx)^p \text{Simp}[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

Rule 156

$\text{Int}[\frac{((e_.) + (f_.)*(x_))^{(p_)}*((g_.) + (h_.)*(x_))}{((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))}, x_Symbol] := \text{Dist}[(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2]]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3940

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^{(n_.)}, x_Symbol] := \text{Dist}[(a^2*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{(m - 1/2)}*(c + d*x)^n/(x*\text{Sqrt}[a - b*x]), x], x, \text{Csc}[e + f*x]], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \sec(e + fx)}}{(c + d \sec(e + fx))^3} dx &= -\frac{(a^2 \tan(e + fx)) \text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(c+dx)^3} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{ad \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} - \frac{(a \tan(e + fx)) \text{Subst}\left(\int \frac{2}{x} dx, x, \sec(e + fx)\right)}{2c(c + d)f\sqrt{a - a \sec(e + fx)}} \\ &= -\frac{ad \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} - \frac{ad(7c + 4d) \tan(e + fx)}{4c^2(c + d)^2 f\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{ad \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} - \frac{ad(7c + 4d) \tan(e + fx)}{4c^2(c + d)^2 f\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{ad \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} - \frac{ad(7c + 4d) \tan(e + fx)}{4c^2(c + d)^2 f\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{ad \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} - \frac{ad(7c + 4d) \tan(e + fx)}{4c^2(c + d)^2 f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{a^{3/2} \sqrt{d} (15c^2 + 20cd + 8d^2) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{4c^3 (c + d)^{5/2} f \sqrt{a - a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 24.49, size = 3070, normalized size = 10.70

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]

[Out]
$$\begin{aligned} & ((d + c\cos[e + fx])^3 \sec[(e + fx)/2] \sec[e + fx]^3 \sqrt{a(1 + \sec[e + fx])}) \cdot \\ & \left(\frac{(-3d(3c + 2d)\sin[(e + fx)/2])}{(4c^3(c + d)^2) - (d^3\sin[(e + fx)/2])} \right) \cdot \\ & \left(\frac{(2c^3(c + d)(d + c\cos[e + fx])^2) + (11cd^2\sin[(e + fx)/2] + 8d^3\sin[(e + fx)/2])}{(4c^3(c + d)^2(d + c\cos[e + fx]))} \right) \cdot \\ & \left(\frac{1}{(c + d\sec[e + fx])^3} - \frac{(\sqrt{3 - 2\sqrt{2}})\cos[(e + fx)/4]^2(d + c\cos[e + fx])^3}{(c(8c^2 + 9cd + 4d^2)\text{EllipticF}[\text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] - 16(c + d)^3\text{EllipticPi}[-3 + 2\sqrt{2}, \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + d(15c^2 + 20cd + 8d^2)(\text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(3c + 2\sqrt{2})\sqrt{c(c - d)} - d)), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + \text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(-3c + 2\sqrt{2})\sqrt{c(c - d)} + d), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}])]} \right) \cdot \\ & \left(\frac{1}{(c + d)^2(d + c\cos[e + fx])} + \frac{(d\cos[(e + fx)/2]\sqrt{\sec[e + fx]})}{(8c(c + d)^2(d + c\cos[e + fx])} + \frac{(\cos[(3(e + fx))/2]\sqrt{\sec[e + fx]})}{(2(c + d)^2(d + c\cos[e + fx])} + \frac{(d\cos[(3(e + fx))/2]\sqrt{\sec[e + fx]})}{(c(c + d)^2(d + c\cos[e + fx])} + \frac{(d^2\cos[(3(e + fx))/2]\sqrt{\sec[e + fx]})}{(2c^2(c + d)^2(d + c\cos[e + fx]))} \right) \cdot \\ & \left(\frac{1}{\sqrt{a(1 + \sec[e + fx])}\sqrt{1 + (-3 + 2\sqrt{2})\tan[(e + fx)/4]^2}\sqrt{1 - (3 + 2\sqrt{2})\tan[(e + fx)/4]^2}} \right) \cdot \\ & \left(\frac{1}{(\sqrt{3 - 2\sqrt{2}})(3 + 2\sqrt{2})(c(8c^2 + 9cd + 4d^2)\text{EllipticF}[\text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] - 16(c + d)^3\text{EllipticPi}[-3 + 2\sqrt{2}, \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + d(15c^2 + 20cd + 8d^2)(\text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(3c + 2\sqrt{2})\sqrt{c(c - d)} - d)), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + \text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(-3c + 2\sqrt{2})\sqrt{c(c - d)} + d), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}])]} \right) \cdot \\ & \left(\frac{1}{(8c^3(c + d)^3\sqrt{1 - (3 + 2\sqrt{2})\tan[(e + fx)/4]^2})} - \frac{(\sqrt{3 - 2\sqrt{2}})(-3 + 2\sqrt{2})(c(8c^2 + 9cd + 4d^2)\text{EllipticF}[\text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] - 16(c + d)^3\text{EllipticPi}[-3 + 2\sqrt{2}, \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + d(15c^2 + 20cd + 8d^2)(\text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(3c + 2\sqrt{2})\sqrt{c(c - d)} - d)), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + \text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(-3c + 2\sqrt{2})\sqrt{c(c - d)} + d), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}])]} \right) \cdot \\ & \left(\frac{1}{(8c^3(c + d)^3\sqrt{1 - (3 + 2\sqrt{2})\tan[(e + fx)/4]^2})} + \frac{(\sqrt{3 - 2\sqrt{2}})\cos[(e + fx)/4](c(8c^2 + 9cd + 4d^2)\text{EllipticF}[\text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] - 16(c + d)^3\text{EllipticPi}[-3 + 2\sqrt{2}, \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + d(15c^2 + 20cd + 8d^2)(\text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(3c + 2\sqrt{2})\sqrt{c(c - d)} - d)), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + \text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(-3c + 2\sqrt{2})\sqrt{c(c - d)} + d), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}])]} \right) \cdot \\ & \left(\frac{1}{\sqrt{\sec[e + fx]}\sin[(e + fx)/4]\sqrt{1 + (-3 + 2\sqrt{2})\tan[(e + fx)/4]^2}\sqrt{1 - (3 + 2\sqrt{2})\tan[(e + fx)/4]^2}} \right) \cdot \\ & \left(\frac{1}{(4c^3(c + d)^3 - (\sqrt{3 - 2\sqrt{2}})\cos[(e + fx)/4]^2(c(8c^2 + 9cd + 4d^2)\text{EllipticF}[\text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] - 16(c + d)^3\text{EllipticPi}[-3 + 2\sqrt{2}, \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + d(15c^2 + 20cd + 8d^2)(\text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(3c + 2\sqrt{2})\sqrt{c(c - d)} - d)), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}] + \text{EllipticPi}[-(((-3 + 2\sqrt{2})(c + d))/(-3c + 2\sqrt{2})\sqrt{c(c - d)} + d), \text{ArcSin}[\tan[(e + fx)/4]/\sqrt{3 - 2\sqrt{2}}], 17 - 12\sqrt{2}])]} \right) \end{aligned}$$

$$c*d + 8*d^2)*(EllipticPi[-(((-3 + 2*sqrt(2))*(c + d))/(3*c + 2*sqrt(2)*sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/sqrt[3 - 2*sqrt(2)]], 17 - 12*sqrt(2)] + EllipticPi[(((-3 + 2*sqrt(2))*(c + d))/(-3*c + 2*sqrt(2)*sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/sqrt[3 - 2*sqrt(2)]], 17 - 12*sqrt(2)])*Sec[e + f*x]^(3/2)*Sin[e + f*x]*sqrt[1 + (-3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2]*sqrt[1 - (3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2])/(4*c^3*(c + d)^3) - (sqrt[3 - 2*sqrt(2)]*Cos[(e + f*x)/4]^2*sqrt[Sec[e + f*x]]*sqrt[1 + (-3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2]*sqrt[1 - (3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2]*((c*(8*c^2 + 9*c*d + 4*d^2)*Sec[(e + f*x)/4]^2)/(4*sqrt[3 - 2*sqrt(2)]*sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*sqrt(2))])*sqrt[1 - ((17 - 12*sqrt(2))*Tan[(e + f*x)/4]^2)/(3 - 2*sqrt(2))]) - (4*(c + d)^3*Sec[(e + f*x)/4]^2)/(sqrt[3 - 2*sqrt(2)]*sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*sqrt(2))])*sqrt[1 - ((17 - 12*sqrt(2))*Tan[(e + f*x)/4]^2)/(3 - 2*sqrt(2))])*sqrt[1 - ((17 - 12*sqrt(2))*Tan[(e + f*x)/4]^2)/(3 - 2*sqrt(2))])*(1 - ((-3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2/(3 - 2*sqrt(2)))) + d*(15*c^2 + 20*c*d + 8*d^2)*(Sec[(e + f*x)/4]^2/(4*sqrt[3 - 2*sqrt(2)]*sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*sqrt(2))])*sqrt[1 - ((17 - 12*sqrt(2))*Tan[(e + f*x)/4]^2)/(3 - 2*sqrt(2))])*(1 + ((-3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2)/((3 - 2*sqrt(2))*(3*c + 2*sqrt(2)*sqrt[c*(c - d)] - d))) + Sec[(e + f*x)/4]^2/(4*sqrt[3 - 2*sqrt(2)]*sqrt[1 - Tan[(e + f*x)/4]^2/(3 - 2*sqrt(2))])*sqrt[1 - ((17 - 12*sqrt(2))*Tan[(e + f*x)/4]^2)/(3 - 2*sqrt(2))])*(1 - ((-3 + 2*sqrt(2))*Tan[(e + f*x)/4]^2)/((3 - 2*sqrt(2))*(-3*c + 2*sqrt(2)*sqrt[c*(c - d)] + d)))))/(2*c^3*(c + d)^3))$$

fricas [B] time = 7.74, size = 2368, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
[Out] [1/8*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d) + 8*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 + (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d) + 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), 1/4*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*
```



```

-27*a^3*sqrt(-a)*c^4*d*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2
*(f*x+exp(1)))^2+4*a^4*sqrt(-a)*d^5-7*a^4*sqrt(-a)*c*d^4-7*a^4*sqrt(-a)*c^
2*d^3+19*a^4*sqrt(-a)*c^3*d^2-9*a^4*sqrt(-a)*c^4*d)/(c*(sqrt(-a*tan(1/2*(f*
x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4-d*(sqrt(-a*tan(1/2*(f*x+e
xp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4+2*a*c*(sqrt(-a*tan(1/2*(f*x+
exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+6*a*d*(sqrt(-a*tan(1/2*(f*x
+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a^2*c-a^2*d)^2/(-8*c^6-8*c
^2*d^4+16*c^4*d^2)+1/4*(-8*a*sqrt(-a)*d^3-20*a*sqrt(-a)*c*d^2-15*a*sqrt(-a)
*c^2*d)*atan(1/2*(c*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f
*x+exp(1))))^2-d*(sqrt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+
exp(1))))^2+a*c+3*a*d)/sqrt(2)/sqrt(-d^2-c*d)/a/sqrt(2)/sqrt(-d^2-c*d)/a/(
4*c^5+4*c^3*d^2+8*c^4*d)-1/4*a*sqrt(-a)*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(
1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sq
rt(-a*tan(1/2*(f*x+exp(1))))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)
)*abs(a)-6*a)/sqrt(2)/c^3/abs(a))*sign(cos(f*x+exp(1)))/f

```

maple [B] time = 19.10, size = 330749, normalized size = 1152.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x)
```

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c + \frac{d}{\cos(e+fx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3,x)
```

[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e+fx)+1)}}{(c+d\sec(e+fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
```

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**3, x)

3.154 $\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^3 dx$

Optimal. Leaf size=241

$$\frac{2a^{5/2}c^3 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^2 \tan(e + fx) (d(24c^2 + 111cd + 52d^2) \sec(e + fx) + 2(36c^3 + 243c^2d + 189cd^2 + 52d^3))}{105f\sqrt{a\sec(e+fx)+a}}$$

[Out] $2/35*a^2*(6*c+13*d)*(c+d*\sec(f*x+e))^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/7*a^2*(c+d*\sec(f*x+e))^3*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/105*a^2*(7*2*c^3+486*c^2*d+378*c*d^2+104*d^3+d*(24*c^2+111*c*d+52*d^2))*\sec(f*x+e)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*a^{(5/2)}*c^3*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)})/a^{(1/2)})*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 153, 147, 63, 206}

$$\frac{2a^2 \tan(e + fx) (d(24c^2 + 111cd + 52d^2) \sec(e + fx) + 2(243c^2d + 36c^3 + 189cd^2 + 52d^3))}{105f\sqrt{a\sec(e+fx)+a}} + \frac{2a^{5/2}c^3 \tan(e + fx)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]`

[Out] `(2*a^(5/2)*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(6*c + 13*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 111*c*d + 52*d^2))*Sec[e + f*x]*Tan[e + f*x])/(105*f*Sqrt[a + a*Sec[e + f*x]])`

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 147

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))*((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m + 2) - b*d*(f*g + e*h))*(m + n + 3) - b*d*f*h*(m + n + 2)*x*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), Int[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]`

Rule 153

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /`

$$4\pi/x/2)^2(2*(((-1/216090000*(108045000*\sqrt{2})*a^5*c^3*\text{sign}(\cos(f*x+\exp(1))))+78204000*\sqrt{2})*a^5*d^3*\text{sign}(\cos(f*x+\exp(1))))+259308000*\sqrt{2})*a^5*c*d^2*\text{sign}(\cos(f*x+\exp(1))))+432180000*\sqrt{2})*a^5*c^2*d*\text{sign}(\cos(f*x+\exp(1))))*\tan(1/2*(f*x+\exp(1)))^2-1/147000*(-220500*\sqrt{2})*a^5*c^3*\text{sign}(\cos(f*x+\exp(1)))-186200*\sqrt{2})*a^5*d^3*\text{sign}(\cos(f*x+\exp(1)))-617400*\sqrt{2})*a^5*c*d^2*\text{sign}(\cos(f*x+\exp(1)))-1029000*\sqrt{2})*a^5*c^2*d*\text{sign}(\cos(f*x+\exp(1))))*\tan(1/2*(f*x+\exp(1)))^2+1/3087000*(-4630500*\sqrt{2})*a^5*c^3*\text{sign}(\cos(f*x+\exp(1)))-4116000*\sqrt{2})*a^5*d^3*\text{sign}(\cos(f*x+\exp(1)))-18522000*\sqrt{2})*a^5*c*d^2*\text{sign}(\cos(f*x+\exp(1)))-24696000*\sqrt{2})*a^5*c^2*d*\text{sign}(\cos(f*x+\exp(1))))*\tan(1/2*(f*x+\exp(1)))^2+1/14700*(7350*\sqrt{2})*a^5*c^3*\text{sign}(\cos(f*x+\exp(1)))+14700*\sqrt{2})*a^5*d^3*\text{sign}(\cos(f*x+\exp(1)))+44100*\sqrt{2})*a^5*c*d^2*\text{sign}(\cos(f*x+\exp(1)))+44100*\sqrt{2})*a^5*c^2*d*\text{sign}(\cos(f*x+\exp(1)))))/\sqrt{-a*\tan(1/2*(f*x+\exp(1)))^2+a}/(-a*\tan(1/2*(f*x+\exp(1)))^2+a)^3*\tan(1/2*(f*x+\exp(1)))-1/2*a^2*\sqrt{-a}*c^3*\text{sign}(\cos(f*x+\exp(1)))*\ln(\text{abs}(2*(\sqrt{-a*\tan(1/2*(f*x+\exp(1)))^2+a}-\sqrt{-a})*\tan(1/2*(f*x+\exp(1))))^2-4*\sqrt{2}*abs(a)-6*a)/abs(2*(\sqrt{-a*\tan(1/2*(f*x+\exp(1)))^2+a}-\sqrt{-a})*\tan(1/2*(f*x+\exp(1))))^2+4*\sqrt{2}*abs(a)-6*a))/abs(a))/f$$

maple [B] time = 1.85, size = 539, normalized size = 2.24

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(105\sqrt{2} (\cos^3(fx+e)) \sin(fx+e) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{7}{2}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e)\sqrt{2}}{2\cos(fx+e)} \right) \right) c^3 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x)

[Out] 1/840/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(105*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3+315*2^(1/2)*cos(f*x+e)^2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3+315*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3+105*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3*sin(f*x+e)-1680*cos(f*x+e)^4*c^3-8400*cos(f*x+e)^4*c^2*d-6048*cos(f*x+e)^4*c*d^2-1664*cos(f*x+e)^4*d^3+1680*cos(f*x+e)^3*c^3+6720*cos(f*x+e)^3*c^2*d+3024*cos(f*x+e)^3*c*d^2+832*cos(f*x+e)^3*d^3+1680*cos(f*x+e)^2*c^2*d+2016*cos(f*x+e)^2*c*d^2+208*cos(f*x+e)^2*d^3+1008*cos(f*x+e)*c*d^2+384*cos(f*x+e)*d^3+240*d^3)/cos(f*x+e)^3/sin(f*x+e)*a

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{a}{\cos(e+fx)} \right)^{3/2} \left(c + \frac{d}{\cos(e+fx)} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3, x)`

[Out] `int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^{\frac{3}{2}} (c + d \sec(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**3, x)`

[Out] `Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3, x)`

3.155 $\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^2 dx$

Optimal. Leaf size=176

$$\frac{2a^{5/2}c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}} + \frac{2a^2 \tan(e + fx) (2(6c^2 + 25cd + 9d^2) + d(4c + 9d) \sec(e + fx))}{15f\sqrt{a \sec(e + fx) + a}}$$

[Out] $2/5*a^2*(c+d*\sec(f*x+e))^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/15*a^2*(12*c^2+50*c*d+18*d^2+d*(4*c+9*d)*\sec(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*a^{(5/2)}*c^2*\arctanh((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 153, 147, 63, 206}

$$\frac{2a^2 \tan(e + fx) (2(6c^2 + 25cd + 9d^2) + d(4c + 9d) \sec(e + fx))}{15f\sqrt{a \sec(e + fx) + a}} + \frac{2a^{5/2}c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a \sec(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(3/2)}*(c + d*\text{Sec}[e + f*x])^2, x]$

[Out] $(2*a^{(5/2)}*c^2*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/((f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^2*(c + d*\text{Sec}[e + f*x])^2*\text{Tan}[e + f*x])/(5*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^2*(2*(6*c^2 + 25*c*d + 9*d^2) + d*(4*c + 9*d)*\text{Sec}[e + f*x])*\text{Tan}[e + f*x])/(15*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 147

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.)), x_Symbol] := -\text{Simp}[(a*d*f*h*(n+2) + b*c*f*h*(m+2) - b*d*(f*g + e*h)*(m+n+3) - b*d*f*h*(m+n+2)*x)*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}/(b^2*d^2*(m+n+2)*(m+n+3)), x] + \text{Dist}[(a^2*d^2*f*h*(n+1)*(n+2) + a*b*d*(n+1)*(2*c*f*h*(m+1) - d*(f*g + e*h)*(m+n+3)) + b^2*(c^2*f*h*(m+1)*(m+2) - c*d*(f*g + e*h)*(m+1)*(m+n+3) + d^2*e*g*(m+n+2)*(m+n+3))]/(b^2*d^2*(m+n+2)*(m+n+3)), \text{Int}[(a + b*x)^m*(c + d*x)^n, x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, n\}, x] \&\& \text{NeQ}[m+n+2, 0] \&\& \text{NeQ}[m+n+3, 0]$

Rule 153

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.)), x_Symbol] := \text{Simp}[(h*(a + b*x)^m*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)}/(d*f*(m+n+p+2)), x] + \text{Dist}[1/(d*f*(m+n+p+2)), \text{Int}[(a + b*x)^{(m-1)}*(c + d*x)^n*(e + f*x)^p*\text{Simp}[a*d*f*g*(m+n+p+2) - h*(b*c*e*m + a*(d*e*(n+1) + c*f*(p+1))) + (b*d*f*g*(m+n+p+2) + h*(a*d*f*m - b*(d*e*(m+n+1) + c*f*(m+p+1)))]*x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, n, p\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[m+n+p+2, 0]$

2, 0] && IntegerQ[m]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^2 dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)(c+dx)^2}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2a^2(c + d \sec(e + fx))^2 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}} + \frac{(2a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)(c+dx)^2}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{5f\sqrt{a - a \sec(e + fx)}}$$

$$= \frac{2a^2(c + d \sec(e + fx))^2 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2(2(6c^2 + 25cd + 9d^2))}{15f\sqrt{a - a \sec(e + fx)}}$$

$$= \frac{2a^2(c + d \sec(e + fx))^2 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2(2(6c^2 + 25cd + 9d^2))}{15f\sqrt{a - a \sec(e + fx)}}$$

$$= \frac{2a^{5/2}c^2 \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{2a^2(c + d \sec(e + fx))^2 \tan(e + fx)}{5f\sqrt{a + a \sec(e + fx)}}$$

Mathematica [A] time = 1.33, size = 145, normalized size = 0.82

$$\frac{a \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left(2 \sin\left(\frac{1}{2}(e + fx)\right) \left((15c^2 + 50cd + 18d^2) \cos(2(e + fx)) + 15c\right)\right)}{30f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
 [Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(15*c^2 + 50*c*d + 24*d^2 + 2*d*(10*c + 9*d)*Cos[e + f*x] + (15*c^2 + 50*c*d + 18*d^2)*Cos[2*(e + f*x)]*Sin[(e + f*x)/2]))/(30*f)

maple [B] time = 1.67, size = 382, normalized size = 2.17

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(15 (\cos^2(fx+e)) \sin(fx+e) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{5}{2}} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \right) \sqrt{2} c^2 + 30$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x)

[Out] -1/60/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(15*cos(f*x+e)^2*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*c^2+30*cos(f*x+e)*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*c^2+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^2*sin(f*x+e)+120*cos(f*x+e)^3*c^2+400*cos(f*x+e)^3*c*d+144*cos(f*x+e)^3*d^2-120*cos(f*x+e)^2*c^2-320*cos(f*x+e)^2*c*d-72*cos(f*x+e)^2*d^2-80*cos(f*x+e)*c*d-48*cos(f*x+e)*d^2-24*d^2)/cos(f*x+e)^2/sin(f*x+e)*a

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{3/2} \left(c + \frac{d}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^{\frac{3}{2}} (c + d \sec(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**2,x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2, x)

3.156 $\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx)) dx$

Optimal. Leaf size=105

$$\frac{2a^{3/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2a^2(3c+4d) \tan(e+fx)}{3f\sqrt{a \sec(e+fx)+a}} + \frac{2ad \tan(e+fx)\sqrt{a \sec(e+fx)+a}}{3f}$$

[Out] $2*a^{(3/2)}*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f+2/3*a^2*(3*c+4*d)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/3*a*d*(a+a*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f$

Rubi [A] time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^2(3c+4d) \tan(e+fx)}{3f\sqrt{a \sec(e+fx)+a}} + \frac{2a^{3/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2ad \tan(e+fx)\sqrt{a \sec(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(3/2)}*(c + d*\text{Sec}[e + f*x]),x]$

[Out] $(2*a^{(3/2)}*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/f + (2*a^2*(3*c + 4*d)*\text{Tan}[e + f*x])/((3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a*d*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Tan}[e + f*x]))/(3*f)$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/(\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2])), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_ + (d_)*(x_)]*(b_ + (a_))], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/(\text{Sqrt}[a + b*\text{Csc}[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3792

$\text{Int}[\text{csc}[(e_ + (f_)*(x_)]*\text{Sqrt}[\text{csc}[(e_ + (f_)*(x_)]*(b_ + (a_))], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cot}[e + f*x])/((f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]])), x] /; \text{FreeQ}\{a, b, e, f, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3915

$\text{Int}[\text{Sqrt}[\text{csc}[(e_ + (f_)*(x_)]*(b_ + (a_))]*(\text{csc}[(e_ + (f_)*(x_)]*(d_ + (c_))), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[d, \text{Int}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Csc}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3917

$\text{Int}[(\text{csc}[(e_ + (f_)*(x_)]*(b_ + (a_)))^{(m_)}*(\text{csc}[(e_ + (f_)*(x_)]*(d_ + (c_))), x_Symbol] \rightarrow -\text{Simp}[(b*d*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m-1)})/(f*m), x] + \text{Dist}[1/m, \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m-1)}*\text{Simp}[a*c*m + (b*c*m + a*d*(2*m-1))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[2*m]$

Rubi steps

$$\begin{aligned}
\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx)) dx &= \frac{2ad\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{3f} + \frac{2}{3} \int \sqrt{a + a \sec(e + fx)} \left(\right. \\
&= \frac{2ad\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{3f} + (ac) \int \sqrt{a + a \sec(e + fx)} \left(\right. \\
&= \frac{2a^2(3c + 4d) \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}} + \frac{2ad\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{3f} \\
&= \frac{2a^{3/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{f} + \frac{2a^2(3c + 4d) \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}} + \frac{2ad\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 102, normalized size = 0.97

$$\frac{a \sec\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left(2 \sin\left(\frac{1}{2}(e + fx)\right) ((3c + 5d) \cos(e + fx) + d) + 3\sqrt{2}c \sin^{-1}\left(\sqrt{\frac{a \cos(e + fx) + a}{\cos(e + fx) + 1}}\right)\right)}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]), x]

[Out] (a*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(3*Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(3/2) + 2*(d + (3*c + 5*d)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(3*f)

fricas [A] time = 0.51, size = 316, normalized size = 3.01

$$\frac{3 \left(ac \cos^2(fx + e) + ac \cos(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right) + 2}{3 \left(f \cos^2(fx + e) + f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)), x, algorithm="fricas")

[Out] [1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e)))]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2*(2/sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)/(-a*tan(1/2*(f*x+exp(1)))^2+a))*(-1/324*(162*sqrt(2)*a^3*c*sign(cos(f*x+exp(1)))+216*sqrt(2)*a^3*d*sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2-1/18*(-9*sqrt(2)*a^3*c*sign(cos(f*x+exp(1)))-18*sqrt(2)*a^3*d*sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))-1/2*a^2*sqrt(-a)*c*sign(cos(f*x+exp(1)))*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a)/abs(a))/f

maple [B] time = 1.54, size = 237, normalized size = 2.26

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(3 \sin(fx+e) \cos(fx+e) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{3}{2}} c + 3 \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x)

[Out] 1/6/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c+3*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*c*sin(f*x+e)-12*cos(f*x+e)^2*c-20*cos(f*x+e)^2*d+12*c*cos(f*x+e)+16*d*cos(f*x+e)+4*d)/cos(f*x+e)/sin(f*x+e)*a

maxima [B] time = 1.40, size = 998, normalized size = 9.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] 1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))) + 1 - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))))

```

tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
+ 1) + a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1))*cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 4*(a*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - (a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(a))*c/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*f)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{3/2} \left(c + \frac{d}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^{3/2} (c + d \sec(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x)), x)

$$3.157 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{c+d \sec(e+fx)} dx$$

Optimal. Leaf size=110

$$\frac{2a^{3/2}(c-d) \tan^{-1}\left(\frac{\sqrt{a}\sqrt{d} \tan(e+fx)}{\sqrt{c+d}\sqrt{a \sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}} + \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf}$$

[Out] $2a^{3/2} \arctan(a^{1/2} \tan(fx+e)/(a+a \sec(fx+e))^{1/2})/c/f+2a^{3/2} (c-d) \arctan(a^{1/2} d^{1/2} \tan(fx+e)/(c+d)^{1/2} (a+a \sec(fx+e))^{1/2})/c/f/d^{1/2}/(c+d)^{1/2}$

Rubi [A] time = 0.24, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3927, 3774, 203, 3967, 205}

$$\frac{2a^{3/2}(c-d) \tan^{-1}\left(\frac{\sqrt{a}\sqrt{d} \tan(e+fx)}{\sqrt{c+d}\sqrt{a \sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}} + \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{cf}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]

[Out] $(2a^{3/2} \text{ArcTan}[(\text{Sqrt}[a] \text{Tan}[e + fx])/\text{Sqrt}[a + a \text{Sec}[e + fx]])/(c*f) + (2a^{3/2} (c-d) \text{ArcTan}[(\text{Sqrt}[a] \text{Sqrt}[d] \text{Tan}[e + fx])/(\text{Sqrt}[c + d] \text{Sqrt}[a + a \text{Sec}[e + fx]])])/(c \text{Sqrt}[d] \text{Sqrt}[c + d] * f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3927

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[a/c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[(b*c - a*d)/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])

Rule 3967

Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{3/2}}{c + d \sec(e + fx)} dx &= \frac{a \int \sqrt{a + a \sec(e + fx)} dx}{c} + \frac{(ac - ad) \int \frac{\sec(e+fx) \sqrt{a+a \sec(e+fx)}}{c+d \sec(e+fx)} dx}{c} \\
&= \frac{(2a^2) \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} - \frac{(2a^2(c-d)) \text{Subst}\left(\int \frac{1}{ac+ad+dx^2} dx, x, \frac{d \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} \\
&= \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{cf} + \frac{2a^{3/2}(c-d) \tan^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e+fx)}{\sqrt{c+d} \sqrt{a+a \sec(e+fx)}}\right)}{c\sqrt{d} \sqrt{c+d} f}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 135, normalized size = 1.23

$$\frac{\sqrt{2} a \sqrt{\cos(e + fx)} \sec\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(\sqrt{d} \sqrt{c + d} \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) + (c - d) \tan^{-1}\left(\frac{\sqrt{d} \sqrt{c + d} \sin\left(\frac{1}{2}(e + fx)\right)}{\sqrt{a+a \sec(e+fx)}}\right) \right)}{c\sqrt{d} f \sqrt{c+d}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]`

```
[Out] (Sqrt[2]*a*(Sqrt[d]*Sqrt[c + d]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (c - d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]])])*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]/(c*Sqrt[d]*Sqrt[c + d]*f)
```

fricas [A] time = 1.63, size = 731, normalized size = 6.65

$$\frac{(ac - ad) \sqrt{-\frac{a}{cd+d^2}} \log\left(\frac{2(cd+d^2) \sqrt{-\frac{a}{cd+d^2}} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + (ac+2ad) \cos(fx+e)^2 - ad + (ac+ad) \cos(fx+e)}{c \cos(fx+e)^2 + (c+d) \cos(fx+e) + d}\right) - \sqrt{-\frac{a}{cd+d^2}}}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")`

```
[Out] [-(a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*a^(3/2)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), -(2*(a*c - a*d)*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
```


maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sec(fx + e) + a)^{\frac{3}{2}}}{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{c + \frac{d}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x)), x)

$$3.158 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=229

$$\frac{a^{5/2} (c^2 - 3cd - 2d^2) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}} \right)}{c^2 \sqrt{d} f (c + d)^{3/2} \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2a^{5/2} \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}} \right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{1}{cf(c$$

[Out] $a^2(c-d)*\tan(f*x+e)/c/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)+2*a^{(5/2)*\arctanh((a-a*\sec(f*x+e))^{(1/2)/a^{(1/2)})*\tan(f*x+e)/c^2/f/(a-a*\sec(f*x+e))^{(1/2)/(a+a*\sec(f*x+e))^{(1/2)+a^{(5/2)*(c^2-3*c*d-2*d^2)*\arctanh(d^{(1/2)*(a-a*\sec(f*x+e))^{(1/2)/a^{(1/2)/(c+d)^{(1/2)}*\tan(f*x+e)/c^2/(c+d)^{(3/2)/f/d^{(1/2)/(a-a*\sec(f*x+e))^{(1/2)/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.25, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 151, 156, 63, 206, 208}

$$\frac{a^{5/2} (c^2 - 3cd - 2d^2) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}} \right)}{c^2 \sqrt{d} f (c + d)^{3/2} \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2a^{5/2} \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}} \right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{1}{cf(c$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]

[Out] $(2*a^{(5/2)*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/(c^2*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (a^{(5/2)*(c^2 - 3*c*d - 2*d^2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(c^2*\text{Sqrt}[d]*(c + d)^{(3/2)*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (a^2*(c - d)*\text{Tan}[e + f*x])/(c*(c + d)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x]))$

Rule 63

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 151

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

Rule 156

Int((((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)))/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3940

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^2} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}(c+dx)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} - \frac{(a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{a^2(c+d)+}{x\sqrt{a-a}}\right)}{c(c + d)f\sqrt{a - a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} - \frac{(a^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-a}}\right)}{c^2 f\sqrt{a - a \sec(e + fx)}\sqrt{a - a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} + \frac{(2a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\right)}{c^2 f\sqrt{a - a \sec(e + fx)}\sqrt{a - a \sec(e + fx)}} \\ &= \frac{2a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^2 f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{a^{5/2} (c^2 - 3cd - 2d^2) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a}}{\sqrt{a}}\right)}{c^2 \sqrt{d} (c + d)^{3/2} f\sqrt{a - a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 24.92, size = 2862, normalized size = 12.50

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]

[Out] ((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^3*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(3/2)*(((c - d)*Sin[(e + f*x)/2])/(2*c^2*(c + d)) + (-(c*d*Sin[(e + f*x)/2]) + d^2*Sin[(e + f*x)/2])/(2*c^2*(c + d)*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e + f*x])^2) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e + f*x])^2*(c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2])*Sqrt[c*(c - d)] -

d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[(-3 + 2*Sqrt[2])*(c + d)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])) * Sec[(e + f*x)/2]^3 * ((Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(c + d)*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*(c + d)*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c*(c + d)*(d + c*Cos[e + f*x])))) * Sec[e + f*x] * (a*(1 + Sec[e + f*x]))^(3/2) * Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2] * Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) / (c^2*(c + d)^2*f*(c + d*Sec[e + f*x])^2 * ((Sqrt[3 - 2*Sqrt[2]]*(3 + 2*Sqrt[2])) * (c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[(-3 + 2*Sqrt[2])*(c + d)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])) * Sqrt[Sec[e + f*x]] * Tan[(e + f*x)/4] * Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) / (4*c^2*(c + d)^2 * Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) - (Sqrt[3 - 2*Sqrt[2]]*(-3 + 2*Sqrt[2]) * (c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[(-3 + 2*Sqrt[2])*(c + d)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])) * Sqrt[Sec[e + f*x]] * Tan[(e + f*x)/4] * Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) / (4*c^2*(c + d)^2 * Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) + (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4] * (c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[(-3 + 2*Sqrt[2])*(c + d)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])) * Sqrt[Sec[e + f*x]] * Sin[(e + f*x)/4] * Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2] * Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) / (2*c^2*(c + d)^2 - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2 * (c*(3*c + d)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 4*(c + d)^2*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^2 - 3*c*d - 2*d^2)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[(-3 + 2*Sqrt[2])*(c + d)/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])) * Sqrt[Sec[e + f*x]]^(3/2) * Sin[e + f*x] * Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2] * Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) / (2*c^2*(c + d)^2 - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2 * Sqrt[Sec[e + f*x]] * Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2] * Sqrt[1 - (3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2]) * ((c*(3*c + d)*Sec[(e + f*x)/4]^2) / (4*Sqrt[3 - 2*Sqrt[2]] * Sqrt[1 - Tan[(e + f*x)/4]^2 / (3 - 2*Sqrt[2])]) * Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2) / (3 - 2*Sqrt[2])]) - ((c + d)^2 * Sec[(e + f*x)/4]^2) / (Sqrt[3 - 2*Sqrt[2]] * Sqrt[1 - Tan[(e + f*x)/4]^2 / (3 - 2*Sqrt[2])]) * Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2) / (3 - 2*Sqrt[2])]) * (1 - ((-3 + 2*Sqrt[2])*Tan[(e + f*x)/4]^2) / (3 - 2*Sqrt[2]))) - (c^2 - 3*c*d - 2*d^2) * (Sec[(e + f*x)/4]^2 / (4*Sqrt[3 - 2*Sqrt[2]] * Sqrt[1 - Tan[(e + f*x)/4]^2 / (3 - 2*Sqrt[2])]) * Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2) / (3 - 2*Sqrt[2])]) * (1 + ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2) / ((3 - 2*Sqrt[2])*(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)))) + Sec[(e + f*x)/4]^2 / (4*Sqrt[3 - 2*Sqrt[2]] * Sqrt[1 - Tan[(e + f*x)/4]^2 / (3 - 2*Sqrt[2])]) * Sqrt[1 - ((17 - 12*Sqrt[2])*Tan[(e + f*x)/4]^2) / (3 - 2*Sqrt[2])]) * (1 - ((-3 + 2*Sqrt[2])*(c + d)*Tan[(e + f*x)/4]^2)

$$\frac{1}{((3 - 2\sqrt{2}) \cdot (-3c + 2\sqrt{2}\sqrt{c(c-d)} + d))^{1/2}} \cdot \frac{1}{(c^2(c+d)^2)}$$

fricas [A] time = 7.40, size = 1640, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
[Out] [1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*
c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x +
e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*co
s(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*
cos(f*x + e) + d)) + 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a
*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2
+ (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), 1/2*(2*(
a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - 4*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*
d + a*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3
+ (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a
*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqr
t(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(
c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x + e
)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), ((a*
c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(
f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(a
/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + (a*c*d + a*d^2 + (a*c^2 + a*
c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 +
c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d
+ c^2*d^2)*f), ((a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2
*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*
cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 2*(a*c*d
+ a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f
*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e)))))/((c^4 + c^3*d)*f*cos(f*x + e)^2 + (c^4 + 2*c^3*d
+ c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
```


[Out] `int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \left(\sec(e + fx) + 1\right)\right)^{\frac{3}{2}}}{\left(c + d \sec(e + fx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)`

[Out] `Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**2, x)`

$$3.159 \quad \int \frac{(a+a \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^3} dx$$

Optimal. Leaf size=310

$$\frac{2a^{5/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{c^3 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{a^{5/2} (3c^3 - 15c^2d - 20cd^2 - 8d^3) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{4c^3 \sqrt{d} f (c+d)^{5/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

[Out] 1/2*a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)+1/4*a^2*(3*c^2-7*c*d-4*d^2)*tan(f*x+e)/c^2/(c+d)^2/f/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*a^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+1/4*a^(5/2)*(3*c^2-15*c*d-20*c*d^2-8*d^3)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2))/(c+d)^(1/2))*tan(f*x+e)/c^3/(c+d)^(5/2)/f/d^(1/2)/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)

Rubi [A] time = 0.34, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 151, 156, 63, 206, 208}

$$\frac{a^2 (3c^2 - 7cd - 4d^2) \tan(e+fx)}{4c^2 f (c+d)^2 \sqrt{a \sec(e+fx)+a} (c+d \sec(e+fx))} + \frac{a^{5/2} (-15c^2d + 3c^3 - 20cd^2 - 8d^3) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{4c^3 \sqrt{d} f (c+d)^{5/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]

[Out] (2*a^(5/2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^(5/2)*(3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(4*c^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^2*(c - d)*Tan[e + f*x])/(2*c*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (a^2*(3*c^2 - 7*c*d - 4*d^2)*Tan[e + f*x])/(4*c^2*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 151

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

Rule 156

Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +

$f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2]]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 3940

$\text{Int}[(\text{csc}[e_ + (f_)*(x_)]*(b_ + (a_))^{(m_)}*(\text{csc}[e_ + (f_)*(x_)]*(d_ + (c_))^{(n_)}), x_Symbol] := \text{Dist}[(a^2*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{(m - 1/2)}*(c + d*x)^n/(x*\text{Sqrt}[a - b*x]), x], x, \text{Csc}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{IntegerQ}[m - 1/2]$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^3} dx &= -\frac{(a^2 \tan(e + fx)) \text{Subst}\left(\int \frac{a+ax}{x\sqrt{a-ax}(c+dx)^3} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} - \frac{(a \tan(e + fx)) \text{Subst}\left(\int \frac{2a^2(c - d)}{x\sqrt{a - a \sec(e + fx)}} dx, x, \sec(e + fx)\right)}{2c(c + d)f\sqrt{a - a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} + \frac{a^2(3c^2 - 7cd - 4d^2)}{4c^2(c + d)^2f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} + \frac{a^2(3c^2 - 7cd - 4d^2)}{4c^2(c + d)^2f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{a^2(c - d) \tan(e + fx)}{2c(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} + \frac{a^2(3c^2 - 7cd - 4d^2)}{4c^2(c + d)^2f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{a^{5/2}(3c^3 - 15c^2d - 20cd^2 - 8d^3) \tan(e + fx)}{4c^3 \sqrt{d} (c + d)^{5/2} f \sqrt{a - a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 24.70, size = 3166, normalized size = 10.21

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]

[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^3*Sec[e + f*x]^2*(a*(1 + Sec[e + f*x]))^(3/2)*(-1/8*((-5*c^2 + 7*c*d + 6*d^2)*Sin[(e + f*x)/2])/(c^3*(c + d)^3))

$$\begin{aligned}
& 2) + (c*d^2*\sin[(e + f*x)/2] - d^3*\sin[(e + f*x)/2])/(4*c^3*(c + d)*(d + c* \\
& \cos[e + f*x])^2) + (-7*c^2*d*\sin[(e + f*x)/2] + 7*c*d^2*\sin[(e + f*x)/2] + \\
& 8*d^3*\sin[(e + f*x)/2])/(8*c^3*(c + d)^2*(d + c*\cos[e + f*x]))/(f*(c + d* \\
& \sec[e + f*x])^3) - (\sqrt{3 - 2*\sqrt{2}}*\cos[(e + f*x)/4]^2*(d + c*\cos[e + f \\
& *x])^3*(c*(11*c^2 + 9*c*d + 4*d^2)*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 \\
& - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - 16*(c + d)^3*\text{EllipticPi}[-3 + 2*\sqrt{2}], \\
& \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - (3*c^3 - 1 \\
& 5*c^2*d - 20*c*d^2 - 8*d^3)*(\text{EllipticPi}[-(((-3 + 2*\sqrt{2})*(c + d))/(3*c + \\
& 2*\sqrt{2})*\sqrt{c*(c - d)} - d)), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2} \\
& }], 17 - 12*\sqrt{2}] + \text{EllipticPi}[((-3 + 2*\sqrt{2})*(c + d))/(-3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} + d), \\
& \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 \\
& - 12*\sqrt{2}]))*\sec[(e + f*x)/2]^3*((7*\cos[(e + f*x)/2]*\sqrt{\sec[e + f*x]}) \\
&)/(16*(c + d)^2*(d + c*\cos[e + f*x])) + (d*\cos[(e + f*x)/2]*\sqrt{\sec[e + f \\
& *x]})/(16*c*(c + d)^2*(d + c*\cos[e + f*x])) + (\cos[(3*(e + f*x))/2]*\sqrt{\sec \\
& [e + f*x]})/(4*(c + d)^2*(d + c*\cos[e + f*x])) + (d*\cos[(3*(e + f*x))/2]*\sqrt{ \\
& \sec[e + f*x]})/(2*c*(c + d)^2*(d + c*\cos[e + f*x])) + (d^2*\cos[(3*(e + f \\
& *x))/2]*\sqrt{\sec[e + f*x]})/(4*c^2*(c + d)^2*(d + c*\cos[e + f*x]))*\sec[e + \\
& f*x]^2*(a*(1 + \sec[e + f*x]))^(3/2)*\sqrt{1 + (-3 + 2*\sqrt{2})*\text{Tan}[(e + f*x) \\
&]/4]^2*\sqrt{1 - (3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2})/(4*c^3*(c + d)^3*f*(c \\
& + d*\sec[e + f*x])^3*((\sqrt{3 - 2*\sqrt{2}}*(3 + 2*\sqrt{2})*(c*(11*c^2 + 9*c \\
& *d + 4*d^2)*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12 \\
& *\sqrt{2}] - 16*(c + d)^3*\text{EllipticPi}[-3 + 2*\sqrt{2}], \text{ArcSin}[\text{Tan}[(e + f*x)/4] \\
&]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8* \\
& d^3)*(\text{EllipticPi}[-(((-3 + 2*\sqrt{2})*(c + d))/(3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} - d)), \\
& \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] + \\
& \text{EllipticPi}[((-3 + 2*\sqrt{2})*(c + d))/(-3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} + d), \\
& \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}]))*\sqrt{\sec[e + f*x]}* \\
& \text{Tan}[(e + f*x)/4]*\sqrt{1 + (-3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2} \\
&)/(16*c^3*(c + d)^3*\sqrt{1 - (3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2}) - (\sqrt{3 - 2*\sqrt{2}} \\
&)*(-3 + 2*\sqrt{2})*(c*(11*c^2 + 9*c*d + 4*d^2)*\text{EllipticF}[\text{ArcSin} \\
& [\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - 16*(c + d)^3*\text{EllipticPi} \\
& [-3 + 2*\sqrt{2}], \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - \\
& 12*\sqrt{2}] - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*(\text{EllipticPi}[-(((-3 + 2*\sqrt{2})*(c + d))/(3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} - d)), \\
& \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] + \text{EllipticPi}[((-3 + 2*\sqrt{2})* \\
& (c + d))/(-3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} + d), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/ \\
& \sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}]))*\sqrt{\sec[e + f*x]}* \\
& \text{Tan}[(e + f*x)/4]*\sqrt{1 - (3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2})/(16*c^3*(c + d)^3*\sqrt{1 + \\
& (-3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2}) + (\sqrt{3 - 2*\sqrt{2}}*\cos[(e + f*x)/4] \\
&)*(c*(11*c^2 + 9*c*d + 4*d^2)*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2 \\
& *\sqrt{2}}], 17 - 12*\sqrt{2}] - 16*(c + d)^3*\text{EllipticPi}[-3 + 2*\sqrt{2}], \text{ArcS} \\
& \text{in}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - (3*c^3 - 15*c^ \\
& 2*d - 20*c*d^2 - 8*d^3)*(\text{EllipticPi}[-(((-3 + 2*\sqrt{2})*(c + d))/(3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} - d)), \\
& \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] + \text{EllipticPi} \\
& [(((-3 + 2*\sqrt{2})*(c + d))/(-3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} + d), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], \\
& 17 - 12*\sqrt{2}]))*\sqrt{\sec[e + f*x]}*\sin[(e + f*x)/4]*\sqrt{1 + (-3 + 2*\sqrt{2})* \\
& \text{Tan}[(e + f*x)/4]^2}* \\
& \sqrt{1 - (3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2})/(8*c^3*(c + d)^3) - (\sqrt{3 - 2*\sqrt{2}}*\cos[(e + f*x)/4]^2*(c*(11*c^2 + 9*c*d + 4*d^2)* \\
& \text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - 16*(c + d)^3*\text{EllipticPi}[-3 + 2*\sqrt{2}], \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*(\text{EllipticPi}[-(((-3 + 2*\sqrt{2})*(c + d))/(3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} - d)), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}] + \text{EllipticPi} \\
& [(((-3 + 2*\sqrt{2})*(c + d))/(-3*c + 2*\sqrt{2})*\sqrt{c*(c - d)} + d), \text{ArcSin}[\text{Tan}[(e + f*x)/4]/\sqrt{3 - 2*\sqrt{2}}], 17 - 12*\sqrt{2}]))*\sec[e + f*x]^(\\
& 3/2)*\sin[e + f*x]*\sqrt{1 + (-3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2}* \\
& \sqrt{1 - (3 + 2*\sqrt{2})*\text{Tan}[(e + f*x)/4]^2})/(8*c^3*(c + d)^3) - (\sqrt{3 - 2*\sqrt{2}} \\
&)*\cos[(e + f*x)/4]^2*\sqrt{\sec[e + f*x]}*\sqrt{1 + (-3 + 2*\sqrt{2})*\text{Tan}[(e + f
\end{aligned}$$

$$\begin{aligned} & *x)/4]^2 * \text{Sqrt}[1 - (3 + 2*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4]^2] * ((c*(11*c^2 + 9*c*d \\ & + 4*d^2)* \text{Sec}[(e + f*x)/4]^2)/(4*\text{Sqrt}[3 - 2*\text{Sqrt}[2]]*\text{Sqrt}[1 - \text{Tan}[(e + f*x)/ \\ & 4]^2/(3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4]^2)/(3 - \\ & 2*\text{Sqrt}[2])]) - (4*(c + d)^3 * \text{Sec}[(e + f*x)/4]^2)/(\text{Sqrt}[3 - 2*\text{Sqrt}[2]]*\text{Sqrt}[1 \\ & - \text{Tan}[(e + f*x)/4]^2/(3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2])* \text{Tan}[(e + \\ & f*x)/4]^2)/(3 - 2*\text{Sqrt}[2])]) * (1 - ((-3 + 2*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4]^2)/(3 \\ & - 2*\text{Sqrt}[2])) - (3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3) * (\text{Sec}[(e + f*x)/4]^2/ \\ & (4*\text{Sqrt}[3 - 2*\text{Sqrt}[2]]*\text{Sqrt}[1 - \text{Tan}[(e + f*x)/4]^2/(3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 \\ & - ((17 - 12*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4]^2)/(3 - 2*\text{Sqrt}[2])]) * (1 + ((-3 + 2*\text{Sqr} \\ & t[2])* (c + d) * \text{Tan}[(e + f*x)/4]^2)/((3 - 2*\text{Sqrt}[2]) * (3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * \\ & (c - d)] - d)))) + \text{Sec}[(e + f*x)/4]^2/(4*\text{Sqrt}[3 - 2*\text{Sqrt}[2]]*\text{Sqrt}[1 - \text{Tan}[(\\ & e + f*x)/4]^2/(3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2])* \text{Tan}[(e + f*x)/4] \\ & ^2)/(3 - 2*\text{Sqrt}[2])]) * (1 - ((-3 + 2*\text{Sqrt}[2])* (c + d) * \text{Tan}[(e + f*x)/4]^2)/((3 \\ & - 2*\text{Sqrt}[2]) * (-3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c*(c - d)] + d)))))))/(4*c^3*(c + d)^3 \\ &))) \end{aligned}$$

fricas [B] time = 20.44, size = 2729, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
[Out] [-1/8*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a
*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d
- 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 2
7*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c
*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 -
a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) +
d)) - 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*co
s(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2
+ (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*((5*
a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2
- 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*
d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*
d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/8*(16*(a*c^2*d
^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e)^3 + (
a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e)^2 + (2*a*c^3*d +
5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3*a*c^3*
d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^
3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 -
48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55
*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((
2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d
)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - 2*((5*a*c^
4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d - 7*a*c^2*d^2 - 4*
a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)
)/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4*c^6*d + 5*c^5*d^2
+ 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 + 4*c^4*d^3 + c^3*d^4)
*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f), -1/4*((3*a*c^3*d^2 -
15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*a*c^4*d - 20*a*c^3*d^2
- 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*d - 50*a*c^3*d^2 - 48*a*
c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d - 27*a*c^3*d^2 - 55*a*c^2
*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c +
```



```

1120*a^2*sqrt(-a)*c^4*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*
(f*x+exp(1))))^6*sign(cos(f*x+exp(1)))+14855280471424563298789490688*a^2*sq
rt(-a)*d^4*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1)
)))^6*sign(cos(f*x+exp(1)))+18569100589280704123486863360*a^3*sqrt(-a)*c^4*
(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(
cos(f*x+exp(1)))-94083442985688900892333441024*a^3*sqrt(-a)*d^4*(sqrt(-a*ta
n(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(f*x+exp
(1)))+18569100589280704123486863360*a^4*sqrt(-a)*c^4*(sqrt(-a*tan(1/2*(f*x+
exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))+44565
841414273689896368472064*a^4*sqrt(-a)*d^4*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+
a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))+3713820117856140
824697372672*a^5*sqrt(-a)*c*d^3*sign(cos(f*x+exp(1)))+136173404321391830238
90366464*a^5*sqrt(-a)*c^2*d^2*sign(cos(f*x+exp(1)))-18569100589280704123486
863360*a^5*sqrt(-a)*c^3*d*sign(cos(f*x+exp(1)))-123794003928538027489912422
4*a^2*sqrt(-a)*c*d^3*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(
f*x+exp(1))))^6*sign(cos(f*x+exp(1)))-35900261139276027972074602496*a^2*sq
rt(-a)*c^2*d^2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp
(1))))^6*sign(cos(f*x+exp(1)))+16093220510709943573688614912*a^2*sqrt(-a)*c
^3*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^6*
sign(cos(f*x+exp(1)))-167121905303526337111381770240*a^3*sqrt(-a)*c*d^3*(sq
rt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos
(f*x+exp(1)))+40852021296417549071671099392*a^3*sqrt(-a)*c^2*d^2*(sqrt(-a*t
an(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(f*x+ex
p(1)))+43327901374988309621469347840*a^3*sqrt(-a)*c^3*d*(sqrt(-a*tan(1/2*(f
*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(f*x+exp(1)))+35
900261139276027972074602496*a^4*sqrt(-a)*c*d^3*(sqrt(-a*tan(1/2*(f*x+exp(1)
))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))-10770078341
7828083916223807488*a^4*sqrt(-a)*c^2*d^2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a
)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))+86655802749976619
24293869568*a^4*sqrt(-a)*c^3*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)
*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))/(c*(sqrt(-a*tan(1/2*(f*x+e
xp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4-d*(sqrt(-a*tan(1/2*(f*x+exp(
1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4+2*a*c*(sqrt(-a*tan(1/2*(f*x+exp
(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+6*a*d*(sqrt(-a*tan(1/2*(f*x+ex
p(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a^2*c-a^2*d)^2/(-495176015714
1521099596496896*sqrt(2)*c^5+4951760157141521099596496896*sqrt(2)*c^2*d^3+4
951760157141521099596496896*sqrt(2)*c^3*d^2-4951760157141521099596496896*sq
rt(2)*c^4*d)+1/19807040628566084398385987584*(36779892980781332600600328651
607420406849738932406452224*a^2*sqrt(-a)*c^3*sign(cos(f*x+exp(1)))-98079714
615416886934934209737619787751599303819750539264*a^2*sqrt(-a)*d^3*sign(cos(
f*x+exp(1)))-245199286538542217337335524344049469378998259549376348160*a^2*
sqrt(-a)*c*d^2*sign(cos(f*x+exp(1)))-18389946490390666300300164325803710203
4248694662032261120*a^2*sqrt(-a)*c^2*d*sign(cos(f*x+exp(1)))*atan(1/2*(c*(
sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-d*(sq
rt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a*c+3*a*d
)/sqrt(2)/sqrt(-d^2-c*d)/a)/sqrt(2)/sqrt(-d^2-c*d)/a/(495176015714152109959
6496896*sqrt(2)*c^5+4951760157141521099596496896*sqrt(2)*c^3*d^2+9903520314
283042199192993792*sqrt(2)*c^4*d)-1/4*a^2*sqrt(-a)*sign(cos(f*x+exp(1)))*ln
(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^
2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*
tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a))/c^3/abs(a))/f

```

maple [B] time = 11.53, size = 234091, normalized size = 755.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sec(f*x+e))^{3/2}/(c+d*\sec(f*x+e))^3,x)$

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}}{\left(c + \frac{d}{\cos(e+fx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sec(e+fx)+1)\right)^{3/2}}{(c+d\sec(e+fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**3, x)

3.160 $\int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^3 dx$

Optimal. Leaf size=336

$$\frac{2a^{7/2}c^3 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{a-a\sec(e+fx)}} - \frac{2(c^3 + 12c^2d + 24cd^2 + 12d^3) \tan(e + fx) (a^3 - a^3 \sec(e + fx))}{3f\sqrt{a\sec(e+fx)+a}} + \frac{2a^3}{f\sqrt{a\sec(e+fx)+a}}$$

[Out] $2*a^3*(3*c^3+12*c^2*d+12*c*d^2+4*d^3)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/5*a*d*(3*c^2+15*c*d+13*d^2)*(a-a*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-6/7*d^2*(c+2*d)*(a-a*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/9*d^3*(a-a*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}-2/3*(c^3+12*c^2*d+24*c*d^2+12*d^3)*(a^3-a^3*\sec(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*a^{(7/2)}*c^3*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3940, 180, 63, 206}

$$\frac{2(12c^2d + c^3 + 24cd^2 + 12d^3) \tan(e + fx) (a^3 - a^3 \sec(e + fx))}{3f\sqrt{a\sec(e+fx)+a}} + \frac{2a^3(12c^2d + 3c^3 + 12cd^2 + 4d^3) \tan(e + fx)}{f\sqrt{a\sec(e+fx)+a}} + \frac{2a^3}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sec}[e + f*x])^{(5/2)}*(c + d*\text{Sec}[e + f*x])^3, x]$

[Out] $(2*a^3*(3*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^{(7/2)}*c^3*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a*d*(3*c^2 + 15*c*d + 13*d^2)*(a - a*\text{Sec}[e + f*x])^{(1/2)}*\text{Tan}[e + f*x])/(5*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (6*d^2*(c + 2*d)*(a - a*\text{Sec}[e + f*x])^{(1/2)}*\text{Tan}[e + f*x])/(7*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*d^3*(a - a*\text{Sec}[e + f*x])^{(1/2)}*\text{Tan}[e + f*x])/(9*a*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*(c^3 + 12*c^2*d + 24*c*d^2 + 12*d^3)*(a^3 - a^3*\text{Sec}[e + f*x])*\text{Tan}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 180

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.))^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, n\}, x] \&\& \text{IntegersQ}[p, q]$

Rule 206

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^3 dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^2(c+dx)^3}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{a^2(3c^3+12c^2d+12cd^2+4d^3)}{\sqrt{a-ax}} + \frac{a^2c^3}{x\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^3(3c^3 + 12c^2d + 12cd^2 + 4d^3) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2ad(3c^2 + 15cd + 5d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^3(3c^3 + 12c^2d + 12cd^2 + 4d^3) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2ad(3c^2 + 15cd + 5d^2) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^3(3c^3 + 12c^2d + 12cd^2 + 4d^3) \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2a^{7/2}c^3 \tanh^{-1}\left(\frac{\sec(e + fx) - \sqrt{a - a \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}}\right)}{f\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [A] time = 6.50, size = 286, normalized size = 0.85

$$\frac{a^2 \sec\left(\frac{1}{2}(e + fx)\right) \sec^4(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left(2520\sqrt{2}c^3 \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) \cos^{\frac{9}{2}}(e + fx) + 2520\sqrt{2}c^3 \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) \cos^{\frac{7}{2}}(e + fx) + 2520\sqrt{2}c^3 \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) \cos^{\frac{5}{2}}(e + fx) + 2520\sqrt{2}c^3 \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) \cos^{\frac{3}{2}}(e + fx) + 2520\sqrt{2}c^3 \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) \cos^{\frac{1}{2}}(e + fx)\right)}{(2520f)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]

[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*(2520*Sqrt[2]*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(9/2) + 2*(2520*c^3 + 8883*c^2*d + 8370*c*d^2 + 2908*d^3 + (630*c^3 + 5292*c^2*d + 7290*c*d^2 + 2792*d^3)*Cos[e + f*x] + 4*(840*c^3 + 2898*c^2*d + 2610*c*d^2 + 803*d^3)*Cos[2*(e + f*x)] + 210*c^3*Cos[3*(e + f*x)] + 1764*c^2*d*Cos[3*(e + f*x)] + 2070*c*d^2*Cos[3*(e + f*x)] + 584*d^3*Cos[3*(e + f*x)] + 840*c^3*Cos[4*(e + f*x)] + 2709*c^2*d*Cos[4*(e + f*x)] + 2070*c*d^2*Cos[4*(e + f*x)] + 584*d^3*Cos[4*(e + f*x)]*Sin[(e + f*x)/2]))/(2520*f)

fricas [A] time = 0.52, size = 620, normalized size = 1.85

$$\frac{315 \left(a^2 c^3 \cos^5(fx + e) + a^2 c^3 \cos^4(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e)}{\cos(fx + e) + 1} \right)}{(2520f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
[Out] [1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(35*a
^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*cos(
f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)*c
os(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e)^
2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*(3
15*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (35
*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*co
s(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d^3)
*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x + e
)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>
(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign
: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2*(2*((( (-1/12759898410000*(-14886548145
000*sqrt(2)*a^7*c^3*sign(cos(f*x+exp(1)))-8425583712000*sqrt(2)*a^7*d^3*sig
n(cos(f*x+exp(1)))-29165482080000*sqrt(2)*a^7*c*d^2*sign(cos(f*x+exp(1)))-4
0831674912000*sqrt(2)*a^7*c^2*d*sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1))
)^2-1/125023500*(625117500*sqrt(2)*a^7*c^3*sign(cos(f*x+exp(1)))+371498400*
sqrt(2)*a^7*d^3*sign(cos(f*x+exp(1)))+1285956000*sqrt(2)*a^7*c*d^2*sign(cos
(f*x+exp(1)))+1800338400*sqrt(2)*a^7*c^2*d*sign(cos(f*x+exp(1))))*tan(1/2*
(f*x+exp(1)))^2+1/27005076000*(216040608000*sqrt(2)*a^7*c^3*sign(cos(f*x+ex
p(1)))+140426395200*sqrt(2)*a^7*d^3*sign(cos(f*x+exp(1)))+486091368000*sqrt
(2)*a^7*c*d^2*sign(cos(f*x+exp(1)))+680527915200*sqrt(2)*a^7*c^2*d*sign(cos
(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2+1/1350253800*(-7651438200*sqrt(2)*a
^7*c^3*sign(cos(f*x+exp(1)))-5401015200*sqrt(2)*a^7*d^3*sign(cos(f*x+exp(1))
```

) - 21604060800*sqrt(2)*a^7*c*d^2*sign(cos(f*x+exp(1))) - 27005076000*sqrt(2)*a^7*c^2*d*sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2 + 1/3572100*(5358150*sqrt(2)*a^7*c^3*sign(cos(f*x+exp(1))) + 7144200*sqrt(2)*a^7*d^3*sign(cos(f*x+exp(1))) + 21432600*sqrt(2)*a^7*c*d^2*sign(cos(f*x+exp(1))) + 21432600*sqrt(2)*a^7*c^2*d*sign(cos(f*x+exp(1))))/sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)/(-a*tan(1/2*(f*x+exp(1)))^2+a)^4*tan(1/2*(f*x+exp(1))) - 1/2*a^3*sqrt(-a)*c^3*sign(cos(f*x+exp(1)))*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a))/abs(a))/f

maple [B] time = 1.91, size = 677, normalized size = 2.01

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(315\sqrt{2} \sin(fx+e) (\cos^4(fx+e)) \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e)\sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{9}{2}} c^3 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x)

[Out] -1/5040/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(315*2^(1/2)*sin(f*x+e)*cos(f*x+e)^4*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*c^3+1260*2^(1/2)*sin(f*x+e)*cos(f*x+e)^3*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*c^3+1890*2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*c^3+1260*2^(1/2)*sin(f*x+e)*cos(f*x+e)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*c^3+315*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)*c^3*sin(f*x+e)+26880*cos(f*x+e)^5*c^3+86688*cos(f*x+e)^5*c^2*d+66240*cos(f*x+e)^5*c*d^2+18688*cos(f*x+e)^5*d^3-23520*cos(f*x+e)^4*c^3-58464*cos(f*x+e)^4*c^2*d-33120*cos(f*x+e)^4*c*d^2-9344*cos(f*x+e)^4*d^3-3360*cos(f*x+e)^3*c^3-22176*cos(f*x+e)^3*c^2*d-15840*cos(f*x+e)^3*c*d^2-2336*cos(f*x+e)^3*d^3-6048*cos(f*x+e)^2*c^2*d-12960*cos(f*x+e)^2*c*d^2-2848*cos(f*x+e)^2*d^3-4320*cos(f*x+e)*c*d^2-3040*cos(f*x+e)*d^3-1120*d^3)/cos(f*x+e)^4/sin(f*x+e)*a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c + \frac{d}{\cos(e + fx)} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^{\frac{5}{2}} (c + d \sec(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**3,x)

[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3, x)

3.161 $\int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^2 dx$

Optimal. Leaf size=258

$$\frac{2a^{7/2}c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{2(c^2 + 8cd + 8d^2) \tan(e + fx) (a^3 - a^3 \sec(e + fx))}{3f\sqrt{a\sec(e+fx)+a}} + \frac{2a^3(c + 2d)}{f\sqrt{a}}$$

[Out] $2*a^3*(c+2*d)*(3*c+2*d)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/5*a*d*(2*c+5*d)*(a-a*\sec(f*x+e))^{2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2/7*d^2*(a-a*\sec(f*x+e))^{3*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2/3*(c^2+8*c*d+8*d^2)*(a^3-a^3*\sec(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*a^{(7/2)*c^2*\arctanh((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.18, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3940, 180, 63, 206}

$$\frac{2(c^2 + 8cd + 8d^2) \tan(e + fx) (a^3 - a^3 \sec(e + fx))}{3f\sqrt{a\sec(e+fx)+a}} + \frac{2a^{7/2}c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^3(c + 2d)}{f\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]

[Out] $(2*a^3*(c + 2*d)*(3*c + 2*d)*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a^{(7/2)*c^2*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/(f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*a*d*(2*c + 5*d)*(a - a*\text{Sec}[e + f*x])^{2*\text{Tan}[e + f*x]})/(5*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*d^2*(a - a*\text{Sec}[e + f*x])^{3*\text{Tan}[e + f*x]})/(7*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*(c^2 + 8*c*d + 8*d^2)*(a^3 - a^3*\text{Sec}[e + f*x])*\text{Tan}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 63

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d

$x)^n/(x\sqrt{a-bx}), x], x, \text{Csc}[e+fx]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{IntegerQ}[m - 1/2]$

Rubi steps

$$\begin{aligned} \int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^2 dx &= -\frac{(a^2 \tan(e + fx)) \text{Subst}\left(\int \frac{(a+ax)^2(c+dx)^2}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \text{Subst}\left(\int \left(\frac{a^2(c+2d)(3c+2d)}{\sqrt{a-ax}} + \frac{a^2c^2}{x\sqrt{a-ax}} - a(c^2 + 8cd + 5d^2)\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad(2c+5d)(a-a\sec(e+fx))}{5f\sqrt{a+a\sec(e+fx)}} \\ &= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2ad(2c+5d)(a-a\sec(e+fx))}{5f\sqrt{a+a\sec(e+fx)}} \\ &= \frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}c^2 \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \end{aligned}$$

Mathematica [A] time = 2.67, size = 191, normalized size = 0.74

$$\frac{a^2 \sec\left(\frac{1}{2}(e + fx)\right) \sec^3(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left(4 \sin\left(\frac{1}{2}(e + fx)\right) \left((420c^2 + 987cd + 465d^2) \cos(e + fx) + (420c^2 + 987cd + 465d^2) \cos(e + fx) + (35c^2 + 196cd + 145d^2) \cos(2(e + fx)) + 140c^2 \cos(3(e + fx)) + 301cd \cos(3(e + fx)) + 115d^2 \cos(3(e + fx))\right) \sin\left(\frac{1}{2}(e + fx)\right)\right)}{(420f)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]

[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 4*(35*c^2 + 196*c*d + 145*d^2 + (420*c^2 + 987*c*d + 465*d^2)*Cos[e + f*x] + (35*c^2 + 196*c*d + 115*d^2)*Cos[2*(e + f*x)] + 140*c^2*Cos[3*(e + f*x)] + 301*c*d*Cos[3*(e + f*x)] + 115*d^2*Cos[3*(e + f*x)])*Sin[(e + f*x)/2])/ (420*f)

fricas [A] time = 0.54, size = 500, normalized size = 1.94

$$\left[\frac{105 \left(a^2 c^2 \cos^4(fx + e) + a^2 c^2 \cos^3(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e)}{\cos(fx + e) + 1} \right)}{\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a

```

^2*d^2 + 2*(140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a
^2*c^2 + 196*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*
d^2)*cos(f*x + e)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f
*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4 +
a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a^2*d^2 + 2*(140*a^2*c^2 + 3
01*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a^2*c^2 + 196*a^2*c*d + 115*
a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*d^2)*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x
+ e)^3)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)*2*(2*((-1/216090
000*(252105000*sqrt(2)*a^6*c^2*sign(cos(f*x+exp(1)))+164640000*sqrt(2)*a^6*
d^2*sign(cos(f*x+exp(1)))+460992000*sqrt(2)*a^6*c*d*sign(cos(f*x+exp(1))))*
tan(1/2*(f*x+exp(1)))^2-1/147000*(-563500*sqrt(2)*a^6*c^2*sign(cos(f*x+exp(
1)))-392000*sqrt(2)*a^6*d^2*sign(cos(f*x+exp(1)))-1097600*sqrt(2)*a^6*c*d*s
ign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2-1/3087000*(12862500*sqrt(2)*
a^6*c^2*sign(cos(f*x+exp(1)))+10290000*sqrt(2)*a^6*d^2*sign(cos(f*x+exp(1))
)+28812000*sqrt(2)*a^6*c*d*sign(cos(f*x+exp(1))))*tan(1/2*(f*x+exp(1)))^2-
1/14700*(-22050*sqrt(2)*a^6*c^2*sign(cos(f*x+exp(1)))-29400*sqrt(2)*a^6*d^2
*sign(cos(f*x+exp(1)))-58800*sqrt(2)*a^6*c*d*sign(cos(f*x+exp(1))))/sqrt(-
a*tan(1/2*(f*x+exp(1)))^2+a)/(-a*tan(1/2*(f*x+exp(1)))^2+a)^3*tan(1/2*(f*x+
exp(1))-1/2*a^3*sqrt(-a)*c^2*sign(cos(f*x+exp(1)))*ln(abs(2*(sqrt(-a*tan(1
/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2-4*sqrt(2)*abs(a)-6*
a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))
)^2+4*sqrt(2)*abs(a)-6*a))/abs(a))/f

maple [B] time = 1.80, size = 504, normalized size = 1.95

$$\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(105 \sin(fx+e) (\cos^3(fx+e)) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) \sqrt{2}}{2\cos(fx+e)} \right) \left(-\frac{2\cos(fx+e)}{1+\cos(fx+e)} \right)^{\frac{7}{2}} c^2 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x)

[Out] 1/840/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(105*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^2+315*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^2+315*sin(f*x+e)*cos(f*x+e)*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^2+105*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^2+105*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*c^2*2*sin(f*x+e)-4480*cos(f*x+e)^4*c^2-9632*cos(f*x+e)^4*c*d-3680*cos(f*x+e)^4*d^2+3920*cos(f*x+e)^3*c^2+6496*cos(f*x+e)^3*c*d+1840*cos(f*x+e)^3*d^2+560*cos(f*x+e)^2*c^2+2464*cos(f*x+e)^2*c*d+880*cos(f*x+e)^2*d^2+672*cos(f*x+e)*c*d+720*cos(f*x+e)*d^2+240*d^2/sin(f*x+e)/cos(f*x+e)^3*a^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c + \frac{d}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e + fx) + 1))^{5/2} (c + d \sec(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**2,x)

[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2, x)

3.162 $\int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx)) dx$

Optimal. Leaf size=142

$$\frac{2a^{5/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2a^3(35c+32d) \tan(e+fx)}{15f\sqrt{a \sec(e+fx)+a}} + \frac{2a^2(5c+8d) \tan(e+fx)\sqrt{a \sec(e+fx)+a}}{15f} + \frac{2ad}{f}$$

[Out] $2*a^{(5/2)}*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f+2/5*a*d*(a+a*\sec(f*x+e))^{(3/2)}*\tan(f*x+e)/f+2/15*a^3*(35*c+32*d)*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2/15*a^2*(5*c+8*d)*(a+a*\sec(f*x+e))^{(1/2)}*\tan(f*x+e)/f$

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3917, 3915, 3774, 203, 3792}

$$\frac{2a^3(35c+32d) \tan(e+fx)}{15f\sqrt{a \sec(e+fx)+a}} + \frac{2a^2(5c+8d) \tan(e+fx)\sqrt{a \sec(e+fx)+a}}{15f} + \frac{2a^{5/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{f} + \frac{2ad}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]

[Out] $(2*a^{(5/2)}*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e+f*x])/\text{Sqrt}[a+a*\text{Sec}[e+f*x]])/f + (2*a^3*(35*c+32*d)*\text{Tan}[e+f*x])/(15*f*\text{Sqrt}[a+a*\text{Sec}[e+f*x]]) + (2*a^2*(5*c+8*d)*\text{Sqrt}[a+a*\text{Sec}[e+f*x]]*\text{Tan}[e+f*x])/(15*f) + (2*a*d*(a+a*\text{Sec}[e+f*x])^{(3/2)}*\text{Tan}[e+f*x])/(5*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3915

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] := Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 3917

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m-1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m-1)*Simp[a*c*m + (b*c*m + a*d*(2*m-1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},

x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx)) dx &= \frac{2ad(a + a \sec(e + fx))^{3/2} \tan(e + fx)}{5f} + \frac{2}{5} \int (a + a \sec(e + fx))^{3/2} dx \\
 &= \frac{2a^2(5c + 8d)\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{15f} + \frac{2ad(a + a \sec(e + fx))^{3/2}}{15f} \\
 &= \frac{2a^2(5c + 8d)\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{15f} + \frac{2ad(a + a \sec(e + fx))^{3/2}}{15f} \\
 &= \frac{2a^3(35c + 32d) \tan(e + fx)}{15f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2(5c + 8d)\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{15f} \\
 &= \frac{2a^{5/2}c \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{f} + \frac{2a^3(35c + 32d) \tan(e + fx)}{15f\sqrt{a + a \sec(e + fx)}} + \frac{2a^2(5c + 8d)\sqrt{a + a \sec(e + fx)} \tan(e + fx)}{15f}
 \end{aligned}$$

Mathematica [A] time = 1.03, size = 128, normalized size = 0.90

$$\frac{a^2 \sec\left(\frac{1}{2}(e + fx)\right) \sec^2(e + fx) \sqrt{a(\sec(e + fx) + 1)} \left(2 \sin\left(\frac{1}{2}(e + fx)\right) (2(5c + 14d) \cos(e + fx) + (40c + 43d) \cos(2(e + fx)))\right)}{30f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]

[Out] (a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(40*c + 49*d + 2*(5*c + 14*d)*Cos[e + f*x] + (40*c + 43*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*f)

fricas [A] time = 0.46, size = 390, normalized size = 2.75

$$\frac{15 \left(a^2 c \cos^3(fx + e) + a^2 c \cos^2(fx + e) \right) \sqrt{-a} \log \left(\frac{2a \cos^2(fx + e) - 2\sqrt{-a} \sqrt{\frac{a \cos(fx + e) + a}{\cos(fx + e)}} \cos(fx + e) \sin(fx + e) + a \cos(fx + e) - a}{\cos(fx + e) + 1} \right)}{15 \left(f \cos^3(fx + e) + f \cos^2(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c + 14*a^2*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c

[In] integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot (30 \cdot (\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{3/4} \cdot a^{5/2} \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) - 2 \cdot (\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot ((12 \cdot a^2 \cdot \cos(\frac{3}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \cdot \sin(2fx + 2e) - 3 \cdot a^2 \cdot \sin(2fx + 2e) - 4 \cdot (3 \cdot a^2 \cdot \cos(2fx + 2e) + 4 \cdot a^2) \cdot \sin(\frac{3}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \cdot \cos(\frac{3}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) + (12 \cdot a^2 \cdot \sin(2fx + 2e) \cdot \sin(\frac{3}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 3 \cdot a^2 \cdot \cos(2fx + 2e) - a^2 + 4 \cdot (3 \cdot a^2 \cdot \cos(2fx + 2e) + 4 \cdot a^2) \cdot \cos(\frac{3}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \cdot \sin(\frac{3}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))) \cdot \sqrt{a} + 3 \cdot ((a^2 \cdot \cos(2fx + 2e))^2 + a^2 \cdot \sin(2fx + 2e)^2 + 2 \cdot a^2 \cdot \cos(2fx + 2e) + a^2) \cdot \arctan2((\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot (\cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) - \cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))), (\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot (\cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cdot \cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + 1) - (a^2 \cdot \cos(2fx + 2e))^2 + a^2 \cdot \sin(2fx + 2e)^2 + 2 \cdot a^2 \cdot \cos(2fx + 2e) + a^2) \cdot \arctan2((\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot (\cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) - \cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e))))), (\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot (\cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cdot \cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e)))) + \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)))) - 1) - (a^2 \cdot \cos(2fx + 2e))^2 + a^2 \cdot \sin(2fx + 2e)^2 + 2 \cdot a^2 \cdot \cos(2fx + 2e) + a^2) \cdot \arctan2((\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)), (\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot \cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) + 1) + (a^2 \cdot \cos(2fx + 2e))^2 + a^2 \cdot \sin(2fx + 2e)^2 + 2 \cdot a^2 \cdot \cos(2fx + 2e) + a^2) \cdot \arctan2((\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot \sin(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)), (\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1)^{1/4} \cdot \cos(\frac{1}{2} \arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) - 1)) \cdot \sqrt{a} \cdot c / ((\cos(2fx + 2e))^2 + \sin(2fx + 2e)^2 + 2 \cdot \cos(2fx + 2e) + 1) \cdot f)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\cos(e + fx)} \right)^{5/2} \left(c + \frac{d}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)

[Out] int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec(e + fx) + 1 \right) \right)^{5/2} \left(c + d \sec(e + fx) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)

```
[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x)), x)
```

$$3.163 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{c+d \sec(e+fx)} dx$$

Optimal. Leaf size=203

$$\frac{2a^{7/2}(c-d)^2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a \sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f\sqrt{c+d}\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} + \frac{2a^{7/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{cf\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} + \frac{2a^3 \tan(e+fx)}{df\sqrt{a \sec(e+fx)+a}}$$

[Out] $2a^3 \tan(f*x+e)/d/f/(a+a*\sec(f*x+e))^{(1/2)}+2a^{(7/2)}*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/c/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-2a^{(7/2)}*(c-d)^2*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*\tan(f*x+e)/c/d^{(3/2)}/f/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 180, 63, 206, 208}

$$\frac{2a^{7/2}(c-d)^2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a \sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}f\sqrt{c+d}\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} + \frac{2a^{7/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{cf\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} + \frac{2a^3 \tan(e+fx)}{df\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]), x]

[Out] $(2*a^3*\operatorname{Tan}[e+f*x])/(d*f*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]])+(2*a^{(7/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]/\operatorname{Sqrt}[a]]*\operatorname{Tan}[e+f*x])/(c*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]])-(2*a^{(7/2)}*(c-d)^2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c+d])]*\operatorname{Tan}[e+f*x])/(c*d^{(3/2)}*\operatorname{Sqrt}[c+d]*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]])$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n, x], x, (a+b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c-a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a+b*x)^m*(c+d*x)^n*(e+f*x)^p*(g+h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(
d_.) + (c_.))^(n_.), x_Symbol] :=> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sec(e + fx))^{5/2}}{c + d \sec(e + fx)} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{a^2}{d\sqrt{a-ax}} + \frac{a^2}{cx\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^3 \tan(e + fx)}{df\sqrt{a + a \sec(e + fx)}} - \frac{(a^4 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{cf\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \\ &= \frac{2a^3 \tan(e + fx)}{df\sqrt{a + a \sec(e + fx)}} + \frac{(2a^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a - a \sec(e + fx)}\right)}{cf\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2a^3 \tan(e + fx)}{df\sqrt{a + a \sec(e + fx)}} + \frac{2a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{cf\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{2a^{7/2}(c - d)}{cd^{3/2}\sqrt{c + d}} \end{aligned}$$

Mathematica [C] time = 6.55, size = 343, normalized size = 1.69

$$\cos^{\frac{3}{2}}(e + fx) \sec^5\left(\frac{1}{2}(e + fx)\right) (a(\sec(e + fx) + 1))^{5/2} (c \cos(e + fx) + d) \left(-\frac{16d(c-d)^2 \sin^3\left(\frac{1}{2}(e+fx)\right) (c \cos(e+fx)+d) {}_2F_1\left(2, \frac{5}{2}, \frac{7}{2}, \frac{-2d \sec(e+fx) \sin(e+fx)/2}{(c+d)^3 \cos^2(e+fx)}\right)}{(c+d)^3 \cos^2(e+fx)} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^(3/2)*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e +
f*x]))^(5/2)*((10*(c - d)^2*(c + 3*d + 2*c*Cos[e + f*x])*Csc[(e + f*x)/2]*
(-ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]] + Sqrt[-((d*(-1 + Sec[e +
f*x]))/(c + d))]))/(d*(c + d)*Sqrt[Cos[e + f*x]]*Sqrt[-((d*(-1 + Sec[e +
f*x]))/(c + d))]) + (20*(3*c - d)*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] - (
16*(c - d)^2*d*(d + c*Cos[e + f*x])*Hypergeometric2F1[2, 5/2, 7/2, (-2*d*Se
c[e + f*x]*Sin[(e + f*x)/2]^2)/(c + d)]*Sin[(e + f*x)/2]^3)/((c + d)^3*Cos[
e + f*x]^(5/2)) + 10*c*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(
e + f*x)/2])/Sqrt[Cos[e + f*x]])))/(40*c^2*f*(c + d*Sec[e + f*x]))
```

fricas [A] time = 5.11, size = 1140, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```


$$\frac{\sqrt{-c*d}/a/c/d+1/4*a^2*\sqrt{-a}*sign(\cos(f*x+\exp(1)))*\ln(\text{abs}(\sqrt{-a*\tan(1/2*(f*x+\exp(1))}^2+a)-\sqrt{-a}*\tan(1/2*(f*x+\exp(1))})^2+a*(2*\sqrt{2}-3)))/c-1/4*a^2*\sqrt{-a}*sign(\cos(f*x+\exp(1)))*\ln(\text{abs}(\sqrt{-a*\tan(1/2*(f*x+\exp(1))}^2+a)-\sqrt{-a}*\tan(1/2*(f*x+\exp(1))})^2+a*(-2*\sqrt{2}-3))/c)/f}$$

maple [B] time = 1.50, size = 1487, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x)

[Out]
$$\begin{aligned} & -1/2/f*(2*(d/(c-d))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2})* \\ & \sin(f*x+e)/\cos(f*x+e)*2^{1/2})*((c+d)*(c-d))^{1/2}*2^{1/2}*(-2*\cos(f*x+e)/(\\ & 1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)-\ln(2*((d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+ \\ & \cos(f*x+e)))^{1/2}*2^{1/2}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+ \\ & e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)+((c+d)*(c-d))^{1/2}*\cos(f*x+e)-c*\sin(\\ & f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2}))/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)-c* \\ & \cos(f*x+e)+d*\cos(f*x+e)+c-d))*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}* \\ & c^2*\sin(f*x+e)+2*\ln(2*((d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2} \\ & *2^{1/2}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)) \\ &)^{1/2}*d*\sin(f*x+e)+((c+d)*(c-d))^{1/2}*\cos(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+ \\ & e)-((c+d)*(c-d))^{1/2}))/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)-c*\cos(f*x+e)+d*\cos(\\ & f*x+e)+c-d))*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*c*d*\sin(f*x+e)-\ln \\ & (2*((d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*2^{1/2}*c*\sin(f*x+ \\ & e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+ \\ & e)+((c+d)*(c-d))^{1/2}*\cos(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2} \\ &)/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)-c*\cos(f*x+e)+d*\cos(f*x+e)+c-d))*2^{1/2} \\ & *(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d^2*\sin(f*x+e)+\ln(-2*((d/(c-d))^{1/2} \\ &)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*2^{1/2}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d) \\ &))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*s \\ & \sin(f*x+e)-((c+d)*(c-d))^{1/2}*\cos(f*x+e)+((c+d)*(c-d))^{1/2}))/(((c+d)*(c-d) \\ &)^{1/2}*\sin(f*x+e)+c*\cos(f*x+e)-d*\cos(f*x+e)-c+d))*2^{1/2}*(-2*\cos(f*x+e)/(\\ & 1+\cos(f*x+e)))^{1/2}*c^2*\sin(f*x+e)-2*\ln(-2*((d/(c-d))^{1/2}*(-2*\cos(f*x+e) \\ & / (1+\cos(f*x+e)))^{1/2}*2^{1/2}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos \\ & (f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d) \\ & *(c-d))^{1/2}*\cos(f*x+e)+((c+d)*(c-d))^{1/2}))/(((c+d)*(c-d))^{1/2}*\sin(f*x+ \\ & e)+c*\cos(f*x+e)-d*\cos(f*x+e)-c+d))*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2} \\ & *c*d*\sin(f*x+e)+\ln(-2*((d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2} \\ & *2^{1/2}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x \\ & +e)))^{1/2}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2}*\cos(\\ & f*x+e)+((c+d)*(c-d))^{1/2}))/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)+c*\cos(f*x+e)-d* \\ & \cos(f*x+e)-c+d))*2^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d^2*\sin(f*x+e \\ &)+4*(d/(c-d))^{1/2}*((c+d)*(c-d))^{1/2}*c*\cos(f*x+e)-4*(d/(c-d))^{1/2}*((c+ \\ & d)*(c-d))^{1/2}*c*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}/\sin(f*x+e)*a^2/c/((c \\ & +d)*(c-d))^{1/2}/d/(d/(c-d))^{1/2} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sec(fx + e) + a)^{\frac{5}{2}}}{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{c + \frac{d}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)), x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \left(\sec(e + fx) + 1\right)\right)^{5/2}}{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)), x)

[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x)), x)

$$3.164 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=329

$$\frac{2a^{7/2}(c-d)\sqrt{c+d} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a \sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2 d^{3/2} f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2a^{7/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{c^2 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{a^{7/2}}{cd^{3/2} f}$$

[Out] $-a^3(c-d)^2 \tan(f*x+e)/c/d/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)} + 2*a^{(7/2)}*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/c^2/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)} - a^{(7/2)}*(c-d)^2*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*\tan(f*x+e)/c/d^{(3/2)}/(c+d)^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)} + 2*a^{(7/2)}*(c-d)*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*(c+d)^{(1/2)}*\tan(f*x+e)/c^2/d^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{2a^{7/2}(c-d)\sqrt{c+d} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a \sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2 d^{3/2} f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2a^{7/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{c^2 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{a^{7/2}}{cd^{3/2} f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]

[Out] $(2*a^{(7/2)}*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^{(7/2)}*(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d]])*Tan[e + f*x])/(c*d^{(3/2)}*(c + d)^{(3/2)}*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^{(7/2)}*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d]])*Tan[e + f*x])/(c^2*d^{(3/2)}*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Tan[e + f*x])/(c*d*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,

g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3940

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^2} dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{a^2}{c^2x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)^2} + \frac{a^2(c^2-d^2)}{c^2d\sqrt{a-ax}(c+dx)}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a^4 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^2f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{(a^4(c - d)^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{c+dx} dx, x, \sec(e + fx)\right)}{cdf\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{a^3(c - d)^2 \tan(e + fx)}{cd(c + d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} + \frac{(2a^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{c+dx} dx, x, \sec(e + fx)\right)}{c^2f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^2f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{2a^{7/2}(c - d)\sqrt{c + d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a \sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^2f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} - \frac{a^{7/2}(c - d)^2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a \sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{cd^{3/2}(c + d)^{3/2}f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

Mathematica [A] time = 3.64, size = 280, normalized size = 0.85

$$\sqrt{\cos(e + fx)} \sec^5\left(\frac{1}{2}(e + fx)\right) (a(\sec(e + fx) + 1))^{5/2} (c \cos(e + fx) + d)^2 \left[\frac{4\sqrt{2}(c-d) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{d} \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}} - \frac{(c-d)^2 \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d}\sqrt{c+d}} \right] - \frac{8c^2 f(c + d \sec(e + fx))}{\dots}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c + \frac{d}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2,x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a(\sec(e+fx) + 1)\right)^{5/2}}{(c + d \sec(e+fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)

[Out] Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**2, x)

$$3.165 \quad \int \frac{(a+a \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^3} dx$$

Optimal. Leaf size=536

$$\frac{2a^{7/2}\sqrt{d} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^3f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^{7/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^{7/2}(c-d) \tan(e+fx)}{c^2d^{3/2}f\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}$$

[Out] $-1/2*a^3*(c-d)^2*\tan(f*x+e)/c/d/(c+d)/f/(c+d*\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{1/2}+a^3*(c-d)*\tan(f*x+e)/c^2/d/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}-3/4*a^3*(c-d)^2*\tan(f*x+e)/c/d/(c+d)^2/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}+2*a^{7/2}*\operatorname{arctanh}((a-a*\sec(f*x+e))^{1/2}/a^{1/2})*\tan(f*x+e)/c^3/f/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}-3/4*a^{7/2}*(c-d)^2*\operatorname{arctanh}(d^{1/2}*(a-a*\sec(f*x+e))^{1/2}/a^{1/2}/(c+d)^{1/2})*\tan(f*x+e)/c/d^{3/2}/(c+d)^{5/2}/f/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}+a^{7/2}*(c-d)*\operatorname{arctanh}(d^{1/2}*(a-a*\sec(f*x+e))^{1/2}/a^{1/2}/(c+d)^{1/2})*\tan(f*x+e)/c^2/d^{3/2}/f/(c+d)^{1/2}/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}-2*a^{7/2}*\operatorname{arctanh}(d^{1/2}*(a-a*\sec(f*x+e))^{1/2}/a^{1/2}/(c+d)^{1/2})*d^{1/2}*\tan(f*x+e)/c^3/f/(c+d)^{1/2}/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.49, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{a^{7/2}(c-d) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2d^{3/2}f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^3(c-d) \tan(e+fx)}{c^2df\sqrt{a\sec(e+fx)+a}(c+d \sec(e+fx))} - \frac{2a^{7/2}\sqrt{d} \tan(e+fx)}{c^3f\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]

[Out] $(2*a^{7/2}*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(c^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (3*a^{7/2}*(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d]])*Tan[e + f*x])/(4*c*d^{3/2}*(c + d)^{5/2}*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^{7/2}*(c - d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d]])*Tan[e + f*x])/(c^2*d^{3/2}*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^{7/2}*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d]])*Tan[e + f*x])/(c^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^3*(c - d)^2*Tan[e + f*x])/(2*c*d*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2) + (a^3*(c - d)*Tan[e + f*x])/(c^2*d*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])) - (3*a^3*(c - d)^2*Tan[e + f*x])/(4*c*d*(c + d)^2*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 180

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.))^{(q_.)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, n\}, x\} \&\& \text{IntegersQ}[p, q]$

Rule 206

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rule 3940

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{(n_.)}, x_Symbol] :> \text{Dist}[(a^2*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{(m - 1/2)}*(c + d*x)^n]/(x*\text{Sqrt}[a - b*x]), x], x, \text{Csc}[e + f*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{IntegerQ}[m - 1/2]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^3} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^2}{x\sqrt{a-ax}(c+dx)^3} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{a^2}{c^3 x\sqrt{a-ax}} - \frac{a^2(c-d)^2}{cd\sqrt{a-ax}(c+dx)^3} + \frac{a^2(c^2-d^2)}{c^2 d\sqrt{a-ax}(c+dx)^2} - \frac{a^2 d}{c^3\sqrt{a-ax}(c+dx)}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^4 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^3 f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{(a^4(c-d)^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{cdf\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{a^3(c-d)^2 \tan(e + fx)}{2cd(c+d)f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))^2} + \frac{a^3(c-d) \tan(e + fx)}{c^2 d f\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} \\
&= \frac{2a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} - \frac{2a^{7/2} \sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(e + fx)}{c^3 \sqrt{c+d} f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= \frac{2a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{a^{7/2}(c-d) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(e + fx)}{c^2 d^{3/2} \sqrt{c+d} f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= \frac{2a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} - \frac{3a^{7/2}(c-d)^2 \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(e + fx)}{4cd^{3/2}(c+d)^{5/2} f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 26.11, size = 3344, normalized size = 6.24

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3, x]

[Out] ((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^5*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*(-1/16*((c^3 - 12*c^2*d + 5*c*d^2 + 6*d^3)*Sin[(e + f*x)/2]))/(c^3*d*(c + d)^2) + (-c^2*d*Sin[(e + f*x)/2]) + 2*c*d^2*Sin[(e + f*x)/2] - d^3*Sin[(e + f*x)/2))/(8*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (3*c^3*Sin[(e + f*x)/2] - 14*c^2*d*Sin[(e + f*x)/2] + 3*c*d^2*Sin[(e + f*x)/2] + 8*d^3*Sin[(e + f*x)/2))/(16*c^3*(c + d)^2*(d + c*Cos[e + f*x])))/(f*(c + d*Sec[e + f*x])^3) - (Sqrt[3 - 2*Sqrt[2]]*Cos[(e + f*x)/4]^2*(d + c*Cos[e + f*x])^3*(c*(c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*EllipticF[ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 16*d*(c + d)^3*EllipticPi[-3 + 2*Sqrt[2], ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*(EllipticPi[-(((-3 + 2*Sqrt[2])*(c + d))/(3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] - d)), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] + EllipticPi[((-3 + 2*Sqrt[2])*(c + d))/(-3*c + 2*Sqrt[2]*Sqrt[c*(c - d)] + d), ArcSin[Tan[(e + f*x)/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]]))*Sec[(e + f*x)/2]^5*((7*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(16*(c + d)^2*(d + c*Cos[e + f*x])) + (c*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(32*d*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(32*c*(c + d)^2*(d + c*Cos[e + f*x])) + (Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(8*(c + d)^2*(d + c*Cos[e + f*x])) + (d*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(4*c*(c + d)^2*(d + c*Cos[e + f*x])) + (d^2*Cos[(3*(e + f*x))/2]*Sqrt[Sec[e + f*x]])/(8*c^2*(c + d)^2*(d + c*Cos[e + f*x])))*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[1 + (-3 + 2*Sqr

$$\begin{aligned}
& t[2]) * \text{Tan}[(e + f*x)/4]^2 * \text{Sqrt}[1 - (3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x)/4]^2] / (8 * \\
& c^3 * d * (c + d)^3 * f * (c + d * \text{Sec}[e + f*x])^3 * ((\text{Sqrt}[3 - 2*\text{Sqrt}[2]] * (3 + 2*\text{Sqrt}[2]) * \\
& (c * (c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3) * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x)/4] / \\
& \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - 16*d * (c + d)^3 * \text{EllipticPi}[-3 + 2* \\
& \text{Sqrt}[2], \text{ArcSin}[\text{Tan}[(e + f*x)/4] / \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - (\\
& c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4) * (\text{EllipticPi}[-(((-3 + 2*\text{Sqrt}[2]) * \\
& (c + d)) / (3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] - d)), \text{ArcSin}[\text{Tan}[(e + f*x) / \\
& 4] / \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 + 2*\text{Sqrt}[2]) * (c \\
& + d)) / (-3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt} \\
& [3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2])) * \text{Sqrt}[\text{Sec}[e + f*x]] * \text{Tan}[(e + f*x) / 4] * \text{Sqrt} \\
& [1 + (-3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2] / (32 * c^3 * d * (c + d)^3 * \text{Sqrt}[1 - (\\
& 3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2] - (\text{Sqrt}[3 - 2*\text{Sqrt}[2]] * (-3 + 2*\text{Sqrt}[2]) \\
& * (c * (c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3) * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt} \\
& [3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - 16*d * (c + d)^3 * \text{EllipticPi}[-3 + 2*\text{Sqrt} \\
& [2], \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - (c^4 \\
& + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4) * (\text{EllipticPi}[-(((-3 + 2*\text{Sqrt}[2]) * \\
& (c + d)) / (3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] - d)), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \\
& \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 + 2*\text{Sqrt}[2]) * (c + \\
& d)) / (-3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 \\
& - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2])) * \text{Sqrt}[\text{Sec}[e + f*x]] * \text{Tan}[(e + f*x) / 4] * \text{Sqrt} \\
& [1 - (3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2] / (32 * c^3 * d * (c + d)^3 * \text{Sqrt}[1 + (-3 + \\
& 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2] + (\text{Sqrt}[3 - 2*\text{Sqrt}[2]] * \text{Cos}[(e + f*x) / 4] * (c \\
& * (c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3) * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[\\
& 3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - 16*d * (c + d)^3 * \text{EllipticPi}[-3 + 2*\text{Sqrt}[2] \\
&], \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - (c^4 + \\
& 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4) * (\text{EllipticPi}[-(((-3 + 2*\text{Sqrt}[2]) * (c \\
& + d)) / (3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] - d)), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt} \\
& [3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 + 2*\text{Sqrt}[2]) * (c + d)) \\
& / (-3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 - 2 \\
& * \text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2])) * \text{Sqrt}[\text{Sec}[e + f*x]] * \text{Sin}[(e + f*x) / 4] * \text{Sqrt}[1 + \\
& (-3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2 * \text{Sqrt}[1 - (3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2] \\
&) / (16 * c^3 * d * (c + d)^3) - (\text{Sqrt}[3 - 2*\text{Sqrt}[2]] * \text{Cos}[(e + f*x) / 4]^2 * (c * \\
& (c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3) * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 \\
& - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - 16*d * (c + d)^3 * \text{EllipticPi}[-3 + 2*\text{Sqrt}[2] \\
&], \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] - (c^4 + 1 \\
& 0 * c^3 * d - 15 * c^2 * d^2 - 20 * c * d^3 - 8 * d^4) * (\text{EllipticPi}[-(((-3 + 2*\text{Sqrt}[2]) * (c \\
& + d)) / (3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] - d)), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt} \\
& [3 - 2*\text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2]] + \text{EllipticPi}[((-3 + 2*\text{Sqrt}[2]) * (c + d)) / \\
& (-3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] + d), \text{ArcSin}[\text{Tan}[(e + f*x) / 4] / \text{Sqrt}[3 - 2* \\
& \text{Sqrt}[2]]], 17 - 12*\text{Sqrt}[2])) * \text{Sec}[e + f*x]^(3/2) * \text{Sin}[e + f*x] * \text{Sqrt}[1 + (-3 \\
& + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2 * \text{Sqrt}[1 - (3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2] \\
&) / (16 * c^3 * d * (c + d)^3) - (\text{Sqrt}[3 - 2*\text{Sqrt}[2]] * \text{Cos}[(e + f*x) / 4]^2 * \text{Sqrt}[\text{Sec} \\
& [e + f*x]] * \text{Sqrt}[1 + (-3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2 * \text{Sqrt}[1 - (3 + 2*\text{Sqrt} \\
& [2]) * \text{Tan}[(e + f*x) / 4]^2] * ((c * (c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3) * \text{Sec}[(e + \\
& f*x) / 4]^2) / (4 * \text{Sqrt}[3 - 2*\text{Sqrt}[2]] * \text{Sqrt}[1 - \text{Tan}[(e + f*x) / 4]^2 / (3 - 2*\text{Sqrt}[2] \\
&)]) * \text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2) / (3 - 2*\text{Sqrt}[2])]) - (4 * \\
& d * (c + d)^3 * \text{Sec}[(e + f*x) / 4]^2) / (\text{Sqrt}[3 - 2*\text{Sqrt}[2]] * \text{Sqrt}[1 - \text{Tan}[(e + f*x) \\
& / 4]^2 / (3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2) / (3 - \\
& 2*\text{Sqrt}[2])]) * (1 - ((-3 + 2*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2) / (3 - 2*\text{Sqrt}[2])) - \\
& (c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4) * (\text{Sec}[(e + f*x) / 4]^2 / (4 * \text{Sqrt} \\
& [3 - 2*\text{Sqrt}[2]] * \text{Sqrt}[1 - \text{Tan}[(e + f*x) / 4]^2 / (3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 - ((1 \\
& 7 - 12*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2) / (3 - 2*\text{Sqrt}[2])]) * (1 + ((-3 + 2*\text{Sqrt}[2]) \\
& * (c + d) * \text{Tan}[(e + f*x) / 4]^2) / ((3 - 2*\text{Sqrt}[2]) * (3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - \\
& d)] - d))) + \text{Sec}[(e + f*x) / 4]^2 / (4 * \text{Sqrt}[3 - 2*\text{Sqrt}[2]] * \text{Sqrt}[1 - \text{Tan}[(e + f \\
& *x) / 4]^2 / (3 - 2*\text{Sqrt}[2])]) * \text{Sqrt}[1 - ((17 - 12*\text{Sqrt}[2]) * \text{Tan}[(e + f*x) / 4]^2) / (\\
& 3 - 2*\text{Sqrt}[2])]) * (1 - ((-3 + 2*\text{Sqrt}[2]) * (c + d) * \text{Tan}[(e + f*x) / 4]^2) / ((3 - 2* \\
& \text{Sqrt}[2]) * (-3*c + 2*\text{Sqrt}[2] * \text{Sqrt}[c * (c - d)] + d)))))) / (8 * c^3 * d * (c + d)^3))
\end{aligned}$$

fricas [A] time = 40.75, size = 3351, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
[Out] [-1/8*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^
2*d^6 + (a^2*c^6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c
^2*d^4)*cos(f*x + e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c
^3*d^3 - 48*a^2*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*
a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*c
os(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2
*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 +
(c + d)*cos(f*x + e) + d)) - 8*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*
c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^
3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^
2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*((a^2*c^5 - 12*a^2
*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2*c^4*d + 10*a^
2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d^3)*f*cos(f*x
+ e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*x + e)^2 + (2*
c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (c^5*d^3 + 2*c^
4*d^4 + c^3*d^5)*f), -1/8*(16*(a^2*c^2*d^3 + 2*a^2*c*d^4 + a^2*d^5 + (a^2*c
^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*c^4*d + 4*a^2*c^3
*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^2*c^3*d^2 + 5*a^2
*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^4
*d^2 + 10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^
6 + 10*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x
+ e)^3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2
*c^2*d^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 2
0*a^2*c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sq
rt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x +
e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
+ e) + d)) + 2*((a^2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*c
os(f*x + e)^2 - (a^2*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d
+ 2*c^6*d^2 + c^5*d^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 +
2*c^4*d^4)*f*cos(f*x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5
)*f*cos(f*x + e) + (c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 +
10*a^2*c^3*d^3 - 15*a^2*c^2*d^4 - 20*a^2*c*d^5 - 8*a^2*d^6 + (a^2*c^6 + 10
*a^2*c^5*d - 15*a^2*c^4*d^2 - 20*a^2*c^3*d^3 - 8*a^2*c^2*d^4)*cos(f*x + e)^
3 + (a^2*c^6 + 12*a^2*c^5*d + 5*a^2*c^4*d^2 - 50*a^2*c^3*d^3 - 48*a^2*c^2*d
^4 - 16*a^2*c*d^5)*cos(f*x + e)^2 + (2*a^2*c^5*d + 21*a^2*c^4*d^2 - 20*a^2*
c^3*d^3 - 55*a^2*c^2*d^4 - 36*a^2*c*d^5 - 8*a^2*d^6)*cos(f*x + e))*sqrt(a/(
c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) - 4*(a^2*c^2*d^3 + 2*a^2*c*d^4 +
a^2*d^5 + (a^2*c^4*d + 2*a^2*c^3*d^2 + a^2*c^2*d^3)*cos(f*x + e)^3 + (a^2*
c^4*d + 4*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 2*a^2*c*d^4)*cos(f*x + e)^2 + (2*a^
2*c^3*d^2 + 5*a^2*c^2*d^3 + 4*a^2*c*d^4 + a^2*d^5)*cos(f*x + e))*sqrt(-a)*l
og((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + ((a^
2*c^5 - 12*a^2*c^4*d + 5*a^2*c^3*d^2 + 6*a^2*c^2*d^3)*cos(f*x + e)^2 - (a^2
*c^4*d + 10*a^2*c^3*d^2 - 7*a^2*c^2*d^3 - 4*a^2*c*d^4)*cos(f*x + e))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/((c^7*d + 2*c^6*d^2 + c^5*d
^3)*f*cos(f*x + e)^3 + (c^7*d + 4*c^6*d^2 + 5*c^5*d^3 + 2*c^4*d^4)*f*cos(f*
x + e)^2 + (2*c^6*d^2 + 5*c^5*d^3 + 4*c^4*d^4 + c^3*d^5)*f*cos(f*x + e) + (
c^5*d^3 + 2*c^4*d^4 + c^3*d^5)*f), -1/4*((a^2*c^4*d^2 + 10*a^2*c^3*d^3 - 15
```



```
(cos(f*x+exp(1)))-87042659012253300578844672*a^5*sqrt(-a)*d^4*(sqrt(-a*tan(
1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1
)))+2417851639229258349412352*a^6*sqrt(-a)*c^3*d^3*sign(cos(f*x+exp(1)))-3626
7774588438875241185280*a^6*sqrt(-a)*c^2*d^2*sign(cos(f*x+exp(1)))+265963680
31521841843535872*a^6*sqrt(-a)*c^3*d*sign(cos(f*x+exp(1)))-2659636803152184
1843535872*a^3*sqrt(-a)*c*d^3*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*
tan(1/2*(f*x+exp(1))))^6*sign(cos(f*x+exp(1)))+60446290980731458735308800*a
^3*sqrt(-a)*c^2*d^2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f
*x+exp(1))))^6*sign(cos(f*x+exp(1)))-2417851639229258349412352*a^3*sqrt(-a)
*c^3*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^
6*sign(cos(f*x+exp(1)))+510166695877373511726006272*a^4*sqrt(-a)*c*d^3*(sq
r(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(
f*x+exp(1)))+200681686056028443001225216*a^4*sqrt(-a)*c^2*d^2*(sqrt(-a*tan(
1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(f*x+exp(1
)))+41103477866897391940009984*a^4*sqrt(-a)*c^3*d*(sqrt(-a*tan(1/2*(f*x+exp
(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^4*sign(cos(f*x+exp(1)))-15716035
6549901792711802880*a^5*sqrt(-a)*c*d^3*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-
sqrt(-a)*tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))+1813388729421943762
05926400*a^5*sqrt(-a)*c^2*d^2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*
tan(1/2*(f*x+exp(1))))^2*sign(cos(f*x+exp(1)))+70117697537648492132958208*a
^5*sqrt(-a)*c^3*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x
+exp(1))))^2*sign(cos(f*x+exp(1)))/(c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-
sqrt(-a)*tan(1/2*(f*x+exp(1))))^4-d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sq
rt(-a)*tan(1/2*(f*x+exp(1))))^4+2*a*c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sq
rt(-a)*tan(1/2*(f*x+exp(1))))^2+6*a*d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-s
qrt(-a)*tan(1/2*(f*x+exp(1))))^2+a^2*c-a^2*d)^2/(-9671406556917033397649408
*sqrt(2)*c^2*d^3-19342813113834066795298816*sqrt(2)*c^3*d^2-967140655691703
3397649408*sqrt(2)*c^4*d)+1/38685626227668133590597632*(4676805239458889338
2517914646921056628989841375232*a^3*sqrt(-a)*c^4*sign(cos(f*x+exp(1)))-3741
44419156711147060143317175368453031918731001856*a^3*sqrt(-a)*d^4*sign(cos(f
*x+exp(1)))-935361047891777867650358292938421132579796827504640*a^3*sqrt(-a
)*c*d^3*sign(cos(f*x+exp(1)))-701520785918833400737768719703815849434847620
628480*a^3*sqrt(-a)*c^2*d^2*sign(cos(f*x+exp(1)))+4676805239458889338251791
46469210566289898413752320*a^3*sqrt(-a)*c^3*d*sign(cos(f*x+exp(1)))*atan(1
/2*(c*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2
-d*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1))))^2+a*
c+3*a*d)/sqrt(2)/sqrt(-d^2-c*d)/a/sqrt(2)/sqrt(-d^2-c*d)/a/(96714065569170
33397649408*sqrt(2)*c^3*d^3+19342813113834066795298816*sqrt(2)*c^4*d^2+9671
406556917033397649408*sqrt(2)*c^5*d)-1/4*a^3*sqrt(-a)*sign(cos(f*x+exp(1)))
*ln(abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-a)*tan(1/2*(f*x+exp(1)
)))^2-4*sqrt(2)*abs(a)-6*a)/abs(2*(sqrt(-a*tan(1/2*(f*x+exp(1)))^2+a)-sqrt(-
a)*tan(1/2*(f*x+exp(1))))^2+4*sqrt(2)*abs(a)-6*a))/c^3/abs(a))/f
```

maple [B] time = 8.15, size = 209489, normalized size = 390.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x)

[Out] result too large to display

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}}{\left(c + \frac{d}{\cos(e+fx)}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3,x)

[Out] int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)

[Out] Timed out

$$3.166 \quad \int \frac{(c+d \sec(e+fx))^3}{\sqrt{a+a \sec(e+fx)}} dx$$

Optimal. Leaf size=258

$$\frac{2\sqrt{a} c^3 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2d^2(3c-d) \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}} - \frac{\sqrt{2} \sqrt{a} (c-d)^3 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

[Out] $2*(3*c-d)*d^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*d^3*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}-2/3*d^3*(1-\sec(f*x+e))*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*c^3*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*a^{(1/2)}*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c-d)^3*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*a^{(1/2)}*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 180, 63, 206, 43}

$$\frac{2\sqrt{a} c^3 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2d^2(3c-d) \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}} - \frac{\sqrt{2} \sqrt{a} (c-d)^3 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]], x]

[Out] $(2*(3*c - d)*d^2*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}) + (2*d^3*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}) - (2*d^3*(1 - \sec[e + f*x])*\tan[e + f*x])/(3*f*\sqrt{a + a*\sec[e + f*x]}) + (2*\sqrt{a}*c^3*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/\sqrt{a}]*\tan[e + f*x])/(f*\sqrt{a - a*\sec[e + f*x]}*\sqrt{a + a*\sec[e + f*x]}) - (\sqrt{2}*\sqrt{a}*(c - d)^3*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/(\sqrt{2}*\sqrt{a})]*\tan[e + f*x])/(f*\sqrt{a - a*\sec[e + f*x]}*\sqrt{a + a*\sec[e + f*x]})$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 206


```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 3940

```
Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{(c + d \sec(e + fx))^3}{\sqrt{a + a \sec(e + fx)}} dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{(3c-d)d^2}{a\sqrt{a-ax}} + \frac{c^3}{ax\sqrt{a-ax}} + \frac{d^3x}{a\sqrt{a-ax}} - \frac{(c-d)^3}{a(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2(3c - d)d^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{(ac^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2(3c - d)d^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{(2c^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a - a \sec(e + fx)}\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2(3c - d)d^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2d^3 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{2d^3(1 - \sec(e + fx)) \tan(e + fx)}{3f\sqrt{a + a \sec(e + fx)}}$$

Mathematica [C] time = 7.24, size = 787, normalized size = 3.05

$$2\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}\cos\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\left(-\frac{(c-d)^3\csc^5\left(\frac{1}{2}(e+fx)\right)\left(-12\sin^8\left(\frac{1}{2}(e+fx)\right)\right)}{\dots}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((2*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2])/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) - (4*c^2*(c + 3*d)*Sin[(e + f*x)/2]^3)/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) + (4*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2])/(3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) + (c^3*Csc[(e + f*x)/2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((4*Sin[(e + f*x)/2]^4)/(1 - 2*Sin[(e + f*x)/2]^2) - (6*Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + (3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/3 - ((c - d)^3*Csc[(e + f*x)/2]^5*(-12*Cos[(e + f*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2])
```


2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.77index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.91, size = 907, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)

[Out] $\frac{1}{6}f \frac{(a(1+\cos(fx+e))/\cos(fx+e))^{1/2} (3(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \operatorname{arctanh}(1/2(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e)/\cos(fx+e))^2)^{1/2} \sin(fx+e) \cos(fx+e) 2^{1/2} c^3 + 3(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} 2^{1/2} \operatorname{arctanh}(1/2(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e)/\cos(fx+e))^2)^{1/2} c^3 \sin(fx+e) + 3(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) \sin(fx+e) \cos(fx+e) c^3 - 9(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) \sin(fx+e) \cos(fx+e) c^2 d + 9(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) \sin(fx+e) \cos(fx+e) c d^2 - 3(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) \sin(fx+e) \cos(fx+e) d^3 + 3(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) c^3 \sin(fx+e) - 9(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) \sin(fx+e) \cos(fx+e) c^2 d^2 + 9(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) c d^2 \sin(fx+e) - 3(-2\cos(fx+e)/(1+\cos(fx+e)))^{3/2} \ln(-(-2\cos(fx+e)/(1+\cos(fx+e)))^{1/2} \sin(fx+e) + \cos(fx+e) - 1)/\sin(fx+e)) d^3 \sin(fx+e) - 36 \cos(fx+e)^2 c d^2 + 4 \cos(fx+e)^2 d^3 + 36 \cos(fx+e) c d^2 - 8 \cos(fx+e) d^3 + 4 d^3 / \sin(fx+e) / \cos(fx+e) / a$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^3}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)

[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^3}{\sqrt{a(\sec(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2), x)

[Out] Integral((c + d*sec(e + f*x))**3/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.167 \quad \int \frac{(c+d \sec(e+fx))^2}{\sqrt{a+a \sec(e+fx)}} dx$$

Optimal. Leaf size=183

$$\frac{2\sqrt{a}c^2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} + \frac{2d^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

[Out] $2*d^2*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^{(1/2)}+2*c^2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*a^{(1/2)}*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c-d)^2*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*a^{(1/2)}*\tan(f*x+e)/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3940, 180, 63, 206}

$$\frac{2\sqrt{a}c^2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{f\sqrt{a-a \sec(e+fx)}\sqrt{a \sec(e+fx)+a}} + \frac{2d^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]

[Out] $(2*d^2*\tan[e + f*x])/(f*\sqrt{a + a*\sec[e + f*x]}) + (2*\sqrt{a}*c^2*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/\sqrt{a}]*\tan[e + f*x])/(f*\sqrt{a - a*\sec[e + f*x]}) - (\sqrt{2}*\sqrt{a}*(c - d)^2*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/(\sqrt{2}*\sqrt{a})]*\tan[e + f*x])/(f*\sqrt{a - a*\sec[e + f*x]}) + \frac{2d^2 \tan(e+fx)}{f\sqrt{a \sec(e+fx)+a}}$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^n/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

&& IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(c + d \sec(e + fx))^2}{\sqrt{a + a \sec(e + fx)}} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{d^2}{a\sqrt{a-ax}} + \frac{c^2}{ax\sqrt{a-ax}} - \frac{(c-d)^2}{a(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} - \frac{(ac^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{(a(c-d)^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-x} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{(2c^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a - a \sec(e + fx)}\right)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^2 \tan(e + fx)}{f\sqrt{a + a \sec(e + fx)}} + \frac{2\sqrt{a} c^2 \tanh^{-1}\left(\frac{\sqrt{a - a \sec(e + fx)}}{\sqrt{a}}\right) \tan(e + fx)}{f\sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{\sqrt{2} \sqrt{a} (c - d)^2 \tan(e + fx)}{f\sqrt{a - a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 2.50, size = 295, normalized size = 1.61

$$2 \cos\left(\frac{1}{2}(e + fx)\right) \cos^{\frac{3}{2}}(e + fx) (c + d \sec(e + fx))^2 \left(c^2 \left(\sqrt{2} \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) - \frac{2 \sin\left(\frac{1}{2}(e + fx)\right)}{\sqrt{\cos(e + fx)}} \right) - \frac{(c-d)^2 \sin\left(\frac{1}{2}(e + fx)\right)}{\sqrt{\cos(e + fx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]^(3/2)*(c + d*Sec[e + f*x])^2*(-1/2*((c - d)^2*Sqrt[-1 + Cos[e + f*x]]*(2 + Cos[e + f*x])*Csc[(e + f*x)/2]^3*(-2*ArcTanh[Sqrt[-(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]] + Sqrt[2 - 2*Sec[e + f*x]]))/Sqrt[2] + (4*c*d*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] + c^2*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]]) - ((c - d)^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]*Sin[(e + f*x)/2]*Sin[e + f*x]^2)/(10*Cos[e + f*x]^(5/2)))/(f*(d + c*Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 3.35, size = 481, normalized size = 2.63

$$4d^2 \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sin(fx+e) + \sqrt{2} (ac^2 - 2acd + ad^2 + (ac^2 - 2acd + ad^2) \cos(fx+e)) \sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx+e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(2)*
(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-1/
a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*
x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^
2 + 2*cos(f*x + e) + 1)) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos
(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f*cos(f*x + e
) + a*f), (2*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(
c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^2 - 2*a*c*d + a*d^2
+ (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*c
os(f*x + e) + a*f)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableUnabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
```


$$\frac{(1+\cos(f*x+e))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1}{\sin(f*x+e)}*c*d*\sin(f*x+e)+(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*d^2*\sin(f*x+e)+2*\cos(f*x+e)*d^2-2*d^2)/\sin(f*x+e)/a$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^2}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^2}{\sqrt{a (\sec(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral((c + d*sec(e + f*x))**2/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.168 \quad \int \frac{c+d \sec(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx$$

Optimal. Leaf size=91

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{\sqrt{2}(c-d) \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f}$$

[Out] $2*c*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/f/a^{(1/2)}-(c-d)*\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/f/a^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3920, 3774, 203, 3795}

$$\frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{\sqrt{2}(c-d) \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a} f}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]], x]

[Out] $(2*c*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(\text{Sqrt}[a]*f) - (\text{Sqrt}[2]*(c - d)*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(\text{Sqrt}[a]*f))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c + d \sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx &= \frac{c \int \sqrt{a + a \sec(e + fx)} dx}{a} - (c - d) \int \frac{\sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx \\
&= \frac{(2c) \text{Subst} \left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} + \frac{(2(c-d)) \text{Subst} \left(\int \frac{1}{2a+x^2} dx, x, -\frac{a \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{f} \\
&= \frac{2c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{a} f} - \frac{\sqrt{2} (c-d) \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}} \right)}{\sqrt{a} f}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 92, normalized size = 1.01

$$\frac{2 \cos\left(\frac{1}{2}(e + fx)\right) \left((d - c) \tan^{-1} \left(\frac{\sin\left(\frac{1}{2}(e + fx)\right)}{\sqrt{\cos(e + fx)}} \right) + \sqrt{2} c \sin^{-1} \left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)}{f \sqrt{\cos(e + fx)} \sqrt{a(\sec(e + fx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (2*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (-c + d)*ArcTan[Sin[(e + f*x)/2]/Sqrt[Cos[e + f*x]]])*Cos[(e + f*x)/2]/(f*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 1.23, size = 314, normalized size = 3.45

$$\frac{\sqrt{2}(ac - ad)\sqrt{-\frac{1}{a}} \log \left(\frac{2\sqrt{2} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{-\frac{1}{a}} \cos(fx+e) \sin(fx+e) - 3 \cos(fx+e)^2 - 2 \cos(fx+e) + 1}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right) + 2\sqrt{-a} c \log \left(\frac{2a \cos(fx+e)}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/2*(sqrt(2)*(a*c - a*d)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*c*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), -(2*sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c - a*d)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Warning, assuming -2*a+a is positive. Hint: run assume to make assump
tions on a variableWarning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableUnable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep
```


*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.18index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.50, size = 194, normalized size = 2.13

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \left(c \ln \left(\frac{-\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) + \cos(fx+e) - 1}{\sin(fx+e)} \right) - d \ln \left(\frac{-\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e) + \cos(fx+e) - 1}{\sin(fx+e)} \right) \right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)

[Out] -1/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(c*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))-d*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))+c*2^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))/a

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + \frac{d}{\cos(e+fx)}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)

[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec(e + fx)}{\sqrt{a (\sec(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral((c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.169 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c+d \sec(e+fx))} dx$$

Optimal. Leaf size=166

$$\frac{2d^{3/2} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{d} \tan(e+fx)}{\sqrt{c+d} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}} - \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

[Out] $2*\arctan(a^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)})/c/f/a^{(1/2)}-\arctan(1/2*a^{(1/2)}*\tan(f*x+e)*2^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/(c-d)/f/a^{(1/2)}+2*d^{(3/2)}*\arctan(a^{(1/2)}*d^{(1/2)}*\tan(f*x+e)/(c+d)^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)})/c/(c-d)/f/a^{(1/2)}/(c+d)^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3929, 3920, 3774, 203, 3795, 3967, 205}

$$\frac{2d^{3/2} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{d} \tan(e+fx)}{\sqrt{c+d} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}} - \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)} + \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]

[Out] $(2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])/\text{Sqrt}[a + a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*c*f) - (\text{Sqrt}[2]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Tan}[e + f*x])]/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(\text{Sqrt}[a]*(c - d)*f) + (2*d^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Tan}[e + f*x])]/(\text{Sqrt}[c + d]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])]/(\text{Sqrt}[a]*c*(c - d)*\text{Sqrt}[c + d]*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre

$eQ[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3929

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))), x_Symbol] := Dist[1/(c*(b*c - a*d)), Int[(b*c - a*d - b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d^2/(c*(b*c - a*d)), Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)])/(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)}(c + d \sec(e + fx))} dx = \frac{\int \frac{ac - ad - ad \sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx}{ac(c - d)} + \frac{d^2 \int \frac{\sec(e + fx)\sqrt{a + a \sec(e + fx)}}{c + d \sec(e + fx)} dx}{ac(c - d)}$$

$$= \frac{\int \sqrt{a + a \sec(e + fx)} dx}{ac} - \frac{\int \frac{\sec(e + fx)}{\sqrt{a + a \sec(e + fx)}} dx}{c - d} - \frac{(2d^2) \text{Subst}\left(\int \frac{1}{a + x^2} dx, x, -\frac{a \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{cf}$$

$$= \frac{2d^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e + fx)}{\sqrt{c + d} \sqrt{a + a \sec(e + fx)}}\right)}{\sqrt{a} c(c - d) \sqrt{c + d} f} - \frac{2 \text{Subst}\left(\int \frac{1}{a + x^2} dx, x, -\frac{a \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{cf}$$

$$= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{a + a \sec(e + fx)}}\right)}{\sqrt{a} cf} - \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \tan(e + fx)}{\sqrt{2} \sqrt{a + a \sec(e + fx)}}\right)}{\sqrt{a}(c - d)f} + \frac{2d^{3/2}}{cf}$$

Mathematica [C] time = 39.13, size = 431980, normalized size = 2602.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 18.62, size = 1050, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*a*d*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e) + 1)))/sqrt(a) + 2*d^2*sqrt(2)*sqrt(a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/sqrt(a) + 2*d^2*sqrt(2)*sqrt(a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/sqrt(a)]
```

$$\begin{aligned}
& x + e)) * \cos(f*x + e) * \sin(f*x + e) + (c + 2*d) * \cos(f*x + e)^2 + (c + d) * \cos(f*x + e) - d) / (c * \cos(f*x + e)^2 + (c + d) * \cos(f*x + e) + d) + 2 * \sqrt{-a} * (c - d) * \log((2*a * \cos(f*x + e)^2 + 2 * \sqrt{-a} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) * \sin(f*x + e) + a * \cos(f*x + e) - a) / (\cos(f*x + e) + 1)) / ((a * c^2 - a * c * d) * f), \\
& -1/2 * (\sqrt{2} * a * c * \sqrt{-1/a} * \log(-(2 * \sqrt{2} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \sqrt{-1/a} * \cos(f*x + e) * \sin(f*x + e) - 3 * \cos(f*x + e)^2 - 2 * \cos(f*x + e) + 1) / (\cos(f*x + e)^2 + 2 * \cos(f*x + e) + 1)) + 4 * a * d * \sqrt{d / (a * c + a * d)} * \arctan((c + d) * \sqrt{d / (a * c + a * d)} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) / (d * \sin(f*x + e))) + 2 * \sqrt{-a} * (c - d) * \log((2*a * \cos(f*x + e)^2 + 2 * \sqrt{-a} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) * \sin(f*x + e) + a * \cos(f*x + e) - a) / (\cos(f*x + e) + 1)) / ((a * c^2 - a * c * d) * f), \\
& -(a * d * \sqrt{-d / (a * c + a * d)} * \log((2 * (c + d) * \sqrt{-d / (a * c + a * d)}) * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) * \sin(f*x + e) + (c + 2*d) * \cos(f*x + e)^2 + (c + d) * \cos(f*x + e) - d) / (c * \cos(f*x + e)^2 + (c + d) * \cos(f*x + e) + d) - \sqrt{2} * \sqrt{a} * c * \arctan(\sqrt{2} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) / (\sqrt{a} * \sin(f*x + e))) + 2 * \sqrt{a} * (c - d) * \arctan(\sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) / (\sqrt{a} * \sin(f*x + e))) / ((a * c^2 - a * c * d) * f), \\
& -(2 * a * d * \sqrt{d / (a * c + a * d)} * \arctan((c + d) * \sqrt{d / (a * c + a * d)} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) / (d * \sin(f*x + e))) - \sqrt{2} * \sqrt{a} * c * \arctan(\sqrt{2} * \sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) / (\sqrt{a} * \sin(f*x + e))) + 2 * \sqrt{a} * (c - d) * \arctan(\sqrt{(a * \cos(f*x + e) + a) / \cos(f*x + e)}) * \cos(f*x + e) / (\sqrt{a} * \sin(f*x + e))) / ((a * c^2 - a * c * d) * f)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
UT:sage2:=int(sage0,x); OUTPUT: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2*pi/x/2) Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2) Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Warning, integratio
n of abs or sign assumes constant sign by intervals (correct if the argumen
t is real): Check [abs(cos(f*t_nostep+exp(1)))] Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/
2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable t
o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2) Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2) Warning, assuming -2*a+a is positive. Hint: run assume to make assump

$$\begin{aligned} & f*x+e)/(1+\cos(f*x+e))^{(1/2)}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*(d/(c-d)) \\ & ^{(1/2)}*c+2^{(1/2)}*\ln(2*((d/(c-d))^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)} \\ & *2^{(1/2)}*c*\sin(f*x+e)-2^{(1/2)}*(d/(c-d))^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)) \\ &)^{(1/2)}*d*\sin(f*x+e)+((c+d)*(c-d))^{(1/2)}*\cos(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+ \\ & e)-((c+d)*(c-d))^{(1/2)}))/(((c+d)*(c-d))^{(1/2)}*\sin(f*x+e)-c*\cos(f*x+e)+d*\cos(\\ & f*x+e)+c-d))*d^2-2^{(1/2)}*\ln(-2*((d/(c-d))^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos(f*x+e) \\ &))^{(1/2)}*2^{(1/2)}*c*\sin(f*x+e)-2^{(1/2)}*(d/(c-d))^{(1/2)}*(-2*\cos(f*x+e)/(1+\cos \\ & s(f*x+e)))^{(1/2)}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{(1/2)} \\ & *cos(f*x+e)+((c+d)*(c-d))^{(1/2)}))/(((c+d)*(c-d))^{(1/2)}*\sin(f*x+e)+c*\cos(f*x+ \\ & e)-d*\cos(f*x+e)-c+d))*d^2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*(a*(1+\cos(f \\ & *x+e))/\cos(f*x+e))^{(1/2)}/(d/(c-d))^{(1/2)}/(c-d)/c/((c+d)*(c-d))^{(1/2)}/a \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec(fx + e) + a} (d \sec(fx + e) + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c + \frac{d}{\cos(e+fx)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)

$$3.170 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=416

$$\frac{2\sqrt{a}d^{3/2}(2c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{d^2\tan(e+fx)}{cf(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} + \dots$$

[Out] $d^2 \tan(f*x+e)/c/(c^2-d^2)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)+2*\arctan((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*a^{(1/2)}*\tan(f*x+e)/c^2/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)+d^{(3/2)}*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*a^{(1/2)}*\tan(f*x+e)/c/(c-d)/(c+d)^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-\arctanh(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*a^{(1/2)}*\tan(f*x+e)/(c-d)^2/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)+2*(2*c-d)*d^{(3/2)}*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*a^{(1/2)}*\tan(f*x+e)/c^2/(c-d)^2/f/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.42, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{d^2 \tan(e+fx)}{cf(c^2-d^2)\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))} + \frac{2\sqrt{a}d^{3/2}(2c-d)\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{c^2f(c-d)^2\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2), x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/((c^2*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (\text{Sqrt}[2]*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])]*\text{Tan}[e + f*x])/((c - d)^2*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (\text{Sqrt}[a]*d^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/((c - d)*(c + d)^{(3/2)}*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*\text{Sqrt}[a]*(2*c - d)*d^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/((c^2*(c - d)^2*\text{Sqrt}[c + d]*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (d^2*\text{Tan}[e + f*x])/((c*(c^2 - d^2)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])))$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180


```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^(m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^n/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^2} dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax} (a+ax)(c+dx)^2} dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{ac^2x \sqrt{a-ax}} - \frac{1}{a(c-d)^2(1+x) \sqrt{a-ax}} + \frac{1}{ac^2x \sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{(a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

$$= \frac{d^2 \tan(e + fx)}{c(c^2 - d^2) f \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))} + \frac{(2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{\sqrt{2} \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{(c - d)^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

$$= \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{\sqrt{2} \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{(c - d)^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}$$

Mathematica [C] time = 36.13, size = 473385, normalized size = 1137.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
 g, integration of abs or sign assumes constant sign by intervals (correct i
 f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
 tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
 ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
 i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
 *pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
 able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
 n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
 /2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
 ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
 eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
 >(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
 o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
 sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableUnabl
 e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
 /2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
 k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
 ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
 nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable


```

*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discon
tinuities at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs
or sign assumes constant sign by intervals (correct if the argument is rea
l):Check [abs(t_nostep^2-1)]Evaluation time: 1.13index.cc index_m i_lex_is_
greater Error: Bad Argument Value

```

maple [B] time = 5.17, size = 117715, normalized size = 282.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec(fx + e) + a} (d \sec(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \left(c + \frac{d}{\cos(e+fx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(e + fx) + 1)} (c + d \sec(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2), x)

$$3.171 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} (c+d \sec(e+fx))^3} dx$$

Optimal. Leaf size=653

$$\frac{2\sqrt{a} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{c^3 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{\sqrt{a} d^{3/2} (2c-d) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{c^2 f (c-d)^2 (c+d)^{3/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{1}{c^2 f (c-d)^2 (c+d)^{3/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

[Out] $1/2*d^2*\tan(f*x+e)/c/(c^2-d^2)/f/(c+d*\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{(1/2)+3/4*d^2*\tan(f*x+e)/c/(c-d)/(c+d)^2/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}+(2*c-d)*d^2*\tan(f*x+e)/c^2/(c-d)^2/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)+2*\arctanh((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*a^{(1/2)*\tan(f*x+e)/c^3/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)+3/4*d^{(3/2)*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)})*a^{(1/2)*\tan(f*x+e)/c/(c-d)/(c+d)^{(5/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}+(2*c-d)*d^{(3/2)*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)})*a^{(1/2)*\tan(f*x+e)/c^2/(c-d)^2/(c+d)^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-a \operatorname{rctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)*2^{(1/2)}/a^{(1/2)})*2^{(1/2)*a^{(1/2)*\tan(f*x+e)/(c-d)^3/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)+2*d^{(3/2)}*(3*c^2-3*c*d+d^2)*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)})*a^{(1/2)*\tan(f*x+e)/c^3/(c-d)^3/f/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.65, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{d^2(2c-d) \tan(e+fx)}{c^2 f (c-d)^2 (c+d) \sqrt{a \sec(e+fx)+a} (c+d \sec(e+fx))} + \frac{d^2 \tan(e+fx)}{2cf (c^2-d^2) \sqrt{a \sec(e+fx)+a} (c+d \sec(e+fx))^2} + \frac{1}{c^2 f (c-d)^2 (c+d)^{3/2} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3), x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/(c^3*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (\text{Sqrt}[2]*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])]*\text{Tan}[e + f*x])/((c - d)^3*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (3*\text{Sqrt}[a]*d^{(3/2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(4*c*(c - d)*(c + d)^{(5/2)*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (\text{Sqrt}[a]*(2*c - d)*d^{(3/2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(c^2*(c - d)^2*(c + d)^{(3/2)*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*\text{Sqrt}[a]*d^{(3/2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(c^3*(c - d)^3*\text{Sqrt}[c + d]*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (d^2*\text{Tan}[e + f*x])/(2*c*(c^2 - d^2)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])^2) + (3*d^2*\text{Tan}[e + f*x])/(4*c*(c - d)*(c + d)^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])) + ((2*c - d)*d^2*\text{Tan}[e + f*x])/(c^2*(c - d)^2*(c + d)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x]))$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I

ntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*sqrt[a + b*Csc[e + f*x]]*sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*sqrt[a - b*x]), x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^3} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax} (a+ax)(c+dx)^3} dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{ac^3 x \sqrt{a-ax}} - \frac{1}{a(c-d)^3 (1+x) \sqrt{a-ax}} + \frac{1}{ac(c-d)^3}\right) dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{(a \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(c-d)^3 (1+x) \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{(c-d)^3 \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= \frac{d^2 \tan(e + fx)}{2c(c^2 - d^2) f \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^2} + \frac{a \tan(e + fx)}{c^2(c-d)} \\
&= \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{\sqrt{2} \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{(c-d)^3 f \sqrt{a - a \sec(e + fx)}} \\
&= \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{\sqrt{2} \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{(c-d)^3 f \sqrt{a - a \sec(e + fx)}} \\
&= \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} - \frac{\sqrt{2} \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{(c-d)^3 f \sqrt{a - a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 38.99, size = 654358, normalized size = 1002.08

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3), x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)

$$3.172 \quad \int \frac{(c+d \sec(e+fx))^3}{(a+a \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=324

$$\frac{2c^3 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a} f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{(c-d)^3 \tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a}} - \frac{(c-d)^3 \tan(e+fx) \tan(e+fx)}{2\sqrt{2} \sqrt{a} f \sqrt{a-a \sec(e+fx)}}$$

[Out] $2*d^3*\tan(f*x+e)/a/f/(a+a*\sec(f*x+e))^{(1/2)}-1/2*(c-d)^3*\tan(f*x+e)/a/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}+2*c^3*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/4*(c-d)^3*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f*2^{(1/2)}/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c-d)^2*(c+2*d)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*\tan(f*x+e)/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.24, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 180, 63, 206, 51}

$$\frac{2c^3 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a} f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{(c-d)^3 \tan(e+fx)}{2af(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a}} - \frac{(c-d)^3 \tan(e+fx) \tan(e+fx)}{2\sqrt{2} \sqrt{a} f \sqrt{a-a \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $(2*d^3*\tan[e+f*x])/(a*f*\sqrt{a+a*\sec[e+f*x]}) - ((c-d)^3*\tan[e+f*x])/(2*a*f*(1+\sec[e+f*x])*\sqrt{a+a*\sec[e+f*x]}) + (2*c^3*\operatorname{ArcTanh}[\sqrt{a-a*\sec[e+f*x]}/\sqrt{a}]*\tan[e+f*x])/(f*\sqrt{a-a*\sec[e+f*x]}) - ((c-d)^3*\operatorname{ArcTanh}[\sqrt{a-a*\sec[e+f*x]}/\sqrt{a}]*\tan[e+f*x])/(2*\sqrt{2}*\sqrt{a}*\sqrt{a-a*\sec[e+f*x]}) - (c-d)^2*(c+2*d)*\operatorname{ArcTanh}[\sqrt{a-a*\sec[e+f*x]}/\sqrt{a}]*\tan[e+f*x]/(a*\sqrt{a-a*\sec[e+f*x]})$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^(m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,

g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(c + d \sec(e + fx))^3}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{d^3}{a^2\sqrt{a-ax}} + \frac{c^3}{a^2x\sqrt{a-ax}} - \frac{(c-d)^3}{a^2(1+x)^2\sqrt{a-ax}} - \frac{(c-d)^2(c+2d)}{a^2(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} - \frac{(c^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{(2c^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} - \frac{(c - d)^3 \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2c^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} - \frac{(c - d)^3 \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2c^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} \\ &= \frac{2d^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} - \frac{(c - d)^3 \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2c^3 \tan(e + fx)}{af\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 6.57, size = 856, normalized size = 2.64

$$2 \cos^3\left(\frac{1}{2}(e + fx)\right) (c + d \sec(e + fx))^3 \sqrt{\frac{1}{1 - 2 \sin^2\left(\frac{1}{2}(e + fx)\right)}} \sqrt{1 - 2 \sin^2\left(\frac{1}{2}(e + fx)\right)} \left(\frac{2\left(2\sqrt{2} \sin^{-1}\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right)\right) \sin\left(\frac{1}{2}(e + fx)\right)}{\sqrt{a + a \sec(e + fx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]

[Out] (2*Cos[(e + f*x)/2]^3*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((-3*(c - d)^3*ArcTan[(1 - 2*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 + (3*(c - d)^3*ArcTan[(1 + 2*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 - (4*c^2*(c - 3*d)*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2] + ((c - d)^3*(1 - 2*Sin[(e + f*x)/2]))/(4*(1 + Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) - ((c - d)^3*(1 + 2*Sin[(e + f*x)/2]))/(4*(1 - Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) - ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 - Sin[(e + f*x)/2]) + ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 + Sin[(e + f*x)/2]) - (2*c^3*(-(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]) + 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/(1 - 2*Sin[(e + f*x)/2]^2) - ((c - d)^2*(11*c + d)*Sin[(e + f*x)/2]*((2*Cos[(e + f*x)/2]^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + 5*Csc[(e + f*x)/2]^4*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^2*(3 - 2*Sin[(e + f*x)/2]^2)*(-ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]] + Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]))/(10*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(3/2)*(a*(1 + Sec[e + f*x]))^(3/2))

fricas [A] time = 34.52, size = 701, normalized size = 2.16

$$\sqrt{2} \left(5c^3 - 3c^2d - 9cd^2 + 7d^3 + (5c^3 - 3c^2d - 9cd^2 + 7d^3) \cos(fx + e) \right)^2 + 2(5c^3 - 3c^2d - 9cd^2 + 7d^3) \cos(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), 1/4*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 2*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
 /2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
 t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
 pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
 le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant si
 gn by intervals (correct if the argument is real):Check [abs(cos(f*t_nostep
 +exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is po
 sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
 ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableWarning, assuming -2*a+a is positive. Hint: run assume to make ass
 umptions on a variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nost
 ep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
 eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
 _nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
 /t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
 e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
 (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
 >(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/
 2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
 step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
 nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
 o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
 pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
 4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
 nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
 gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
 p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
 tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
 heck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/
 t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
 pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab


```

(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) wer
e not checkedUnable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t
_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/
2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep^2-1)]Evaluation time: 1.8index.cc index_m i_lex_is_greater Error:
Bad Argument Value

```

maple [B] time = 1.88, size = 957, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x)
```

```

[Out] 1/4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(4*2^(1/2)*cos(f*
x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*sin(f*x+e)*c^3+4*2^(1/2)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*c^3*sin(f*x+e)+5*cos(f*x+e)*(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^3-3*cos(f*x+e)*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+
e)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^2*d-9*cos(f*x+e)*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c*d^2+7*cos(f*x+e)*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos
(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*d^3+5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f
*x+e))*c^3*sin(f*x+e)-3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2*d*sin
(f*x+e)-9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2*sin(f*x+e)+7*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^3*sin(f*x+e)-2*cos(f*x+e)^2*c^3+6*c
os(f*x+e)^2*c^2*d-6*cos(f*x+e)^2*c*d^2+10*cos(f*x+e)^2*d^3+2*c^3*cos(f*x+e)

```

$-6*\cos(f*x+e)*c^2*d+6*\cos(f*x+e)*c*d^2-2*\cos(f*x+e)*d^3-8*d^3)/\sin(f*x+e)^3/a^2$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^3}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)

[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^3}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)

[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(3/2), x)

$$3.173 \quad \int \frac{(c+d \sec(e+fx))^2}{(a+a \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=290

$$\frac{\sqrt{2} (c^2 - d^2) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}} \right)}{\sqrt{a} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2c^2 \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}} \right)}{\sqrt{a} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} - \frac{(c}{2af(\sec(e +$$

[Out] $-1/2*(c-d)^2*\tan(f*x+e)/a/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}+2*c^2*\arctan(\sqrt{a-a*\sec(f*x+e)}/\sqrt{a})/\sqrt{a}*\tan(f*x+e)/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/4*(c-d)^2*\arctan(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/f*2^{(1/2)}/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c^2-d^2)*\arctan(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*\tan(f*x+e)/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 180, 63, 206, 51}

$$\frac{\sqrt{2} (c^2 - d^2) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}} \right)}{\sqrt{a} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2c^2 \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}} \right)}{\sqrt{a} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} - \frac{(c}{2af(\sec(e +$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2), x]

[Out] $-((c - d)^2*\tan[e + f*x])/(2*a*f*(1 + \sec[e + f*x])*sqrt[a + a*\sec[e + f*x]]) + (2*c^2*\arctan(sqrt[a - a*\sec[e + f*x]]/sqrt[a])*tan[e + f*x])/(sqrt[a]*f*sqrt[a - a*\sec[e + f*x]]*sqrt[a + a*\sec[e + f*x]]) - ((c - d)^2*\arctan(sqrt[a - a*\sec[e + f*x]]/(sqrt[2]*sqrt[a]))*tan[e + f*x])/(2*sqrt[2]*sqrt[a]*f*sqrt[a - a*\sec[e + f*x]]*sqrt[a + a*\sec[e + f*x]]) - (sqrt[2]*(c^2 - d^2)*\arctan(sqrt[a - a*\sec[e + f*x]]/(sqrt[2]*sqrt[a]))*tan[e + f*x])/(sqrt[a]*f*sqrt[a - a*\sec[e + f*x]]*sqrt[a + a*\sec[e + f*x]])$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegerQ[p, q]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_)^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_)^(n_)), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned}
 \int \frac{(c + d \sec(e + fx))^2}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
 &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{c^2}{a^2x\sqrt{a-ax}} - \frac{(c-d)^2}{a^2(1+x)^2\sqrt{a-ax}} + \frac{-c^2+d^2}{a^2(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
 &= -\frac{(c^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{((c - d)^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
 &= -\frac{(c - d)^2 \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{(2c^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sec(e + fx)\right)}{af\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
 &= -\frac{(c - d)^2 \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{\sqrt{a} f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
 &= -\frac{(c - d)^2 \tan(e + fx)}{2af(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right) \tan(e + fx)}{\sqrt{a} f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}}
 \end{aligned}$$

Mathematica [C] time = 27.75, size = 16153, normalized size = 55.70

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2), x]

[Out] Result too large to show

fricas [A] time = 12.90, size = 620, normalized size = 2.14

$$\left[\frac{4(c^2 - 2cd + d^2) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - \sqrt{2} \left((5c^2 - 2cd - 3d^2) \cos(fx+e)^2 + 5c^2 - 2cd \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))]/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
```



```

sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)U
nable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check si
gn: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_noste
p/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*p
i/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unab
le to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Discontinuities at zeroes of cos(f*t_nostep+exp(1)) were not checked
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Warning, integration of abs or sign assumes constant sign by intervals
(correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time:
1.42index.cc index_m i_lex_is_greater Error: Bad Argument Value

```

maple [B] time = 1.55, size = 756, normalized size = 2.61

$$\frac{\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \left(-4\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \operatorname{arctanh}\left(\frac{\sqrt{\frac{2\cos(fx+e)}{1+\cos(fx+e)}} \sin(fx+e)\sqrt{2}}{2\cos(fx+e)}}\right) \sqrt{2} \sin(fx+e) c^2 \cos(fx+e) - 5\sqrt{\dots} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2), x)

```
[Out] 1/4/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-4*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*sin(f*x+e)*c^2*cos(f*x+e)-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c^2*cos(f*x+e)+2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*c*d*cos(f*x+e)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*d^2*cos(f*x+e)-4*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*arctanh(1/2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/cos(f*x+e)*2^(1/2))*2^(1/2)*c^2*sin(f*x+e)-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2*sin(f*x+e)+2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d*sin(f*x+e)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d^2*sin(f*x+e)+2*cos(f*x+e)^2*c^2-4*cos(f*x+e)^2*c*d+2*cos(f*x+e)^2*d^2-2*cos(f*x+e)*c^2+4*cos(f*x+e)*c*d-2*cos(f*x+e)*d^2)/(1+cos(f*x+e))/sin(f*x+e)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec(fx + e) + c)^2}{(a \sec(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^2}{\left(a + \frac{a}{\cos(e+fx)}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
```

```
[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^2}{(a (\sec(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(3/2), x)
```


$$3.174 \quad \int \frac{c+d \sec(e+fx)}{(a+a \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=127

$$-\frac{(5c-d) \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} + \frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2} f} - \frac{(c-d) \tan(e+fx)}{2f(a \sec(e+fx)+a)^{3/2}}$$

[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-1/4*(5*c-d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)-1/2*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3922, 3920, 3774, 203, 3795}

$$-\frac{(5c-d) \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} + \frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}}\right)}{a^{3/2} f} - \frac{(c-d) \tan(e+fx)}{2f(a \sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2), x]

[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/(a^(3/2)*f) - ((5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]])/(2*Sqrt[2]*a^(3/2)*f) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x]

)]^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
 [a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int \frac{c + d \sec(e + fx)}{(a + a \sec(e + fx))^{3/2}} dx &= -\frac{(c - d) \tan(e + fx)}{2f(a + a \sec(e + fx))^{3/2}} - \frac{\int \frac{-2ac + \frac{1}{2}a(c-d) \sec(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx}{2a^2} \\ &= -\frac{(c - d) \tan(e + fx)}{2f(a + a \sec(e + fx))^{3/2}} + \frac{c \int \sqrt{a + a \sec(e + fx)} dx}{a^2} - \frac{(5c - d) \int \frac{\sec(e+fx)}{\sqrt{a+a \sec(e+fx)}} dx}{4a} \\ &= -\frac{(c - d) \tan(e + fx)}{2f(a + a \sec(e + fx))^{3/2}} - \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{af} + \frac{(5c - d) \operatorname{Subst}\left(\int \frac{1}{a+x^2} dx, x, -\frac{a \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{af} \\ &= \frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{3/2}f} - \frac{(5c - d) \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{2\sqrt{2} a^{3/2}f} - \frac{(c - d) \tan(e + fx)}{2f(a + a \sec(e + fx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 26.70, size = 10105, normalized size = 79.57

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2), x]

[Out] Result too large to show

fricas [B] time = 3.18, size = 548, normalized size = 4.31

$$\left[\frac{4(c-d) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - \sqrt{2} \left((5c-d) \cos(fx+e)^2 + 2(5c-d) \cos(fx+e) + 5c-d \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] [-1/8*(4*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*c - d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) + 5*c - d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sq

```
rt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*
sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)
>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint: run assume t
o make assumptions on a variableWarning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableUnable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to chec
k sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_n
ostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t
_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nost
ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_no
step/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
```


mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + \frac{d}{\cos(e+fx)}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)

[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec(e + fx)}{\left(a (\sec(e + fx) + 1)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2), x)

[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(3/2), x)

$$3.175 \int \frac{1}{(a+a \sec(e+fx))^{3/2}(c+d \sec(e+fx))} dx$$

Optimal. Leaf size=394

$$\frac{2d^{5/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{\sqrt{a} c f (c-d)^2 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{\tan(e+fx)}{2af(c-d)(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a}}$$

[Out] $-1/2*\tan(f*x+e)/a/(c-d)/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)+2*\arctanh((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/c/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/4*\arctanh(1/2*(a-a*\sec(f*x+e))^{(1/2)*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/(c-d)/f*2^{(1/2)}/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c-2*d)*\arctanh(1/2*(a-a*\sec(f*x+e))^{(1/2)*2^{(1/2)}/a^{(1/2)})*2^{(1/2)*\tan(f*x+e)/(c-d)^2/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-2*d^{(5/2)*\arctanh(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)}/(c+d)^{(1/2)})*\tan(f*x+e)/c/(c-d)^2/f/a^{(1/2)}/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{2d^{5/2} \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{\sqrt{a} c f (c-d)^2 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{\tan(e+fx)}{2af(c-d)(\sec(e+fx)+1)\sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]

[Out] $-\text{Tan}[e + f*x]/(2*a*(c - d)*f*(1 + \text{Sec}[e + f*x])* \text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*c*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (\text{Sqrt}[2]*(c - 2*d)*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*(c - d)^2*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])]*\text{Tan}[e + f*x])/(2*\text{Sqrt}[2]*\text{Sqrt}[a]*(c - d)*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*d^{(5/2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*c*(c - d)^2*\text{Sqrt}[c + d]*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]])$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^(m)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]
```

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2}(c + d \sec(e + fx))} dx = -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{a^2cx\sqrt{a-ax}} - \frac{1}{a^2(c-d)(1+x)^2\sqrt{a-ax}} + \frac{1}{a^2(c+d)(1-x)^2\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{\tan(e + fx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{cf\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{((c - 2d) \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{(c - d)\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}}$$

$$= -\frac{\tan(e + fx)}{2a(c - d)f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{(2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(1+x)^2\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{acf\sqrt{a - a \sec(e + fx)}}$$

$$= -\frac{\tan(e + fx)}{2a(c - d)f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2 \tanh^{-1}\left(\frac{\sec(e + fx) - \sqrt{a - a \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}}\right)}{\sqrt{a}cf\sqrt{a - a \sec(e + fx)}}$$

$$= -\frac{\tan(e + fx)}{2a(c - d)f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2 \tanh^{-1}\left(\frac{\sec(e + fx) - \sqrt{a - a \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}}\right)}{\sqrt{a}cf\sqrt{a - a \sec(e + fx)}}$$

Mathematica [C] time = 35.21, size = 378865, normalized size = 961.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```


[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(2
 *pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
 ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
 step/2)Warning, integration of abs or sign assumes constant sign by interval
 s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
 : (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
 2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
 p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
 ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
 nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
 t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
 e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
 (-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
 tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
 ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi
 i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
 run assume to make assumptions on a variableWarning, assuming -2*a+a is po
 sitive. Hint: run assume to make assumptions on a variableWarning, assuming
 -2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
 ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
 variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
 o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
 pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
 4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
 nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
 gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
 p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos

$$2) * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(-2 * ((d / (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * 2^{(1/2)} * c * \sin(f * x + e) - 2^{(1/2)} * (d / (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * d * \sin(f * x + e) - c * \sin(f * x + e) + d * \sin(f * x + e) - ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e) + ((c + d) * (c - d))^{(1/2)}) / (((c + d) * (c - d))^{(1/2)} * \sin(f * x + e) + c * \cos(f * x + e) - d * \cos(f * x + e) - c + d) * \sin(f * x + e) * \cos(f * x + e) * d^3 + 5 * ((c + d) * (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(-(-(-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \sin(f * x + e) + \cos(f * x + e) - 1) / \sin(f * x + e)) * \sin(f * x + e) * \cos(f * x + e) * (d / (c - d))^{(1/2)} * c^2 - 9 * ((c + d) * (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(-(-(-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \sin(f * x + e) + \cos(f * x + e) - 1) / \sin(f * x + e)) * \sin(f * x + e) * \cos(f * x + e) * (d / (c - d))^{(1/2)} * c * d - 2 * 2^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(2 * ((d / (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * 2^{(1/2)} * c * \sin(f * x + e) - 2^{(1/2)} * (d / (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * d * \sin(f * x + e) + ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e) - c * \sin(f * x + e) + d * \sin(f * x + e) - ((c + d) * (c - d))^{(1/2)}) / (((c + d) * (c - d))^{(1/2)} * \sin(f * x + e) - c * \cos(f * x + e) + d * \cos(f * x + e) + c - d) * \sin(f * x + e) * d^3 + 2 * 2^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(-2 * ((d / (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * 2^{(1/2)} * c * \sin(f * x + e) - 2^{(1/2)} * (d / (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * d * \sin(f * x + e) - c * \sin(f * x + e) + d * \sin(f * x + e) - ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e) + ((c + d) * (c - d))^{(1/2)}) / (((c + d) * (c - d))^{(1/2)} * \sin(f * x + e) + c * \cos(f * x + e) - d * \cos(f * x + e) - c + d) * \sin(f * x + e) * d^3 + 5 * ((c + d) * (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(-(-(-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \sin(f * x + e) + \cos(f * x + e) - 1) / \sin(f * x + e)) * \sin(f * x + e) * (d / (c - d))^{(1/2)} * c^2 - 9 * ((c + d) * (c - d))^{(1/2)} * (-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \ln(-(-(-2 * \cos(f * x + e) / (1 + \cos(f * x + e)))^{(1/2)} * \sin(f * x + e) + \cos(f * x + e) - 1) / \sin(f * x + e)) * \sin(f * x + e) * (d / (c - d))^{(1/2)} * c * d - 2 * ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e)^2 * (d / (c - d))^{(1/2)} * c^2 + 2 * ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e)^2 * (d / (c - d))^{(1/2)} * c * d + 2 * ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e) * (d / (c - d))^{(1/2)} * c^2 - 2 * ((c + d) * (c - d))^{(1/2)} * \cos(f * x + e) * (d / (c - d))^{(1/2)} * c * d) * (a * (1 + \cos(f * x + e)) / \cos(f * x + e))^{(1/2)} / \sin(f * x + e)^3 / (d / (c - d))^{(1/2)} / (c - d)^2 / c / ((c + d) * (c - d))^{(1/2)} / a^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sec(fx + e) + a\right)^{\frac{3}{2}} \left(d \sec(fx + e) + c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e + fx)}\right)^{\frac{3}{2}} \left(c + \frac{d}{\cos(e + fx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))),x)

[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \left(\sec(e + fx) + 1\right)\right)^{\frac{3}{2}} \left(c + d \sec(e + fx)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)

[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))), x)

3.176 $\int \frac{1}{(a+a \sec(e+fx))^{3/2}(c+d \sec(e+fx))^2} dx$

Optimal. Leaf size=560

$$\frac{2d^{5/2}(3c-d) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{\sqrt{a} c^2 f (c-d)^3 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a} c^2 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

[Out] $-1/2*\tan(f*x+e)/a/(c-d)^2/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}-d^3*\tan(f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}+2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/c^2/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-d^{(5/2)}*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)})*\tan(f*x+e)/c/(c-d)^2/(c+d)^{(3/2)}/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/4*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/(c-d)^2/f*2^{(1/2)}/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c-3*d)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)})*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*\tan(f*x+e)/(c-d)^3/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-2*(3*c-d)*d^{(5/2)}*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)})*\tan(f*x+e)/c^2/(c-d)^3/f/a^{(1/2)}/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.51, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{2d^{5/2}(3c-d) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{\sqrt{a} c^2 f (c-d)^3 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} + \frac{2 \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a} c^2 f \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2), x]`
 [Out] `-Tan[e + f*x]/(2*a*(c - d)^2*f*(1 + Sec[e + f*x])*Sqrt[a + a*Sec[e + f*x]]) + (2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]]*Tan[e + f*x])/(Sqrt[a]*c^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (Sqrt[2]*(c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(Sqrt[a]*(c - d)^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Tan[e + f*x])/(2*Sqrt[2]*Sqrt[a]*(c - d)^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*(3*c - d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])]*Tan[e + f*x])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d^3*Tan[e + f*x])/(a*c*(c - d)^2*(c + d)*f*Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]))`

Rule 51

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])))] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +`

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 180

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}*((e_.) + (f_.)*(x_)^{(p_)}*((g_.) + (h_.)*(x_)^{(q_)}), x_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, n\}, x] \ \&\& \ \text{IntegersQ}[p, q]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 3940

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^{(n_.)}, x_Symbol] \ :> \ \text{Dist}[(a^2*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(a + b*x)^{(m - 1/2)}*(c + d*x)^n]/(x*\text{Sqrt}[a - b*x]), x], x, \text{Csc}[e + f*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{IntegerQ}[m - 1/2]$

Rubi steps

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2}(c + d \sec(e + fx))^2} dx = -\frac{(a^2 \tan(e + fx)) \text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^2(c+dx)^2} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ = -\frac{(a^2 \tan(e + fx)) \text{Subst}\left(\int \left(\frac{1}{a^2 c^2 x \sqrt{a-ax}} - \frac{1}{a^2 (c-d)^2 (1+x)^2 \sqrt{a-ax}} + \dots\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}} \\ = -\frac{\tan(e + fx) \text{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{c^2 f \sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{((c - 3d) \tan(e + fx)) \text{Subst}\left(\int \frac{1}{(1+x)^2 \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{(c - d)\sqrt{a - a \sec(e + fx)}} \\ = -\frac{\tan(e + fx)}{2a(c - d)^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{ac(c - d)^2 \sqrt{a - a \sec(e + fx)}} \\ = -\frac{\tan(e + fx)}{2a(c - d)^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2 \tan(e + fx)}{\sqrt{a} c^2 f \sqrt{a - a \sec(e + fx)}} \\ = -\frac{\tan(e + fx)}{2a(c - d)^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} + \frac{2 \tan(e + fx)}{\sqrt{a} c^2 f \sqrt{a - a \sec(e + fx)}}$$

Mathematica [C] time = 38.10, size = 582620, normalized size = 1040.39

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
 gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
 e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
 *pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
 *pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
 /2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
 x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
 sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
 step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
 nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable t
 o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
 pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assume
 s constant sign by intervals (correct if the argument is real):Check [abs(c
 os(f*t_nostep+exp(1)))]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nos
 tep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to c
 heck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/
 t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*
 pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unab
 le to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
 (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2
)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
 /2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to chec
 k sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_n
 ostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
 _nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2
 *pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
 (-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
 Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming


```

nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/
t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable
to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (
4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>
(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2
)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check
sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nos
tstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to
check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi
i/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2
*pi/t_nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Un
able to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sig
n: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep
/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nost
ep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to ch
eck sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t
_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi
/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unabl
e to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign:
(4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)
>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/
2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check
sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nos
tstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_
nstep/2)Unable to check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable t
o check sign: (4*pi/t_nstep/2)>(-4*pi/t_nstep/2)Unable to check sign: (4*
pi/t_nstep/2)>(-4*pi/t_nstep/2)Discontinuities at zeroes of cos(f*t_noste
p+exp(1)) were not checkedUnable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_
nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable t
o check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*
pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-
2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)U
nable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check si
gn: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nste
p/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nos
tstep/2)Unable to check sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to c
heck sign: (2*pi/t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/
t_nstep/2)>(-2*pi/t_nstep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by in
tervals (correct if the argument is real):Check [abs(t_nstep^2-1)]Evaluati
on time: 1.44index.cc index_m i_lex_is_greater Error: Bad Argument Value

```

maple [B] time = 5.45, size = 164796, normalized size = 294.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima"
)
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{3/2} \left(c + \frac{d}{\cos(e+fx)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2), x)

[Out] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a(\sec(e+fx)+1)\right)^{3/2} (c+d\sec(e+fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2, x)

[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2), x)

3.177 $\int \frac{1}{(a+a \sec(e+fx))^{3/2}(c+d \sec(e+fx))^3} dx$

Optimal. Leaf size=802

$$\frac{(3c - d) \tan(e + fx)d^3}{ac^2(c - d)^3(c + d)f\sqrt{\sec(e + fx)a + a}(c + d \sec(e + fx))} - \frac{3 \tan(e + fx)d^3}{4ac(c^2 - d^2)^2 f\sqrt{\sec(e + fx)a + a}(c + d \sec(e + fx))}$$

[Out] $-1/2*\tan(f*x+e)/a/(c-d)^3/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}-1/2*d^3*\tan(f*x+e)/a/c/(c-d)^2/(c+d)/f/(c+d*\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{(1/2)}-(3*c-d)*d^3*\tan(f*x+e)/a/c^2/(c-d)^3/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}-3/4*d^3*\tan(f*x+e)/a/c/(c^2-d^2)^2/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}+2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/c^3/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-3/4*d^{(5/2)}*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*\tan(f*x+e)/c/(c-d)^2/(c+d)^{(5/2)}/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(3*c-d)*d^{(5/2)}*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*\tan(f*x+e)/c^2/(c-d)^3/(c+d)^{(3/2)}/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/4*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/(c-d)^3/f*2^{(1/2)}/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-(c-4*d)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*\tan(f*x+e)/(c-d)^4/f/a^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-2*d^{(5/2)}*(6*c^2-4*c*d+d^2)*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})/(c+d)^{(1/2)}*\tan(f*x+e)/c^3/(c-d)^4/f/a^{(1/2)}/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.80, antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{(3c - d) \tan(e + fx)d^3}{ac^2(c - d)^3(c + d)f\sqrt{\sec(e + fx)a + a}(c + d \sec(e + fx))} - \frac{3 \tan(e + fx)d^3}{4ac(c^2 - d^2)^2 f\sqrt{\sec(e + fx)a + a}(c + d \sec(e + fx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((a + a*\text{Sec}[e + f*x])^{(3/2)}*(c + d*\text{Sec}[e + f*x])^3), x]$

[Out] $-\text{Tan}[e + f*x]/(2*a*(c - d)^3*f*(1 + \text{Sec}[e + f*x])* \text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/\text{Sqrt}[a]]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*c^3*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (\text{Sqrt}[2]*(c - 4*d)*\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*(c - d)^4*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (\text{ArcTanh}[\text{Sqrt}[a - a*\text{Sec}[e + f*x]]/(\text{Sqrt}[2]*\text{Sqrt}[a])]*\text{Tan}[e + f*x])/(2*\text{Sqrt}[2]*\text{Sqrt}[a]*(c - d)^3*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (3*d^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(4*\text{Sqrt}[a]*c*(c - d)^2*(c + d)^{(5/2)}*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - ((3*c - d)*d^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*c^2*(c - d)^3*(c + d)^{(3/2)}*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (2*d^{(5/2)}*(6*c^2 - 4*c*d + d^2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/(\text{Sqrt}[a]*c^3*(c - d)^4*\text{Sqrt}[c + d]*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) - (d^3*\text{Tan}[e + f*x])/(2*a*c*(c - d)^2*(c + d)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])^2) - ((3*c - d)*d^3*\text{Tan}[e + f*x])/(a*c^2*(c - d)^3*(c + d)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])) - (3*d^3*\text{Tan}[e + f*x])/(4*a*c*(c^2 - d^2)^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x]))$

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
g, h, m, n}, x] && IntegerQ[p, q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*sqrt[a + b*Csc[
e + f*x]]*sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^3} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax} (a+ax)^2 (c+dx)^3} dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{a^2 c^3 x \sqrt{a-ax}} - \frac{1}{a^2 (c-d)^3 (1+x)^2 \sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{c^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{((c - 4a) \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{2ac(c - d) \sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{2a(c - d)^3 f (1 + \sec(e + fx)) \sqrt{a + a \sec(e + fx)}} - \frac{2ac(c - d) \tan(e + fx)}{2ac(c - d) \sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{2a(c - d)^3 f (1 + \sec(e + fx)) \sqrt{a + a \sec(e + fx)}} + \frac{2ac(c - d) \tan(e + fx)}{\sqrt{a} c^3 f} \\
&= -\frac{\tan(e + fx)}{2a(c - d)^3 f (1 + \sec(e + fx)) \sqrt{a + a \sec(e + fx)}} + \frac{2ac(c - d) \tan(e + fx)}{\sqrt{a} c^3 f} \\
&= -\frac{\tan(e + fx)}{2a(c - d)^3 f (1 + \sec(e + fx)) \sqrt{a + a \sec(e + fx)}} + \frac{2ac(c - d) \tan(e + fx)}{\sqrt{a} c^3 f}
\end{aligned}$$

Mathematica [C] time = 41.23, size = 776222, normalized size = 967.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)


```
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities
at zeroes of cos(f*t_nostep+exp(1)) were not checkedUnable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nost
ep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to ch
eck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t
_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi
/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign:
(2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Ev
aluation time: 1.92index.cc index_m i_lex_is_greater Error: Bad Argument Va
lue
```

maple [B] time = 31.94, size = 480553, normalized size = 599.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(c+d\sec(e+fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3), x)
```


ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3940

Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_)]^(n_.), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]

Rubi steps

$$\begin{aligned} \int \frac{(c + d \sec(e + fx))^3}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^3}{x\sqrt{a-ax}(a+ax)^3} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{c^3}{a^3x\sqrt{a-ax}} - \frac{(c-d)^3}{a^3(1+x)^3\sqrt{a-ax}} - \frac{(c-d)^2(c+2d)}{a^3(1+x)^2\sqrt{a-ax}} + \frac{-c^3+d^3}{a^3(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{af\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{((c-d)^3 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(1+x)^3\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{af\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^3 \tan(e + fx)}{4a^2f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{(c-d)^2(c+2d) \tan(e + fx)}{2a^2f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^3 \tan(e + fx)}{4a^2f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{3(c-d)^3 \tan(e + fx)}{16a^2f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^3 \tan(e + fx)}{4a^2f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{3(c-d)^3 \tan(e + fx)}{16a^2f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^3 \tan(e + fx)}{4a^2f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{3(c-d)^3 \tan(e + fx)}{16a^2f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 29.39, size = 21194, normalized size = 44.15

Result too large to show

Warning: Unable to verify antiderivative.


```
8*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*sin(f*x+e)*d^3+9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c^2*d*sin(f*x+e)+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c*d^2*sin(f*x+e)+18*cos(f*x+e)^3*d^3+8*cos(f*x+e)^2*d^3-26*cos(f*x+e)*d^3-8*cos(f*x+e)^2*c^3-22*c^3*cos(f*x+e)+30*cos(f*x+e)^3*c^3)/(1+cos(f*x+e))/sin(f*x+e)^3/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^3}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^3}{(a(\sec(e + fx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)

[Out] Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(5/2), x)

$$3.179 \quad \int \frac{(c+d \sec(e+fx))^2}{(a+a \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=468

$$\frac{(c^2 - d^2) \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2c^2 \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{a}}\right)}{a^{3/2} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} - \frac{\sqrt{2} c^2 \tan(e + fx)}{a^{3/2} f \sqrt{a - a \sec(e + fx)}}$$

[Out] $-1/4*(c-d)^2*\tan(f*x+e)/a^2/f/(1+\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{(1/2)}-3/16*(c-d)^2*\tan(f*x+e)/a^2/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}-1/2*(c^2-d^2)*\tan(f*x+e)/a^2/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)}+2*c^2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-3/32*(c-d)^2*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/f*2^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-1/4*(c^2-d^2)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/f*2^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}-c^2*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*\tan(f*x+e)/a^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3940, 180, 63, 206, 51}

$$\frac{(c^2 - d^2) \tan(e + fx)}{2a^2 f (\sec(e + fx) + 1) \sqrt{a \sec(e + fx) + a}} - \frac{(c^2 - d^2) \tan(e + fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}}\right)}{2\sqrt{2} a^{3/2} f \sqrt{a - a \sec(e + fx)} \sqrt{a \sec(e + fx) + a}} + \frac{2c^2 \tan(e + fx)}{a^{3/2} f \sqrt{a - a \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2), x]

[Out] $-((c - d)^2*\tan[e + f*x])/(4*a^2*f*(1 + \sec[e + f*x])^2*\sqrt{a + a*\sec[e + f*x]}) - (3*(c - d)^2*\tan[e + f*x])/(16*a^2*f*(1 + \sec[e + f*x])*\sqrt{a + a*\sec[e + f*x]}) - ((c^2 - d^2)*\tan[e + f*x])/(2*a^2*f*(1 + \sec[e + f*x])*\sqrt{a + a*\sec[e + f*x]}) + (2*c^2*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/\sqrt{a}]*\tan[e + f*x])/(a^{(3/2)}*f*\sqrt{a - a*\sec[e + f*x]}*\sqrt{a + a*\sec[e + f*x]}) - (\sqrt{2}*c^2*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/(\sqrt{2}*\sqrt{a})]*\tan[e + f*x])/(a^{(3/2)}*f*\sqrt{a - a*\sec[e + f*x]}*\sqrt{a + a*\sec[e + f*x]}) - (3*(c - d)^2*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/(\sqrt{2}*\sqrt{a})]*\tan[e + f*x])/(16*\sqrt{2}*a^{(3/2)}*f*\sqrt{a - a*\sec[e + f*x]}*\sqrt{a + a*\sec[e + f*x]}) - ((c^2 - d^2)*\operatorname{ArcTanh}[\sqrt{a - a*\sec[e + f*x]}/(\sqrt{2}*\sqrt{a})]*\tan[e + f*x])/(2*\sqrt{2}*a^{(3/2)}*f*\sqrt{a - a*\sec[e + f*x]}*\sqrt{a + a*\sec[e + f*x]})$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^(n + 1)/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned} \int \frac{(c + d \sec(e + fx))^2}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^2}{x\sqrt{a-ax}(a+ax)^3} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{c^2}{a^3x\sqrt{a-ax}} - \frac{(c-d)^2}{a^3(1+x)^3\sqrt{a-ax}} + \frac{-c^2+d^2}{a^3(1+x)^2\sqrt{a-ax}} - \frac{c^2}{a^3(1+x)\sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{af\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{(c^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(1+x)^3\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{af\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^2 \tan(e + fx)}{4a^2 f(1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{(c^2 - d^2) \tan(e + fx)}{2a^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^2 \tan(e + fx)}{4a^2 f(1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{3(c-d)^2 \tan(e + fx)}{16a^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^2 \tan(e + fx)}{4a^2 f(1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{3(c-d)^2 \tan(e + fx)}{16a^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \\ &= -\frac{(c-d)^2 \tan(e + fx)}{4a^2 f(1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{3(c-d)^2 \tan(e + fx)}{16a^2 f(1 + \sec(e + fx))\sqrt{a + a \sec(e + fx)}} \end{aligned}$$

Mathematica [C] time = 27.95, size = 16249, normalized size = 34.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2), x]
```


step/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.84index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 1.64, size = 1133, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x)

[Out]
$$\begin{aligned} & -1/32/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*(32*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*\cos(f*x+e)^2*2^{1/2}*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*c^2+64*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*2^{1/2}*\sin(f*x+e)*c^2*\cos(f*x+e)+43*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*c^2-6*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*c*d-5*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*\cos(f*x+e)^2*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d^2+32*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\operatorname{arctanh}(1/2*(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)/\cos(f*x+e)*2^{1/2})*2^{1/2}*c^2*\sin(f*x+e)+86*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*\sin(f*x+e)*c^2*\cos(f*x+e)-12*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*\sin(f*x+e)*c*d*\cos(f*x+e)-10*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*\sin(f*x+e)*d^2*\cos(f*x+e)+43*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*c^2*\sin(f*x+e)-6*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*c*d*\sin(f*x+e)-5*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-(-(-2*\cos(f*x+e)/(1+\cos(f*x+e))))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*d^2*\sin(f*x+e)-30*\cos(f*x+e)^3*c^2+28*\cos(f*x+e)^3*c*d+2*\cos(f*x+e)^3*d^2+8*\cos(f*x+e)^2*c^2-16*\cos(f*x+e)^2*c*d+8*\cos(f*x+e)^2*d^2+22*\cos(f*x+e)*c^2-12*\cos(f*x+e)*c*d-10*\cos(f*x+e)*d^2)/(1+\cos(f*x+e))^2/\sin(f*x+e)/a^3 \end{aligned}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)}\right)^2}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)

[Out] int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^2}{(a(\sec(e + fx) + 1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2), x)

[Out] Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(5/2), x)

$$3.180 \quad \int \frac{c+d \sec(e+fx)}{(a+a \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=164

$$\frac{(43c - 3d) \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}} \right)}{16\sqrt{2} a^{5/2} f} + \frac{2c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}} \right)}{a^{5/2} f} - \frac{(11c - 3d) \tan(e+fx)}{16af(a \sec(e+fx) + a)^{3/2}} - \frac{(c-d) \tan(e+fx)}{4f(a \sec(e+fx) + a)}$$

[Out] 2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-1/32*(43*c-3*d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)-1/4*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-1/16*(11*c-3*d)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)

Rubi [A] time = 0.26, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3922, 3920, 3774, 203, 3795}

$$\frac{(43c - 3d) \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a}} \right)}{16\sqrt{2} a^{5/2} f} + \frac{2c \tan^{-1} \left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a}} \right)}{a^{5/2} f} - \frac{(11c - 3d) \tan(e+fx)}{16af(a \sec(e+fx) + a)^{3/2}} - \frac{(c-d) \tan(e+fx)}{4f(a \sec(e+fx) + a)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2), x]

[Out] (2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]])/(a^(5/2)*f) - ((43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((c - d)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2)) - ((11*c - 3*d)*Tan[e + f*x])/(16*a*f*(a + a*Sec[e + f*x])^(3/2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])

$x]^{m+1}/(b*f*(2*m + 1)), x] + \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*\text{Simp}[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*\text{Csc}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[m, -1] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[2*m]$

Rubi steps

$$\begin{aligned} \int \frac{c + d \sec(e + fx)}{(a + a \sec(e + fx))^{5/2}} dx &= -\frac{(c - d) \tan(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} - \frac{\int \frac{-4ac + \frac{3}{2}a(c-d) \sec(e+fx)}{(a+a \sec(e+fx))^{3/2}} dx}{4a^2} \\ &= -\frac{(c - d) \tan(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} - \frac{(11c - 3d) \tan(e + fx)}{16af(a + a \sec(e + fx))^{3/2}} + \frac{\int \frac{8a^2c - \frac{1}{4}a^2(11c-3d) \sec(e+fx)}{\sqrt{a+a \sec(e+fx)}}}{8a^4} \\ &= -\frac{(c - d) \tan(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} - \frac{(11c - 3d) \tan(e + fx)}{16af(a + a \sec(e + fx))^{3/2}} + \frac{c \int \sqrt{a + a \sec(e + fx)}}{a^3} \\ &= -\frac{(c - d) \tan(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} - \frac{(11c - 3d) \tan(e + fx)}{16af(a + a \sec(e + fx))^{3/2}} - \frac{(2c) \text{Subst}\left(\int \frac{1}{a+x^2} dx\right)}{a^2 f} \\ &= \frac{2c \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}}\right)}{a^{5/2} f} - \frac{(43c - 3d) \tan^{-1}\left(\frac{\sqrt{a} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(c - d) \tan(e + fx)}{4f(a + a \sec(e + fx))^{5/2}} \end{aligned}$$

Mathematica [C] time = 26.95, size = 11243, normalized size = 68.55

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2), x]

[Out] Result too large to show

fricas [B] time = 6.36, size = 670, normalized size = 4.09

$$\sqrt{2} \left((43c - 3d) \cos^3(fx + e) + 3(43c - 3d) \cos^2(fx + e) + 3(43c - 3d) \cos(fx + e) + 43c - 3d \right) \sqrt{-a} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2), x, algorithm="fricas")

[Out] $[1/64*(\text{sqrt}(2))*((43*c - 3*d)*\cos(f*x + e)^3 + 3*(43*c - 3*d)*\cos(f*x + e)^2 + 3*(43*c - 3*d)*\cos(f*x + e) + 43*c - 3*d)*\text{sqrt}(-a)*\log((2*\text{sqrt}(2))*\text{sqrt}(-a)*\text{sqrt}((a*\cos(f*x + e) + a)/\cos(f*x + e))*\cos(f*x + e)*\sin(f*x + e) + 3*a*\cos(f*x + e)^2 + 2*a*\cos(f*x + e) - a)/(\cos(f*x + e)^2 + 2*\cos(f*x + e) + 1)) - 64*(c*\cos(f*x + e)^3 + 3*c*\cos(f*x + e)^2 + 3*c*\cos(f*x + e) + c)*\text{sqrt}(-a)*\log((2*a*\cos(f*x + e)^2 + 2*\text{sqrt}(-a))*\text{sqrt}((a*\cos(f*x + e) + a)/\cos(f*x + e))*\cos(f*x + e)*\sin(f*x + e) + a*\cos(f*x + e) - a)/(\cos(f*x + e) + 1)) - 4*((15*c - 7*d)*\cos(f*x + e)^2 + (11*c - 3*d)*\cos(f*x + e))*\text{sqrt}((a*\cos(f$

```
*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos
s(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*c - 3*d)*c
os(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2 + 3*(43*c - 3*d)*cos(f*x + e)
+ 43*c - 3*d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
)))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*
x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c - 7*d)*cos(f*x
+ e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*co
s(f*x + e) + a^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(
-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_no
step/2)Warning, integration of abs or sign assumes constant sign by interval
s (correct if the argument is real):Check [abs(cos(f*t_nostep+exp(1)))]Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unabl
e to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>
(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2
)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check
sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nos
tep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_n
ostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, assuming -2*a+a is positive. Hint:
run assume to make assumptions on a variableWarning, assuming -2*a+a is po
sitive. Hint: run assume to make assumptions on a variableWarning, assuming
-2*a+a is positive. Hint: run assume to make assumptions on a variableWarn
ing, assuming -2*a+a is positive. Hint: run assume to make assumptions on a
variableUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-
4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)U
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
```


*x+e))*d*cos(f*x+e)+43*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*c-3*sin(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*ln(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*d-30*cos(f*x+e)^3*c+14*cos(f*x+e)^3*d+8*cos(f*x+e)^2*c-8*cos(f*x+e)^2*d+22*c*cos(f*x+e)-6*d*cos(f*x+e))/(1+cos(f*x+e))^2/sin(f*x+e)/a^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{(a \sec(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + \frac{d}{\cos(e+fx)}}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)

[Out] int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec(e + fx)}{(a(\sec(e + fx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)

[Out] Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(5/2), x)

$$3.181 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))} dx$$

Optimal. Leaf size=592

$$\frac{\sqrt{2} (c^2 - 3cd + 3d^2) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}} \right)}{a^{3/2} f (c-d)^3 \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx) + a}} + \frac{2d^{7/2} \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}} \right)}{a^{3/2} c f (c-d)^3 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx) + a}}$$

[Out] $-1/4*\tan(f*x+e)/a^2/(c-d)/f/(1+\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{1/2}-1/2*(c-2*d)*\tan(f*x+e)/a^2/(c-d)^2/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}-3/16*\tan(f*x+e)/a^2/(c-d)/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{1/2}+2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{1/2}/a^{1/2})*\tan(f*x+e)/a^{3/2}/c/f/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}-1/4*(c-2*d)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*\tan(f*x+e)/a^{3/2}/(c-d)^2/f*2^{1/2}/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}-3/32*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*\tan(f*x+e)/a^{3/2}/(c-d)/f*2^{1/2}/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}-(c^2-3*c*d+3*d^2)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*2^{1/2}*\tan(f*x+e)/a^{3/2}/(c-d)^3/f/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}+2*d^{7/2}*\operatorname{arctanh}(d^{1/2}*(a-a*\sec(f*x+e))^{1/2}/a^{1/2}/(c+d)^{1/2})*\tan(f*x+e)/a^{3/2}/c/(c-d)^3/f/(c+d)^{1/2}/(a-a*\sec(f*x+e))^{1/2}/(a+a*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.46, antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{\sqrt{2} (c^2 - 3cd + 3d^2) \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}} \right)}{a^{3/2} f (c-d)^3 \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx) + a}} + \frac{2d^{7/2} \tan(e + fx) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}} \right)}{a^{3/2} c f (c-d)^3 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]

[Out] $-\operatorname{Tan}[e + f*x]/(4*a^2*(c - d)*f*(1 + \operatorname{Sec}[e + f*x])^2*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - ((c - 2*d)*\operatorname{Tan}[e + f*x])/(2*a^2*(c - d)^2*f*(1 + \operatorname{Sec}[e + f*x])*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - (3*\operatorname{Tan}[e + f*x])/(16*a^2*(c - d)*f*(1 + \operatorname{Sec}[e + f*x])*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) + (2*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]/\operatorname{Sqrt}[a]]*\operatorname{Tan}[e + f*x])/(a^{3/2}*c*f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - ((c - 2*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]*\operatorname{Tan}[e + f*x])/(2*\operatorname{Sqrt}[2]*a^{3/2}*(c - d)^2*f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - (3*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]*\operatorname{Tan}[e + f*x])/(16*\operatorname{Sqrt}[2]*a^{3/2}*(c - d)*f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) - (\operatorname{Sqrt}[2]*(c^2 - 3*c*d + 3*d^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]*\operatorname{Tan}[e + f*x])/(a^{3/2}*(c - d)^3*f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]]) + (2*d^{7/2}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c + d])]*\operatorname{Tan}[e + f*x])/(a^{3/2}*c*(c - d)^3*\operatorname{Sqrt}[c + d]*f*\operatorname{Sqrt}[a - a*\operatorname{Sec}[e + f*x]]*\operatorname{Sqrt}[a + a*\operatorname{Sec}[e + f*x]])$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
)^(p_))*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}(a+ax)^3(c+dx)} dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{a^3cx\sqrt{a-ax}} - \frac{1}{a^3(c-d)(1+x)^3\sqrt{a-ax}} + \frac{1}{a^3(c-d)(1+x)^3\sqrt{a+ax}}\right) dx, x, \sec(e + fx)\right)}{f\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{acf\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} + \frac{((c - 2d) \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(1+x)^3\sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{a(c-d)\sqrt{a - a \sec(e + fx)}\sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2(c-d)f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2(c-d)\sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2(c-d)f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2(c-d)\sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2(c-d)f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2(c-d)\sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2(c-d)f(1 + \sec(e + fx))^2\sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2(c-d)\sqrt{a - a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 37.00, size = 486155, normalized size = 821.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
```


o check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep^2-1)]Evaluation time: 1.52index.c c index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 2.05, size = 3860, normalized size = 6.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x)

[Out]
$$-1/32/f*(-1+\cos(f*x+e))^{2*(115*(d/(c-d))^{1/2}*\sin(f*x+e)*((c+d)*(c-d))^{1/2}*\cos(f*x+e)^{2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}}*\ln(-(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\sin(f*x+e)+\cos(f*x+e)-1)/\sin(f*x+e))*c*d^{2-16*2^{1/2}}*\sin(f*x+e)*\cos(f*x+e)^{2*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}}*\ln(-2*((d/(c-d))^{1/2})^{1/2})^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2})^{2^{1/2}}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2}*\cos(f*x+e)+((c+d)*(c-d))^{1/2})/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)+c*\cos(f*x+e)-d*\cos(f*x+e)-c+d))*d^4+32*2^{1/2}*\sin(f*x+e)*\cos(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(2*((d/(c-d))^{1/2})^{1/2})^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2})^{2^{1/2}}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)+((c+d)*(c-d))^{1/2}*\cos(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2})/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)-c*\cos(f*x+e)+d*\cos(f*x+e)+c-d))*d^4-30*(d/(c-d))^{1/2}*((c+d)*(c-d))^{1/2}*\cos(f*x+e)^3*c^3+8*(d/(c-d))^{1/2}*((c+d)*(c-d))^{1/2}*\cos(f*x+e)^2*c^3+22*(d/(c-d))^{1/2}*((c+d)*(c-d))^{1/2}*\cos(f*x+e)*c^3-32*2^{1/2}*\sin(f*x+e)*\cos(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-2*((d/(c-d))^{1/2})^{1/2})^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2})^{2^{1/2}}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2}*\cos(f*x+e)+((c+d)*(c-d))^{1/2})/(((c+d)*(c-d))^{1/2}*\sin(f*x+e)+c*\cos(f*x+e)-d*\cos(f*x+e)-c+d))*d^4-16*2^{1/2}*\sin(f*x+e)*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*\ln(-2*((d/(c-d))^{1/2})^{1/2})^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2})^{2^{1/2}}*c*\sin(f*x+e)-2^{1/2}*(d/(c-d))^{1/2}*(-2*\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*d*\sin(f*x+e)-c*\sin(f*x+e)+d*\sin(f*x+e)-((c+d)*(c-d))^{1/2}*\cos(f*x+e)+((c+d)*(c-d))^{1/2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(fx + e) + a)^{\frac{5}{2}} (d \sec(fx + e) + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(1/((a*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + \frac{a}{\cos(e+fx)}\right)^{5/2} \left(c + \frac{d}{\cos(e+fx)}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))),x)

[Out] int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a(\sec(e + fx) + 1))^{\frac{5}{2}} (c + d \sec(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)

[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))), x)

$$3.182 \quad \int \frac{1}{(a+a \sec(e+fx))^{5/2}(c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=756

$$\frac{2d^{7/2}(4c-d) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{a^{3/2}c^2 f(c-d)^4 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{\sqrt{2} (c^2-4cd+6d^2) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}}\right)}{a^{3/2} f(c-d)^4 \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

[Out] $-1/4*\tan(f*x+e)/a^2/(c-d)^2/f/(1+\sec(f*x+e))^2/(a+a*\sec(f*x+e))^{(1/2)-1/2}*(c-3*d)*\tan(f*x+e)/a^2/(c-d)^3/f/(1+\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)-3/16}*\tan(f*x+e)/a^2/c/(c-d)^3/(c+d)/f/(c+d*\sec(f*x+e))/(a+a*\sec(f*x+e))^{(1/2)+2*\operatorname{arctanh}((a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/c^2/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)+d^{(7/2)}*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)}/(c+d)^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/c/(c-d)^3/(c+d)^{(3/2)}/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)-1/4}*(c-3*d)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/(c-d)^3/f*2^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)-3/32}*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/(c-d)^2/f*2^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)-(c^2-4*c*d+6*d^2)*\operatorname{arctanh}(1/2*(a-a*\sec(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*2^{(1/2)}*\tan(f*x+e)/a^{(3/2)}/(c-d)^4/f/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)+2*(4*c-d)*d^{(7/2)}*\operatorname{arctanh}(d^{(1/2)}*(a-a*\sec(f*x+e))^{(1/2)}/a^{(1/2)}/(c+d)^{(1/2)})*\tan(f*x+e)/a^{(3/2)}/c^2/(c-d)^4/f/(c+d)^{(1/2)}/(a-a*\sec(f*x+e))^{(1/2)}/(a+a*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.63, antiderivative size = 756, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3940, 180, 63, 206, 51, 208}

$$\frac{2d^{7/2}(4c-d) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{a-a \sec(e+fx)}}{\sqrt{a} \sqrt{c+d}}\right)}{a^{3/2}c^2 f(c-d)^4 \sqrt{c+d} \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}} - \frac{\sqrt{2} (c^2-4cd+6d^2) \tan(e+fx) \tanh^{-1}\left(\frac{\sqrt{a-a \sec(e+fx)}}{\sqrt{2} \sqrt{a}}\right)}{a^{3/2} f(c-d)^4 \sqrt{a-a \sec(e+fx)} \sqrt{a \sec(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2), x]

[Out] $-\operatorname{Tan}[e+f*x]/(4*a^2*(c-d)^2*f*(1+\operatorname{Sec}[e+f*x])^2*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) - ((c-3*d)*\operatorname{Tan}[e+f*x])/(2*a^2*(c-d)^3*f*(1+\operatorname{Sec}[e+f*x])* \operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) - (3*\operatorname{Tan}[e+f*x])/(16*a^2*(c-d)^2*f*(1+\operatorname{Sec}[e+f*x])* \operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) + (2*\operatorname{ArcTanh}[\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]/\operatorname{Sqrt}[a]]*\operatorname{Tan}[e+f*x])/(a^{(3/2)}*c^2*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) - ((c-3*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]* \operatorname{Tan}[e+f*x])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*(c-d)^3*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) - (3*\operatorname{ArcTanh}[\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]* \operatorname{Tan}[e+f*x])/(16*\operatorname{Sqrt}[2]*a^{(3/2)}*(c-d)^2*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) - (\operatorname{Sqrt}[2]*(c^2-4*c*d+6*d^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a])]* \operatorname{Tan}[e+f*x])/(a^{(3/2)}*(c-d)^4*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) + (d^{(7/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c+d])]* \operatorname{Tan}[e+f*x])/(a^{(3/2)}*c*(c-d)^3*(c+d)^{(3/2)}*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) + (2*(4*c-d)*d^{(7/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c+d])]* \operatorname{Tan}[e+f*x])/(a^{(3/2)}*c^2*(c-d)^4*\operatorname{Sqrt}[c+d]*f*\operatorname{Sqrt}[a-a*\operatorname{Sec}[e+f*x]]*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]) + (d^4*\operatorname{Tan}[e+f*x])/(a^2*c*(c-d)^3*(c+d)*f*\operatorname{Sqrt}[a+a*\operatorname{Sec}[e+f*x]]*(c+d*\operatorname{Sec}[e+f*x]))$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(

```

m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 180

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
g, h, m, n}, x] && IntegersQ[p, q]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 3940

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*
x)^n/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& IntegerQ[m - 1/2]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^2} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax} (a+ax)^3 (c+dx)^2} dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{a^3 c^2 x \sqrt{a-ax}} - \frac{1}{a^3 (c-d)^2 (1+x)^3 \sqrt{a-ax}} + \dots\right) dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{a c^2 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{((c - 3d) \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{a (c - d)^2 \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4 a^2 (c - d)^2 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2 a^2 (c - d)^2 f (1 + \sec(e + fx))^2 \sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4 a^2 (c - d)^2 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2 a^2 (c - d)^2 f (1 + \sec(e + fx))^2 \sqrt{a - a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4 a^2 (c - d)^2 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2 a^2 (c - d)^2 f (1 + \sec(e + fx))^2 \sqrt{a - a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 40.04, size = 688080, normalized size = 910.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
```


Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2), x)
```


$$a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (2*d^{(7/2)}*(10*c^2 - 5*c*d + d^2)*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[a - a*\text{Sec}[e + f*x]])/(\text{Sqrt}[a]*\text{Sqrt}[c + d])]*\text{Tan}[e + f*x])/ (a^{(3/2)}*c^3*(c - d)^5*\text{Sqrt}[c + d]*f*\text{Sqrt}[a - a*\text{Sec}[e + f*x]]*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]) + (d^4*\text{Tan}[e + f*x])/(2*a^2*c*(c - d)^3*(c + d)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])^2) + (3*d^4*\text{Tan}[e + f*x])/(4*a^2*c*(c - d)^3*(c + d)^2*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x])) + ((4*c - d)*d^4*\text{Tan}[e + f*x])/(a^2*c^2*(c - d)^4*(c + d)*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*(c + d*\text{Sec}[e + f*x]))$$
Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*x
)^(m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
g, h, m, n}, x] && IntegerQ[p, q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3940

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] := Dist[(a^2*Cot[e + f*x])/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*
x)^n)/(x*Sqrt[a - b*x]), x], x, Csc[e + f*x], x] /; FreeQ[{a, b, c, d, e,
f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& IntegerQ[m - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^3} dx &= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax} (a+ax)^3 (c+dx)^3} dx, x, \sec(e + fx)\right)}{f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{(a^2 \tan(e + fx)) \operatorname{Subst}\left(\int \left(\frac{1}{a^3 c^3 x \sqrt{a-ax}} - \frac{1}{a^3 (c-d)^3 (1+x)^3 \sqrt{a-ax}}\right) dx, x, \sec(e + fx)\right)}{a^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{a^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} + \frac{((c - d) \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{(1+x)^3 \sqrt{a-ax}} dx, x, \sec(e + fx)\right)}{a^3 f \sqrt{a - a \sec(e + fx)} \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} \\
&= -\frac{\tan(e + fx)}{4a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}} - \frac{\tan(e + fx)}{2a^2 (c - d)^3 f (1 + \sec(e + fx))^2 \sqrt{a + a \sec(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 43.44, size = 893714, normalized size = 894.61

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3),x)`

[Out] `\text{Hanged}`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a(\sec(e + fx) + 1)\right)^{\frac{5}{2}} (c + d \sec(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)`

[Out] `Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3), x)`

3.184 $\int \sqrt{a + a \sec(e + fx)} \sqrt{c + d \sec(e + fx)} dx$

Optimal. Leaf size=123

$$\frac{2\sqrt{a} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a} \sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{f}$$

[Out] $2*\arctan(a^{(1/2)}*c^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))}^{(1/2)})*a^{(1/2)}*c^{(1/2)}/f+2*\operatorname{arctanh}(a^{(1/2)}*d^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))}^{(1/2)})*a^{(1/2)}*d^{(1/2)}/f$

Rubi [A] time = 0.34, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3932, 3934, 203, 3980, 206}

$$\frac{2\sqrt{a} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a} \sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]

[Out] $(2*\text{Sqrt}[a]*\text{Sqrt}[c]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])]/f + (2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])])/f$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3932

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(d_) + (c_)], x_Symbol] := Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]], x], x] + Dist[d, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/Sqrt[c + d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3934

Int[Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(d_) + (c_)], x_Symbol] := Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3980

Int[(csc[(e_) + (f_)*(x_)]*Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)])/Sqrt[csc[(e_) + (f_)*(x_)]*(d_) + (c_)], x_Symbol] := Dist[(-2*b)/f, Subst

[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \sqrt{a + a \sec(e + fx)} \sqrt{c + d \sec(e + fx)} dx = c \int \frac{\sqrt{a + a \sec(e + fx)}}{\sqrt{c + d \sec(e + fx)}} dx + d \int \frac{\sec(e + fx) \sqrt{a + a \sec(e + fx)}}{\sqrt{c + d \sec(e + fx)}} dx$$

$$\stackrel{(2ac) \text{ Subst}}{=} \frac{\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}}{f} \quad (2)$$

$$= \frac{2\sqrt{a} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a} \sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \tanh(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f}$$

Mathematica [A] time = 18.42, size = 240, normalized size = 1.95

$$\frac{2 \cot(e + fx) \sqrt{a(\sec(e + fx) + 1)} \sqrt{c + d \sec(e + fx)} \left(\sqrt{c^2 \sin^2(e + fx)} \sqrt{c - c \cos(e + fx)} \tan^{-1} \left(\frac{\sqrt{c(\cos(e + fx) + 1)}}{\sqrt{c - c \cos(e + fx)}} \right) \right)}{f \sqrt{c(\cos(e + fx) + 1)} \sqrt{c - c \cos(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]

[Out] (-2*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]]*(-2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c - c*Cos[e + f*x]])]*Sqrt[c*(1 + Cos[e + f*x])]*Sin[(e + f*x)/2]^2 + ArcTan[(Sqrt[c*(1 + Cos[e + f*x])]*Sqrt[d + c*Cos[e + f*x]])/Sqrt[c^2*Sin[e + f*x]^2]]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[c^2*Sin[e + f*x]^2]))/(f*Sqrt[c*(1 + Cos[e + f*x])]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])

fricas [A] time = 1.47, size = 806, normalized size = 6.55

$$\frac{\sqrt{ad} \log \left(\frac{2 \sqrt{ad} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)+d}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + (ac-ad) \cos(fx+e)^2 + 2ad + (ac+ad) \cos(fx+e)}{\cos(fx+e)^2 + \cos(fx+e)} \right) + \sqrt{-ac} \log \left(\frac{2ac \cos(fx+e) + (ac+ad) \cos(fx+e)}{\cos(fx+e)^2 + \cos(fx+e)} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [(sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*x + e))) + sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/f, -(2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e)))]

```

- sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)*
cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(f*
x + e))))/f, -(2*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*
x + e)))) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) +
1)))/f, -2*(sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))
) + sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/f]

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

maple [B] time = 2.24, size = 1562, normalized size = 12.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)
*cos(f*x+e)*(-1+cos(f*x+e))*((c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c
*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*s
in(f*x+e)-d*sin(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e)))*c^2*d-2*(c-d)^(1/2)
*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*sin(f
*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c-d)/(-1+cos(f*x+
e)+sin(f*x+e)))*c*d^2+(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*
x+e))/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*sin(f*x+
e)-d*sin(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e))*d^3-(c-d)^(1/2)*ln(-2*(2^(
1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)+c*cos
(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-sin(f*x+e)-1+cos(f*x+
e))*c^2*d+2*(c-d)^(1/2)*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+
cos(f*x+e))))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-d*sin(
f*x+e)-c+d)/(-sin(f*x+e)-1+cos(f*x+e))*c*d^2-(c-d)^(1/2)*ln(-2*(2^(1/2)*(-
d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*sin(f*x+e)+c*cos(f*x+e)
-d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(-sin(f*x+e)-1+cos(f*x+e))*d^
3+ln(((2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*c
os(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*2^(1/2)*(-d)^(1/2)*c^3-
3*ln(((2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*c
os(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*2^(1/2)*(-d)^(1/2)*c^2*
d+3*ln(((2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c
*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*2^(1/2)*(-d)^(1/2)*c*
d^2-ln(((2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c
*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*2^(1/2)*(-d)^(1/2)*d^
3-ln(-((c-d)^(1/2)*cos(f*x+e)-sin(f*x+e)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)
)))^(1/2)-(c-d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^(1/2)*c^3+3*ln(-((c-d)^(1/2)
*cos(f*x+e)-sin(f*x+e)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)-(c-d)^(1/
2))/sin(f*x+e))*2^(1/2)*(-d)^(1/2)*c^2*d-3*ln(-((c-d)^(1/2)*cos(f*x+e)-sin(
f*x+e)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e))))^(1/2)-(c-d)^(1/2))/sin(f*x+e))*
```

$$2^{1/2}(-d)^{1/2}cd^2 + \ln\left(-\left((c-d)^{1/2}\cos(fx+e) - \sin(fx+e)\right)\left(-2(d+c\cos(fx+e))/(1+\cos(fx+e))\right)^{1/2} - (c-d)^{1/2}/\sin(fx+e)\right) 2^{1/2}(-d)^{1/2}d^3 - 2\left(-\left((c-d)^4c\right)^{1/2}\arctan\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\left(-2(d+c\cos(fx+e))/(1+\cos(fx+e))\right)^{1/2}\right)(c-d)^2c^2^{1/2}/\left(-\left((c-d)^4c\right)^{1/2}\right)(c-d)^{1/2}(-d)^{1/2}/\sin(fx+e)^2/\left(-2(d+c\cos(fx+e))/(1+\cos(fx+e))\right)^{1/2}/(c-d)^{1/2}2^{1/2}/(-d)^{1/2}/(c^2-2cd+d^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{a}{\cos(e + fx)}} \sqrt{c + \frac{d}{\cos(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)

[Out] int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sec(e + fx) + 1)} \sqrt{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)), x)

$$3.185 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx$$

Optimal. Leaf size=61

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{c} f}$$

[Out] $2*\arctan(a^{(1/2)*c^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))^{(1/2)}}*a^{(1/2)}/f/c^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {3934, 203}

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{c} f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])]/(\text{Sqrt}[c]*f)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3934

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx &= -\frac{(2a) \text{Subst}\left(\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f} \\ &= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{c} f} \end{aligned}$$

Mathematica [A] time = 0.22, size = 102, normalized size = 1.67

$$\frac{\sqrt{2} \sec\left(\frac{1}{2}(e+fx)\right) \sqrt{a(\sec(e+fx)+1)} \sqrt{c \cos(e+fx)+d} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c \cos(e+fx)+d}}\right)}{\sqrt{c} f \sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(Sqrt[c]*f*Sqrt[c + d*Sec[e + f*x]])

fricas [A] time = 0.59, size = 206, normalized size = 3.38

$$\frac{\sqrt{-\frac{a}{c}} \log \left(\frac{2c\sqrt{-\frac{a}{c}} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)+d}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - 2ac \cos(fx+e)^2 + ac - ad - (ac+ad) \cos(fx+e)}{\cos(fx+e)+1} \right)}{f}, 2\sqrt{\frac{a}{c}} \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1))/f, -2*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e)))/f]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec(fx + e) + a}}{\sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)

maple [B] time = 1.96, size = 189, normalized size = 3.10

$$\frac{2\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \cos(fx+e) (-1 + \cos(fx+e)) \sqrt{\frac{d+c \cos(fx+e)}{\cos(fx+e)}} \arctan \left(\frac{(-1+\cos(fx+e))(c-d)^2 c \sqrt{2}}{\sin(fx+e) \sqrt{\frac{2(d+c \cos(fx+e))}{1+\cos(fx+e)}} \sqrt{-(c-d)^4 c}} \right)}{f \sin(fx+e)^2 \sqrt{-\frac{2(d+c \cos(fx+e))}{1+\cos(fx+e)}} (c^2 - 2cd + d^2) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)

[Out] -2/f*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*arctan((-1+cos(f*x+e))/sin(f*x+e)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*2^(1/2)*(-(c-d)^4*c)^(1/2)/(c^2-2*c*d+d^2)/c

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more details)Is d-c zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\sqrt{c + \frac{d}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e+fx)+1)}}{\sqrt{c+d\sec(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(c + d*sec(e + f*x)), x)

$$3.186 \quad \int \frac{\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{c^{3/2} f} - \frac{2ad \tan(e+fx)}{cf(c+d)\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}$$

[Out] $2*\arctan(a^{(1/2)}*c^{(1/2)}*\tan(f*x+e)/(a+a*\sec(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))^{(1/2)}}*a^{(1/2)}/c^{(3/2)}/f-2*a*d*\tan(f*x+e)/c/(c+d)/f/(a+a*\sec(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3939, 3934, 203, 3987, 37}

$$\frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{c^{3/2} f} - \frac{2ad \tan(e+fx)}{cf(c+d)\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2), x]

[Out] $(2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Tan}[e + f*x])/(\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])]/(c^{(3/2)}*f) - (2*a*d*\text{Tan}[e + f*x])/((c*(c + d))*f*\text{Sqrt}[a + a*\text{Sec}[e + f*x]]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])$

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp [((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3934

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)], x_Symbol] := Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3939

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(3/2), x_Symbol] := Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n]/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c + d \sec(e + fx))^{3/2}} dx = \frac{\int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx}{c} - \frac{d \int \frac{\sec(e+fx)\sqrt{a+a \sec(e+fx)}}{(c+d \sec(e+fx))^{3/2}} dx}{c}$$

$$= -\frac{(2a) \operatorname{Subst}\left(\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}\sqrt{c+d \sec(e+fx)}}\right) + (a^2 d \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}\sqrt{c+d \sec(e+fx)}}\right)}{cf} + \frac{(a^2 d \tan(e + fx)) \operatorname{Subst}\left(\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)}\sqrt{c+d \sec(e+fx)}}\right)}{cf\sqrt{a - a \sec(e + fx)}}$$

$$= \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{c} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)}\sqrt{c+d \sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2ad \tan(e + fx)}{c(c + d)f\sqrt{a + a \sec(e + fx)}\sqrt{c + d \sec(e + fx)}}$$

Mathematica [A] time = 0.96, size = 135, normalized size = 1.22

$$\frac{\sec\left(\frac{1}{2}(e + fx)\right) \sqrt{a(\sec(e + fx) + 1)} \left(2\sqrt{c} d \sin\left(\frac{1}{2}(e + fx)\right) - \sqrt{2}(c + d)^{3/2} \sin^{-1}\left(\frac{\sqrt{2}\sqrt{c} \sin\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c+d}}\right)\right) \sqrt{\frac{c \cos(e + fx)}{c+d}}}{c^{3/2} f (c + d) \sqrt{c + d \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2), x]

[Out] -((Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])*(-(Sqrt[2]*(c + d)^(3/2)*ArcSin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)]) + 2*Sqrt[c]*d*Sin[(e + f*x)/2]))/(c^(3/2)*(c + d)*f*Sqrt[c + d*Sec[e + f*x]])

fricas [B] time = 0.60, size = 517, normalized size = 4.66

$$\frac{2d \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)+d}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) - \left((c^2 + cd) \cos(fx+e)^2 + cd + d^2 + (c^2 + 2cd + d^2) \sin^2(fx+e) \right)}{(c^3 + c^2d)f \cos(fx+e)^2 + (c^3 + 2c^2d) \sin^2(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2), x, algorithm="fricas")

[Out] [-(2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c + a*d)*sin(f*x + e)))]

$d \cdot \cos(fx + e) / (\cos(fx + e) + 1) / ((c^3 + c^2d)fx \cos(fx + e)^2 + (c^3 + 2c^2d + cd^2)fx \cos(fx + e) + (c^2d + cd^2)f), -2(d \sqrt{(a \cos(fx + e) + a) / \cos(fx + e)} \sqrt{(c \cos(fx + e) + d) / \cos(fx + e)} \cos(fx + e) \sin(fx + e) + ((c^2 + cd) \cos(fx + e)^2 + cd + d^2 + (c^2 + 2cd + d^2) \cos(fx + e)) \sqrt{a/c} \arctan(\sqrt{a/c} \sqrt{(a \cos(fx + e) + a) / \cos(fx + e)} \sqrt{(c \cos(fx + e) + d) / \cos(fx + e)} \cos(fx + e) / (a \sin(fx + e)))) / ((c^3 + c^2d)fx \cos(fx + e)^2 + (c^3 + 2c^2d + cd^2)fx \cos(fx + e) + (c^2d + cd^2)f)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sec(fx + e) + a}}{(d \sec(fx + e) + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)

maple [B] time = 2.05, size = 377, normalized size = 3.40

$$\left(\sqrt{2} \sqrt{-(c-d)^4} c \arctan \left(\frac{(-1+\cos(fx+e))(c-d)^2 c \sqrt{2}}{\sin(fx+e) \sqrt{\frac{2(d+c \cos(fx+e))}{1+\cos(fx+e)}} \sqrt{-(c-d)^4} c} \right) \sqrt{\frac{2(d+c \cos(fx+e))}{1+\cos(fx+e)}} c \sin(fx+e) + \sqrt{2} \sqrt{-(c-d)^4} c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x)

[Out] $-1/f * (2^{1/2} * (-(c-d)^4 * c)^{1/2} * \arctan((-1+\cos(f*x+e))/\sin(f*x+e)/(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e))))^{1/2} * (c-d)^2 * c * 2^{1/2} / (-(c-d)^4 * c)^{1/2}) * (-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2} * c * \sin(f*x+e) + 2^{1/2} * (-(c-d)^4 * c)^{1/2} * \arctan((-1+\cos(f*x+e))/\sin(f*x+e)/(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e))))^{1/2} * (c-d)^2 * c * 2^{1/2} / (-(c-d)^4 * c)^{1/2}) * (-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2} * d * \sin(f*x+e) - 2 * c^3 * d * \cos(f*x+e) + 4 * \cos(f*x+e) * c^2 * d^2 - 2 * \cos(f*x+e) * c * d^3 + 2 * c^3 * d - 4 * c^2 * d^2 + 2 * c * d^3) * \cos(f*x+e) * (a * (1 + \cos(f*x+e)) / \cos(f*x+e))^{1/2} * ((d+c*\cos(f*x+e))/\cos(f*x+e))^{1/2} / (d+c*\cos(f*x+e)) / \sin(f*x+e) / (c+d) / c^2 / (c^2 - 2 * c * d + d^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more details) Is d-c zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{a}{\cos(e+fx)}}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)`

[Out] `int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e + fx) + 1)}}{(c + d \sec(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2), x)`

[Out] `Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**(3/2), x)`

$$3.187 \quad \int \frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{a+a \sec(e+fx)}} dx$$

Optimal. Leaf size=141

$$\frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} f} - \frac{\sqrt{2} \sqrt{c-d} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} f}$$

[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*c^(1/2)/f/a^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/2)

Rubi [A] time = 0.37, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3935, 3934, 203, 3983}

$$\frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} f} - \frac{\sqrt{2} \sqrt{c-d} \tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]/(Sqrt[a]*f)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3934

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3935

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[a/c, Int[Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]

Rule 3983

Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])], x_Symbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[

$b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{a+a \sec(e+fx)}} dx &= \frac{c \int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx}{a} + (-c+d) \int \frac{\sec(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}} dx \\ &= \frac{(2c) \text{Subst}\left(\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f} + \frac{(2(c-d)) \text{Subst}\left(\int \frac{1}{2+(acx^2)} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f} \\ &= \frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} f} - \frac{\sqrt{2} \sqrt{c-d} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c-d} \tan(e+fx)}{\sqrt{2} \sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} f} \end{aligned}$$

Mathematica [A] time = 14.62, size = 184, normalized size = 1.30

$$\frac{2 \cos\left(\frac{1}{2}(e+fx)\right) \sqrt{c+d \sec(e+fx)} \left(\frac{\sqrt{2} \sqrt{c} \sqrt{c+d} \sin^{-1}\left(\frac{\sqrt{2} \sqrt{c} \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right) \sqrt{\frac{c \cos(e+fx)+d}{c+d}}}{\sqrt{c \cos(e+fx)+d}} + \sqrt{d-c} \tanh^{-1}\left(\frac{\sqrt{d-c} \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c \cos(e+fx)+d}}\right) \right)}{f \sqrt{a(\sec(e+fx)+1)} \sqrt{c \cos(e+fx)+d}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]

[Out] (2*Cos[(e + f*x)/2]*(Sqrt[-c + d]*ArcTanh[(Sqrt[-c + d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]) + (Sqrt[2]*Sqrt[c]*Sqrt[c + d]*ArcSin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)]/Sqrt[d + c*Cos[e + f*x]])*Sqrt[c + d*Sec[e + f*x]])/(f*Sqrt[d + c*Cos[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])])

fricas [A] time = 1.00, size = 883, normalized size = 6.26

$$\left[\frac{\sqrt{2} \sqrt{-\frac{c-d}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{-\frac{c-d}{a}} \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)+d}{\cos(fx+e)}} \cos(fx+e) \sin(fx+e) + (3c-d) \cos(fx+e)^2 + 2(c+d) \cos(fx+e) - c + 3d}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)}{2f} + 2 \sqrt{-\frac{c-d}{a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-2*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x + e) + c - d)/(cos(f*x + e) + 1))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-2*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x + e) + c - d)/(cos(f*x + e) + 1))/f]


```
s(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x
+ e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e
) + 1)) - 4*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(c*sin(f*x + e))))
/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)/((c - d)*sin(f*x + e))) - sqrt(-c/a)*log(-(2*sqrt(-c/a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x + e) + c - d)/(cos(
f*x + e) + 1)))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
+ e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + 2*sqrt(c/a)*arctan(sqrt(c/a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
e))*cos(f*x + e)/(c*sin(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec(fx + e) + c}}{\sqrt{a \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

maple [B] time = 1.88, size = 494, normalized size = 3.50

$$2\sqrt{\frac{d+c \cos(fx+e)}{\cos(fx+e)}} \sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}} \cos(fx + e) (-1 + \cos(fx + e)) \left(\ln \left(-\frac{\sqrt{c-d} \cos(fx+e) - \sin(fx+e) \sqrt{-\frac{2(d+c \cos(fx+e))}{1+\cos(fx+e)}}}{\sin(fx+e)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(1+cos(f*x+e))/cos(f*x+e))^(1/2
)*cos(f*x+e)*(-1+cos(f*x+e))*(ln(-((c-d)^(1/2)*cos(f*x+e)-sin(f*x+e))*(-2*(d
+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-(c-d)^(1/2))/sin(f*x+e))*c^3-3*ln(-((c
-d)^(1/2)*cos(f*x+e)-sin(f*x+e))*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-
(c-d)^(1/2))/sin(f*x+e))*c^2*d+3*ln(-((c-d)^(1/2)*cos(f*x+e)-sin(f*x+e))*(-2
*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-(c-d)^(1/2))/sin(f*x+e))*c*d^2-ln(-
((c-d)^(1/2)*cos(f*x+e)-sin(f*x+e))*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/
2)-(c-d)^(1/2))/sin(f*x+e))*d^3+2^(1/2)*(-(c-d)^4*c)^(1/2)*arctan((-1+cos(f
*x+e))/sin(f*x+e)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^2*c*2^(1
/2)/(-(c-d)^4*c)^(1/2))*(c-d)^(1/2))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+c
os(f*x+e)))^(1/2)/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxim
a")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more details) Is d-c positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + \frac{d}{\cos(e+fx)}}}{\sqrt{a + \frac{a}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)

[Out] int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sec(e + fx)}}{\sqrt{a(\sec(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2), x)

[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)

$$3.188 \quad \int \frac{1}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}} dx$$

Optimal. Leaf size=141

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}} \right)}{\sqrt{a} \sqrt{c} f} - \frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt{a} \sqrt{c-d} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}} \right)}{\sqrt{a} f \sqrt{c-d}}$$

[Out] 2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/f/a^(1/2)/c^(1/2)-arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)

Rubi [A] time = 0.34, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {3938, 3934, 203, 3983}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}} \right)}{\sqrt{a} \sqrt{c} f} - \frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt{a} \sqrt{c-d} \tan(e+fx)}{\sqrt{2} \sqrt{a \sec(e+fx)+a} \sqrt{c+d \sec(e+fx)}} \right)}{\sqrt{a} f \sqrt{c-d}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] (2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/((Sqrt[a]*Sqrt[c - d]*f))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3934

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)], x_Symbol] := Dist[(-2*a)/f, Subst[Int[1/(1 + a*c*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3938

Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)]), x_Symbol] := Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3983

Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)]), x_Symbol] := Dist[(-2*a)/(b*f), Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ

b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)} \sqrt{c + d \sec(e + fx)}} dx = \frac{\int \frac{\sqrt{a+a \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx}{a} - \int \frac{\sec(e + fx)}{\sqrt{a + a \sec(e + fx)} \sqrt{c + d \sec(e + fx)}} dx$$

$$= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{1+acx^2} dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+acx^2} dx, x, \frac{\tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{f}$$

$$= \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c} \tan(e+fx)}{\sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} \sqrt{c} f} - \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{2} \sqrt{a+a \sec(e+fx)} \sqrt{c+d \sec(e+fx)}}\right)}{\sqrt{a} \sqrt{c-d} f}$$

Mathematica [A] time = 0.35, size = 171, normalized size = 1.21

$$\frac{2 \cos\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{c \cos(e + fx) + d} \left(\sqrt{2} \sqrt{c-d} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c \cos(e+fx)+d}}\right) - \sqrt{c} \tan^{-1}\left(\frac{\sqrt{c-d} \sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c \cos(e+fx)+d}}\right) \right)}{\sqrt{c} f \sqrt{c-d} \sqrt{a(\sec(e + fx) + 1)} \sqrt{c + d \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] (2*(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]] - Sqrt[c]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(Sqrt[c]*Sqrt[c - d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])

fricas [A] time = 1.69, size = 913, normalized size = 6.48

$$\left[\frac{\sqrt{2} ac \sqrt{-\frac{1}{ac-ad}} \log\left(\frac{2 \sqrt{2} (c-d) \sqrt{\frac{a \cos(fx+e)+a}{\cos(fx+e)}} \sqrt{\frac{c \cos(fx+e)+d}{\cos(fx+e)}} \sqrt{-\frac{1}{ac-ad}} \cos(fx+e) \sin(fx+e) + (3c-d) \cos(fx+e)^2 + 2(c+d) \cos(fx+e) - c + 3d}{\cos(fx+e)^2 + 2 \cos(fx+e) + 1} \right)}{2 acf} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/(a*c*f), 1/

$$2\sqrt{2}ac\sqrt{-1/(ac - ad)}\log((2\sqrt{2}(c - d)\sqrt{(a\cos(fx + e) + a)/\cos(fx + e)}\sqrt{(c\cos(fx + e) + d)/\cos(fx + e)}\sqrt{-1/(ac - ad)}\cos(fx + e)\sin(fx + e) + (3c - d)\cos(fx + e)^2 + 2(c + d)\cos(fx + e) - c + 3d)/(\cos(fx + e)^2 + 2\cos(fx + e) + 1)) - 4\sqrt{ac}\arctan(\sqrt{ac}\sqrt{(a\cos(fx + e) + a)/\cos(fx + e)}\sqrt{(c\cos(fx + e) + d)/\cos(fx + e)}\cos(fx + e)/(a\sin(fx + e))))/(acf), (\sqrt{2}ac\arctan(\sqrt{2}\sqrt{(a\cos(fx + e) + a)/\cos(fx + e)}\sqrt{(c\cos(fx + e) + d)/\cos(fx + e)}\cos(fx + e)/(\sqrt{ac - ad}\sin(fx + e)))/\sqrt{ac - ad} - \sqrt{-ac}\log((2ac\cos(fx + e)^2 + 2\sqrt{-ac}\sqrt{(a\cos(fx + e) + a)/\cos(fx + e)}\sqrt{(c\cos(fx + e) + d)/\cos(fx + e)}\cos(fx + e)\sin(fx + e) - ac + ad + (ac + ad)\cos(fx + e))/(\cos(fx + e) + 1)))/(acf), (\sqrt{2}ac\arctan(\sqrt{2}\sqrt{(a\cos(fx + e) + a)/\cos(fx + e)}\sqrt{(c\cos(fx + e) + d)/\cos(fx + e)}\cos(fx + e)/(\sqrt{ac - ad}\sin(fx + e)))/\sqrt{ac - ad} - 2\sqrt{ac}\arctan(\sqrt{ac}\sqrt{(a\cos(fx + e) + a)/\cos(fx + e)}\sqrt{(c\cos(fx + e) + d)/\cos(fx + e)}\cos(fx + e)/(a\sin(fx + e))))/(acf)]$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

maple [B] time = 2.13, size = 424, normalized size = 3.01

$$2\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\cos(fx+e)(-1+\cos(fx+e))\left(\sqrt{2}\sqrt{-(c-d)^4c}\arctan\left(\frac{(-1+\cos(fx+e))}{\sin(fx+e)\sqrt{-\frac{2(d+c)}{1+c}}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)

[Out]
$$-2/f*(a*(1+\cos(f*x+e))/\cos(f*x+e))^{1/2}*((d+c*\cos(f*x+e))/\cos(f*x+e))^{1/2}*\cos(f*x+e)*(-1+\cos(f*x+e))*(2^{1/2})*(-(c-d)^4*c)^{1/2}*\arctan((-1+\cos(f*x+e))/\sin(f*x+e)/(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2}*(c-d)^2*c*2^{1/2})/(-(c-d)^4*c)^{1/2}*(c-d)^{1/2}+\ln(-((c-d)^{1/2}*\cos(f*x+e)-\sin(f*x+e))*(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2}-(c-d)^{1/2})/\sin(f*x+e)*c^3-2*\ln(-((c-d)^{1/2}*\cos(f*x+e)-\sin(f*x+e))*(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2}-(c-d)^{1/2})/\sin(f*x+e)*c^2*d+\ln(-((c-d)^{1/2}*\cos(f*x+e)-\sin(f*x+e))*(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2}-(c-d)^{1/2})/\sin(f*x+e))*c*d^2)/\sin(f*x+e)^2/(-2*(d+c*\cos(f*x+e))/(1+\cos(f*x+e)))^{1/2}/a/(c-d)^{1/2}/(c^2-2*c*d+d^2)/c$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{a}{\cos(e+fx)}} \sqrt{c + \frac{d}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)

[Out] int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a(\sec(e+fx)+1)} \sqrt{c+d\sec(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))), x)

$$3.189 \quad \int \frac{a+b \sec(e+fx)}{c+d \sec(e+fx)} dx$$

Optimal. Leaf size=67

$$\frac{2(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}} \right)}{cf\sqrt{c-d}\sqrt{c+d}} + \frac{ax}{c}$$

[Out] $ax/c + 2*(-a*d+b*c)*\operatorname{arctanh}((c-d)^{(1/2)}*\tan(1/2*e+1/2*f*x)/(c+d)^{(1/2)})/c/f/(c-d)^{(1/2)}/(c+d)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3919, 3831, 2659, 208}

$$\frac{2(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}} \right)}{cf\sqrt{c-d}\sqrt{c+d}} + \frac{ax}{c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sec}[e + f*x])/(c + d*\operatorname{Sec}[e + f*x]), x]$

[Out] $(a*x)/c + (2*(b*c - a*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c - d]*\operatorname{Tan}[(e + f*x)/2])/\operatorname{Sqrt}[c + d]])/(c*\operatorname{Sqrt}[c - d]*\operatorname{Sqrt}[c + d]*f)$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] \text{ ; FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a_ + (b_)*\sin[\operatorname{Pi}/2 + (c_.) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \operatorname{With}[e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x], \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x]] \text{ ; FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_)*(x_)]/(csc[(e_.) + (f_)*(x_)]*(b_.) + (a_)), x_Symbol] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[1/(1 + (a*\operatorname{Sin}[e + f*x])/b), x], x] \text{ ; FreeQ}\{a, b, e, f\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3919

$\operatorname{Int}[(csc[(e_.) + (f_)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_)*(x_)]*(b_.) + (a_)), x_Symbol] \rightarrow \operatorname{Simp}[(c*x)/a, x] - \operatorname{Dist}[(b*c - a*d)/a, \operatorname{Int}[\operatorname{Csc}[e + f*x]/(a + b*\operatorname{Csc}[e + f*x]), x], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec(e + fx)}{c + d \sec(e + fx)} dx &= \frac{ax}{c} - \frac{(-bc + ad) \int \frac{\sec(e+fx)}{c+d \sec(e+fx)} dx}{c} \\
&= \frac{ax}{c} - \frac{(-bc + ad) \int \frac{1}{1 + \frac{c \cos(e+fx)}{d}} dx}{cd} \\
&= \frac{ax}{c} + \frac{(2(bc - ad)) \text{Subst} \left(\int \frac{1}{1 + \frac{c}{d} + (1 - \frac{c}{d})x^2} dx, x, \tan \left(\frac{1}{2}(e + fx) \right) \right)}{cdf} \\
&= \frac{ax}{c} + \frac{2(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c-d} \tan \left(\frac{1}{2}(e+fx) \right)}{\sqrt{c+d}} \right)}{c\sqrt{c-d}\sqrt{c+d}f}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 68, normalized size = 1.01

$$\frac{\frac{2(ad-bc) \tanh^{-1} \left(\frac{(d-c) \tan \left(\frac{1}{2}(e+fx) \right)}{\sqrt{c^2-d^2}} \right)}{\sqrt{c^2-d^2}} + a(e+fx)}{cf}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]

[Out] (a*(e + f*x) + (2*(-(b*c) + a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2])/(c*f)

fricas [A] time = 0.48, size = 250, normalized size = 3.73

$$\left[\frac{2(ac^2 - ad^2)fx - (bc - ad)\sqrt{c^2 - d^2} \log \left(\frac{2cd \cos(fx+e) - (c^2 - 2d^2) \cos(fx+e)^2 - 2\sqrt{c^2 - d^2} (d \cos(fx+e) + c) \sin(fx+e) + 2c^2 - d^2}{c^2 \cos(fx+e)^2 + 2cd \cos(fx+e) + d^2} \right)}{2(c^3 - cd^2)f}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(2*(a*c^2 - a*d^2)*f*x - (b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)))/((c^3 - c*d^2)*f), ((a*c^2 - a*d^2)*f*x + (b*c - a*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e)))/((c^3 - c*d^2)*f)]

giac [B] time = 0.28, size = 278, normalized size = 4.15

$$\frac{\left(\sqrt{-c^2+d^2} a(c-2d)|-c+d| + \sqrt{-c^2+d^2} b|c|-c+d| - \sqrt{-c^2+d^2} a|c||-c+d| + \sqrt{-c^2+d^2} b|c||-c+d| \right) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)}{\sqrt{\frac{d + \sqrt{(c+d)(c-d)+d^2}}{c-d}}} \right) \right)}{(c^2 - 2cd + d^2)c^2 + (c^2d - 2cd^2 + d^3)|c|} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] ((sqrt(-c^2 + d^2)*a*(c - 2*d)*abs(-c + d) + sqrt(-c^2 + d^2)*b*c*abs(-c + d) - sqrt(-c^2 + d^2)*a*abs(c)*abs(-c + d) + sqrt(-c^2 + d^2)*b*abs(c)*abs(-c + d))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sqrt(-(d + sqrt((c + d)*(c - d) + d^2))/(c - d))))/((c^2 - 2*c*d + d^2)*c^2 + (c^2*d - 2*c*d^2 + d^3)*abs(c)) + (a*c + b*c - 2*a*d + a*abs(c) - b*abs(c))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sqrt(-(d - sqrt((c + d)*(c - d) + d^2))/(c - d))))/(c^2 - d*abs(c))/f

maple [A] time = 0.74, size = 113, normalized size = 1.69

$$-\frac{2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right) da}{fc\sqrt{(c+d)(c-d)}} + \frac{2 \operatorname{arctanh}\left(\frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right) b}{f\sqrt{(c+d)(c-d)}} + \frac{2a \operatorname{arctan}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)

[Out] -2/f/c/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*d*a+2/f/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*b+2/f*a/c*arctan(tan(1/2*e+1/2*f*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 2.68, size = 573, normalized size = 8.55

$$\frac{b c^2 \ln\left(\frac{c \sin\left(\frac{e}{2} + \frac{fx}{2}\right) - d \sin\left(\frac{e}{2} + \frac{fx}{2}\right) + \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sqrt{c^2 - d^2}}{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)}\right)}{f(c^2 - d^2)^{3/2}} - \frac{b d^2 \ln\left(\frac{c \sin\left(\frac{e}{2} + \frac{fx}{2}\right) - d \sin\left(\frac{e}{2} + \frac{fx}{2}\right) + \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \sqrt{c^2 - d^2}}{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)}\right)}{f(c^2 - d^2)^{3/2}} + \frac{2 a c \operatorname{atan}\left(\frac{\sin\left(\frac{e}{2} + \frac{fx}{2}\right)}{\cos\left(\frac{e}{2} + \frac{fx}{2}\right)}\right)}{f(c^2 - d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x)),x)

[Out] (b*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (b*d^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (b*log((c*cos(e/2 + (f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) - (a*c*d*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*a*d^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)) + (a*d^3*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)^(3/2)) + (a*d*log((c*cos(e/2 + (f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(c*f*(c^2 - d^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(e + fx)}{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)

[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)

$$3.190 \quad \int \frac{a+b \sec(e+fx)}{(c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=123

$$-\frac{d(bc-ad)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))} + \frac{2(-2ac^2d+ad^3+bc^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}} + \frac{ax}{c^2}$$

[Out] a*x/c^2+2*(-2*a*c^2*d+a*d^3+b*c^3)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f-d*(-a*d+b*c)*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))

Rubi [A] time = 0.25, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3923, 3919, 3831, 2659, 208}

$$\frac{2(-2ac^2d+ad^3+bc^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2f(c-d)^{3/2}(c+d)^{3/2}} - \frac{d(bc-ad)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))} + \frac{ax}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]

[Out] (a*x)/c^2 + (2*(b*c^3 - 2*a*c^2*d + a*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) - (d*(b*c - a*d)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2))

), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\int \frac{a + b \sec(e + fx)}{(c + d \sec(e + fx))^2} dx = -\frac{d(bc - ad) \tan(e + fx)}{c(c^2 - d^2) f(c + d \sec(e + fx))} - \frac{\int \frac{-a(c^2 - d^2) - c(bc - ad) \sec(e + fx)}{c + d \sec(e + fx)} dx}{c(c^2 - d^2)}$$

$$= \frac{ax}{c^2} - \frac{d(bc - ad) \tan(e + fx)}{c(c^2 - d^2) f(c + d \sec(e + fx))} + \frac{(c^2(bc - ad) - ad(c^2 - d^2)) \int \frac{\sec(e + fx)}{c + d \sec(e + fx)} dx}{c^2(c^2 - d^2)}$$

$$= \frac{ax}{c^2} - \frac{d(bc - ad) \tan(e + fx)}{c(c^2 - d^2) f(c + d \sec(e + fx))} + \frac{(c^2(bc - ad) - ad(c^2 - d^2)) \int \frac{1}{1 + \frac{c \cos(e + fx)}{d}} dx}{c^2 d(c^2 - d^2)}$$

$$= \frac{ax}{c^2} - \frac{d(bc - ad) \tan(e + fx)}{c(c^2 - d^2) f(c + d \sec(e + fx))} + \frac{(2(c^2(bc - ad) - ad(c^2 - d^2))) \text{Subst}\left(\int \frac{1}{1 + \frac{c \cos(e + fx)}{d}} dx\right)}{c^2 d(c^2 - d^2)}$$

$$= \frac{ax}{c^2} + \frac{2(bc^3 - 2ac^2d + ad^3) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc - ad) \tan(e + fx)}{c(c^2 - d^2) f(c + d \sec(e + fx))}$$

Mathematica [A] time = 0.66, size = 155, normalized size = 1.26

$$\frac{-cd(bc - ad) \sin(e + fx) + ad(c^2 - d^2)(e + fx) + ac(c^2 - d^2)(e + fx) \cos(e + fx)}{c \cos(e + fx) + d} - \frac{2(ad(d^2 - 2c^2) + bc^3) \tanh^{-1}\left(\frac{(d - c) \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{\sqrt{c^2 - d^2}}$$

$$c^2 f(c - d)(c + d)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2, x]

[Out] ((-2*(b*c^3 + a*d*(-2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (a*d*(c^2 - d^2)*(e + f*x) + a*c*(c^2 - d^2)*(e + f*x)*Cos[e + f*x] - c*d*(b*c - a*d)*Sin[e + f*x])/(d + c*Cos[e + f*x])/(c^2*(c - d)*(c + d)*f)

fricas [B] time = 0.50, size = 561, normalized size = 4.56

$$\frac{2(ac^5 - 2ac^3d^2 + acd^4)fx \cos(fx + e) + 2(ac^4d - 2ac^2d^3 + ad^5)fx - (bc^3d - 2ac^2d^2 + ad^4 + (bc^4 - 2ac^3d + \dots))}{2((c^7 - 2c^5d^2 + \dots))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/2*(2*(a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + 2*(a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f*x - (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -

$$2*d^2*\cos(f*x + e)^2 - 2*\sqrt{c^2 - d^2}*(d*\cos(f*x + e) + c)*\sin(f*x + e) + 2*c^2 - d^2)/(c^2*\cos(f*x + e)^2 + 2*c*d*\cos(f*x + e) + d^2)) - 2*(b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*\sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*\cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*\cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f*x + (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*\cos(f*x + e))*\sqrt{-c^2 + d^2}*\arctan(-\sqrt{-c^2 + d^2}*(d*\cos(f*x + e) + c)/((c^2 - d^2)*\sin(f*x + e)))) - (b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*\sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*\cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f)]$$

giac [A] time = 0.58, size = 209, normalized size = 1.70

$$\frac{2(bc^3 - 2ac^2d + ad^3) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2c+2d) + \arctan \left(-\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2+d^2}} \right) \right)}{(c^4 - c^2d^2)\sqrt{-c^2+d^2}} + \frac{(fx+e)a}{c^2} + \frac{2(bcd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - ad^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right))}{(c^3 - cd^2) \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}$$

f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac")

[Out] (2*(b*c^3 - 2*a*c^2*d + a*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) + (f*x + e)*a/c^2 + 2*(b*c*d*tan(1/2*f*x + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f

maple [B] time = 0.77, size = 328, normalized size = 2.67

$$\frac{2d^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) a}{fc(c^2 - d^2) \left(\left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) c - \left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) d - c - d \right)} + \frac{2d \tan\left(\frac{e}{2} + \frac{fx}{2}\right) b}{f(c^2 - d^2) \left(\left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) c - \left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) d - c - d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)

[Out] -2/f/c*d^2/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)*a+2/f*d/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)*b-4/f/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a*d+2/f/c^2/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a*d^3+2/f*c/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*b+2/f*a/c^2*arctan(tan(1/2*e+1/2*f*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more details)Is 4*c^2-4*d^2 positive or negative?


```

*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (((32*tan(e/2 + (f*x)/2)*(a^2*c^6
+ 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c
^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d
^3 - c^3*d^2) + (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
- b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^
4*d^2) - (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3
- 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c
^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*
c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^8 -
c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^
3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) - (((32*tan(e/2 + (
f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5*d - 5*a^2*
c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c^3*d^3))/(c^
4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6
*d^3 + a*c^7*d^2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c
^6 - c^3*d^3 - c^4*d^2) + (32*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2
)*(a*d^3 + b*c^3 - 2*a*c^2*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4
- 4*c^7*d^3 - 2*c^8*d^2))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6
+ 3*c^4*d^4 - 3*c^6*d^2)))*((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*
a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*((c + d)^3*(c - d)^3)^(1
/2)*(a*d^3 + b*c^3 - 2*a*c^2*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*
((c + d)^3*(c - d)^3)^(1/2)*(a*d^3 + b*c^3 - 2*a*c^2*d)*2i)/(f*(c^8 - c^2*d
^6 + 3*c^4*d^4 - 3*c^6*d^2)) - (2*tan(e/2 + (f*x)/2)*(a*d^2 - b*c*d))/(f*(c
+ d)*(c*d - c^2)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(e + fx)}{(c + d \sec(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)

[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**2, x)

$$3.191 \quad \int \frac{a+b \sec(e+fx)}{(c+d \sec(e+fx))^3} dx$$

Optimal. Leaf size=204

$$\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2} - \frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{2c^2f(c^2-d^2)^2(c+d\sec(e+fx))} + \frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}}$$

[Out] a*x/c^3+(b*c^3*(2*c^2+d^2)-a*d*(6*c^4-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f-1/2*d*(-a*d+b*c)*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2-1/2*d*(-5*a*c^2*d+2*a*d^3+3*b*c^3)*tan(f*x+e)/c^2/(c^2-d^2)^2/f/(c+d*sec(f*x+e))

Rubi [A] time = 0.51, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3923, 4060, 3919, 3831, 2659, 208}

$$\frac{(bc^3(2c^2+d^2)-ad(-5c^2d^2+6c^4+2d^4))\operatorname{tanh}^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3f(c-d)^{5/2}(c+d)^{5/2}} - \frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{2c^2f(c^2-d^2)^2(c+d\sec(e+fx))} - 2c$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]

[Out] (a*x)/c^3 + ((b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTan h[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) - (d*(b*c - a*d)*Tan[e + f*x])/(2*c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*d^3)*Tan[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3923


```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a + b \sec(e + fx)}{(c + d \sec(e + fx))^3} dx = -\frac{d(bc - ad) \tan(e + fx)}{2c(c^2 - d^2) f(c + d \sec(e + fx))^2} - \frac{\int \frac{-2a(c^2 - d^2) - 2c(bc - ad) \sec(e + fx) + d(bc - ad) \sec^2(e + fx)}{(c + d \sec(e + fx))^2} dx}{2c(c^2 - d^2)}$$

$$= -\frac{d(bc - ad) \tan(e + fx)}{2c(c^2 - d^2) f(c + d \sec(e + fx))^2} - \frac{d(3bc^3 - 5ac^2d + 2ad^3) \tan(e + fx)}{2c^2(c^2 - d^2)^2 f(c + d \sec(e + fx))} + \frac{\int \frac{2a}{c + d \sec(e + fx)} dx}{2c^2(c^2 - d^2)}$$

$$= \frac{ax}{c^3} - \frac{d(bc - ad) \tan(e + fx)}{2c(c^2 - d^2) f(c + d \sec(e + fx))^2} - \frac{d(3bc^3 - 5ac^2d + 2ad^3) \tan(e + fx)}{2c^2(c^2 - d^2)^2 f(c + d \sec(e + fx))} + \frac{\int \frac{2a}{c + d \sec(e + fx)} dx}{2c^2(c^2 - d^2)}$$

$$= \frac{ax}{c^3} - \frac{d(bc - ad) \tan(e + fx)}{2c(c^2 - d^2) f(c + d \sec(e + fx))^2} - \frac{d(3bc^3 - 5ac^2d + 2ad^3) \tan(e + fx)}{2c^2(c^2 - d^2)^2 f(c + d \sec(e + fx))} + \frac{\int \frac{2a}{c + d \sec(e + fx)} dx}{2c^2(c^2 - d^2)}$$

$$= \frac{ax}{c^3} - \frac{d(bc - ad) \tan(e + fx)}{2c(c^2 - d^2) f(c + d \sec(e + fx))^2} - \frac{d(3bc^3 - 5ac^2d + 2ad^3) \tan(e + fx)}{2c^2(c^2 - d^2)^2 f(c + d \sec(e + fx))} + \frac{\int \frac{2a}{c + d \sec(e + fx)} dx}{2c^2(c^2 - d^2)}$$

$$= \frac{ax}{c^3} + \frac{(2bc^5 - 6ac^4d + bc^3d^2 + 5ac^2d^3 - 2ad^5) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{\int \frac{2a}{c + d \sec(e + fx)} dx}{2c^2(c^2 - d^2)}$$

Mathematica [A] time = 1.42, size = 267, normalized size = 1.31

$$\frac{\sec^2(e + fx)(a + b \sec(e + fx))(c \cos(e + fx) + d) \left(\frac{cd(-6ac^2d + 3ad^3 + 4bc^3 - bcd^2) \sin(e + fx)(c \cos(e + fx) + d)}{(c-d)^2(c+d)^2} - \frac{2(ad(-6c^4 + 5c^2d + d^2)) \sin(e + fx)}{(c-d)^2(c+d)^2} \right)}{2c^3 f(a \cos(e + fx) + b) \dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```


$$x + 1/2*e) + 6*a*c^3*d^2*\tan(1/2*f*x + 1/2*e) - 3*b*c^3*d^2*\tan(1/2*f*x + 1/2*e) + 5*a*c^2*d^3*\tan(1/2*f*x + 1/2*e) + b*c^2*d^3*\tan(1/2*f*x + 1/2*e) - 3*a*c*d^4*\tan(1/2*f*x + 1/2*e) - 2*a*d^5*\tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*\tan(1/2*f*x + 1/2*e)^2 - d*\tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f$$

maple [B] time = 0.77, size = 1063, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)

[Out]
$$-6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a-1/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^3/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a+2/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^4/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a+4/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d-c-d)^2*d/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*b+1/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*b+6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a-1/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^3/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a-2/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^4/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a-4/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*b+1/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2*d^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*b-6/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d)))^(1/2))*a*d+5/f/c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d)))^(1/2))*a*d^3-2/f/c^3/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d)))^(1/2))*a*d^5+2/f*c^2/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d)))^(1/2))*b+1/f/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d)))^(1/2))*b*d^2+2/f*a/c^3*\operatorname{arctan}(\tan(1/2*e+1/2*f*x))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see 'assume?' for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 11.35, size = 6909, normalized size = 33.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^3,x)

[Out]
$$(2*a*\operatorname{atan}(((a*((8*\tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 +$$

$$\begin{aligned}
& 8*a*b*c^7*d^3)/(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 \\
& - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a*c^{15} + 4*b*c^{15} - 4*a*c^6*d^9 + 2*a \\
& *c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^{10}*d^5 + 6*a*c^{11}*d^4 + 34*a \\
& *c^{12}*d^3 - 8*a*c^{13}*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^{12}*d^3 - 6*b*c \\
& ^{13}*d^2 - 12*a*c^{14}*d - 4*b*c^{14}*d)))/(c^{12}*d + c^{13} - c^6*d^7 - c^7*d^6 + 3 \\
& *c^8*d^5 + 3*c^9*d^4 - 3*c^{10}*d^3 - 3*c^{11}*d^2) - (a*\tan(e/2 + (f*x)/2)*(8* \\
& c^{15}*d - 8*c^6*d^{10} + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^{10}*d^6 + 4 \\
& 8*c^{11}*d^5 + 32*c^{12}*d^4 - 32*c^{13}*d^3 - 8*c^{14}*d^2)*8i)/(c^3*(c^{10}*d + c^1 \\
& 1 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))) *1i \\
&)/c^3)/c^3 + (a*((8*\tan(e/2 + (f*x)/2)*(4*a^2*c^{10} + 8*a^2*d^{10} + 4*b^2*c^ \\
& 10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c \\
& ^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 \\
& + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^ \\
& 5 + 8*a*b*c^7*d^3))/(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7* \\
& d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((8*(4*a*c^{15} + 4*b*c^{15} - 4*a*c^6*d^9 + \\
& 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^{10}*d^5 + 6*a*c^{11}*d^4 + 3 \\
& 4*a*c^{12}*d^3 - 8*a*c^{13}*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^{12}*d^3 - 6* \\
& b*c^{13}*d^2 - 12*a*c^{14}*d - 4*b*c^{14}*d)))/(c^{12}*d + c^{13} - c^6*d^7 - c^7*d^6 \\
& + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^{10}*d^3 - 3*c^{11}*d^2) + (a*\tan(e/2 + (f*x)/2)* \\
& (8*c^{15}*d - 8*c^6*d^{10} + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^{10}*d^6 \\
& + 48*c^{11}*d^5 + 32*c^{12}*d^4 - 32*c^{13}*d^3 - 8*c^{14}*d^2)*8i)/(c^3*(c^{10}*d + \\
& c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))) \\
& *1i)/c^3)/c^3)/((16*(4*a^3*d^9 + 4*a*b^2*c^9 - 4*a^2*b*c^9 - 2*a^3*c*d^8 + \\
& 12*a^3*c^8*d - 18*a^3*c^2*d^7 + 13*a^3*c^3*d^6 + 36*a^3*c^4*d^5 - 26*a^3*c^ \\
& ^5*d^4 - 34*a^3*c^6*d^3 + 24*a^3*c^7*d^2 + a*b^2*c^5*d^4 + 4*a*b^2*c^7*d^2 \\
& - 2*a^2*b*c^2*d^7 - 2*a^2*b*c^3*d^6 + 2*a^2*b*c^4*d^5 + 2*a^2*b*c^6*d^3 + 6 \\
& *a^2*b*c^7*d^2 - 20*a^2*b*c^8*d))/(c^{12}*d + c^{13} - c^6*d^7 - c^7*d^6 + 3*c^ \\
& 8*d^5 + 3*c^9*d^4 - 3*c^{10}*d^3 - 3*c^{11}*d^2) - (a*((8*\tan(e/2 + (f*x)/2)*(4 \\
& *a^2*c^{10} + 8*a^2*d^{10} + 4*b^2*c^{10} - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^ \\
& 2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + \\
& 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9 \\
& *d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^{10}*d + c^{11} - c^4*d \\
& ^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a \\
& *c^{15} + 4*b*c^{15} - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - \\
& 36*a*c^{10}*d^5 + 6*a*c^{11}*d^4 + 34*a*c^{12}*d^3 - 8*a*c^{13}*d^2 - 2*b*c^8*d^7 \\
& + 2*b*c^9*d^6 + 6*b*c^{12}*d^3 - 6*b*c^{13}*d^2 - 12*a*c^{14}*d - 4*b*c^{14}*d)))/(c \\
& ^{12}*d + c^{13} - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^{10}*d^3 - 3*c \\
& ^{11}*d^2) - (a*\tan(e/2 + (f*x)/2)*(8*c^{15}*d - 8*c^6*d^{10} + 8*c^7*d^9 + 32*c^ \\
& 8*d^8 - 32*c^9*d^7 - 48*c^{10}*d^6 + 48*c^{11}*d^5 + 32*c^{12}*d^4 - 32*c^{13}*d^3 \\
& - 8*c^{14}*d^2)*8i)/(c^3*(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c \\
& ^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))) *1i)/c^3) *1i)/c^3 + (a*((8*\tan(e/2 + (f*x) \\
& /2)*(4*a^2*c^{10} + 8*a^2*d^{10} + 4*b^2*c^{10} - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32* \\
& a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6 \\
& *d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a \\
& *b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^{10}*d + c^{11} - \\
& c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((\\
& 8*(4*a*c^{15} + 4*b*c^{15} - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9 \\
& *d^6 - 36*a*c^{10}*d^5 + 6*a*c^{11}*d^4 + 34*a*c^{12}*d^3 - 8*a*c^{13}*d^2 - 2*b*c^ \\
& 8*d^7 + 2*b*c^9*d^6 + 6*b*c^{12}*d^3 - 6*b*c^{13}*d^2 - 12*a*c^{14}*d - 4*b*c^{14} \\
& *d))/(c^{12}*d + c^{13} - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^{10}*d^3 \\
& - 3*c^{11}*d^2) + (a*\tan(e/2 + (f*x)/2)*(8*c^{15}*d - 8*c^6*d^{10} + 8*c^7*d^9 + \\
& 32*c^8*d^8 - 32*c^9*d^7 - 48*c^{10}*d^6 + 48*c^{11}*d^5 + 32*c^{12}*d^4 - 32*c^1 \\
& 3*d^3 - 8*c^{14}*d^2)*8i)/(c^3*(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 \\
& + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2))) *1i)/c^3) *1i)/c^3))/((c^3*f) - ((\tan \\
& (e/2 + (f*x)/2)^3*(2*a*d^4 - 6*a*c^2*d^2 + b*c^2*d^2 - a*c*d^3 + 4*b*c^3*d) \\
&))/((c^2*d - c^3)*(c + d)^2) + (\tan(e/2 + (f*x)/2)*(2*a*d^4 - 6*a*c^2*d^2 - \\
& b*c^2*d^2 + a*c*d^3 + 4*b*c^3*d))/((c + d)*(c^4 - 2*c^3*d + c^2*d^2)))/((f*(\\
& 2*c*d - \tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + \tan(e/2 + (f*x)/2)^4*(c^2 - \\
& 2*c*d + d^2) + c^2 + d^2)) + (\operatorname{atan}((((8*\tan(e/2 + (f*x)/2)*(4*a^2*c^{10} + 8
\end{aligned}$$


```

*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c
+ d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a
*c^4*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11
*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10 - 8*
a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^4*d^6
- 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2 + b^2*c
^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d^5 + 8*a
*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3
*c^8*d^3 - 3*c^9*d^2) - (((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9 + 2*a*c^7*d
^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4 + 34*a*c^12*
d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3 - 6*b*c^13*d^
2 - 12*a*c^14*d - 4*b*c^14*d)))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d
^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2 + (f*x)/2))*((c + d)^
5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d
)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^
6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^1
0 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*
d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5
*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 - 6*a*c^4*d)
)/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2))
)*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 + b*c^3*d^2 -
6*a*c^4*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*
c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*b*c^5 - 2*a*d^5 + 5*a*c^2*d^3 +
b*c^3*d^2 - 6*a*c^4*d)*1i)/(f*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 +
10*c^9*d^4 - 5*c^11*d^2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(e + fx)}{(c + d \sec(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)

[Out] Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**3, x)

$$3.192 \quad \int \frac{(a+b \sec(e+fx))^2}{(c+d \sec(e+fx))^2} dx$$

Optimal. Leaf size=133

$$\frac{a^2 x}{c^2} + \frac{(bc-ad)^2 \sin(e+fx)}{cf(c^2-d^2)(c \cos(e+fx)+d)} + \frac{2(bc-ad)(2ac^2-ad^2-bcd) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2 f(c-d)^{3/2}(c+d)^{3/2}}$$

[Out] a^2*x/c^2+2*(-a*d+b*c)*(2*a*c^2-a*d^2-b*c*d)*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f+(-a*d+b*c)^2*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3941, 2790, 2735, 2659, 208}

$$\frac{a^2 x}{c^2} + \frac{(bc-ad)^2 \sin(e+fx)}{cf(c^2-d^2)(c \cos(e+fx)+d)} + \frac{2(bc-ad)(2ac^2-ad^2-bcd) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2 f(c-d)^{3/2}(c+d)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2, x]

[Out] (a^2*x)/c^2 + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^2*(c - d)^(3/2)*(c + d)^(3/2)*f) + ((b*c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2790

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) + c^2*(m + 2)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 3941

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^2} dx &= \int \frac{(b + a \cos(e + fx))^2}{(d + c \cos(e + fx))^2} dx \\ &= \frac{(bc - ad)^2 \sin(e + fx)}{c(c^2 - d^2) f(d + c \cos(e + fx))} - \frac{\int \frac{-c(2abc - (a^2 + b^2)d) - a^2(c^2 - d^2) \cos(e + fx)}{d + c \cos(e + fx)} dx}{c(c^2 - d^2)} \\ &= \frac{a^2 x}{c^2} + \frac{(bc - ad)^2 \sin(e + fx)}{c(c^2 - d^2) f(d + c \cos(e + fx))} + \frac{(c^2(2abc - (a^2 + b^2)d) - a^2 d(c^2 - d^2)) \int}{c^2(c^2 - d^2)} \\ &= \frac{a^2 x}{c^2} + \frac{(bc - ad)^2 \sin(e + fx)}{c(c^2 - d^2) f(d + c \cos(e + fx))} + \frac{(2(c^2(2abc - (a^2 + b^2)d) - a^2 d(c^2 - d^2)) \int}{c^2(c^2 - d^2)} \\ &= \frac{a^2 x}{c^2} + \frac{2(bc - ad)(2ac^2 - bcd - ad^2) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc - ad)^2 \sin}{c(c^2 - d^2) f(d + c} \end{aligned}$$

Mathematica [A] time = 0.79, size = 136, normalized size = 1.02

$$\frac{2(a^2(2c^2d - d^3) - 2abc^3 + b^2c^2d) \tanh^{-1}\left(\frac{(d-c) \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + a^2(e+fx) + \frac{c(bc-ad)^2 \sin(e+fx)}{(c-d)(c+d)(c \cos(e+fx)+d)}}{c^2 f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]

[Out] (a^2*(e + f*x) + (2*(-2*a*b*c^3 + b^2*c^2*d + a^2*(2*c^2*d - d^3))*ArcTanh[(-c + d)*Tan[(e + f*x)/2]]/Sqrt[c^2 - d^2]))/(c^2 - d^2)^(3/2) + (c*(b*c - a*d)^2*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x]))/(c^2*f)

fricas [B] time = 0.53, size = 671, normalized size = 5.05

$$\left[\frac{2(a^2c^5 - 2a^2c^3d^2 + a^2cd^4)fx \cos(fx + e) + 2(a^2c^4d - 2a^2c^2d^3 + a^2d^5)fx + (2abc^3d + a^2d^4 - (2a^2 + b^2)c^2d^2}{c^2 f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fricas")

[Out] [1/2*(2*(a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + 2*(a^2*c^4*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^3

$$3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*\sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*\cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*\cos(f*x + e) + (a^2*c^4*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*\cos(f*x + e))*\sqrt{-c^2 + d^2}*\arctan(-\sqrt{-c^2 + d^2}*(d*\cos(f*x + e) + c)/((c^2 - d^2)*\sin(f*x + e))) + (b^2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*\sin(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*\cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f]$$

giac [A] time = 0.30, size = 246, normalized size = 1.85

$$\frac{(f_{x+e})a^2}{c^2} + \frac{2(2abc^3 - 2a^2c^2d - b^2c^2d + a^2d^3) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2c+2d) + \arctan \left(-\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2+d^2}} \right) \right)}{(c^4 - c^2d^2)\sqrt{-c^2+d^2}} - \frac{2(b^2c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 2abd)}{(c^3 - cd^2) \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")

[Out] ((f*x + e)*a^2/c^2 + 2*(2*a*b*c^3 - 2*a^2*c^2*d - b^2*c^2*d + a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) - 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e) + a^2*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*e))^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d))/f

maple [B] time = 0.78, size = 462, normalized size = 3.47

$$\frac{2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) a^2 d^2}{f c (c^2 - d^2) \left(\left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) c - \left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) d - c - d \right)} + \frac{4 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) a b d}{f (c^2 - d^2) \left(\left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) c - \left(\tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) \right) d - c - d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)

[Out] -2/f/c/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)*a^2*d^2+4/f/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)*a*b*d-2/f*c/(c^2-d^2)*tan(1/2*e+1/2*f*x)/(tan(1/2*e+1/2*f*x)^2*c-tan(1/2*e+1/2*f*x)^2*d-c-d)*b^2-4/f/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d+2/f/c^2/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*d^3+4/f*c/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a*b-2/f/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*b^2*d+2/f*a^2/c^2*arctan(tan(1/2*e+1/2*f*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 10.20, size = 4934, normalized size = 37.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^2,x)

[Out]
$$\begin{aligned} & (2*a^2*atan(((a^2*((32*\tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d)))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2)))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a^2*\tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2))) * i) / c^2)) / c^2 + (a^2*((32*\tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d)))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2)))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*\tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2))) * i) / c^2)) / c^2) / ((64*(a^6*d^5 - 2*a^5*b*c^5 - a^6*c*d^4 + 2*a^6*c^4*d + 4*a^4*b^2*c^5 - 3*a^6*c^2*d^3 + 2*a^6*c^3*d^2 - 4*a^3*b^3*c^4*d - a^4*b^2*c*d^4 + a^4*b^2*c^4*d + 2*a^5*b*c^2*d^3 + 2*a^5*b*c^3*d^2 + a^2*b^4*c^3*d^2 - a^4*b^2*c^2*d^3 + 3*a^4*b^2*c^3*d^2 - 6*a^5*b*c^4*d)) / (c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*((32*\tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d)) / (c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2)) / (c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (a^2*\tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i) / (c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2))) * i) / c^2) * i) / c^2) - (a^2*((32*\tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d)) / (c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2)) / (c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a^2*\tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i) / (c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2))) * i) / c^2) * i) / c^2)) / (c^2*f) + (atan((((a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2))*((32*\tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d)) / (c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2)) / (c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (32*\tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)) / ((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2$$

```

*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*
d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2
*a*c^2 + b*c*d)*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2) + ((a*d - b*c)*
((c + d)^3*(c - d)^3)^(1/2))*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 -
2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 +
3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*
b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*
d^2) - (((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d
^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*
a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2)))/(c^5*d + c^6 - c^3*d^3 - c^4*d^
2) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2
- 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3
- 2*c^8*d^2)))/((c^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^
4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 +
b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2*a*c^2 + b*c*d)
*1i)/(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))/((64*(a^6*d^5 - 2*a^5*b*c^5 -
a^6*c*d^4 + 2*a^6*c^4*d + 4*a^4*b^2*c^5 - 3*a^6*c^2*d^3 + 2*a^6*c^3*d^2 -
4*a^3*b^3*c^4*d - a^4*b^2*c*d^4 + a^4*b^2*c^4*d + 2*a^5*b*c^2*d^3 + 2*a^5*b
*c^3*d^2 + a^2*b^4*c^3*d^2 - a^4*b^2*c^2*d^3 + 3*a^4*b^2*c^3*d^2 - 6*a^5*b*
c^4*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + ((a*d - b*c)*((c + d)^3*(c - d)
^3)^(1/2))*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^
4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^
4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*
b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) + (((32*(2*a^
2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 +
b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*
c^6*d^3 + 2*a*b*c^7*d^2)))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) - (32*tan(e/2 +
(f*x)/2)*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)
*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)))/((c
^4*d + c^5 - c^2*d^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(
a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^
2*d^6 + 3*c^4*d^4 - 3*c^6*d^2))*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 +
3*c^4*d^4 - 3*c^6*d^2) - ((a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2))*((32*tan
(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d^5 - 2*a^4*c^5*d + 4*a^2*b^
2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c^4*d^2 + b^4*c^4*d^2 + 4*a^3
*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d - 8*a^3*
b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) - (((32*(2*a^2*c^8*d - a^2*c^9
+ b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2
*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^
7*d^2)))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (32*tan(e/2 + (f*x)/2)*(a*d - b
*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*(2*c^9*d - 2*c^4*
d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3 - 2*c^8*d^2)))/((c^4*d + c^5 - c^2*d
^3 - c^3*d^2)*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)))*(a*d - b*c)*((c + d)
^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d))/(c^8 - c^2*d^6 + 3*c^4*d^4
- 3*c^6*d^2))*(a*d - b*c)*((c + d)^3*(c - d)^3)^(1/2)*(a*d^2 - 2*a*c^2 + b*c*d)*2
i)/(f*(c^8 - c^2*d^6 + 3*c^4*d^4 - 3*c^6*d^2)) - (2*tan(e/2 + (f*x)/2)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))/(f*(c + d)*(c*d - c^2)*(c + d - tan(e/2 + (f*x
)/2)^2*(c - d)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)

[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**2, x)

$$3.193 \quad \int \frac{(a+b \sec(e+fx))^2}{(c+d \sec(e+fx))^3} dx$$

Optimal. Leaf size=237

$$\frac{(a^2(6c^4d - 5c^2d^3 + 2d^5) - 2abc^3(2c^2 + d^2) + 3b^2c^4d) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3 f(c-d)^{5/2}(c+d)^{5/2}} + \frac{a^2 x}{c^3} - \frac{(bc-ad)(3ad(2c^2 - d^2))}{2c^2 f(c^2 - d^2)}$$

[Out] a^2*x/c^3 - (3*b^2*c^4*d - 2*a*b*c^3*(2*c^2+d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5)) * arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f - 1/2*d*(-a*d+b*c)^2*sin(f*x+e)/c^2/(c^2-d^2)/f/(d+c*cos(f*x+e))^2 - 1/2*(-a*d+b*c)*(3*a*d*(2*c^2-d^2) - b*c*(2*c^2+d^2))*sin(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))

Rubi [A] time = 0.80, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3941, 2988, 3021, 2735, 2659, 208}

$$\frac{(a^2(-5c^2d^3 + 6c^4d + 2d^5) - 2abc^3(2c^2 + d^2) + 3b^2c^4d) \tanh^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3 f(c-d)^{5/2}(c+d)^{5/2}} + \frac{a^2 x}{c^3} - \frac{(bc-ad)(3ad(2c^2 - d^2))}{2c^2 f(c^2 - d^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3, x]

[Out] (a^2*x)/c^3 - ((3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) - (d*(b*c - a*d)^2*Sin[e + f*x])/(2*c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - ((b*c - a*d)*(3*a*d*(2*c^2 - d^2) - b*c*(2*c^2 + d^2))*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2988

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -

$2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 - b^2, 0] \&\& NeQ[c^2 - d^2, 0] \&\& LtQ[n, -1]$

Rule 3021

$Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] \&\& LtQ[m, -1] \&\& NeQ[a^2 - b^2, 0]$

Rule 3941

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] \&\& NeQ[b*c - a*d, 0] \&\& IntegerQ[m] \&\& IntegerQ[n] \&\& LeQ[-2, m + n, 0]$

Rubi steps

$$\int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^3} dx = \int \frac{\cos(e + fx)(b + a \cos(e + fx))^2}{(d + c \cos(e + fx))^3} dx$$

$$= -\frac{d(bc - ad)^2 \sin(e + fx)}{2c^2 (c^2 - d^2) f (d + c \cos(e + fx))^2} - \frac{\int \frac{-2c(bc - ad)^2 + (b^2c^2d - 2abc(2c^2 - d^2) + a^2(2c^2d - d^3)) \cos(e + fx)}{(d + c \cos(e + fx))^2} dx}{2c^2 (c^2 - d^2)}$$

$$= -\frac{d(bc - ad)^2 \sin(e + fx)}{2c^2 (c^2 - d^2) f (d + c \cos(e + fx))^2} - \frac{(bc - ad) (3ad (2c^2 - d^2) - bc (2c^2 + d^2))}{2c^2 (c^2 - d^2)^2 f (d + c \cos(e + fx))}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc - ad)^2 \sin(e + fx)}{2c^2 (c^2 - d^2) f (d + c \cos(e + fx))^2} - \frac{(bc - ad) (3ad (2c^2 - d^2) - bc (2c^2 + d^2))}{2c^2 (c^2 - d^2)^2 f (d + c \cos(e + fx))}$$

$$= \frac{a^2x}{c^3} - \frac{d(bc - ad)^2 \sin(e + fx)}{2c^2 (c^2 - d^2) f (d + c \cos(e + fx))^2} - \frac{(bc - ad) (3ad (2c^2 - d^2) - bc (2c^2 + d^2))}{2c^2 (c^2 - d^2)^2 f (d + c \cos(e + fx))}$$

$$= \frac{a^2x}{c^3} + \frac{(4abc^5 - 6a^2c^4d - 3b^2c^4d + 2abc^3d^2 + 5a^2c^2d^3 - 2a^2d^5) \tanh^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{e+fx}{2}\right)}{\sqrt{c+d}} \right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f}$$

Mathematica [B] time = 2.04, size = 493, normalized size = 2.08

$$\sec(e + fx)(a + b \sec(e + fx))^2(c \cos(e + fx) + d) \left(\frac{4(a^2(6c^4d - 5c^2d^3 + 2d^5) - 2abc^3(2c^2 + d^2) + 3b^2c^4d)(c \cos(e + fx) + d)^2 \tanh^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{e+fx}{2}\right)}{\sqrt{c+d}} \right)}{(c^2 - d^2)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]

[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^2*((4*(3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 - d^2)^(5/2) + (2*a^2*c^6*e - 6*a^2*c^2*d^4*e + 4*a^2*d^6*e + 2*a^2*c^6*f*x - 6*a^2*c^2*d^4*f*x + 4*a^2*d^6*f*x + 8*a^2*c*d*(c^2 - d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^2*c^2*(c^2 - d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^2*c^5*d*Sin[e + f*x] - 12*a*b*c^4*d^2*Sin[e + f*x] + 10*a^2*c^3*d^3*Sin[e + f*x] + 4*b^2*c^3*d^3*Sin[e + f*x] - 4*a^2*c*d^5*Sin[e + f*x] + 2*b^2*c^6*Sin[2*(e + f*x)] - 8*a*b*c^5*d*Sin[2*(e + f*x)] + 6*a^2*c^4*d^2*Sin[2*(e + f*x)] + b^2*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^2*c^2*d^4*Sin[2*(e + f*x)]))/(c^2 - d^2)^2)/(4*c^3*f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^3)

fricas [B] time = 0.58, size = 1409, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(4*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*x + e)^2 + 8*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*cos(f*x + e) + 4*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*x - (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*cos(f*x + e) + 2*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*x + (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^2*c^7*d - 6*a*b*c^6*d^2 + 6*a*b*c^4*d^4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^5 + (2*b^2*c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*d^6 + (6*a^2 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)]

giac [B] time = 0.46, size = 686, normalized size = 2.89

$$\frac{(4abc^5 - 6a^2c^4d - 3b^2c^4d + 2abc^3d^2 + 5a^2c^2d^3 - 2a^2d^5) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2c+2d) + \arctan \left(-\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2+d^2}} \right) \right)}{(c^7 - 2c^5d^2 + c^3d^4) \sqrt{-c^2+d^2}} + \frac{(fx+e)a^2}{c^3} - \frac{2b^2c^5 \tan}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="giac")

[Out]
$$\frac{((4ab^2c^5 - 6a^2c^4d - 3b^2c^4d + 2ab^2c^3d^2 + 5a^2c^2d^3 - 2a^2d^5)(\pi \operatorname{floor}(\frac{1}{2}(fx + e))/\pi + \frac{1}{2}) \operatorname{sgn}(-2c + 2d) + \arctan(\frac{-c \tan(\frac{1}{2}fx + \frac{1}{2}e) - d \tan(\frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-c^2 + d^2}})) / ((c^7 - 2c^5d^2 + c^3d^4) \sqrt{-c^2 + d^2}) + (fx + e)a^2/c^3 - (2b^2c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 8ab^2c^4d \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - b^2c^4d \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 6a^2c^3d^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 6ab^2c^3d^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + b^2c^3d^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 5a^2c^2d^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 2ab^2c^2d^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 2b^2c^2d^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 3a^2c^2d^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 2a^2d^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 2b^2c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 8ab^2c^4d \tan(\frac{1}{2}fx + \frac{1}{2}e) - b^2c^4d \tan(\frac{1}{2}fx + \frac{1}{2}e) - 6a^2c^3d^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 6ab^2c^3d^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) - b^2c^3d^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 5a^2c^2d^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 2ab^2c^2d^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 2b^2c^2d^3 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 3a^2c^2d^4 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 2a^2d^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)) / ((c^6 - 2c^4d^2 + c^2d^4)(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - d \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c - d)^2)}{f}$$

maple [B] time = 0.74, size = 1593, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)

[Out]
$$\begin{aligned} & -6/f / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 a^2 d^2 - 1/f/c / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 a^2 d^3 + 2/f/c^2 / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 a^2 d^4 + 8/f*c / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 a^2 b d^2 + 2/f / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 a^2 b d^3 - 2/f*c^2 / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 b^2 - 1/f*c / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c - d) / (c^2 + 2cd + d^2) \tan(\frac{1}{2}e + \frac{1}{2}fx)^3 b^2 d^2 + 6/f / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) a^2 d^2 - 1/f/c / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) a^2 d^3 - 2/f/c^2 / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) a^2 d^4 - 8/f*c / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) a^2 b d^2 + 2/f / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) a^2 b d^3 + 2/f*c^2 / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) b^2 - 1/f*c / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) b^2 d^2 + 2/f / (\tan(\frac{1}{2}e + \frac{1}{2}fx)^2 c - \tan(\frac{1}{2}e + \frac{1}{2}fx)^2 d - c - d)^2 / (c + d) / (c - d)^2 \tan(\frac{1}{2}e + \frac{1}{2}fx) b^2 d^3 - 6/f*c / (c^4 - 2c^2 d^2 + d^4) / ((c + d)(c - d))^{1/2} \operatorname{arctanh}(\tan(\frac{1}{2}e + \frac{1}{2}fx) * (c - d) / ((c + d)(c - d))^{1/2}) a^2 d^5 + 5/f/c / (c^4 - 2c^2 d^2 + d^4) / ((c + d)(c - d))^{1/2} \operatorname{arctanh}(\tan(\frac{1}{2}e + \frac{1}{2}fx) * (c - d) / ((c + d)(c - d))^{1/2}) a^2 d^3 - 2/f/c^3 / (c^4 - 2c^2 d^2 + d^4) / ((c + d)(c - d))^{1/2} \operatorname{arctanh}(\tan(\frac{1}{2}e + \frac{1}{2}fx) * (c - d) / ((c + d)(c - d))^{1/2}) a^2 d^5 + 4/f*c^2 / (c^4 - 2c^2 d^2 + d^4) / ((c + d)(c - d))^{1/2} \operatorname{arctanh}(\tan(\frac{1}{2}e + \frac{1}{2}fx) * (c - d) / ((c + d)(c - d))^{1/2}) a^2 b d^2 - 3/f*c / (c^4 - 2c^2 d^2 + d^4) / ((c + d)(c - d))^{1/2} \operatorname{arctanh}(\tan(\frac{1}{2}e + \frac{1}{2}fx) * (c - d) / ((c + d)(c - d))^{1/2}) b^2 d^2 + 2/f a^2 / c^3 \operatorname{arctan}(\tan(\frac{1}{2}e + \frac{1}{2}fx)) \end{aligned}$$


```

- 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 - 12*a*
b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3 + 12*a^2
*b^2*c^4*d^6 - 26*a^2*b^2*c^6*d^4 + 52*a^2*b^2*c^8*d^2 - 24*a*b^3*c^9*d - 4
8*a^3*b*c^9*d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4
- 3*c^8*d^3 - 3*c^9*d^2) - (((c + d)^5*(c - d)^5)^(1/2))*((8*(4*a^2*c^15 - 1
2*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^
7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5 + 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*
a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^
12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b
*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c
^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (4*tan(e/2 + (f
*x)/2))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d -
5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*
d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 3
2*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c
^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7
*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*
a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 -
10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2
*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^
2))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)
)))*((c + d)^5*(c - d)^5)^(1/2)*(2*a^2*d^5 + 6*a^2*c^4*d + 3*b^2*c^4*d - 5*
a^2*c^2*d^3 - 4*a*b*c^5 - 2*a*b*c^3*d^2)*1i)/(f*(c^13 - c^3*d^10 + 5*c^5*d^
8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)

[Out] Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**3, x)

$$3.194 \quad \int \frac{(a+b \sec(e+fx))^2}{(c+d \sec(e+fx))^4} dx$$

Optimal. Leaf size=377

$$\frac{(-a^2 d^2 (34c^4 - 28c^2 d^2 + 9d^4) + 2abcd (18c^4 - 5c^2 d^2 + 2d^4) - (b^2 (6c^6 + 10c^4 d^2 - c^2 d^4))) \sin(e + fx) (a^2 (8c^6 + 10c^4 d^2 - c^2 d^4))}{6c^3 f (c^2 - d^2)^3 (c \cos(e + fx) + d)}$$

[Out] $a^2 x/c^4 - (b^2 c^4 d (4c^2 + d^2) - a b (4c^7 + 6c^5 d^2) + a^2 (8c^6 d - 8c^4 d^3 + 7c^2 d^5 - 2d^7)) \operatorname{arctanh}((c-d)^{1/2} \tan(1/2 e + 1/2 f x) / (c+d)^{1/2}) / c^4 / (c-d)^{7/2} / (c+d)^{7/2} / f + 1/3 d^2 (b+a \cos(fx+e))^2 \sin(fx+e) / c / (c^2 - d^2) / f / (d+c \cos(fx+e))^3 - 1/6 d (-a d + b c) (-8 a^2 c^2 d + 3 a^2 d^3 + 6 b c^3 - b c d^2) \sin(fx+e) / c^3 / (c^2 - d^2)^2 / f / (d+c \cos(fx+e))^2 - 1/6 (2 a b c d (18 c^4 - 5 c^2 d^2 + 2 d^4) - a^2 d^2 (34 c^4 - 28 c^2 d^2 + 9 d^4) - b^2 (6 c^6 + 10 c^4 d^2 - c^2 d^4)) \sin(fx+e) / c^3 / (c^2 - d^2)^3 / f / (d+c \cos(fx+e))$

Rubi [A] time = 2.00, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3941, 3048, 3031, 3021, 2735, 2659, 208}

$$\frac{(-a^2 d^2 (-28c^2 d^2 + 34c^4 + 9d^4) + 2abcd (-5c^2 d^2 + 18c^4 + 2d^4) + b^2 (- (10c^4 d^2 - c^2 d^4 + 6c^6))) \sin(e + fx) (a^2 (8c^6 + 10c^4 d^2 - c^2 d^4))}{6c^3 f (c^2 - d^2)^3 (c \cos(e + fx) + d)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b \operatorname{Sec}[e + f x])^2 / (c + d \operatorname{Sec}[e + f x])^4, x]$

[Out] $(a^2 x) / c^4 - ((b^2 c^4 d (4c^2 + d^2) - a b (4c^7 + 6c^5 d^2) + a^2 (8c^6 d - 8c^4 d^3 + 7c^2 d^5 - 2d^7)) \operatorname{ArcTanh}[\operatorname{Sqrt}[c - d] \operatorname{Tan}[(e + f x) / 2] / \operatorname{Sqrt}[c + d]]) / (c^4 (c - d)^{7/2} (c + d)^{7/2} f) + (d^2 (b + a \cos[e + f x])^2 \sin[e + f x]) / (3c (c^2 - d^2) f (d + c \cos[e + f x])^3) - (d (b c - a d) (6 b c^3 - 8 a^2 c^2 d - b c d^2 + 3 a^2 d^3) \sin[e + f x]) / (6 c^3 (c^2 - d^2)^2 f (d + c \cos[e + f x])^2) - ((2 a b c d (18 c^4 - 5 c^2 d^2 + 2 d^4) - a^2 d^2 (34 c^4 - 28 c^2 d^2 + 9 d^4) - b^2 (6 c^6 + 10 c^4 d^2 - c^2 d^4)) \sin[e + f x]) / (6 c^3 (c^2 - d^2)^3 f (d + c \cos[e + f x]))$

Rule 208

$\text{Int}[(a + (b + (c + d x)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\operatorname{Rt}[-(a/b), 2] \operatorname{ArcTanh}[x / \operatorname{Rt}[-(a/b), 2]]) / a, x] / ; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 2659

$\text{Int}[(a + (b + (c + d x)^2)^{-1}), x_Symbol] \rightarrow \text{With}\{e = \text{FreeFactors}[\operatorname{Tan}[(c + d x) / 2], x], \text{Dist}[(2 e) / d, \text{Subst}[\text{Int}[1 / (a + b + (a - b) e^2 x^2), x], x, \operatorname{Tan}[(c + d x) / 2] / e], x]\} / ; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2735

$\text{Int}[(a + (b + (c + d x)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(b x) / d, x] - \text{Dist}[(b c - a d) / d, \text{Int}[1 / (c + d \operatorname{Sin}[e + f x]), x], x] / ; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b c - a d, 0]$

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
+ f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]^(n_))*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :=
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_.))^(n_), x_Symbol] := Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^4} dx &= \int \frac{\cos^2(e + fx)(b + a \cos(e + fx))^2}{(d + c \cos(e + fx))^4} dx \\
&= \frac{d^2(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))^3} + \frac{\int \frac{(b + a \cos(e + fx))(-d(3bc - 2ad) + (3bc^2 - 3acd - bd^2) \cos(e + fx))}{(d + c \cos(e + fx))^3}}{3c(c^2 - d^2)} \\
&= \frac{d^2(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))^3} - \frac{d(bc - ad)(6bc^3 - 8ac^2d - bcd^2 + 3ad^3) \sin(e + fx)}{6c^3(c^2 - d^2)^2 f(d + c \cos(e + fx))^2} \\
&= \frac{d^2(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))^3} - \frac{d(bc - ad)(6bc^3 - 8ac^2d - bcd^2 + 3ad^3) \sin(e + fx)}{6c^3(c^2 - d^2)^2 f(d + c \cos(e + fx))^2} \\
&= \frac{a^2x}{c^4} + \frac{d^2(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))^3} - \frac{d(bc - ad)(6bc^3 - 8ac^2d - bcd^2 + 3ad^3)}{6c^3(c^2 - d^2)^2 f(d + c \cos(e + fx))} \\
&= \frac{a^2x}{c^4} + \frac{d^2(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))^3} - \frac{d(bc - ad)(6bc^3 - 8ac^2d - bcd^2 + 3ad^3)}{6c^3(c^2 - d^2)^2 f(d + c \cos(e + fx))} \\
&= \frac{a^2x}{c^4} + \frac{(4abc^7 - 8a^2c^6d - 4b^2c^6d + 6abc^5d^2 + 8a^2c^4d^3 - b^2c^4d^3 - 7a^2c^2d^5 + 2a^2d^7) \sin(e + fx)}{c^4(c - d)^{7/2}(c + d)^{7/2}f}
\end{aligned}$$

Mathematica [A] time = 3.45, size = 438, normalized size = 1.16

$$\sec^2(e + fx)(a + b \sec(e + fx))^2(c \cos(e + fx) + d) \left(\frac{c(a^2d^2(36c^4 - 32c^2d^2 + 11d^4) - 2abcd(18c^4 - 5c^2d^2 + 2d^4) + b^2(6c^6 + 10c^4d^2 - c^2d^4)) \sin(e + fx)}{(c^2 - d^2)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]

[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^2*(6*a^2*(e + f*x)*(d + c*Cos[e + f*x])^3 + (6*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2 + 3*d^2) + a^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2])*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^(7/2) + (2*c*d^2*(b*c - a*d)^2*Sin[e + f*x])/(c^2 - d^2) - (c*d*(a^2*d^2*(12*c^2 - 7*d^2) + b^2*(6*c^4 - c^2*d^2) + a*b*(-18*c^3*d + 8*c*d^3))*(d + c*Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d^4) + a^2*d^2*(36*c^4 - 32*c^2*d^2 + 11*d^4) + b^2*(6*c^6 + 10*c^4*d^2 - c^2*d^4))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3)/(6*c^4*f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^4)

fricas [B] time = 0.73, size = 2362, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fricas")

```
[Out] [1/12*(12*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*c^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^2*c^9*d^2 - 4*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e) + 12*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*d^11)*f*x - 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10 - 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5 + 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2*d^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)*f), 1/6*(6*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 18*(a^2*c^9*d^2 - 4*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e) + 6*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*d^11)*f*x + 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10 - 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14*a*b*c^6*d^5 + 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 11*b^2)*c^7*d^4 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d + 46*a*b*c^8*d^3 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a^2 + b^2)*c^9*d^2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*b*c^5*d^6 - 5*a^2*c^2*d^9 + (20*a^2 + 7*b^2)*c^8*d^3 - 5*(7*a^2 + 2*b^2)*c^6*d^5 + (20*a^2 + b^2)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)*f)]
```

giac [B] time = 0.46, size = 1253, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a*b*c^7 - 8*a^2*c^6*d - 4*b^2*c^6*d + 6*a*b*c^5*d^2 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 - 7*a^2*c^2*d^5 + 2*a^2*d^7)*(pi*floor(1/2*(f*x + e)/pi +
```

$$\begin{aligned} & \frac{1}{2} * \operatorname{sgn}(-2*c + 2*d) + \arctan\left(\frac{-(c*\tan(1/2*f*x + 1/2*e) - d*\tan(1/2*f*x + 1/2*e))}{\sqrt{-c^2 + d^2}}\right) / \left((c^{10} - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6) * \sqrt{-c^2 + d^2} \right) + 3*(f*x + e)*a^2/c^4 - (6*b^2*c^8*\tan(1/2*f*x + 1/2*e)^5 - 36*a*b*c^7*d*\tan(1/2*f*x + 1/2*e)^5 - 6*b^2*c^7*d*\tan(1/2*f*x + 1/2*e)^5 + 36*a^2*c^6*d^2*\tan(1/2*f*x + 1/2*e)^5 + 54*a*b*c^6*d^2*\tan(1/2*f*x + 1/2*e)^5 + 12*b^2*c^6*d^2*\tan(1/2*f*x + 1/2*e)^5 - 60*a^2*c^5*d^3*\tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^5*d^3*\tan(1/2*f*x + 1/2*e)^5 - 27*b^2*c^5*d^3*\tan(1/2*f*x + 1/2*e)^5 - 6*a^2*c^4*d^4*\tan(1/2*f*x + 1/2*e)^5 + 6*a*b*c^4*d^4*\tan(1/2*f*x + 1/2*e)^5 + 12*b^2*c^4*d^4*\tan(1/2*f*x + 1/2*e)^5 + 45*a^2*c^3*d^5*\tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^3*d^5*\tan(1/2*f*x + 1/2*e)^5 + 3*b^2*c^3*d^5*\tan(1/2*f*x + 1/2*e)^5 - 6*a^2*c^2*d^6*\tan(1/2*f*x + 1/2*e)^5 - 15*a^2*c*d^7*\tan(1/2*f*x + 1/2*e)^5 + 6*a^2*d^8*\tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c^8*\tan(1/2*f*x + 1/2*e)^3 + 72*a*b*c^7*d*\tan(1/2*f*x + 1/2*e)^3 - 72*a^2*c^6*d^2*\tan(1/2*f*x + 1/2*e)^3 - 16*b^2*c^6*d^2*\tan(1/2*f*x + 1/2*e)^3 - 64*a*b*c^5*d^3*\tan(1/2*f*x + 1/2*e)^3 + 116*a^2*c^4*d^4*\tan(1/2*f*x + 1/2*e)^3 + 28*b^2*c^4*d^4*\tan(1/2*f*x + 1/2*e)^3 - 8*a*b*c^3*d^5*\tan(1/2*f*x + 1/2*e)^3 - 56*a^2*c^2*d^6*\tan(1/2*f*x + 1/2*e)^3 + 12*a^2*d^8*\tan(1/2*f*x + 1/2*e)^3 + 6*b^2*c^8*\tan(1/2*f*x + 1/2*e) - 36*a*b*c^7*d*\tan(1/2*f*x + 1/2*e) + 6*b^2*c^7*d*\tan(1/2*f*x + 1/2*e) + 36*a^2*c^6*d^2*\tan(1/2*f*x + 1/2*e) - 54*a*b*c^6*d^2*\tan(1/2*f*x + 1/2*e) + 12*b^2*c^6*d^2*\tan(1/2*f*x + 1/2*e) + 60*a^2*c^5*d^3*\tan(1/2*f*x + 1/2*e) - 12*a*b*c^5*d^3*\tan(1/2*f*x + 1/2*e) + 27*b^2*c^5*d^3*\tan(1/2*f*x + 1/2*e) - 6*a^2*c^4*d^4*\tan(1/2*f*x + 1/2*e) - 6*a*b*c^4*d^4*\tan(1/2*f*x + 1/2*e) + 12*b^2*c^4*d^4*\tan(1/2*f*x + 1/2*e) - 45*a^2*c^3*d^5*\tan(1/2*f*x + 1/2*e) - 12*a*b*c^3*d^5*\tan(1/2*f*x + 1/2*e) - 3*b^2*c^3*d^5*\tan(1/2*f*x + 1/2*e) - 6*a^2*c^2*d^6*\tan(1/2*f*x + 1/2*e) + 15*a^2*c*d^7*\tan(1/2*f*x + 1/2*e) + 6*a^2*d^8*\tan(1/2*f*x + 1/2*e)) / \left((c^9 - 3*c^7*d^2 + 3*c^5*d^4 - c^3*d^6) * (c*\tan(1/2*f*x + 1/2*e)^2 - d*\tan(1/2*f*x + 1/2*e)^2 - c - d)^3 \right) / f \end{aligned}$$

maple [B] time = 0.80, size = 3293, normalized size = 8.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((a+b*\sec(f*x+e))^2/(c+d*\sec(f*x+e))^4,x)$

[Out]
$$\begin{aligned} & \frac{2/f*a^2/c^4*\arctan(\tan(1/2*e+1/2*f*x))+6/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^2*d^4-1/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^2*d^5-8/3/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a*b*d^3+4/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a*b*d^3-2/f/c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a*b*d^3-2/f/c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*b^2*d^6/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*b^2*d^2+6/f*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^{1/2}*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{1/2})*a*b*d^2-12/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^2*d^2+6/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^2*d^4+1/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^2*d^5-2/f/c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^2*d^6-2/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*b^2*d^6/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*b^2*d^2+24/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2) \end{aligned}$$

$$\frac{1}{(c^2+2cd+d^2)\tan(1/2e+1/2fx)^3a^2d^2-44/3f/c/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c^2-2cd+d^2)/(c^2+2cd+d^2)\tan(1/2e+1/2fx)^3a^2d^4+4/f/c^3/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c^2-2cd+d^2)/(c^2+2cd+d^2)\tan(1/2e+1/2fx)^3a^2d^6+28/3f*c/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c^2-2cd+d^2)/(c^2+2cd+d^2)\tan(1/2e+1/2fx)^3b^2d^2-12/f*c/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c+d)/(c^3-3c^2d+3cd^2-d^3)\tan(1/2e+1/2fx)*a^2d^2+4/f/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c-d)/(c^3+3c^2d+3cd^2+d^3)\tan(1/2e+1/2fx)^5a*b*d^3+6/f*c/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c-d)/(c^3+3c^2d+3cd^2+d^3)*\tan(1/2e+1/2fx)^5a*b*d^2+12/f*c^2/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c-d)/(c^3+3c^2d+3cd^2+d^3)*\tan(1/2e+1/2fx)^5a*b*d+12/f*c^2/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c+d)/(c^3-3c^2d+3cd^2-d^3)\tan(1/2e+1/2fx)*a*b*d+1/f/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c+d)/(c^3-3c^2d+3cd^2-d^3)\tan(1/2e+1/2fx)*b^2d^3-4/f/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c-d)/(c^3+3c^2d+3cd^2+d^3)\tan(1/2e+1/2fx)^5a^2d^3-1/f/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c-d)/(c^3+3c^2d+3cd^2+d^3)\tan(1/2e+1/2fx)^5b^2d^3+4/f/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c+d)/(c^3-3c^2d+3cd^2-d^3)\tan(1/2e+1/2fx)*a^2d^3-8/f*c^2/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*a^2d-4/f*c^2/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*b^2d+2/f/c^4/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*a^2d^7+4/f*c^3/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c^2-2cd+d^2)/(c^2+2cd+d^2)\tan(1/2e+1/2fx)^3b^2-2/f*c^3/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c+d)/(c^3-3c^2d+3cd^2-d^3)\tan(1/2e+1/2fx)*b^2-7/f/c^2/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*a^2d^5+4/f*c^3/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*a*b-2/f*c^3/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c-d)/(c^3+3c^2d+3cd^2+d^3)\tan(1/2e+1/2fx)^5b^2-24/f*c^2/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c^2-2cd+d^2)/(c^2+2cd+d^2)\tan(1/2e+1/2fx)^3a*b*d-6/f*c/(\tan(1/2e+1/2fx)^2c-\tan(1/2e+1/2fx)^2d-c-d)^3/(c+d)/(c^3-3c^2d+3cd^2-d^3)\tan(1/2e+1/2fx)*a*b*d^2+8/f/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*a^2d^3-1/f/(c^6-3c^4d^2+3c^2d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2e+1/2fx)*(c-d)/((c+d)*(c-d))^(1/2))*b^2d^3$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see 'assume?' for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 15.07, size = 12818, normalized size = 34.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^4,x)

[Out] $(2a^2 \operatorname{atan}(((a^2((8 \tan(e/2 + (f*x)/2))(4a^4c^{14} + 8a^4d^{14} - 8a^4c^*d^{13} - 8a^4c^{13}d + 16a^2b^2c^{14} - 48a^4c^2d^{12} + 48a^4c^3d^{11} + 117a^4c^4d^{10} - 120a^4c^5d^9 - 164a^4c^6d^8 + 160a^4c^7d^7 + 156a^4c^8d^6 - 120a^4c^9d^5 - 92a^4c^{10}d^4 + 48a^4c^{11}d^3 + 44a^4c^{12}d^2 + b^4c^8d^6 + 8b^4c^{10}d^4 + 16b^4c^{12}d^2 - 12ab^3c^9d^5 - 56ab^3c^{11}d^3 + 24a^3b^3c^5d^9 - 68a^3b^3c^7d^7 + 40a^3b^3c^9d^5 - 32a^3b^3c^{11}d^3 - 4a^2b^2c^4d^{10} - 2a^2b^2c^6d^8 + 40a^2b^2c^8d^6 - 12a^2b^2c^{10}d^4 + 112a^2b^2c^{12}d^2 - 32ab^3c^{13}d - 64a^3b^3c^{13}d)))/(c^{16}d + c^{17} - c^6d^{11} - c^7d^{10} + 5c^8d^9 + 5c^9d^8 - 10c^{10}d^7 - 10c^{11}d^6 + 10c^{12}d^5 + 10c^{13}d^4 - 5c^{14}d^3 - 5c^{15}d^2) + (a^2((8(4a^2c^{21} - 16a^2c^{20}d - 8b^2c^{20}d - 4a^2c^8d^{13} + 2a^2c^9d^{12} + 26a^2c^{10}d^{11} - 14a^2c^{11}d^{10} - 70a^2c^{12}d^9 + 30a^2c^{13}d^8 + 110a^2c^{14}d^7 - 30a^2c^{15}d^6 - 110a^2c^{16}d^5 + 20a^2c^{17}d^4 + 64a^2c^{18}d^3 - 12a^2c^{19}d^2 - 2b^2c^{11}d^{10} + 2b^2c^{12}d^9 - 2b^2c^{13}d^8 + 2b^2c^{14}d^7 + 18b^2c^{15}d^6 - 18b^2c^{16}d^5 - 22b^2c^{17}d^4 + 22b^2c^{18}d^3 + 8b^2c^{19}d^2 + 8ab^3c^{21} - 8ab^3c^{20}d + 12ab^3c^{12}d^9 - 12ab^3c^{13}d^8 - 28ab^3c^{14}d^7 + 28ab^3c^{15}d^6 + 12ab^3c^{16}d^5 - 12ab^3c^{17}d^4 + 12ab^3c^{18}d^3 - 12ab^3c^{19}d^2)))/(c^{19}d + c^{20} - c^9d^{11} - c^{10}d^{10} + 5c^{11}d^9 + 5c^{12}d^8 - 10c^{13}d^7 - 10c^{14}d^6 + 10c^{15}d^5 + 10c^{16}d^4 - 5c^{17}d^3 - 5c^{18}d^2) - (a^2 \tan(e/2 + (f*x)/2))(8c^{21}d - 8c^8d^{14} + 8c^9d^{13} + 48c^{10}d^{12} - 48c^{11}d^{11} - 120c^{12}d^{10} + 120c^{13}d^9 + 160c^{14}d^8 - 160c^{15}d^7 - 120c^{16}d^6 + 120c^{17}d^5 + 48c^{18}d^4 - 48c^{19}d^3 - 8c^{20}d^2)*8i)/(c^4(c^{16}d + c^{17} - c^6d^{11} - c^7d^{10} + 5c^8d^9 + 5c^9d^8 - 10c^{10}d^7 - 10c^{11}d^6 + 10c^{12}d^5 + 10c^{13}d^4 - 5c^{14}d^3 - 5c^{15}d^2)))*1i)/c^4) + (a^2((8 \tan(e/2 + (f*x)/2))(4a^4c^{14} + 8a^4d^{14} - 8a^4c^*d^{13} - 8a^4c^{13}d + 16a^2b^2c^{14} - 48a^4c^2d^{12} + 48a^4c^3d^{11} + 117a^4c^4d^{10} - 120a^4c^5d^9 - 164a^4c^6d^8 + 160a^4c^7d^7 + 156a^4c^8d^6 - 120a^4c^9d^5 - 92a^4c^{10}d^4 + 48a^4c^{11}d^3 + 44a^4c^{12}d^2 + b^4c^8d^6 + 8b^4c^{10}d^4 + 16b^4c^{12}d^2 - 12ab^3c^9d^5 - 56ab^3c^{11}d^3 + 24a^3b^3c^5d^9 - 68a^3b^3c^7d^7 + 40a^3b^3c^9d^5 - 32a^3b^3c^{11}d^3 - 4a^2b^2c^4d^{10} - 2a^2b^2c^6d^8 + 40a^2b^2c^8d^6 - 12a^2b^2c^{10}d^4 + 112a^2b^2c^{12}d^2 - 32ab^3c^{13}d - 64a^3b^3c^{13}d)))/(c^{16}d + c^{17} - c^6d^{11} - c^7d^{10} + 5c^8d^9 + 5c^9d^8 - 10c^{10}d^7 - 10c^{11}d^6 + 10c^{12}d^5 + 10c^{13}d^4 - 5c^{14}d^3 - 5c^{15}d^2) - (a^2((8(4a^2c^{21} - 16a^2c^{20}d - 8b^2c^{20}d - 4a^2c^8d^{13} + 2a^2c^9d^{12} + 26a^2c^{10}d^{11} - 14a^2c^{11}d^{10} - 70a^2c^{12}d^9 + 30a^2c^{13}d^8 + 110a^2c^{14}d^7 - 30a^2c^{15}d^6 - 110a^2c^{16}d^5 + 20a^2c^{17}d^4 + 64a^2c^{18}d^3 - 12a^2c^{19}d^2 - 2b^2c^{11}d^{10} + 2b^2c^{12}d^9 - 2b^2c^{13}d^8 + 2b^2c^{14}d^7 + 18b^2c^{15}d^6 - 18b^2c^{16}d^5 - 22b^2c^{17}d^4 + 22b^2c^{18}d^3 + 8b^2c^{19}d^2 + 8ab^3c^{21} - 8ab^3c^{20}d + 12ab^3c^{12}d^9 - 12ab^3c^{13}d^8 - 28ab^3c^{14}d^7 + 28ab^3c^{15}d^6 + 12ab^3c^{16}d^5 - 12ab^3c^{17}d^4 + 12ab^3c^{18}d^3 - 12ab^3c^{19}d^2)))/(c^{19}d + c^{20} - c^9d^{11} - c^{10}d^{10} + 5c^{11}d^9 + 5c^{12}d^8 - 10c^{13}d^7 - 10c^{14}d^6 + 10c^{15}d^5 + 10c^{16}d^4 - 5c^{17}d^3 - 5c^{18}d^2) + (a^2 \tan(e/2 + (f*x)/2))(8c^{21}d - 8c^8d^{14} + 8c^9d^{13} + 48c^{10}d^{12} - 48c^{11}d^{11} - 120c^{12}d^{10} + 120c^{13}d^9 + 160c^{14}d^8 - 160c^{15}d^7 - 120c^{16}d^6 + 120c^{17}d^5 + 48c^{18}d^4 - 48c^{19}d^3 - 8c^{20}d^2)*8i)/(c^4(c^{16}d + c^{17} - c^6d^{11} - c^7d^{10} + 5c^8d^9 + 5c^9d^8 - 10c^{10}d^7 - 10c^{11}d^6 + 10c^{12}d^5 + 10c^{13}d^4 - 5c^{14}d^3 - 5c^{15}d^2)))*1i)/c^4)/((16(4a^6d^{13} - 8a^5b^3c^{13} - 2a^6c^3d^{10} + 70a^6c^4d^9 - 34a^6c^5d^8 - 110a^6c^6d^7 + 66a^6c^7d^6 + 110a^6c^8d^5 - 64a^6c^9d^4 - 64a^6c^{10}d^3 + 48a^6c^{11}d^2 - 32a^3b^3c^{12}d + 8a^4b^2c^{12}d + 12a^5b^3c^4d^9 + 12a^5b^3c^5d^8 - 40a^5b^3c^6d^7 - 28a^5b^3c^7d^6 + 28a^5b^3c^8d^5 + 12a^5b^3c^9d^4 - 44a^5b^3c^{10}d^3 + 12a^5b^3c^{11}d^2 + a^2b^4c^7d^6 + 8a^2b^4c^9d^4 + 16a^2b^4c^{11}d^2 - 12a^3b^3c^8d^5 - 56a^3b^3c^{10}d^3 - 2a^4b^2c^3d^{10} - 2a^4b^2c^4d^9 - 2a^4b^2c^6d^8$

$$\begin{aligned}
& ^7 + 22*a^4*b^2*c^7*d^6 + 18*a^4*b^2*c^8*d^5 + 10*a^4*b^2*c^9*d^4 - 22*a^4*b^2*c^10*d^3 + 104*a^4*b^2*c^11*d^2 - 56*a^5*b*c^12*d) / (c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^2*((8*\tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d)) / (c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) + (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2)) / (c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (a^2*\tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i) / (c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i) / c^4 + (a^2*((8*\tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d)) / (c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) - (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2)) / (c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (a^2*\tan(e/2 + (f*x)/2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)*8i) / (c^4*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*1i) / c^4)) / (c^4*f) - ((\tan(e/2 + (f*x)/2))^5*(2*a^2*d^6 + 2*b^2*c^6 - a^2*c*d^5 + 2*b^2*c^5*d - 6*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 12*a^2*c^4*d^2 + b^2*c^3*d^3 + 6*b^2*c^4*d^2 - 12*a*b*c^5*d - 4*a*b*c^3*d^3 - 6*a*b*c^4*d^2)) / ((c^3*d - c^4)*(c + d)^3) + (4*\tan(e/2 + (f*x)/2))^3*(3*a^2*d^6 + 3*b^2*c^6 - 11*a^2*c^2*d^4 + 18*a^2*c^4*d^2 + 7*b^2*c^4*d^2 -
\end{aligned}$$

$$\begin{aligned}
& (18*a*b*c^5*d - 2*a*b*c^3*d^3)/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) + (\tan(e/2 + (f*x)/2)*(2*a^2*d^6 + 2*b^2*c^6 + a^2*c*d^5 - 2*b^2*c^5*d - 6*a^2*c^2*d^4 - 4*a^2*c^3*d^3 + 12*a^2*c^4*d^2 - b^2*c^3*d^3 + 6*b^2*c^4*d^2 - 12*a*b*c^5*d - 4*a*b*c^3*d^3 + 6*a*b*c^4*d^2))/((c + d)*(3*c^5*d - c^6 + c^3*d^3 - 3*c^4*d^2)))/(f*(\tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - \tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - \tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (\operatorname{atan}((((8*\tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d)))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) + (((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2)))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) - (4*\tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d^3 - 8*c^20*d^2)))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/((2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*i)/((2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)) + (((8*\tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*b^3*c^13*d - 64*a^3*b*c^13*d)))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2) - (((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 +
\end{aligned}$$

$$\begin{aligned}
& (12*a*b*c^{16*d^5} - 12*a*b*c^{17*d^4} + 12*a*b*c^{18*d^3} - 12*a*b*c^{19*d^2}) / (c^{19*d} + c^{20} - c^{9*d^{11}} - c^{10*d^{10}} + 5*c^{11*d^9} + 5*c^{12*d^8} - 10*c^{13*d^7} \\
& - 10*c^{14*d^6} + 10*c^{15*d^5} + 10*c^{16*d^4} - 5*c^{17*d^3} - 5*c^{18*d^2}) + (4*\tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^{(1/2)}*(2*a^2*d^7 - 8*a^2*c^6*d - 4 \\
& *b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^{21*d} - 8*c^8*d^{14} + 8*c^9*d^{13} + 48*c^{10*d^{12}} - 48*c^{11*d^{11}} \\
& - 120*c^{12*d^{10}} + 120*c^{13*d^9} + 160*c^{14*d^8} - 160*c^{15*d^7} - 120*c^{16*d^6} + 120*c^{17*d^5} + 48*c^{18*d^4} - 48*c^{19*d^3} - 8*c^{20*d^2})) / ((c^{18} - c^4*d^{14} \\
& + 7*c^6*d^{12} - 21*c^8*d^{10} + 35*c^{10*d^8} - 35*c^{12*d^6} + 21*c^{14*d^4} - 7*c^{16*d^2})*(c^{16*d} + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - \\
& 10*c^{10*d^7} - 10*c^{11*d^6} + 10*c^{12*d^5} + 10*c^{13*d^4} - 5*c^{14*d^3} - 5*c^{15*d^2})) * ((c + d)^7*(c - d)^7)^{(1/2)}*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d \\
& - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)) / (2*(c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} + 35*c^{10*d^8} - 35*c^{12*d^6} \\
& + 21*c^{14*d^4} - 7*c^{16*d^2})) * ((c + d)^7*(c - d)^7)^{(1/2)}*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 \\
& + 6*a*b*c^5*d^2)*i) / (2*(c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} + 35*c^{10*d^8} - 35*c^{12*d^6} + 21*c^{14*d^4} - 7*c^{16*d^2})) / ((16*(4*a^6*d^{13} - \\
& 8*a^5*b*c^{13} - 2*a^6*c*d^{12} + 16*a^6*c^{12*d} + 16*a^4*b^2*c^{13} - 26*a^6*c^2*d^{11} + 11*a^6*c^3*d^{10} + 70*a^6*c^4*d^9 - 34*a^6*c^5*d^8 - 110*a^6*c^6*d^7 \\
& + 66*a^6*c^7*d^6 + 110*a^6*c^8*d^5 - 64*a^6*c^9*d^4 - 64*a^6*c^{10*d^3} + 48*a^6*c^{11*d^2} - 32*a^3*b^3*c^{12*d} + 8*a^4*b^2*c^{12*d} + 12*a^5*b*c^4*d^9 + 1 \\
& 2*a^5*b*c^5*d^8 - 40*a^5*b*c^6*d^7 - 28*a^5*b*c^7*d^6 + 28*a^5*b*c^8*d^5 + 12*a^5*b*c^9*d^4 - 44*a^5*b*c^{10*d^3} + 12*a^5*b*c^{11*d^2} + a^2*b^4*c^7*d^6 \\
& + 8*a^2*b^4*c^9*d^4 + 16*a^2*b^4*c^{11*d^2} - 12*a^3*b^3*c^8*d^5 - 56*a^3*b^3*c^{10*d^3} - 2*a^4*b^2*c^3*d^{10} - 2*a^4*b^2*c^4*d^9 - 2*a^4*b^2*c^6*d^7 + 22 \\
& *a^4*b^2*c^7*d^6 + 18*a^4*b^2*c^8*d^5 + 10*a^4*b^2*c^9*d^4 - 22*a^4*b^2*c^{10*d^3} + 104*a^4*b^2*c^{11*d^2} - 56*a^5*b*c^{12*d})) / (c^{19*d} + c^{20} - c^{9*d^{11}} \\
& - c^{10*d^{10}} + 5*c^{11*d^9} + 5*c^{12*d^8} - 10*c^{13*d^7} - 10*c^{14*d^6} + 10*c^{15*d^5} + 10*c^{16*d^4} - 5*c^{17*d^3} - 5*c^{18*d^2}) - (((8*\tan(e/2 + (f*x)/2)*(4*a^4*c^{14} \\
& + 8*a^4*d^{14} - 8*a^4*c*d^{13} - 8*a^4*c^{13*d} + 16*a^2*b^2*c^{14} - 48*a^4*c^2*d^{12} + 48*a^4*c^3*d^{11} + 117*a^4*c^4*d^{10} - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 \\
& + 160*a^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^{10*d^4} + 48*a^4*c^{11*d^3} + 44*a^4*c^{12*d^2} + b^4*c^8*d^6 + 8*b^4*c^{10*d^4} \\
& + 16*b^4*c^{12*d^2} - 12*a*b^3*c^9*d^5 - 56*a*b^3*c^{11*d^3} + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^{11*d^3} - 4*a^2*b^2*c^4*d^{10} \\
& - 2*a^2*b^2*c^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^{10*d^4} + 112*a^2*b^2*c^{12*d^2} - 32*a*b^3*c^{13*d} - 64*a^3*b*c^{13*d})) / (c^{16*d} + c^{17} - c^6*d^{11} \\
& - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10*d^7} - 10*c^{11*d^6} + 10*c^{12*d^5} + 10*c^{13*d^4} - 5*c^{14*d^3} - 5*c^{15*d^2}) + (((8*(4*a^2*c^{21} - 16*a^2 \\
& *c^{20*d} - 8*b^2*c^{20*d} - 4*a^2*c^8*d^{13} + 2*a^2*c^9*d^{12} + 26*a^2*c^{10*d^{11}} - 14*a^2*c^{11*d^{10}} - 70*a^2*c^{12*d^9} + 30*a^2*c^{13*d^8} + 110*a^2*c^{14*d^7} \\
& - 30*a^2*c^{15*d^6} - 110*a^2*c^{16*d^5} + 20*a^2*c^{17*d^4} + 64*a^2*c^{18*d^3} - 12*a^2*c^{19*d^2} - 2*b^2*c^{11*d^{10}} + 2*b^2*c^{12*d^9} - 2*b^2*c^{13*d^8} + 2*b^2 \\
& *c^{14*d^7} + 18*b^2*c^{15*d^6} - 18*b^2*c^{16*d^5} - 22*b^2*c^{17*d^4} + 22*b^2*c^{18*d^3} + 8*b^2*c^{19*d^2} + 8*a*b*c^{21} - 8*a*b*c^{20*d} + 12*a*b*c^{12*d^9} - 12*a \\
& *b*c^{13*d^8} - 28*a*b*c^{14*d^7} + 28*a*b*c^{15*d^6} + 12*a*b*c^{16*d^5} - 12*a*b*c^{17*d^4} + 12*a*b*c^{18*d^3} - 12*a*b*c^{19*d^2})) / (c^{19*d} + c^{20} - c^{9*d^{11}} - \\
& c^{10*d^{10}} + 5*c^{11*d^9} + 5*c^{12*d^8} - 10*c^{13*d^7} - 10*c^{14*d^6} + 10*c^{15*d^5} + 10*c^{16*d^4} - 5*c^{17*d^3} - 5*c^{18*d^2}) - (4*\tan(e/2 + (f*x)/2)*((c + \\
& d)^7*(c - d)^7)^{(1/2)}*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^{21*d} - 8*c^8*d^{14} \\
& + 8*c^9*d^{13} + 48*c^{10*d^{12}} - 48*c^{11*d^{11}} - 120*c^{12*d^{10}} + 120*c^{13*d^9} + 160*c^{14*d^8} - 160*c^{15*d^7} - 120*c^{16*d^6} + 120*c^{17*d^5} + 48*c^{18*d^4} \\
& - 48*c^{19*d^3} - 8*c^{20*d^2})) / ((c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} + 35*c^{10*d^8} - 35*c^{12*d^6} + 21*c^{14*d^4} - 7*c^{16*d^2})*(c^{16*d} + c^{17} \\
& - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10*d^7} - 10*c^{11*d^6} + 10*c^{12*d^5} + 10*c^{13*d^4} - 5*c^{14*d^3} - 5*c^{15*d^2})) * ((c + d)^7*(c - d \\
&)^7)^{(1/2)}*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c
\end{aligned}$$

```

^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*
c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d
^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d -
7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(
2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 +
21*c^14*d^4 - 7*c^16*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d
^14 - 8*a^4*c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*
a^4*c^3*d^11 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a
^4*c^7*d^7 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c
^11*d^3 + 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2
- 12*a*b^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^
7 + 40*a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c
^6*d^8 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 -
32*a*b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 +
5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d
^4 - 5*c^14*d^3 - 5*c^15*d^2) - (((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*c^
20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*d^1
0 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*d^6
- 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d^2 -
2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 + 18*b^2
*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*b^2*c^1
9*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d^8 - 28*
a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4 + 12*a*b
*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^1
1*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4
- 5*c^17*d^3 - 5*c^18*d^2) + (4*tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1
/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3
- b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2)*(8*c^21*d - 8*c^8*d^14 + 8*c^9*d
^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160*c^14
*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c^19*d
^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^
8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)*(c^16*d + c^17 - c^6*d^11 - c^7
*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 1
0*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*
d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d
^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*
d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(
c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^6*d - 7*a^2*c^2*d^5 + 8*
a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d^2))/(2*(c^18 - c^4*d^14
+ 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c
^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^2*d^7 - 8*a^2*c^6*d - 4*b^2*c^
6*d - 7*a^2*c^2*d^5 + 8*a^2*c^4*d^3 - b^2*c^4*d^3 + 4*a*b*c^7 + 6*a*b*c^5*d
^2)*1i)/(f*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c
^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))*2/(c+d*sec(f*x+e))*4,x)

[Out] Integral((a + b*sec(e + f*x))*2/(c + d*sec(e + f*x))*4, x)

$$3.195 \quad \int \frac{(a+b \sec(e+fx))^3}{(c+d \sec(e+fx))^3} dx$$

Optimal. Leaf size=254

$$\frac{a^3 x}{c^3} - \frac{(bc - ad) \left(- \left(a^2 (6c^4 - 5c^2 d^2 + 2d^4) \right) + 2abcd (4c^2 - d^2) - b^2 c^2 (c^2 + 2d^2) \right) \tanh^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}} \right)}{c^3 f (c-d)^{5/2} (c+d)^{5/2}} + \dots$$

[Out] a^3*x/c^3 - (-a*d+b*c)*(2*a*b*c*d*(4*c^2-d^2)-b^2*c^2*(c^2+2*d^2)-a^2*(6*c^4-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)^2*(b+a*cos(f*x+e))*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))^2+1/2*(-a*d+b*c)^2*(5*a*c^2-2*a*d^2-3*b*c*d)*sin(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))

Rubi [A] time = 1.13, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3941, 2792, 3021, 2735, 2659, 208}

$$\frac{(bc - ad) \left(a^2 \left(- \left(-5c^2 d^2 + 6c^4 + 2d^4 \right) \right) + 2abcd (4c^2 - d^2) - b^2 c^2 (c^2 + 2d^2) \right) \tanh^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}} \right)}{c^3 f (c-d)^{5/2} (c+d)^{5/2}} + \frac{a^3 x}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3, x]

[Out] (a^3*x)/c^3 - ((b*c - a*d)*(2*a*b*c*d*(4*c^2 - d^2) - b^2*c^2*(c^2 + 2*d^2) - a^2*(6*c^4 - 5*c^2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]]/(c^3*(c - d)^(5/2)*(c + d)^(5/2)*f) + ((b*c - a*d)^2*(b + a*Cos[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) + ((b*c - a*d)^2*(5*a*c^2 - 3*b*c*d - 2*a*d^2)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2792

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e

```
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_.))^(n_), x_Symbol] :> Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^3} dx = \int \frac{(b + a \cos(e + fx))^3}{(d + c \cos(e + fx))^3} dx$$

$$= \frac{(bc - ad)^2(b + a \cos(e + fx)) \sin(e + fx)}{2c(c^2 - d^2) f(d + c \cos(e + fx))^2} + \frac{\int \frac{5ab^2c^2 - 4a^2bcd - 2b^3cd + a^3d^2 + (b^3c^2 - 2a^3cd - 4ab^2cd)}{(d + c \cos(e + fx))^3} dx}{2c(c^2 - d^2)}$$

$$= \frac{(bc - ad)^2(b + a \cos(e + fx)) \sin(e + fx)}{2c(c^2 - d^2) f(d + c \cos(e + fx))^2} + \frac{(bc - ad)^2(5ac^2 - 3bcd - 2ad^2) \sin(e + fx)}{2c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))}$$

$$= \frac{a^3x}{c^3} + \frac{(bc - ad)^2(b + a \cos(e + fx)) \sin(e + fx)}{2c(c^2 - d^2) f(d + c \cos(e + fx))^2} + \frac{(bc - ad)^2(5ac^2 - 3bcd - 2ad^2) \sin(e + fx)}{2c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))}$$

$$= \frac{a^3x}{c^3} + \frac{(bc - ad)^2(b + a \cos(e + fx)) \sin(e + fx)}{2c(c^2 - d^2) f(d + c \cos(e + fx))^2} + \frac{(bc - ad)^2(5ac^2 - 3bcd - 2ad^2) \sin(e + fx)}{2c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))}$$

$$= \frac{a^3x}{c^3} + \frac{(bc - ad)(6a^2c^4 + b^2c^4 - 8abc^3d - 5a^2c^2d^2 + 2b^2c^2d^2 + 2abcd^3 + 2a^2d^4) \tan\left(\frac{e + fx}{2}\right)}{c^3(c - d)^{5/2}(c + d)^{5/2}f}$$

Mathematica [B] time = 2.30, size = 517, normalized size = 2.04

$$\frac{2a^3c^6e + 2a^3c^6fx + 6a^3c^4d^2 \sin(2(e+fx)) + 10a^3c^3d^3 \sin(e+fx) + 2a^3(c^3 - cd^2)^2(e+fx) \cos(2(e+fx)) - 3a^3c^2d^4 \sin(2(e+fx)) - 6a^3c^2d^4e - 6a^3c^2d^4fx + 8a^3cd(c^2 - d^2) \sin(e+fx)}{c^3(c - d)^{5/2}(c + d)^{5/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]

[Out]
$$\frac{((-4*(-9*a*b^2*c^4*d + 3*a^2*b*c^3*(2*c^2 + d^2) + b^3*c^3*(c^2 + 2*d^2) + a^3*(-6*c^4*d + 5*c^2*d^3 - 2*d^5))*ArcTanh[(-c + d)*Tan[(e + f*x)/2]]/Sqrt[c^2 - d^2])/(c^2 - d^2)^{(5/2)} + (2*a^3*c^6*e - 6*a^3*c^2*d^4*e + 4*a^3*d^6*e + 2*a^3*c^6*f*x - 6*a^3*c^2*d^4*f*x + 4*a^3*d^6*f*x + 8*a^3*c*d*(c^2 - d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^3*(c^3 - c*d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^3*c^6*Sin[e + f*x] + 6*a*b^2*c^5*d*Sin[e + f*x] - 18*a^2*b*c^4*d^2*Sin[e + f*x] - 8*b^3*c^4*d^2*Sin[e + f*x] + 10*a^3*c^3*d^3*Sin[e + f*x] + 12*a*b^2*c^3*d^3*Sin[e + f*x] - 4*a^3*c*d^5*Sin[e + f*x] + 6*a*b^2*c^6*Sin[2*(e + f*x)] - 12*a^2*b*c^5*d*Sin[2*(e + f*x)] - 3*b^3*c^5*d*Sin[2*(e + f*x)] + 6*a^3*c^4*d^2*Sin[2*(e + f*x)] + 3*a*b^2*c^4*d^2*Sin[2*(e + f*x)] + 3*a^2*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^3*c^2*d^4*Sin[2*(e + f*x)])/(c^2 - d^2)^2*(d + c*Cos[e + f*x])^2)/(4*c^3*f)}$$

fricas [B] time = 0.68, size = 1629, normalized size = 6.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(4*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*f*x*cos(f*x + e)^2 + 8*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7)*f*x*cos \\ & (f*x + e) + 4*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3*d^8)*f*x - \\ & (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 + 3*a*b^2) \\ & *c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2*d^5 + (6* \\ & a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)*c^5*d^2)*c \\ & os(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)*c^6*d - 3* \\ & (2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + e))*sqrt(c \\ & ^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c \\ & ^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e) \\ & ^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7 \\ & - (9*a^2*b + 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3) \\ & *c^4*d^4 - (7*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^3 \\ & *c^2*d^6 - (4*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^3) \\ & *c^5*d^3 - (3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - \\ & 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + \\ & 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f), \\ & 1/2*(2*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)* \\ & f*x*cos(f*x + e)^2 + 4*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7) \\ & *f*x*cos(f*x + e) + 2*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3 \\ & *d^8)*f*x + (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 \\ & + 3*a*b^2)*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2 \\ & *d^5 + (6*a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3) \\ & *c^5*d^2)*cos(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3) \\ & *c^6*d - 3*(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + \\ & e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - \\ & d^2)*sin(f*x + e))) + (b^3*c^8 + 3*a*b^2*c^7*d + 2*a^3*c*d^7 - (9*a^2*b + \\ & 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a^2*b + 4*b^3)*c^4*d^4 - (7 \\ & *a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b*c^3*d^5 + a^3*c^2*d^6 - (4 \\ & *a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 + (5*a^2*b + b^3)*c^5*d^3 - (\\ & 3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2 + 3* \\ & c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4 \\ & *d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)*f)] \end{aligned}$$

giac [B] time = 1.61, size = 852, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")

[Out] $((f*x + e)*a^3/c^3 + (6*a^2*b*c^5 + b^3*c^5 - 6*a^3*c^4*d - 9*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 + 2*b^3*c^3*d^2 + 5*a^3*c^2*d^3 - 2*a^3*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + \arctan(-(c*\tan(1/2*f*x + 1/2*e) - d*\tan(1/2*f*x + 1/2*e))/\sqrt{-c^2 + d^2}))/((c^7 - 2*c^5*d^2 + c^3*d^4)*\sqrt{-c^2 + d^2}) - (6*a*b^2*c^5*\tan(1/2*f*x + 1/2*e)^3 - b^3*c^5*\tan(1/2*f*x + 1/2*e)^3 - 12*a^2*b*c^4*d*\tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^4*d*\tan(1/2*f*x + 1/2*e)^3 - 3*b^3*c^4*d*\tan(1/2*f*x + 1/2*e)^3 + 6*a^3*c^3*d^2*\tan(1/2*f*x + 1/2*e)^3 + 9*a^2*b*c^3*d^2*\tan(1/2*f*x + 1/2*e)^3 + 3*a*b^2*c^3*d^2*\tan(1/2*f*x + 1/2*e)^3 + 4*b^3*c^3*d^2*\tan(1/2*f*x + 1/2*e)^3 - 5*a^3*c^2*d^3*\tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c^2*d^3*\tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^2*d^3*\tan(1/2*f*x + 1/2*e)^3 - 3*a^3*c^2*d^4*\tan(1/2*f*x + 1/2*e)^3 + 2*a^3*d^5*\tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^5*\tan(1/2*f*x + 1/2*e) - b^3*c^5*\tan(1/2*f*x + 1/2*e) + 12*a^2*b*c^4*d*\tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^4*d*\tan(1/2*f*x + 1/2*e) + 3*b^3*c^4*d*\tan(1/2*f*x + 1/2*e) - 6*a^3*c^3*d^2*\tan(1/2*f*x + 1/2*e) + 9*a^2*b*c^3*d^2*\tan(1/2*f*x + 1/2*e) - 3*a*b^2*c^3*d^2*\tan(1/2*f*x + 1/2*e) + 4*b^3*c^3*d^2*\tan(1/2*f*x + 1/2*e) - 5*a^3*c^2*d^3*\tan(1/2*f*x + 1/2*e) - 3*a^2*b*c^2*d^3*\tan(1/2*f*x + 1/2*e) - 6*a*b^2*c^2*d^3*\tan(1/2*f*x + 1/2*e) + 3*a^3*c^2*d^4*\tan(1/2*f*x + 1/2*e) + 2*a^3*d^5*\tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*\tan(1/2*f*x + 1/2*e)^2 - d*\tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f$

maple [B] time = 0.72, size = 2031, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)

[Out] $2/f*a^3/c^3*\arctan(\tan(1/2*e+1/2*f*x))+12/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^2*b*d-12/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a^2*b*d-3/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a*b^2*d+6/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a*b^2-4/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*b^3*d-6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a*b^2*d^2+6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a*b^2*d^2+3/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a^2*b*d^2-3/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a*b^2*d^2-9/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a*b^2*d-1/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^3*d^3+2/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^3*d^4-6/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a*b^2+4/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*b^3*d-1/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a^3*d^3-2/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a^3*d^4-6/f*c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^3*d+1/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*b^3+1/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*$

$$e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*b^3+3/f/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^{(1/2)*\arctanh(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{(1/2)})} * a^2*b*d^2-6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c-d)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^3*d^2+6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^2/(c+d)/(c-d)^2*\tan(1/2*e+1/2*f*x)*a^3*d^2+5/f/c/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^{(1/2)*\arctanh(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{(1/2)})} * a^3*d^3-2/f/c^3/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^{(1/2)*\arctanh(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{(1/2)})} * a^3*d^5+6/f*c^2/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^{(1/2)*\arctanh(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{(1/2)})} * a^2*b+1/f*c^2/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^{(1/2)*\arctanh(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{(1/2)})} * b^3+2/f/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^{(1/2)*\arctanh(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^{(1/2)})} * b^3*d^2$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 14.50, size = 10759, normalized size = 42.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^3,x)

[Out] (atan((((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d)))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + ((a*d - b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d)))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (4*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2))/((c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2)*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/(2*(c^13 - c^3*d^10 + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^11*d^2))*(a*d - b*c)*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a

$$\begin{aligned}
& *d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^{10}*d^6 + 48*c^{11}*d^5 + 32*c^{12}*d^4 - \\
& 32*c^{13}*d^3 - 8*c^{14}*d^2))/((c^{13} - c^3*d^{10} + 5*c^5*d^8 - 10*c^7*d^6 + 10* \\
& c^9*d^4 - 5*c^{11}*d^2)*(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^ \\
& 7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2 \\
& *a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^ \\
& 3*d))/((2*(c^{13} - c^3*d^{10} + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^{11}*d^ \\
& 2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^ \\
& 4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/((2*(c^{13} - \\
& c^3*d^{10} + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^{11}*d^2)) + (((8*tan(e \\
& /2 + (f*x)/2)*(4*a^6*c^{10} + 8*a^6*d^{10} + b^6*c^{10} - 8*a^6*c*d^9 - 8*a^6*c^9 \\
& *d + 12*a^2*b^4*c^{10} + 36*a^4*b^2*c^{10} - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + \\
& 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^ \\
& 8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^ \\
& 9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^ \\
& 6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^ \\
& 3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8* \\
& d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 \\
& + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - ((a*d - b*c)*((8*(4*a^3*c^ \\
& c^{15} + 2*b^3*c^{15} + 12*a^2*b*c^{15} - 12*a^3*c^{14}*d - 2*b^3*c^{14}*d - 4*a^3*c^ \\
& 6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36*a^3*c^{10}*d^5 + \\
& 6*a^3*c^{11}*d^4 + 34*a^3*c^{12}*d^3 - 8*a^3*c^{13}*d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^ \\
& ^9*d^6 + 6*b^3*c^{10}*d^5 - 6*b^3*c^{11}*d^4 + 18*a*b^2*c^9*d^6 - 18*a*b^2*c^{10} \\
& *d^5 - 36*a*b^2*c^{11}*d^4 + 36*a*b^2*c^{12}*d^3 + 18*a*b^2*c^{13}*d^2 - 6*a^2*b* \\
& c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^{12}*d^3 - 18*a^2*b*c^{13}*d^2 - 18*a*b^ \\
& 2*c^{14}*d - 12*a^2*b*c^{14}*d)))/(c^{12}*d + c^{13} - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 \\
& + 3*c^9*d^4 - 3*c^{10}*d^3 - 3*c^{11}*d^2) + (4*tan(e/2 + (f*x)/2)*(a*d - b*c) \\
& *((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^ \\
& ^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*(8*c^{15}*d - 8*c^6*d^{10} + 8* \\
& c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^{10}*d^6 + 48*c^{11}*d^5 + 32*c^{12}*d^4 \\
& - 32*c^{13}*d^3 - 8*c^{14}*d^2))/((c^{13} - c^3*d^{10} + 5*c^5*d^8 - 10*c^7*d^6 + \\
& 10*c^9*d^4 - 5*c^{11}*d^2)*(c^{10}*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3 \\
& *c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 \\
& + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b \\
& *c^3*d))/((2*(c^{13} - c^3*d^{10} + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^{11} \\
& *d^2)))*(a*d - b*c)*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^ \\
& 2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d))/((2*(c^{13} \\
& - c^3*d^{10} + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^{11}*d^2)))*((a*d - \\
& b*c)*((c + d)^5*(c - d)^5)^{(1/2)}*(6*a^2*c^4 + 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^ \\
& ^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*b*c^3*d)*1i)/(f*(c^{13} - c^3*d^{10} \\
& + 5*c^5*d^8 - 10*c^7*d^6 + 10*c^9*d^4 - 5*c^{11}*d^2)) - (2*a^3*atan(((a^3*(\\
& a^3*((8*(4*a^3*c^{15} + 2*b^3*c^{15} + 12*a^2*b*c^{15} - 12*a^3*c^{14}*d - 2*b^3*c^ \\
& ^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*c^9*d^6 - 36 \\
& *a^3*c^{10}*d^5 + 6*a^3*c^{11}*d^4 + 34*a^3*c^{12}*d^3 - 8*a^3*c^{13}*d^2 - 4*b^3*c^ \\
& ^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^{10}*d^5 - 6*b^3*c^{11}*d^4 + 18*a*b^2*c^9*d^6 \\
& - 18*a*b^2*c^{10}*d^5 - 36*a*b^2*c^{11}*d^4 + 36*a*b^2*c^{12}*d^3 + 18*a*b^2*c^1 \\
& 3*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^{12}*d^3 - 18*a^2*b*c^ \\
& 13*d^2 - 18*a*b^2*c^{14}*d - 12*a^2*b*c^{14}*d)))/(c^{12}*d + c^{13} - c^6*d^7 - c^7 \\
& *d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^{10}*d^3 - 3*c^{11}*d^2) - (a^3*tan(e/2 + (f \\
& *x)/2)*(8*c^{15}*d - 8*c^6*d^{10} + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^ \\
& 10*d^6 + 48*c^{11}*d^5 + 32*c^{12}*d^4 - 32*c^{13}*d^3 - 8*c^{14}*d^2)*8i)/(c^3*(c^ \\
& 10*d + c^{11} - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9 \\
& *d^2))*1i)/c^3 + (8*tan(e/2 + (f*x)/2)*(4*a^6*c^{10} + 8*a^6*d^{10} + b^6*c^{10} \\
& - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^{10} + 36*a^4*b^2*c^{10} - 32*a^6*c^ \\
& ^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 \\
& + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^ \\
& 5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5 \\
& *b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + \\
& 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^ \\
& ^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^{10}*d +
\end{aligned}$$

$$\begin{aligned}
& c^{11} - c^4 d^7 - c^5 d^6 + 3c^6 d^5 + 3c^7 d^4 - 3c^8 d^3 - 3c^9 d^2) \\
&)/c^3 - (a^3((a^3((8(4a^3c^{15} + 2b^3c^{15} + 12a^2b^3c^{15} - 12a^3c^{14}d - 2b^3c^{14}d - 4a^3c^6d^9 + 2a^3c^7d^8 + 18a^3c^8d^7 - 4a^3c^9d^6 - 36a^3c^{10}d^5 + 6a^3c^{11}d^4 + 34a^3c^{12}d^3 - 8a^3c^{13}d^2 - 4b^3c^8d^7 + 4b^3c^9d^6 + 6b^3c^{10}d^5 - 6b^3c^{11}d^4 + 18a^2b^2c^9d^6 - 18a^2b^2c^{10}d^5 - 36a^2b^2c^{11}d^4 + 36a^2b^2c^{12}d^3 + 18a^2b^2c^{13}d^2 - 6a^2b^2c^8d^7 + 6a^2b^2c^9d^6 + 18a^2b^2c^{12}d^3 - 18a^2b^2c^{13}d^2 - 18a^2b^2c^{14}d - 12a^2b^2c^{14}d)))/(c^{12}d + c^{13} - c^6d^7 - c^7d^6 + 3c^8d^5 + 3c^9d^4 - 3c^{10}d^3 - 3c^{11}d^2) + (a^3 \tan(e/2 + (f*x)/2) * (8c^{15}d - 8c^6d^{10} + 8c^7d^9 + 32c^8d^8 - 32c^9d^7 - 48c^{10}d^6 + 48c^{11}d^5 + 32c^{12}d^4 - 32c^{13}d^3 - 8c^{14}d^2) * 8i) / (c^3 * (c^{10}d + c^{11} - c^4d^7 - c^5d^6 + 3c^6d^5 + 3c^7d^4 - 3c^8d^3 - 3c^9d^2))) * 1i) / c^3 - (8 \tan(e/2 + (f*x)/2) * (4a^6c^{10} + 8a^6d^{10} + b^6c^{10} - 8a^6c^9d - 8a^6c^9d + 12a^2b^4c^{10} + 36a^4b^2c^{10} - 32a^6c^2d^8 + 32a^6c^3d^7 + 57a^6c^4d^6 - 48a^6c^5d^5 - 52a^6c^6d^4 + 32a^6c^7d^3 + 24a^6c^8d^2 + 4b^6c^6d^4 + 4b^6c^8d^2 - 36a^2b^5c^7d^3 - 120a^3b^3c^9d - 12a^5b^3c^3d^7 + 6a^5b^3c^5d^5 + 24a^5b^3c^7d^3 + 12a^2b^4c^6d^4 + 111a^2b^4c^8d^2 - 8a^3b^3c^3d^7 + 16a^3b^3c^5d^5 - 68a^3b^3c^7d^3 + 36a^4b^2c^4d^6 - 81a^4b^2c^6d^4 + 144a^4b^2c^8d^2 - 18a^2b^5c^9d - 72a^5b^3c^9d) / (c^{10}d + c^{11} - c^4d^7 - c^5d^6 + 3c^6d^5 + 3c^7d^4 - 3c^8d^3 - 3c^9d^2))) / c^3) / ((a^3((a^3((8(4a^3c^{15} + 2b^3c^{15} + 12a^2b^3c^{15} - 12a^3c^{14}d - 2b^3c^{14}d - 4a^3c^6d^9 + 2a^3c^7d^8 + 18a^3c^8d^7 - 4a^3c^9d^6 - 36a^3c^{10}d^5 + 6a^3c^{11}d^4 + 34a^3c^{12}d^3 - 8a^3c^{13}d^2 - 4b^3c^8d^7 + 4b^3c^9d^6 + 6b^3c^{10}d^5 - 6b^3c^{11}d^4 + 18a^2b^2c^9d^6 - 18a^2b^2c^{10}d^5 - 36a^2b^2c^{11}d^4 + 36a^2b^2c^{12}d^3 + 18a^2b^2c^{13}d^2 - 6a^2b^2c^8d^7 + 6a^2b^2c^9d^6 + 18a^2b^2c^{12}d^3 - 18a^2b^2c^{13}d^2 - 18a^2b^2c^{14}d - 12a^2b^2c^{14}d)))/(c^{12}d + c^{13} - c^6d^7 - c^7d^6 + 3c^8d^5 + 3c^9d^4 - 3c^{10}d^3 - 3c^{11}d^2) - (a^3 \tan(e/2 + (f*x)/2) * (8c^{15}d - 8c^6d^{10} + 8c^7d^9 + 32c^8d^8 - 32c^9d^7 - 48c^{10}d^6 + 48c^{11}d^5 + 32c^{12}d^4 - 32c^{13}d^3 - 8c^{14}d^2) * 8i) / (c^3 * (c^{10}d + c^{11} - c^4d^7 - c^5d^6 + 3c^6d^5 + 3c^7d^4 - 3c^8d^3 - 3c^9d^2))) * 1i) / c^3 + (8 \tan(e/2 + (f*x)/2) * (4a^6c^{10} + 8a^6d^{10} + b^6c^{10} - 8a^6c^9d - 8a^6c^9d + 12a^2b^4c^{10} + 36a^4b^2c^{10} - 32a^6c^2d^8 + 32a^6c^3d^7 + 57a^6c^4d^6 - 48a^6c^5d^5 - 52a^6c^6d^4 + 32a^6c^7d^3 + 24a^6c^8d^2 + 4b^6c^6d^4 + 4b^6c^8d^2 - 36a^2b^5c^7d^3 - 120a^3b^3c^9d - 12a^5b^3c^3d^7 + 6a^5b^3c^5d^5 + 24a^5b^3c^7d^3 + 12a^2b^4c^6d^4 + 111a^2b^4c^8d^2 - 8a^3b^3c^3d^7 + 16a^3b^3c^5d^5 - 68a^3b^3c^7d^3 + 36a^4b^2c^4d^6 - 81a^4b^2c^6d^4 + 144a^4b^2c^8d^2 - 18a^2b^5c^9d - 72a^5b^3c^9d) / (c^{10}d + c^{11} - c^4d^7 - c^5d^6 + 3c^6d^5 + 3c^7d^4 - 3c^8d^3 - 3c^9d^2))) * 1i) / c^3 - (16(4a^9d^9 - 12a^8b^3c^9 - 2a^9c^8d^8 + 12a^9c^8d + a^3b^6c^9 + 12a^5b^4c^9 - 2a^6b^3c^9 + 36a^7b^2c^9 - 18a^9c^2d^7 + 13a^9c^3d^6 + 36a^9c^4d^5 - 26a^9c^5d^4 - 34a^9c^6d^3 + 24a^9c^7d^2 - 18a^4b^5c^8d - 118a^6b^3c^8d + 18a^7b^2c^8d - 6a^8b^3c^2d^7 - 6a^8b^3c^3d^6 + 6a^8b^3c^4d^5 + 6a^8b^3c^6d^3 + 18a^8b^3c^7d^2 + 4a^3b^6c^5d^4 + 4a^3b^6c^7d^2 - 36a^4b^5c^6d^3 + 12a^5b^4c^5d^4 + 111a^5b^4c^7d^2 - 4a^6b^3c^2d^7 - 4a^6b^3c^3d^6 + 10a^6b^3c^4d^5 + 6a^6b^3c^5d^4 - 68a^6b^3c^6d^3 + 18a^7b^2c^3d^6 + 18a^7b^2c^4d^5 - 45a^7b^2c^5d^4 - 36a^7b^2c^6d^3 + 126a^7b^2c^7d^2 - 60a^8b^3c^8d) / (c^{12}d + c^{13} - c^6d^7 - c^7d^6 + 3c^8d^5 + 3c^9d^4 - 3c^{10}d^3 - 3c^{11}d^2) + (a^3((a^3((8(4a^3c^{15} + 2b^3c^{15} + 12a^2b^3c^{15} - 12a^3c^{14}d - 2b^3c^{14}d - 4a^3c^6d^9 + 2a^3c^7d^8 + 18a^3c^8d^7 - 4a^3c^9d^6 - 36a^3c^{10}d^5 + 6a^3c^{11}d^4 + 34a^3c^{12}d^3 - 8a^3c^{13}d^2 - 4b^3c^8d^7 + 4b^3c^9d^6 + 6b^3c^{10}d^5 - 6b^3c^{11}d^4 + 18a^2b^2c^9d^6 - 18a^2b^2c^{10}d^5 - 36a^2b^2c^{11}d^4 + 36a^2b^2c^{12}d^3 + 18a^2b^2c^{13}d^2 - 6a^2b^2c^8d^7 + 6a^2b^2c^9d^6 + 18a^2b^2c^{12}d^3 - 18a^2b^2c^{13}d^2 - 18a^2b^2c^{14}d - 12a^2b^2c^{14}d)))/(c^{12}d + c^{13} -
\end{aligned}$$

```

c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) + (a^3
*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^
9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)
*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^
8*d^3 - 3*c^9*d^2))*1i)/c^3 - (8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^
10 + b^6*c^10 - 8*a^6*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^
10 - 32*a^6*c^2*d^8 + 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52
*a^6*c^6*d^4 + 32*a^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*
d^2 - 36*a*b^5*c^7*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5
*d^5 + 24*a^5*b*c^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*
b^3*c^3*d^7 + 16*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6
- 81*a^4*b^2*c^6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*
d))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3
- 3*c^9*d^2))*1i)/c^3)))/(c^3*f) - ((tan(e/2 + (f*x)/2)^3*(2*a^3*d^4 + b^3*
c^4 - 6*a*b^2*c^4 - a^3*c*d^3 + 4*b^3*c^3*d - 6*a^3*c^2*d^2 - 6*a*b^2*c^2*d
^2 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d + 12*a^2*b*c^3*d))/((c^2*d - c^3)*(c +
d)^2) + (tan(e/2 + (f*x)/2)*(2*a^3*d^4 - b^3*c^4 - 6*a*b^2*c^4 + a^3*c*d^3
+ 4*b^3*c^3*d - 6*a^3*c^2*d^2 - 6*a*b^2*c^2*d^2 - 3*a^2*b*c^2*d^2 + 3*a*b^
2*c^3*d + 12*a^2*b*c^3*d))/((c + d)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d -
tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d +
d^2) + c^2 + d^2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)

[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**3, x)

$$3.196 \quad \int \frac{(a+b \sec(e+fx))^3}{(c+d \sec(e+fx))^4} dx$$

Optimal. Leaf size=412

$$\frac{a^3 x (bc - ad) (a^2 (34c^4 d - 28c^2 d^3 + 9d^5) - abc (18c^4 + 17c^2 d^2 - 5d^4) + b^2 c^2 d (13c^2 + 2d^2)) \sin(e + fx)}{c^4} \frac{(a^3 (8c^3 f (c^2 - d^2)^3 (c \cos(e + fx) + d)) \sin(e + fx))}{6c^3 f (c^2 - d^2)^3 (c \cos(e + fx) + d)}$$

[Out] $a^3 x / c^4 - 1/3 d (-a d + b c) (b + a \cos(f x + e))^2 \sin(f x + e) / c / (c^2 - d^2) / f / (d + c \cos(f x + e))^3 + 1/6 (-a d + b c)^2 (-8 a^2 c^2 d + 3 a^2 d^3 + 3 b^2 c^3 + 2 b^2 c d^2) \sin(f x + e) / c^3 / (c^2 - d^2)^2 / f / (d + c \cos(f x + e))^2 - 1/6 (-a d + b c) (b^2 c^2 d (13 c^2 + 2 d^2) - a b^2 c^2 (18 c^4 + 17 c^2 d^2 - 5 d^4) + a^2 (34 c^4 d - 28 c^2 d^3 + 9 d^5)) \sin(f x + e) / c^3 / (c^2 - d^2)^3 / f / (d + c \cos(f x + e)) - (3 a b^2 c^4 d (4 c^2 + d^2) - b^3 c^5 (c^2 + 4 d^2) - a^2 b (6 c^7 + 9 c^5 d^2) + a^3 (8 c^6 d - 8 c^4 d^3 + 7 c^2 d^5 - 2 d^7)) \operatorname{arctanh}((c - d)^{1/2} \tan(1/2 e + 1/2 f x) / (c + d)^{1/2}) / c^4 / (c^2 - d^2)^3 / f / (c - d)^{1/2} / (c + d)^{1/2}$

Rubi [A] time = 1.06, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3941, 2989, 3031, 3021, 2735, 2659, 208}

$$\frac{(bc - ad) (a^2 (-28c^2 d^3 + 34c^4 d + 9d^5) - abc (17c^2 d^2 + 18c^4 - 5d^4) + b^2 c^2 d (13c^2 + 2d^2)) \sin(e + fx)}{c^4} \frac{(-a^2 b (9c^3 f (c^2 - d^2)^3 (c \cos(e + fx) + d)) \sin(e + fx))}{6c^3 f (c^2 - d^2)^3 (c \cos(e + fx) + d)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4, x]

[Out] $(a^3 x) / c^4 - ((3 a b^2 c^4 d (4 c^2 + d^2) - b^3 c^5 (c^2 + 4 d^2) - a^2 b (6 c^7 + 9 c^5 d^2) + a^3 (8 c^6 d - 8 c^4 d^3 + 7 c^2 d^5 - 2 d^7)) \operatorname{ArcTanh}(\operatorname{Sqrt}[c - d] \operatorname{Tan}[(e + f x) / 2]) / \operatorname{Sqrt}[c + d]) / (c^4 \operatorname{Sqrt}[c - d] \operatorname{Sqrt}[c + d]) * (c^2 - d^2)^3 f - (d (b c - a d) (b + a \cos[e + f x])^2 \sin[e + f x]) / (3 c (c^2 - d^2) f (d + c \cos[e + f x])^3) + ((b c - a d)^2 (3 b^2 c^3 - 8 a^2 c^2 d + 2 b^2 c d^2 + 3 a^2 d^3) \sin[e + f x]) / (6 c^3 (c^2 - d^2)^2 f (d + c \cos[e + f x])^2) - ((b c - a d) (b^2 c^2 d (13 c^2 + 2 d^2) - a b^2 c (18 c^4 + 17 c^2 d^2 - 5 d^4) + a^2 (34 c^4 d - 28 c^2 d^3 + 9 d^5)) \sin[e + f x]) / (6 c^3 (c^2 - d^2)^3 f (d + c \cos[e + f x]))$

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]) / ((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)
]*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f
_.)*(x_)^2], x_Symbol] := -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
+ f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))]*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& LtQ[m, -1]
```

Rule 3941

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)])*(d
_.) + (c_.))^(n_), x_Symbol] := Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f
*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^4} dx &= \int \frac{\cos(e + fx)(b + a \cos(e + fx))^3}{(d + c \cos(e + fx))^4} dx \\
&= -\frac{d(bc - ad)(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx))^3} + \frac{\int \frac{(b + a \cos(e + fx))((3bc - 2ad)(bc - ad) - (3a^2 cd + 2d^3))}{(d + c \cos(e + fx))^4} dx}{3c} \\
&= -\frac{d(bc - ad)(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx))^3} + \frac{(bc - ad)^2 (3bc^3 - 8ac^2 d + 2bcd^2 + 3d^3)}{6c^3 (c^2 - d^2)^2 f(d + c \cos(e + fx))^3} \\
&= -\frac{d(bc - ad)(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx))^3} + \frac{(bc - ad)^2 (3bc^3 - 8ac^2 d + 2bcd^2 + 3d^3)}{6c^3 (c^2 - d^2)^2 f(d + c \cos(e + fx))^3} \\
&= \frac{a^3 x}{c^4} - \frac{d(bc - ad)(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx))^3} + \frac{(bc - ad)^2 (3bc^3 - 8ac^2 d + 2bcd^2 + 3d^3)}{6c^3 (c^2 - d^2)^2 f(d + c \cos(e + fx))^3} \\
&= \frac{a^3 x}{c^4} - \frac{d(bc - ad)(b + a \cos(e + fx))^2 \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx))^3} + \frac{(bc - ad)^2 (3bc^3 - 8ac^2 d + 2bcd^2 + 3d^3)}{6c^3 (c^2 - d^2)^2 f(d + c \cos(e + fx))^3} \\
&= \frac{a^3 x}{c^4} - \frac{(3ab^2 c^4 d (4c^2 + d^2) - b^3 c^5 (c^2 + 4d^2) - a^2 b (6c^7 + 9c^5 d^2) + a^3 (8c^6 d - 8c^4 d^3))}{c^4 \sqrt{c - d} \sqrt{c + d} (c^2 - d^2)^3} f
\end{aligned}$$

Mathematica [A] time = 3.78, size = 459, normalized size = 1.11

$$\sec(e + fx)(a + b \sec(e + fx))^3(c \cos(e + fx) + d) \left(6a^3(e + fx)(c \cos(e + fx) + d)^3 + \frac{c(a^3(36c^4 d^2 - 32c^2 d^4 + 11d^6) - 3a^2 b c d^3)}{c^4 \sqrt{c - d} \sqrt{c + d} (c^2 - d^2)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]

[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^3*(6*a^3*(e + f*x)*(d + c*Cos[e + f*x])^3 - (6*(-3*a*b^2*c^4*d*(4*c^2 + d^2) + b^3*c^5*(c^2 + 4*d^2) + a^2*b*(6*c^7 + 9*c^5*d^2) + a^3*(-8*c^6*d + 8*c^4*d^3 - 7*c^2*d^5 + 2*d^7))*ArcTanh[(-c + d)*Tan[(e + f*x)/2]]/Sqrt[c^2 - d^2])*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^(7/2) - (2*c*d*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d^2) + (c*(b*c - a*d)^2*(3*b*c^3 - 12*a*c^2*d + 2*b*c*d^2 + 7*a*d^3)*(d + c*Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-(b^3*c^3*d*(13*c^2 + 2*d^2)) + 3*a*b^2*c^2*(6*c^4 + 10*c^2*d^2 - d^4) - 3*a^2*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d^4) + a^3*(36*c^4*d^2 - 32*c^2*d^4 + 11*d^6))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3)/(6*c^4*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^4)

fricas [B] time = 0.81, size = 2776, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fricas")

```
[Out] [1/12*(12*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*c^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^3*c^9*d^2 - 4*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e) + 12*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*d^11)*f*x + 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 + 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d + 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^2)*c^6*d^4) *cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b^2)*c^9*d^2 + (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (21*a^2*b - 2*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(b^3*c^11 + 6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 + (20*a^3 + 21*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b^2)*c^6*d^5 + (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)*f), 1/6*(6*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*c^3*d^8)*f*x*cos(f*x + e)^3 + 18*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 18*(a^3*c^9*d^2 - 4*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e) + 6*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*d^11)*f*x - 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 + 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d + 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^2)*c^6*d^4)*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^3*c^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 - 11*(3*a^2*b + b^3)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b^3)*c^6*d^5 - (43*a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (18*a*b^2*c^11 + 6*a^2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c^10*d + 12*(3*a^3 + a*b^2)*c^9*d^2 + (69*a^2*b + 11*b^3)*c^8*d^3 - (68*a^3 + 33*a*b^2)*c^7*d^4 - (21*a^2*b - 2*b^3)*c^6*d^5 + (43*a^3 + 3*a*b^2)*c^5*d^6)*cos(f*x + e)^2 + 3*(b^3*c^11 + 6*a*b^2*c^10*d - 5*a^3*c^2*d^9 - (27*a^2*b + 10*b^3)*c^9*d^2 + (20*a^3 + 21*a*b^2)*c^8*d^3 + (24*a^2*b + 7*b^3)*c^7*d^4 - 5*(7*a^3 + 6*a*b^2)*c^6*d^5 + (3*a^2*b + 2*b^3)*c^5*d^6 + (20*a^3 + 3*a*b^2)*c^4*d^7)*cos(f*x + e))*sin(f*x + e))/((c^15 - 4*c^13*d^2 + 6*c^11*d^4 - 4*c^9*d^6 + c^7*d^8)*f*cos(f*x + e)^3 + 3*(c^14*d - 4*c^12*d^3 + 6*c^10*d^5 - 4*c^8*d^7 + c^6*d^9)*f*cos(f*x + e)^2 + 3*(c^13*d^2 - 4*c^11*d^4 + 6*c^9*d^6 - 4*c^7*d^8 + c^5*d^10)*f*cos(f*x + e) + (c^12*d^3 - 4*c^10*d^5 + 6*c^8*d^7 - 4*c^6*d^9 + c^4*d^11)*f)]
```

giac [B] time = 0.93, size = 1639, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="giac")

[Out] $\frac{1}{3} \cdot (3 \cdot (6 \cdot a^2 \cdot b \cdot c^7 + b^3 \cdot c^7 - 8 \cdot a^3 \cdot c^6 \cdot d - 12 \cdot a \cdot b^2 \cdot c^6 \cdot d + 9 \cdot a^2 \cdot b \cdot c^5 \cdot d^2 + 4 \cdot b^3 \cdot c^5 \cdot d^2 + 8 \cdot a^3 \cdot c^4 \cdot d^3 - 3 \cdot a \cdot b^2 \cdot c^4 \cdot d^3 - 7 \cdot a^3 \cdot c^2 \cdot d^5 + 2 \cdot a^3 \cdot d^7) \cdot (\pi \cdot \text{floor}(1/2 \cdot (f \cdot x + e)/\pi + 1/2) \cdot \text{sgn}(-2 \cdot c + 2 \cdot d) + \arctan(-(c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e))/\sqrt{-c^2 + d^2}))) / ((c^{10} - 3 \cdot c^8 \cdot d^2 + 3 \cdot c^6 \cdot d^4 - c^4 \cdot d^6) \cdot \sqrt{-c^2 + d^2}) + 3 \cdot (f \cdot x + e) \cdot a^3 / c^4 - (18 \cdot a \cdot b^2 \cdot c^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 3 \cdot b^3 \cdot c^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 54 \cdot a^2 \cdot b \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 18 \cdot a \cdot b^2 \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 12 \cdot b^3 \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 36 \cdot a^3 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 81 \cdot a^2 \cdot b \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 36 \cdot a \cdot b^2 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 27 \cdot b^3 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 60 \cdot a^3 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 18 \cdot a^2 \cdot b \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 81 \cdot a \cdot b^2 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 12 \cdot b^3 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 6 \cdot a^3 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 9 \cdot a^2 \cdot b \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 36 \cdot a \cdot b^2 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 6 \cdot b^3 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 45 \cdot a^3 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 18 \cdot a^2 \cdot b \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 9 \cdot a \cdot b^2 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 6 \cdot b^3 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 6 \cdot a^3 \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 15 \cdot a^3 \cdot c \cdot d^7 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 + 6 \cdot a^3 \cdot d^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^5 - 36 \cdot a \cdot b^2 \cdot c^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 108 \cdot a^2 \cdot b \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 28 \cdot b^3 \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 72 \cdot a^3 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 48 \cdot a \cdot b^2 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 96 \cdot a^2 \cdot b \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 16 \cdot b^3 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 116 \cdot a^3 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 84 \cdot a \cdot b^2 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 12 \cdot a^2 \cdot b \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 12 \cdot b^3 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 56 \cdot a^3 \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 12 \cdot a^3 \cdot d^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 18 \cdot a \cdot b^2 \cdot c^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 3 \cdot b^3 \cdot c^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 54 \cdot a^2 \cdot b \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 18 \cdot a \cdot b^2 \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 12 \cdot b^3 \cdot c^7 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 36 \cdot a^3 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 81 \cdot a^2 \cdot b \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 36 \cdot a \cdot b^2 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 27 \cdot b^3 \cdot c^6 \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 60 \cdot a^3 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 18 \cdot a^2 \cdot b \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 81 \cdot a \cdot b^2 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 12 \cdot b^3 \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 6 \cdot a^3 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 9 \cdot a^2 \cdot b \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 36 \cdot a \cdot b^2 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 6 \cdot b^3 \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 45 \cdot a^3 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 18 \cdot a^2 \cdot b \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 9 \cdot a \cdot b^2 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 6 \cdot b^3 \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 6 \cdot a^3 \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 15 \cdot a^3 \cdot c \cdot d^7 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 6 \cdot a^3 \cdot d^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)) / ((c^9 - 3 \cdot c^7 \cdot d^2 + 3 \cdot c^5 \cdot d^4 - c^3 \cdot d^6) \cdot (c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - c - d)^3) / f$

maple [B] time = 0.76, size = 4330, normalized size = 10.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x)

[Out] $-12/f \cdot c / (\tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^2 \cdot c - \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^2 \cdot d - c - d)^3 / (c + d) / (c^3 - 3 \cdot c^2 \cdot d + 3 \cdot c \cdot d^2 - d^3) \cdot \tan(1/2 \cdot e + 1/2 \cdot f \cdot x) \cdot a^3 \cdot d^2 - 4/f / (\tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^2 \cdot c - \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^2 \cdot d - c - d)^3 / (c^2 - 2 \cdot c \cdot d + d^2) / (c^2 + 2 \cdot c \cdot d + d^2) \cdot \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^3 \cdot a^2 \cdot b \cdot d^3 + 6/f / (\tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^2 \cdot c - \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^2 \cdot d - c - d)^3 / (c - d) / (c^3 + 3 \cdot c^2 \cdot d + 3 \cdot c \cdot d^2 + d^3) \cdot \tan(1/2 \cdot e + 1/2 \cdot f \cdot x)^5 \cdot a^2 \cdot b \cdot d^3 + 24/f \cdot c / (\tan($

$$\begin{aligned}
& \frac{1}{2}e+1/2f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^3*d^2-44/3/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^3*d^4+4/f/c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^3*d^6+3/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a*b^2*d^3-3/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a*b^2*d^3-6/f*c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a*b^2+6/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*b^3*d-2/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*b^3*d^2-12/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^3*d^2+6/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^3*d^4+1/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^3*d^5-2/f/c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^3*d^6-6/f*c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a*b^2-28/3/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*b^3*d+6/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*b^3*d+2/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*b^3*d^2+9/f*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*b*d^2-12/f*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a*b^2*d+12/f*c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a*b^2+6/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^2*b*d^3+6/f/c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^3*d^4-1/f/c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^3*d^5-2/f/c^3/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^3*d^6+2/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*b^3*d^3-4/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^3*d^3+2/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*b^3*d^3-3/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a*b^2*d^3-8/f*c^2/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^3*d+2/f/c^4/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^2*b+4/f*c/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*b^3*d^2-4/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*b^3*d^3+4/f/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^3*d^3+2/f*a^3/c^4*\operatorname{arctan}(\tan(1/2*e+1/2*f*x))+18/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-
\end{aligned}$$

$$3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^2*b*d+28/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a*b^2*d^2+6/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a*b^2*d-18/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a*b^2*d^2-6/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a*b^2*d-18/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a*b^2*d^2-36/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*\tan(1/2*e+1/2*f*x)^3*a^2*b*d+8/f/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*a^3*d^3+1/f*c^3/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*\operatorname{arctanh}(\tan(1/2*e+1/2*f*x)*(c-d)/((c+d)*(c-d))^(1/2))*b^3+9/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^2*b*d^2+18/f*c^2/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*\tan(1/2*e+1/2*f*x)^5*a^2*b*d-9/f*c/(\tan(1/2*e+1/2*f*x)^2*c-\tan(1/2*e+1/2*f*x)^2*d-c-d)^3/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*\tan(1/2*e+1/2*f*x)*a^2*b*d^2$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 16.09, size = 15647, normalized size = 37.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^4,x)

[Out] ((tan(e/2 + (f*x)/2)^5*(b^3*c^6 - 2*a^3*d^6 - 6*a*b^2*c^6 + a^3*c*d^5 + 6*b^3*c^5*d + 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 - 12*a^3*c^4*d^2 + 2*b^3*c^3*d^3 + 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 - 18*a*b^2*c^4*d^2 + 6*a^2*b*c^3*d^3 + 9*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d + 18*a^2*b*c^5*d))/((c^3*d - c^4)*(c + d)^3) + (4*tan(e/2 + (f*x)/2)^3*(7*b^3*c^5*d - 9*a*b^2*c^6 - 3*a^3*d^6 + 11*a^3*c^2*d^4 - 18*a^3*c^4*d^2 + 3*b^3*c^3*d^3 - 21*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + 27*a^2*b*c^5*d))/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) - (tan(e/2 + (f*x)/2)*(2*a^3*d^6 + b^3*c^6 + 6*a*b^2*c^6 + a^3*c*d^5 - 6*b^3*c^5*d - 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 + 12*a^3*c^4*d^2 - 2*b^3*c^3*d^3 + 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 18*a*b^2*c^4*d^2 - 6*a^2*b*c^3*d^3 + 9*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d - 18*a^2*b*c^5*d))/((c + d)*(3*c^5*d - c^6 + c^3*d^3 - 3*c^4*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) - (2*a^3*a*tan(((a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^14*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^18*d^3 + 2*b^3*c^19*d^2 - 6*a*b^2*c^11*d^10 + 6*a*b^2*c^12*d^9 - 6*a*b^2*c^13*d^8 + 6*a*b^2*c^14*d^8

$$\begin{aligned}
& 7 + 54*a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 - 66*a*b^2*c^{17}*d^4 + 66*a*b^2*c^{18}*d^3 + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12}*d^9 - 18*a^2*b*c^{13}*d^8 - 42*a^2*b*c^{14}*d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b*c^{16}*d^5 - 18*a^2*b*c^{17}*d^4 + 18*a^2*b*c^{18}*d^3 - 18*a^2*b*c^{19}*d^2 - 24*a*b^2*c^{20}*d - 12*a^2*b*c^{20}*d)) / \\
& (c^{19}*d + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5*c^{11}*d^9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 - 10*c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d^4 - 5*c^{17}*d^3 - 5*c^{18}*d^2) - (\\
& a^3*\tan(e/2 + (f*x)/2)*(8*c^{21}*d - 8*c^8*d^{14} + 8*c^9*d^{13} + 48*c^{10}*d^{12} - 48*c^{11}*d^{11} - 120*c^{12}*d^{10} + 120*c^{13}*d^9 + 160*c^{14}*d^8 - 160*c^{15}*d^7 - 120*c^{16}*d^6 + 120*c^{17}*d^5 + 48*c^{18}*d^4 - 48*c^{19}*d^3 - 8*c^{20}*d^2)*8i) \\
& / (c^4*(c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2)) \\
&)*1i)/c^4 + (8*\tan(e/2 + (f*x)/2)*(4*a^6*c^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} - 8*a^6*c^{13}*d + 12*a^2*b^4*c^{14} + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 48*a^6*c^3*d^{11} + 117*a^6*c^4*d^{10} - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^{10}*d^4 + 48*a^6*c^{11}*d^3 + 44*a^6*c^{12}*d^2 + 16*b^6*c^{10}*d^4 + 8*b^6*c^{12}*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^{11}*d^3 - 160*a^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^{10}*d^4 + 210*a^2*b^4*c^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c^{12}*d^2 - 24*a*b^5*c^{13}*d - 96*a^5*b*c^{13}*d)) / (c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2)) / c^4 - (a^3*((a^3*((8*(4*a^3*c^{21} + 2*b^3*c^{21} + 12*a^2*b*c^{21} - 16*a^3*c^{20}*d - 2*b^3*c^{20}*d - 4*a^3*c^8*d^{13} + 2*a^3*c^9*d^{12} + 26*a^3*c^{10}*d^{11} - 14*a^3*c^{11}*d^{10} - 70*a^3*c^{12}*d^9 + 30*a^3*c^{13}*d^8 + 110*a^3*c^{14}*d^7 - 30*a^3*c^{15}*d^6 - 110*a^3*c^{16}*d^5 + 20*a^3*c^{17}*d^4 + 64*a^3*c^{18}*d^3 - 12*a^3*c^{19}*d^2 + 8*b^3*c^{12}*d^9 - 8*b^3*c^{13}*d^8 - 22*b^3*c^{14}*d^7 + 22*b^3*c^{15}*d^6 + 18*b^3*c^{16}*d^5 - 18*b^3*c^{17}*d^4 - 2*b^3*c^{18}*d^3 + 2*b^3*c^{19}*d^2 - 6*a*b^2*c^{11}*d^{10} + 6*a*b^2*c^{12}*d^9 - 6*a*b^2*c^{13}*d^8 + 6*a*b^2*c^{14}*d^7 + 54*a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 - 66*a*b^2*c^{17}*d^4 + 66*a*b^2*c^{18}*d^3 + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12}*d^9 - 18*a^2*b*c^{13}*d^8 - 42*a^2*b*c^{14}*d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b*c^{16}*d^5 - 18*a^2*b*c^{17}*d^4 + 18*a^2*b*c^{18}*d^3 - 18*a^2*b*c^{19}*d^2 - 24*a*b^2*c^{20}*d - 12*a^2*b*c^{20}*d)) / (c^{19}*d + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5*c^{11}*d^9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 - 10*c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d^4 - 5*c^{17}*d^3 - 5*c^{18}*d^2) + (a^3*\tan(e/2 + (f*x)/2)*(8*c^{21}*d - 8*c^8*d^{14} + 8*c^9*d^{13} + 48*c^{10}*d^{12} - 48*c^{11}*d^{11} - 120*c^{12}*d^{10} + 120*c^{13}*d^9 + 160*c^{14}*d^8 - 160*c^{15}*d^7 - 120*c^{16}*d^6 + 120*c^{17}*d^5 + 48*c^{18}*d^4 - 48*c^{19}*d^3 - 8*c^{20}*d^2)*8i) / (c^4*(c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2))) * 1i) / c^4 - (8*\tan(e/2 + (f*x)/2)*(4*a^6*c^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} - 8*a^6*c^{13}*d + 12*a^2*b^4*c^{14} + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 48*a^6*c^3*d^{11} + 117*a^6*c^4*d^{10} - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^{10}*d^4 + 48*a^6*c^{11}*d^3 + 44*a^6*c^{12}*d^2 + 16*b^6*c^{10}*d^4 + 8*b^6*c^{12}*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^{11}*d^3 - 160*a^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^{10}*d^4 + 210*a^2*b^4*c^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c^{12}*d^2 - 24*a*b^5*c^{13}*d - 96*a^5*b*c^{13}*d)) / (c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2)) / c^4 / ((a^3*((a^3*((8*(4*a^3*c^{21} + 2*b^3*c^{21} + 12*a^2*b*c^{21} - 16*a^3*c^{20}*d - 2*b^3*c^{20}*d - 4*a^3*c^8*d^{13} + 2*a^3*c^9*d^{12} + 26*a^3*c^{10}*d^{11} - 14*a^3*c^{11}*d^{10} - 70*a^3*c^{12}*d^9 + 30*a^3*c^{13}*d^8 + 110*a^3*c^{14}*d^7 - 30*a^3*c^{15}*d^6 - 110*a^3*c^{16}*d^5 + 20*a^3*c^{17}*d^4 + 64*a^3*c^{18}*d^3 - 12*a^3*c^{19}*d^2 + 8*b^3*c^{12}*d^9 - 8*b^3*c^{13}*d^8
\end{aligned}$$

$$\begin{aligned}
& 8 - 22*b^3*c^{14}*d^7 + 22*b^3*c^{15}*d^6 + 18*b^3*c^{16}*d^5 - 18*b^3*c^{17}*d^4 - \\
& 2*b^3*c^{18}*d^3 + 2*b^3*c^{19}*d^2 - 6*a*b^2*c^{11}*d^{10} + 6*a*b^2*c^{12}*d^9 - 6 \\
& *a*b^2*c^{13}*d^8 + 6*a*b^2*c^{14}*d^7 + 54*a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 \\
& - 66*a*b^2*c^{17}*d^4 + 66*a*b^2*c^{18}*d^3 + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12} \\
& *d^9 - 18*a^2*b*c^{13}*d^8 - 42*a^2*b*c^{14}*d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b \\
& *c^{16}*d^5 - 18*a^2*b*c^{17}*d^4 + 18*a^2*b*c^{18}*d^3 - 18*a^2*b*c^{19}*d^2 - 24* \\
& a*b^2*c^{20}*d - 12*a^2*b*c^{20}*d)/(c^{19}*d + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5* \\
& c^{11}*d^9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 - 10*c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d \\
& ^4 - 5*c^{17}*d^3 - 5*c^{18}*d^2) - (a^3*tan(e/2 + (f*x)/2)*(8*c^{21}*d - 8*c^8*d \\
& ^{14} + 8*c^9*d^{13} + 48*c^{10}*d^{12} - 48*c^{11}*d^{11} - 120*c^{12}*d^{10} + 120*c^{13}*d \\
& ^9 + 160*c^{14}*d^8 - 160*c^{15}*d^7 - 120*c^{16}*d^6 + 120*c^{17}*d^5 + 48*c^{18}*d \\
& ^4 - 48*c^{19}*d^3 - 8*c^{20}*d^2)*8i)/(c^4*(c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} \\
& + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13} \\
& *d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2))*1i)/c^4 + (8*tan(e/2 + (f*x)/2)*(4*a^6*c \\
& ^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} - 8*a^6*c^{13}*d + 12*a^2*b^4*c^{14} \\
& + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 48*a^6*c^3*d^{11} + 117*a^6*c^4*d^{10} - \\
& 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 12 \\
& 0*a^6*c^9*d^5 - 92*a^6*c^{10}*d^4 + 48*a^6*c^{11}*d^3 + 44*a^6*c^{12}*d^2 + 16*b^6 \\
& *c^{10}*d^4 + 8*b^6*c^{12}*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^{11}*d^3 - 160*a \\
& ^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 4 \\
& 8*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^{10}*d^4 + 210*a^2*b^4*c \\
& ^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300 \\
& *a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c \\
& ^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c^{12}*d^2 - 24*a*b^5*c^{13}*d - 96* \\
& a^5*b*c^{13}*d))/(c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 \\
& - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c \\
& ^{15}*d^2))*1i)/c^4 - (16*(4*a^9*d^{13} - 12*a^8*b*c^{13} - 2*a^9*c*d^{12} + 16*a^9 \\
& *c^{12}*d + a^3*b^6*c^{13} + 12*a^5*b^4*c^{13} - 2*a^6*b^3*c^{13} + 36*a^7*b^2*c^{13} \\
& - 26*a^9*c^2*d^{11} + 11*a^9*c^3*d^{10} + 70*a^9*c^4*d^9 - 34*a^9*c^5*d^8 - 11 \\
& 0*a^9*c^6*d^7 + 66*a^9*c^7*d^6 + 110*a^9*c^8*d^5 - 64*a^9*c^9*d^4 - 64*a^9*c \\
& ^{10}*d^3 + 48*a^9*c^{11}*d^2 - 24*a^4*b^5*c^{12}*d - 158*a^6*b^3*c^{12}*d + 24*a^7 \\
& *b^2*c^{12}*d + 18*a^8*b*c^4*d^9 + 18*a^8*b*c^5*d^8 - 60*a^8*b*c^6*d^7 - 42* \\
& a^8*b*c^7*d^6 + 42*a^8*b*c^8*d^5 + 18*a^8*b*c^9*d^4 - 66*a^8*b*c^{10}*d^3 + 1 \\
& 8*a^8*b*c^{11}*d^2 + 16*a^3*b^6*c^9*d^4 + 8*a^3*b^6*c^{11}*d^2 - 24*a^4*b^5*c^8 \\
& *d^5 - 102*a^4*b^5*c^{10}*d^3 + 9*a^5*b^4*c^7*d^6 + 144*a^5*b^4*c^9*d^4 + 210 \\
& *a^5*b^4*c^{11}*d^2 + 8*a^6*b^3*c^4*d^9 + 8*a^6*b^3*c^5*d^8 - 30*a^6*b^3*c^6* \\
& d^7 - 22*a^6*b^3*c^7*d^6 - 22*a^6*b^3*c^8*d^5 + 18*a^6*b^3*c^9*d^4 - 298*a^6 \\
& *b^3*c^{10}*d^3 - 2*a^6*b^3*c^{11}*d^2 - 6*a^7*b^2*c^3*d^{10} - 6*a^7*b^2*c^4*d^9 \\
& - 6*a^7*b^2*c^6*d^7 + 66*a^7*b^2*c^7*d^6 + 54*a^7*b^2*c^8*d^5 + 3*a^7*b^2 \\
& *c^9*d^4 - 66*a^7*b^2*c^{10}*d^3 + 276*a^7*b^2*c^{11}*d^2 - 84*a^8*b*c^{12}*d))/(\\
& c^{19}*d + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5*c^{11}*d^9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 \\
& - 10*c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d^4 - 5*c^{17}*d^3 - 5*c^{18}*d^2) + (a \\
& ^3*((a^3*((8*(4*a^3*c^{21} + 2*b^3*c^{21} + 12*a^2*b*c^{21} - 16*a^3*c^{20}*d - 2*b \\
& ^3*c^{20}*d - 4*a^3*c^8*d^{13} + 2*a^3*c^9*d^{12} + 26*a^3*c^{10}*d^{11} - 14*a^3*c^{11} \\
& *d^{10} - 70*a^3*c^{12}*d^9 + 30*a^3*c^{13}*d^8 + 110*a^3*c^{14}*d^7 - 30*a^3*c^{15} \\
& *d^6 - 110*a^3*c^{16}*d^5 + 20*a^3*c^{17}*d^4 + 64*a^3*c^{18}*d^3 - 12*a^3*c^{19}*d \\
& ^2 + 8*b^3*c^{12}*d^9 - 8*b^3*c^{13}*d^8 - 22*b^3*c^{14}*d^7 + 22*b^3*c^{15}*d^6 + \\
& 18*b^3*c^{16}*d^5 - 18*b^3*c^{17}*d^4 - 2*b^3*c^{18}*d^3 + 2*b^3*c^{19}*d^2 - 6*a*b \\
& ^2*c^{11}*d^{10} + 6*a*b^2*c^{12}*d^9 - 6*a*b^2*c^{13}*d^8 + 6*a*b^2*c^{14}*d^7 + 54* \\
& a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 - 66*a*b^2*c^{17}*d^4 + 66*a*b^2*c^{18}*d^3 \\
& + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12}*d^9 - 18*a^2*b*c^{13}*d^8 - 42*a^2*b*c^{14} \\
& *d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b*c^{16}*d^5 - 18*a^2*b*c^{17}*d^4 + 18*a^2*b \\
& *c^{18}*d^3 - 18*a^2*b*c^{19}*d^2 - 24*a*b^2*c^{20}*d - 12*a^2*b*c^{20}*d))/(c^{19}*d \\
& + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5*c^{11}*d^9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 - 10 \\
& *c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d^4 - 5*c^{17}*d^3 - 5*c^{18}*d^2) + (a^3*tan \\
& (e/2 + (f*x)/2)*(8*c^{21}*d - 8*c^8*d^{14} + 8*c^9*d^{13} + 48*c^{10}*d^{12} - 48*c^{11} \\
& *d^{11} - 120*c^{12}*d^{10} + 120*c^{13}*d^9 + 160*c^{14}*d^8 - 160*c^{15}*d^7 - 120*c \\
& ^{16}*d^6 + 120*c^{17}*d^5 + 48*c^{18}*d^4 - 48*c^{19}*d^3 - 8*c^{20}*d^2)*8i)/(c^4*(\\
& c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 -
\end{aligned}$$

$$\begin{aligned}
& 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2)))*i)/c \\
& ^4 - (8*\tan(e/2 + (f*x)/2)*(4*a^6*c^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} \\
& - 8*a^6*c^{13}*d + 12*a^2*b^4*c^{14} + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 4 \\
& 8*a^6*c^3*d^{11} + 117*a^6*c^4*d^{10} - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160 \\
& *a^6*c^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^{10}*d^4 + 48*a^6 \\
& *c^{11}*d^3 + 44*a^6*c^{12}*d^2 + 16*b^6*c^{10}*d^4 + 8*b^6*c^{12}*d^2 - 24*a*b^5*c \\
& ^9*d^5 - 102*a*b^5*c^{11}*d^3 - 160*a^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a \\
& ^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 1 \\
& 44*a^2*b^4*c^{10}*d^4 + 210*a^2*b^4*c^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3 \\
& *c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} \\
& - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c \\
& ^{12}*d^2 - 24*a*b^5*c^{13}*d - 96*a^5*b*c^{13}*d)/(c^{16}*d + c^{17} - c^6*d^{11} \\
& - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d \\
& ^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2))*i)/c^4)))/(c^4*f) + (\operatorname{atan}((((\\
& (8*\tan(e/2 + (f*x)/2)*(4*a^6*c^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} - \\
& 8*a^6*c^{13}*d + 12*a^2*b^4*c^{14} + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 48*a^6 \\
& *c^3*d^{11} + 117*a^6*c^4*d^{10} - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c \\
& ^7*d^7 + 156*a^6*c^8*d^6 - 120*a^6*c^9*d^5 - 92*a^6*c^{10}*d^4 + 48*a^6*c^{11} \\
& *d^3 + 44*a^6*c^{12}*d^2 + 16*b^6*c^{10}*d^4 + 8*b^6*c^{12}*d^2 - 24*a*b^5*c^9*d^5 \\
& - 102*a*b^5*c^{11}*d^3 - 160*a^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a^5*b* \\
& c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^ \\
& 2*b^4*c^{10}*d^4 + 210*a^2*b^4*c^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7 \\
& *d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} - 6*a \\
& ^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c^ \\
& ^{12}*d^2 - 24*a*b^5*c^{13}*d - 96*a^5*b*c^{13}*d))/(c^{16}*d + c^{17} - c^6*d^{11} - c^ \\
& 7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + \\
& 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2) + (((8*(4*a^3*c^{21} + 2*b^3*c^{21} + 12 \\
& *a^2*b*c^{21} - 16*a^3*c^{20}*d - 2*b^3*c^{20}*d - 4*a^3*c^8*d^{13} + 2*a^3*c^9*d^{1 \\
& 2} + 26*a^3*c^{10}*d^{11} - 14*a^3*c^{11}*d^{10} - 70*a^3*c^{12}*d^9 + 30*a^3*c^{13}*d^8 \\
& + 110*a^3*c^{14}*d^7 - 30*a^3*c^{15}*d^6 - 110*a^3*c^{16}*d^5 + 20*a^3*c^{17}*d^4 \\
& + 64*a^3*c^{18}*d^3 - 12*a^3*c^{19}*d^2 + 8*b^3*c^{12}*d^9 - 8*b^3*c^{13}*d^8 - 22* \\
& b^3*c^{14}*d^7 + 22*b^3*c^{15}*d^6 + 18*b^3*c^{16}*d^5 - 18*b^3*c^{17}*d^4 - 2*b^3*c \\
& ^{18}*d^3 + 2*b^3*c^{19}*d^2 - 6*a*b^2*c^{11}*d^{10} + 6*a*b^2*c^{12}*d^9 - 6*a*b^2*c \\
& ^{13}*d^8 + 6*a*b^2*c^{14}*d^7 + 54*a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 - 66*a*b \\
& ^2*c^{17}*d^4 + 66*a*b^2*c^{18}*d^3 + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12}*d^9 - \\
& 18*a^2*b*c^{13}*d^8 - 42*a^2*b*c^{14}*d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b*c^{16}*d \\
& ^5 - 18*a^2*b*c^{17}*d^4 + 18*a^2*b*c^{18}*d^3 - 18*a^2*b*c^{19}*d^2 - 24*a*b^2*c \\
& ^{20}*d - 12*a^2*b*c^{20}*d))/(c^{19}*d + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5*c^{11}*d^ \\
& 9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 - 10*c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d^4 - 5* \\
& c^{17}*d^3 - 5*c^{18}*d^2) - (4*\tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^{(1/2)}* \\
& (2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4 \\
& *d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d) \\
& *(8*c^{21}*d - 8*c^8*d^{14} + 8*c^9*d^{13} + 48*c^{10}*d^{12} - 48*c^{11}*d^{11} - 120*c^ \\
& ^{12}*d^{10} + 120*c^{13}*d^9 + 160*c^{14}*d^8 - 160*c^{15}*d^7 - 120*c^{16}*d^6 + 120*c \\
& ^{17}*d^5 + 48*c^{18}*d^4 - 48*c^{19}*d^3 - 8*c^{20}*d^2))/((c^{18} - c^4*d^{14} + 7*c^6 \\
& *d^{12} - 21*c^8*d^{10} + 35*c^{10}*d^8 - 35*c^{12}*d^6 + 21*c^{14}*d^4 - 7*c^{16}*d^2 \\
&)*(c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 \\
& - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2)))*((\\
& c + d)^7*(c - d)^7)^{(1/2)}*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d \\
& - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b \\
& *c^5*d^2 - 12*a*b^2*c^6*d))/(2*(c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} \\
& + 35*c^{10}*d^8 - 35*c^{12}*d^6 + 21*c^{14}*d^4 - 7*c^{16}*d^2)))*((c + d)^7*(c - d \\
&)^7)^{(1/2)}*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 \\
& + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a \\
& *b^2*c^6*d)*i)/(2*(c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} + 35*c^{10}*d^ \\
& 8 - 35*c^{12}*d^6 + 21*c^{14}*d^4 - 7*c^{16}*d^2)) + (((8*\tan(e/2 + (f*x)/2)*(4*a \\
& ^6*c^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} - 8*a^6*c^{13}*d + 12*a^2*b^4* \\
& c^{14} + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 48*a^6*c^3*d^{11} + 117*a^6*c^4*d^{10} \\
& - 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6
\end{aligned}$$

$$\begin{aligned}
& - 120a^6c^9d^5 - 92a^6c^{10}d^4 + 48a^6c^{11}d^3 + 44a^6c^{12}d^2 + 16b^6c^{10}d^4 + 8b^6c^{12}d^2 - 24a^5b^5c^9d^5 - 102a^5b^5c^{11}d^3 - 160a^3b^3c^{13}d + 36a^5b^5c^5d^9 - 102a^5b^5c^7d^7 + 60a^5b^5c^9d^5 \\
& - 48a^5b^5c^{11}d^3 + 9a^2b^4c^8d^6 + 144a^2b^4c^{10}d^4 + 210a^2b^4c^{12}d^2 + 16a^3b^3c^5d^9 - 52a^3b^3c^7d^7 - 4a^3b^3c^9d^5 - 300a^3b^3c^{11}d^3 - 12a^4b^2c^4d^{10} - 6a^4b^2c^6d^8 + 120a^4b^2c^8d^6 - 63a^4b^2c^{10}d^4 + 300a^4b^2c^{12}d^2 - 24a^5b^5c^{13}d - 96a^5b^5c^{13}d) / (c^{16}d + c^{17} - c^6d^{11} - c^7d^{10} + 5c^8d^9 + 5c^9d^8 - 10c^{10}d^7 - 10c^{11}d^6 + 10c^{12}d^5 + 10c^{13}d^4 - 5c^{14}d^3 - 5c^{15}d^2) - (((8(4a^3c^{21} + 2b^3c^{21} + 12a^2b^3c^{21} - 16a^3c^{20}d - 2b^3c^{20}d - 4a^3c^8d^{13} + 2a^3c^9d^{12} + 26a^3c^{10}d^{11} - 14a^3c^{11}d^{10} - 70a^3c^{12}d^9 + 30a^3c^{13}d^8 + 110a^3c^{14}d^7 - 30a^3c^{15}d^6 - 110a^3c^{16}d^5 + 20a^3c^{17}d^4 + 64a^3c^{18}d^3 - 12a^3c^{19}d^2 + 8b^3c^{12}d^9 - 8b^3c^{13}d^8 - 22b^3c^{14}d^7 + 22b^3c^{15}d^6 + 18b^3c^{16}d^5 - 18b^3c^{17}d^4 - 2b^3c^{18}d^3 + 2b^3c^{19}d^2 - 6a^2b^2c^{11}d^{10} + 6a^2b^2c^{12}d^9 - 6a^2b^2c^{13}d^8 + 6a^2b^2c^{14}d^7 + 54a^2b^2c^{15}d^6 - 54a^2b^2c^{16}d^5 - 66a^2b^2c^{17}d^4 + 66a^2b^2c^{18}d^3 + 24a^2b^2c^{19}d^2 + 18a^2b^2c^{12}d^9 - 18a^2b^2c^{13}d^8 - 42a^2b^2c^{14}d^7 + 42a^2b^2c^{15}d^6 + 18a^2b^2c^{16}d^5 - 18a^2b^2c^{17}d^4 + 18a^2b^2c^{18}d^3 - 18a^2b^2c^{19}d^2 - 24a^2b^2c^{20}d - 12a^2b^2c^{20}d)) / (c^{19}d + c^{20} - c^9d^{11} - c^{10}d^{10} + 5c^{11}d^9 + 5c^{12}d^8 - 10c^{13}d^7 - 10c^{14}d^6 + 10c^{15}d^5 + 10c^{16}d^4 - 5c^{17}d^3 - 5c^{18}d^2) + (4\tan(e/2 + (f*x)/2) * ((c + d)^7 * (c - d)^7)^{(1/2)} * (2a^3d^7 + b^3c^7 + 6a^2b^3c^7 - 8a^3c^6d - 7a^3c^2d^5 + 8a^3c^4d^3 + 4b^3c^5d^2 - 3a^2b^2c^4d^3 + 9a^2b^2c^5d^2 - 12a^2b^2c^6d) * (8c^{21}d - 8c^8d^{14} + 8c^9d^{13} + 48c^{10}d^{12} - 48c^{11}d^{11} - 120c^{12}d^{10} + 120c^{13}d^9 + 160c^{14}d^8 - 160c^{15}d^7 - 120c^{16}d^6 + 120c^{17}d^5 + 48c^{18}d^4 - 48c^{19}d^3 - 8c^{20}d^2)) / ((c^{18} - c^4d^{14} + 7c^6d^{12} - 21c^8d^{10} + 35c^{10}d^8 - 35c^{12}d^6 + 21c^{14}d^4 - 7c^{16}d^2) * (c^{16}d + c^{17} - c^6d^{11} - c^7d^{10} + 5c^8d^9 + 5c^9d^8 - 10c^{10}d^7 - 10c^{11}d^6 + 10c^{12}d^5 + 10c^{13}d^4 - 5c^{14}d^3 - 5c^{15}d^2)) * ((c + d)^7 * (c - d)^7)^{(1/2)} * (2a^3d^7 + b^3c^7 + 6a^2b^3c^7 - 8a^3c^6d - 7a^3c^2d^5 + 8a^3c^4d^3 + 4b^3c^5d^2 - 3a^2b^2c^4d^3 + 9a^2b^2c^5d^2 - 12a^2b^2c^6d) / (2 * (c^{18} - c^4d^{14} + 7c^6d^{12} - 21c^8d^{10} + 35c^{10}d^8 - 35c^{12}d^6 + 21c^{14}d^4 - 7c^{16}d^2)) * ((c + d)^7 * (c - d)^7)^{(1/2)} * (2a^3d^7 + b^3c^7 + 6a^2b^3c^7 - 8a^3c^6d - 7a^3c^2d^5 + 8a^3c^4d^3 + 4b^3c^5d^2 - 3a^2b^2c^4d^3 + 9a^2b^2c^5d^2 - 12a^2b^2c^6d) * 1) / (2 * (c^{18} - c^4d^{14} + 7c^6d^{12} - 21c^8d^{10} + 35c^{10}d^8 - 35c^{12}d^6 + 21c^{14}d^4 - 7c^{16}d^2)) / ((16(4a^9d^{13} - 12a^8b^3c^{13} - 2a^9c^3d^{12} + 16a^9c^{12}d + a^3b^6c^{13} + 12a^5b^4c^{13} - 2a^6b^3c^{13} + 36a^7b^2c^{13} - 26a^9c^2d^{11} + 11a^9c^3d^{10} + 70a^9c^4d^9 - 34a^9c^5d^8 - 10a^9c^6d^7 + 66a^9c^7d^6 + 110a^9c^8d^5 - 64a^9c^9d^4 - 64a^9c^{10}d^3 + 48a^9c^{11}d^2 - 24a^4b^5c^{12}d - 158a^6b^3c^{12}d + 24a^7b^2c^{12}d + 18a^8b^3c^4d^9 + 18a^8b^3c^5d^8 - 60a^8b^3c^6d^7 - 42a^8b^3c^7d^6 + 42a^8b^3c^8d^5 + 18a^8b^3c^9d^4 - 66a^8b^3c^{10}d^3 + 18a^8b^3c^{11}d^2 + 16a^3b^6c^9d^4 + 8a^3b^6c^{11}d^2 - 24a^4b^5c^8d^5 - 102a^4b^5c^{10}d^3 + 9a^5b^4c^7d^6 + 144a^5b^4c^9d^4 + 210a^5b^4c^{11}d^2 + 8a^6b^3c^4d^9 + 8a^6b^3c^5d^8 - 30a^6b^3c^6d^7 - 22a^6b^3c^7d^6 - 22a^6b^3c^8d^5 + 18a^6b^3c^9d^4 - 298a^6b^3c^{10}d^3 - 2a^6b^3c^{11}d^2 - 6a^7b^2c^3d^{10} - 6a^7b^2c^4d^9 - 6a^7b^2c^6d^7 + 66a^7b^2c^7d^6 + 54a^7b^2c^8d^5 + 3a^7b^2c^9d^4 - 66a^7b^2c^{10}d^3 + 276a^7b^2c^{11}d^2 - 84a^8b^3c^{12}d)) / (c^{19}d + c^{20} - c^9d^{11} - c^{10}d^{10} + 5c^{11}d^9 + 5c^{12}d^8 - 10c^{13}d^7 - 10c^{14}d^6 + 10c^{15}d^5 + 10c^{16}d^4 - 5c^{17}d^3 - 5c^{18}d^2) - (((8\tan(e/2 + (f*x)/2) * (4a^6c^{14} + 8a^6d^{14} + b^6c^{14} - 8a^6c^3d^{13} - 8a^6c^4d^{13} + 12a^2b^4c^{14} + 36a^4b^2c^{14} - 48a^6c^2d^{12} + 48a^6c^3d^{11} + 117a^6c^4d^{10} - 120a^6c^5d^9 - 164a^6c^6d^8 + 160a^6c^7d^7 + 156a^6c^8d^6 - 120a^6c^9d^5 - 92a^6c^{10}d^4 + 48a^6c^{11}d^3 + 44a^6c^{12}d^2 + 16b^6c^{10}d^4 + 8b^6c^{12}d^2 - 24a^5b^5c^9d^5 -
\end{aligned}$$

$$\begin{aligned}
&^5 - 102*a*b^5*c^{11}*d^3 - 160*a^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a^5*b \\
&*c^7*d^7 + 60*a^5*b*c^9*d^5 - 48*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a \\
&^2*b^4*c^{10}*d^4 + 210*a^2*b^4*c^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^ \\
&7*d^7 - 4*a^3*b^3*c^9*d^5 - 300*a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} - 6* \\
&a^4*b^2*c^6*d^8 + 120*a^4*b^2*c^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c \\
&^{12}*d^2 - 24*a*b^5*c^{13}*d - 96*a^5*b*c^{13}*d))/ (c^{16}*d + c^{17} - c^6*d^{11} - c \\
&^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + \\
&10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2) + (((8*(4*a^3*c^{21} + 2*b^3*c^{21} + 1 \\
&2*a^2*b*c^{21} - 16*a^3*c^{20}*d - 2*b^3*c^{20}*d - 4*a^3*c^8*d^{13} + 2*a^3*c^9*d^ \\
&12 + 26*a^3*c^{10}*d^{11} - 14*a^3*c^{11}*d^{10} - 70*a^3*c^{12}*d^9 + 30*a^3*c^{13}*d^ \\
&8 + 110*a^3*c^{14}*d^7 - 30*a^3*c^{15}*d^6 - 110*a^3*c^{16}*d^5 + 20*a^3*c^{17}*d^4 \\
&+ 64*a^3*c^{18}*d^3 - 12*a^3*c^{19}*d^2 + 8*b^3*c^{12}*d^9 - 8*b^3*c^{13}*d^8 - 22 \\
&*b^3*c^{14}*d^7 + 22*b^3*c^{15}*d^6 + 18*b^3*c^{16}*d^5 - 18*b^3*c^{17}*d^4 - 2*b^3 \\
&*c^{18}*d^3 + 2*b^3*c^{19}*d^2 - 6*a*b^2*c^{11}*d^{10} + 6*a*b^2*c^{12}*d^9 - 6*a*b^2 \\
&*c^{13}*d^8 + 6*a*b^2*c^{14}*d^7 + 54*a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 - 66*a \\
&*b^2*c^{17}*d^4 + 66*a*b^2*c^{18}*d^3 + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12}*d^9 - \\
&18*a^2*b*c^{13}*d^8 - 42*a^2*b*c^{14}*d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b*c^{16} \\
&d^5 - 18*a^2*b*c^{17}*d^4 + 18*a^2*b*c^{18}*d^3 - 18*a^2*b*c^{19}*d^2 - 24*a*b^2* \\
&c^{20}*d - 12*a^2*b*c^{20}*d))/ (c^{19}*d + c^{20} - c^9*d^{11} - c^{10}*d^{10} + 5*c^{11}*d \\
&^9 + 5*c^{12}*d^8 - 10*c^{13}*d^7 - 10*c^{14}*d^6 + 10*c^{15}*d^5 + 10*c^{16}*d^4 - 5 \\
&*c^{17}*d^3 - 5*c^{18}*d^2) - (4*tan(e/2 + (f*x)/2)*(c + d)^7*(c - d)^7)^{(1/2)} \\
&*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c \\
&^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d \\
&)*(8*c^{21}*d - 8*c^8*d^{14} + 8*c^9*d^{13} + 48*c^{10}*d^{12} - 48*c^{11}*d^{11} - 120*c \\
&^{12}*d^{10} + 120*c^{13}*d^9 + 160*c^{14}*d^8 - 160*c^{15}*d^7 - 120*c^{16}*d^6 + 120* \\
&c^{17}*d^5 + 48*c^{18}*d^4 - 48*c^{19}*d^3 - 8*c^{20}*d^2))/ ((c^{18} - c^4*d^{14} + 7*c \\
&^6*d^{12} - 21*c^8*d^{10} + 35*c^{10}*d^8 - 35*c^{12}*d^6 + 21*c^{14}*d^4 - 7*c^{16}*d^ \\
&2)*(c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^{10}*d \\
&^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5*c^{15}*d^2))) * \\
&(c + d)^7*(c - d)^7)^{(1/2)}*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d \\
&- 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2* \\
&b*c^5*d^2 - 12*a*b^2*c^6*d))/ (2*(c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} \\
&+ 35*c^{10}*d^8 - 35*c^{12}*d^6 + 21*c^{14}*d^4 - 7*c^{16}*d^2))) * ((c + d)^7*(c - \\
&d)^7)^{(1/2)}*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 \\
&+ 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12* \\
&a*b^2*c^6*d))/ (2*(c^{18} - c^4*d^{14} + 7*c^6*d^{12} - 21*c^8*d^{10} + 35*c^{10}*d^8 \\
&- 35*c^{12}*d^6 + 21*c^{14}*d^4 - 7*c^{16}*d^2)) + (((8*tan(e/2 + (f*x)/2)*(4*a^6 \\
&*c^{14} + 8*a^6*d^{14} + b^6*c^{14} - 8*a^6*c*d^{13} - 8*a^6*c^{13}*d + 12*a^2*b^4*c^ \\
&14 + 36*a^4*b^2*c^{14} - 48*a^6*c^2*d^{12} + 48*a^6*c^3*d^{11} + 117*a^6*c^4*d^{10} \\
&- 120*a^6*c^5*d^9 - 164*a^6*c^6*d^8 + 160*a^6*c^7*d^7 + 156*a^6*c^8*d^6 - \\
&120*a^6*c^9*d^5 - 92*a^6*c^{10}*d^4 + 48*a^6*c^{11}*d^3 + 44*a^6*c^{12}*d^2 + 16* \\
&b^6*c^{10}*d^4 + 8*b^6*c^{12}*d^2 - 24*a*b^5*c^9*d^5 - 102*a*b^5*c^{11}*d^3 - 160 \\
&*a^3*b^3*c^{13}*d + 36*a^5*b*c^5*d^9 - 102*a^5*b*c^7*d^7 + 60*a^5*b*c^9*d^5 - \\
&48*a^5*b*c^{11}*d^3 + 9*a^2*b^4*c^8*d^6 + 144*a^2*b^4*c^{10}*d^4 + 210*a^2*b^4 \\
&*c^{12}*d^2 + 16*a^3*b^3*c^5*d^9 - 52*a^3*b^3*c^7*d^7 - 4*a^3*b^3*c^9*d^5 - 3 \\
&00*a^3*b^3*c^{11}*d^3 - 12*a^4*b^2*c^4*d^{10} - 6*a^4*b^2*c^6*d^8 + 120*a^4*b^2 \\
&*c^8*d^6 - 63*a^4*b^2*c^{10}*d^4 + 300*a^4*b^2*c^{12}*d^2 - 24*a*b^5*c^{13}*d - 9 \\
&6*a^5*b*c^{13}*d))/ (c^{16}*d + c^{17} - c^6*d^{11} - c^7*d^{10} + 5*c^8*d^9 + 5*c^9*d \\
&^8 - 10*c^{10}*d^7 - 10*c^{11}*d^6 + 10*c^{12}*d^5 + 10*c^{13}*d^4 - 5*c^{14}*d^3 - 5 \\
&*c^{15}*d^2) - (((8*(4*a^3*c^{21} + 2*b^3*c^{21} + 12*a^2*b*c^{21} - 16*a^3*c^{20}*d \\
&- 2*b^3*c^{20}*d - 4*a^3*c^8*d^{13} + 2*a^3*c^9*d^{12} + 26*a^3*c^{10}*d^{11} - 14*a^ \\
&3*c^{11}*d^{10} - 70*a^3*c^{12}*d^9 + 30*a^3*c^{13}*d^8 + 110*a^3*c^{14}*d^7 - 30*a^3 \\
&*c^{15}*d^6 - 110*a^3*c^{16}*d^5 + 20*a^3*c^{17}*d^4 + 64*a^3*c^{18}*d^3 - 12*a^3*c \\
&^{19}*d^2 + 8*b^3*c^{12}*d^9 - 8*b^3*c^{13}*d^8 - 22*b^3*c^{14}*d^7 + 22*b^3*c^{15} \\
&d^6 + 18*b^3*c^{16}*d^5 - 18*b^3*c^{17}*d^4 - 2*b^3*c^{18}*d^3 + 2*b^3*c^{19}*d^2 - \\
&6*a*b^2*c^{11}*d^{10} + 6*a*b^2*c^{12}*d^9 - 6*a*b^2*c^{13}*d^8 + 6*a*b^2*c^{14}*d^7 \\
&+ 54*a*b^2*c^{15}*d^6 - 54*a*b^2*c^{16}*d^5 - 66*a*b^2*c^{17}*d^4 + 66*a*b^2*c^{18} \\
&*d^3 + 24*a*b^2*c^{19}*d^2 + 18*a^2*b*c^{12}*d^9 - 18*a^2*b*c^{13}*d^8 - 42*a^2*b \\
&*c^{14}*d^7 + 42*a^2*b*c^{15}*d^6 + 18*a^2*b*c^{16}*d^5 - 18*a^2*b*c^{17}*d^4 + 18*
\end{aligned}$$

```

a^2*b*c^18*d^3 - 18*a^2*b*c^19*d^2 - 24*a*b^2*c^20*d - 12*a^2*b*c^20*d))/(c
^19*d + c^20 - c^9*d^11 - c^10*d^10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7
- 10*c^14*d^6 + 10*c^15*d^5 + 10*c^16*d^4 - 5*c^17*d^3 - 5*c^18*d^2) + (4*
tan(e/2 + (f*x)/2)*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2
*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*
b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)*(8*c^21*d - 8*c^8*d^14 + 8*
c^9*d^13 + 48*c^10*d^12 - 48*c^11*d^11 - 120*c^12*d^10 + 120*c^13*d^9 + 160
*c^14*d^8 - 160*c^15*d^7 - 120*c^16*d^6 + 120*c^17*d^5 + 48*c^18*d^4 - 48*c
^19*d^3 - 8*c^20*d^2))/((c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^
10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2))*(c^16*d + c^17 - c^6*d^11
- c^7*d^10 + 5*c^8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^
5 + 10*c^13*d^4 - 5*c^14*d^3 - 5*c^15*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2
*a^3*d^7 + b^3*c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*
d^3 + 4*b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d))/
(2*(c^18 - c^4*d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6
+ 21*c^14*d^4 - 7*c^16*d^2)))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*
c^7 + 6*a^2*b*c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5
*d^2 - 3*a*b^2*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d))/(2*(c^18 - c^4*
d^14 + 7*c^6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 -
7*c^16*d^2)))))*((c + d)^7*(c - d)^7)^(1/2)*(2*a^3*d^7 + b^3*c^7 + 6*a^2*b*
c^7 - 8*a^3*c^6*d - 7*a^3*c^2*d^5 + 8*a^3*c^4*d^3 + 4*b^3*c^5*d^2 - 3*a*b^2
*c^4*d^3 + 9*a^2*b*c^5*d^2 - 12*a*b^2*c^6*d)*1i)/(f*(c^18 - c^4*d^14 + 7*c^
6*d^12 - 21*c^8*d^10 + 35*c^10*d^8 - 35*c^12*d^6 + 21*c^14*d^4 - 7*c^16*d^2
))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))*3/(c+d*sec(f*x+e))*4,x)

[Out] Integral((a + b*sec(e + f*x))*3/(c + d*sec(e + f*x))*4, x)

3.197 $\int \frac{(a+b \sec(e+fx))^3}{(c+d \sec(e+fx))^5} dx$

Optimal. Leaf size=622

$$\frac{a^3 x (bc - ad) \left(-a^2 d^2 (58c^4 - 35c^2 d^2 + 12d^4) + 2abcd (32c^4 + c^2 d^2 + 2d^4) - (b^2 (12c^6 + 25c^4 d^2 - 2c^2 d^4)) \right) \sin(e+fx)}{c^5 24c^4 f (c^2 - d^2)^3 (c \cos(e+fx) + d)^2}$$

```
[Out] a^3*x/c^5+1/4*d^2*(b+a*cos(f*x+e))^3*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))^4-1/12*d*(-11*a*c^2*d+4*a*d^3+8*b*c^3-b*c*d^2)*(b+a*cos(f*x+e))^2*sin(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^3-1/24*(-a*d+b*c)*(2*a*b*c*d*(32*c^4+c^2*d^2+2*d^4)-a^2*d^2*(58*c^4-35*c^2*d^2+12*d^4)-b^2*(12*c^6+25*c^4*d^2-2*c^2*d^4))*sin(f*x+e)/c^4/(c^2-d^2)^3/f/(d+c*cos(f*x+e))^2-1/24*(b^3*c^3*d*(68*c^4+39*c^2*d^2-2*d^4)+a^2*b*c*d*(272*c^6+10*c^4*d^2+49*c^2*d^4-16*d^6)-3*a*b^2*c^2*(24*c^6+84*c^4*d^2-5*c^2*d^4+2*d^6)-a^3*(212*c^6*d^2-210*c^4*d^4+139*c^2*d^6-36*d^8))*sin(f*x+e)/c^4/(c^2-d^2)^4/f/(d+c*cos(f*x+e))-1/4*(15*a*b^2*c^6*d*(4*c^2+3*d^2)-3*a^2*b*c^5*(8*c^4+24*c^2*d^2+3*d^4)-b^3*c^5*(4*c^4+27*c^2*d^2+4*d^4)+a^3*(40*c^8*d-40*c^6*d^3+63*c^4*d^5-36*c^2*d^7+8*d^9))*arctanh((c-d)^(1/2)*tan(1/2*e+1/2*f*x)/(c+d)^(1/2))/c^5/(c^2-d^2)^4/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 1.77, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3941, 3048, 3047, 3031, 3021, 2735, 2659, 208}

$$\frac{(a^2bcd(10c^4d^2 + 49c^2d^4 + 272c^6 - 16d^6) + a^3(- (139c^2d^6 - 210c^4d^4 + 212c^6d^2 - 36d^8)) - 3ab^2c^2(84c^4d^2 - 24c^2d^4 + 2d^6)) \sin(e+fx)}{24c^4 f (c^2 - d^2)^4 (c \cos(e+fx) + d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5, x]
```

```
[Out] (a^3*x)/c^5 - ((15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^2*d^2 + 3*d^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*c^6*d^3 + 63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]]/(4*c^5*Sqrt[c - d]*Sqrt[c + d]*(c^2 - d^2)^4*f) + (d^2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^4) - (d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(12*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])^3) - ((b*c - a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*c^2*d^2 + 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(24*c^4*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x])^2) - ((b^3*c^3*d*(68*c^4 + 39*c^2*d^2 - 2*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^4 - 16*d^6) - 3*a*b^2*c^2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*(212*c^6*d^2 - 210*c^4*d^4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/(24*c^4*(c^2 - d^2)^4*f*(d + c*Cos[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

$a - b)e^{2x^2}$, x , $\tan[(c + dx)/2]/e$, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

$\text{Int}[(a + b \sin(e + f x)) / (c + d \sin(e + f x)) x, x] \text{Symbol} \rightarrow \text{Simp}[b x / d, x] - \text{Dist}[(b c - a d) / d, \text{Int}[1 / (c + d \sin(e + f x))], x, x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b c - a d, 0]$

Rule 3021

$\text{Int}[(a + b \sin(e + f x))^m ((A + B \sin(e + f x)) + (C \sin(e + f x))^2) x, x] \text{Symbol} \rightarrow -\text{Simp}[(A b^2 - a b B + a^2 C) \cos(e + f x) (a + b \sin(e + f x))^{m+1} / (b^2 f (m+1) (a^2 - b^2)), x] + \text{Dist}[1 / (b (m+1) (a^2 - b^2)), \text{Int}[(a + b \sin(e + f x))^{m+1} \text{Simp}[b (a A - b B + a C) (m+1) - (A b^2 - a b B + a^2 C + b (A b - a B + b C) (m+1)) \sin(e + f x)], x, x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3031

$\text{Int}[(a + b \sin(e + f x))^m ((c + d \sin(e + f x)) + (f x))^n ((A + B \sin(e + f x)) + (C \sin(e + f x))^2) x, x] \text{Symbol} \rightarrow -\text{Simp}[(b c - a d) (A b^2 - a b B + a^2 C) \cos(e + f x) (a + b \sin(e + f x))^{m+1} / (b^2 f (m+1) (a^2 - b^2)), x] - \text{Dist}[1 / (b^2 (m+1) (a^2 - b^2)), \text{Int}[(a + b \sin(e + f x))^{m+1} \text{Simp}[b (m+1) ((b B - a C) (b c - a d) - A b (a c - b d)) + (b B (a^2 d + b^2 d (m+1) - a b c (m+2)) + (b c - a d) (A b^2 (m+2) + C (a^2 + b^2 (m+1)))] \sin(e + f x) - b C d (m+1) (a^2 - b^2) \sin^2(e + f x)], x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b c - a d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3047

$\text{Int}[(a + b \sin(e + f x))^m ((c + d \sin(e + f x)) + (f x))^n ((A + B \sin(e + f x)) + (C \sin(e + f x)) + (f x))^2) x, x] \text{Symbol} \rightarrow -\text{Simp}[(c^2 C - B c d + A d^2) \cos(e + f x) (a + b \sin(e + f x))^m (c + d \sin(e + f x))^{n+1} / (d f (n+1) (c^2 - d^2)), x] + \text{Dist}[1 / (d (n+1) (c^2 - d^2)), \text{Int}[(a + b \sin(e + f x))^{m-1} (c + d \sin(e + f x))^{n+1} \text{Simp}[A d (b d m + a c (n+1)) + (c C - B d) (b c m + a d (n+1)) - (d (A (a d (n+2) - b c (n+1)) + B (b d (n+1) - a c (n+2))) - C (b c d (n+1) - a (c^2 + d^2 (n+1)))] \sin(e + f x) + b (d (B c - A d) (m + n + 2) - C (c^2 (m+1) + d^2 (n+1))) \sin^2(e + f x)], x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b c - a d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{LtQ}[n, -1]$

Rule 3048

$\text{Int}[(a + b \sin(e + f x))^m ((c + d \sin(e + f x)) + (f x))^n ((A + C \sin(e + f x))^2) x, x] \text{Symbol} \rightarrow -\text{Simp}[(c^2 C + A d^2) \cos(e + f x) (a + b \sin(e + f x))^m (c + d \sin(e + f x))^{n+1} / (d f (n+1) (c^2 - d^2)), x] + \text{Dist}[1 / (d (n+1) (c^2 - d^2)), \text{Int}[(a + b \sin(e + f x))^{m-1} (c + d \sin(e + f x))^{n+1} \text{Simp}[A d (b d m + a c (n+1)) + c C (b c m + a d (n+1)) - (A d (a d (n+2) - b c (n+1)) - C (b c d (n+1) - a (c^2 + d^2 (n+1)))] \sin(e + f x) - b (A d^2 (m + n + 2) + C (c^2 (m+1) + d^2 (n+1))) \sin^2(e + f x)], x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C\}, x] \&\& \text{NeQ}[b c - a d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{LtQ}[n, -1]$

Rule 3941

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]

Rubi steps

$$\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^5} dx = \int \frac{\cos^2(e + fx)(b + a \cos(e + fx))^3}{(d + c \cos(e + fx))^5} dx$$

$$= \frac{d^2(b + a \cos(e + fx))^3 \sin(e + fx)}{4c(c^2 - d^2) f(d + c \cos(e + fx))^4} + \frac{\int \frac{(b + a \cos(e + fx))^2(-d(4bc - 3ad) + (4bc^2 - 4acd - bd^2) \cos(e + fx))}{(d + c \cos(e + fx))^4} dx}{4c(c^2 - d^2)}$$

$$= \frac{d^2(b + a \cos(e + fx))^3 \sin(e + fx)}{4c(c^2 - d^2) f(d + c \cos(e + fx))^4} - \frac{d(8bc^3 - 11ac^2d - bcd^2 + 4ad^3)(b + a \cos(e + fx))}{12c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^3}$$

$$= \frac{d^2(b + a \cos(e + fx))^3 \sin(e + fx)}{4c(c^2 - d^2) f(d + c \cos(e + fx))^4} - \frac{d(8bc^3 - 11ac^2d - bcd^2 + 4ad^3)(b + a \cos(e + fx))}{12c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^3}$$

$$= \frac{d^2(b + a \cos(e + fx))^3 \sin(e + fx)}{4c(c^2 - d^2) f(d + c \cos(e + fx))^4} - \frac{d(8bc^3 - 11ac^2d - bcd^2 + 4ad^3)(b + a \cos(e + fx))}{12c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^3}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b + a \cos(e + fx))^3 \sin(e + fx)}{4c(c^2 - d^2) f(d + c \cos(e + fx))^4} - \frac{d(8bc^3 - 11ac^2d - bcd^2 + 4ad^3)(b + a \cos(e + fx))}{12c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^3}$$

$$= \frac{a^3x}{c^5} + \frac{d^2(b + a \cos(e + fx))^3 \sin(e + fx)}{4c(c^2 - d^2) f(d + c \cos(e + fx))^4} - \frac{d(8bc^3 - 11ac^2d - bcd^2 + 4ad^3)(b + a \cos(e + fx))}{12c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^3}$$

$$= \frac{a^3x}{c^5} - \frac{(15ab^2c^6d(4c^2 + 3d^2) - 3a^2bc^5(8c^4 + 24c^2d^2 + 3d^4) - b^3c^5(4c^4 + 27c^2d^2 + 3d^4))}{4c^5\sqrt{c^2 - d^2} f(d + c \cos(e + fx))^3}$$

Mathematica [B] time = 6.93, size = 1285, normalized size = 2.07

$$\frac{a^3(e + fx) \sec^2(e + fx)(a + b \sec(e + fx))^3(d + c \cos(e + fx))^5}{c^5 f(b + a \cos(e + fx))^3(c + d \sec(e + fx))^5} + \frac{(-4b^3c^9 - 24a^2bc^9 + 40a^3dc^8 + 60ab^2dc^8 - 24a^4c^8 - 4b^4c^8 + 40a^3b^2c^8d - 72a^2b^3c^8d^2 - 27b^4c^8d^3 - 40a^3b^3c^8d^3 + 45a^2b^4c^8d^3 - 9a^2b^5c^8d^4 - 4b^4b^3c^8d^4 + 63a^3b^3c^8d^5 - 36a^3b^3c^8d^7 + 8a^3b^3d^9) \operatorname{ArcTanh}((c + d) \tan((e + fx)/2))}{\sqrt{c^2 - d^2} (d + c \cos(e + fx))^5 \sec^2(e + fx) (a + b \sec(e + fx))^3} / (4c^5 \sqrt{c^2 - d^2} (-c^2 + d^2)^4 f(b + a \cos(e + fx))^3)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]

[Out] (a^3*(e + f*x)*(d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3)/(c^5*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5) + ((-24*a^2*b*c^9 - 4*b^3*c^9 + 40*a^3*c^8*d + 60*a*b^2*c^8*d - 72*a^2*b*c^7*d^2 - 27*b^3*c^7*d^2 - 40*a^3*c^6*d^3 + 45*a*b^2*c^6*d^3 - 9*a^2*b*c^5*d^4 - 4*b^3*c^5*d^4 + 63*a^3*c^4*d^5 - 36*a^3*c^2*d^7 + 8*a^3*d^9)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3)/(4*c^5*Sqrt[c^2 - d^2]*(-c^2 + d^2)^4*f*(b + a*Cos[e + f*x])^3)

$$\begin{aligned}
& c + d*\text{Sec}[e + f*x])^5) + ((d + c*\text{Cos}[e + f*x])* \text{Sec}[e + f*x]^2*(a + b*\text{Sec}[e \\
& + f*x])^3*(b^3*c^3*d^2*\text{Sin}[e + f*x] - 3*a*b^2*c^2*d^3*\text{Sin}[e + f*x] + 3*a^2* \\
& b*c*d^4*\text{Sin}[e + f*x] - a^3*d^5*\text{Sin}[e + f*x]))/(4*c^4*(c^2 - d^2)*f*(b + a*\text{Cos} \\
& \text{os}[e + f*x])^3*(c + d*\text{Sec}[e + f*x])^5) + ((d + c*\text{Cos}[e + f*x])^2*\text{Sec}[e + f* \\
& x]^2*(a + b*\text{Sec}[e + f*x])^3*(-8*b^3*c^5*d*\text{Sin}[e + f*x] + 36*a*b^2*c^4*d^2*\text{S} \\
& \text{in}[e + f*x] - 48*a^2*b*c^3*d^3*\text{Sin}[e + f*x] + b^3*c^3*d^3*\text{Sin}[e + f*x] + 20 \\
& *a^3*c^2*d^4*\text{Sin}[e + f*x] - 15*a*b^2*c^2*d^4*\text{Sin}[e + f*x] + 27*a^2*b*c*d^5* \\
& \text{Sin}[e + f*x] - 13*a^3*d^6*\text{Sin}[e + f*x]))/(12*c^4*(c^2 - d^2)^2*f*(b + a*\text{Cos} \\
& [e + f*x])^3*(c + d*\text{Sec}[e + f*x])^5) + ((d + c*\text{Cos}[e + f*x])^3*\text{Sec}[e + f*x] \\
& ^2*(a + b*\text{Sec}[e + f*x])^3*(12*b^3*c^7*\text{Sin}[e + f*x] - 108*a*b^2*c^6*d*\text{Sin}[e \\
& + f*x] + 216*a^2*b*c^5*d^2*\text{Sin}[e + f*x] + 25*b^3*c^5*d^2*\text{Sin}[e + f*x] - 120 \\
& *a^3*c^4*d^3*\text{Sin}[e + f*x] + 9*a*b^2*c^4*d^3*\text{Sin}[e + f*x] - 165*a^2*b*c^3*d^ \\
& 4*\text{Sin}[e + f*x] - 2*b^3*c^3*d^4*\text{Sin}[e + f*x] + 131*a^3*c^2*d^5*\text{Sin}[e + f*x] \\
& - 6*a*b^2*c^2*d^5*\text{Sin}[e + f*x] + 54*a^2*b*c*d^6*\text{Sin}[e + f*x] - 46*a^3*d^7*\text{S} \\
& \text{in}[e + f*x]))/(24*c^4*(c^2 - d^2)^3*f*(b + a*\text{Cos}[e + f*x])^3*(c + d*\text{Sec}[e + \\
& f*x])^5) + ((d + c*\text{Cos}[e + f*x])^4*\text{Sec}[e + f*x]^2*(a + b*\text{Sec}[e + f*x])^3*(\\
& 72*a*b^2*c^8*\text{Sin}[e + f*x] - 288*a^2*b*c^7*d*\text{Sin}[e + f*x] - 68*b^3*c^7*d*\text{Sin} \\
& [e + f*x] + 240*a^3*c^6*d^2*\text{Sin}[e + f*x] + 252*a*b^2*c^6*d^2*\text{Sin}[e + f*x] + \\
& 24*a^2*b*c^5*d^3*\text{Sin}[e + f*x] - 39*b^3*c^5*d^3*\text{Sin}[e + f*x] - 280*a^3*c^4* \\
& d^4*\text{Sin}[e + f*x] - 15*a*b^2*c^4*d^4*\text{Sin}[e + f*x] - 69*a^2*b*c^3*d^5*\text{Sin}[e + \\
& f*x] + 2*b^3*c^3*d^5*\text{Sin}[e + f*x] + 195*a^3*c^2*d^6*\text{Sin}[e + f*x] + 6*a*b^2 \\
& *c^2*d^6*\text{Sin}[e + f*x] + 18*a^2*b*c*d^7*\text{Sin}[e + f*x] - 50*a^3*d^8*\text{Sin}[e + f* \\
& x]))/(24*c^4*(c^2 - d^2)^4*f*(b + a*\text{Cos}[e + f*x])^3*(c + d*\text{Sec}[e + f*x])^5)
\end{aligned}$$

fricas [B] time = 1.13, size = 4346, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fricas")

[Out] [1/48*(48*(a^3*c^14 - 5*a^3*c^12*d^2 + 10*a^3*c^10*d^4 - 10*a^3*c^8*d^6 + 5*a^3*c^6*d^8 - a^3*c^4*d^10)*f*x*cos(f*x + e)^4 + 192*(a^3*c^13*d - 5*a^3*c^11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^11)*f*x*cos(f*x + e)^3 + 288*(a^3*c^12*d^2 - 5*a^3*c^10*d^4 + 10*a^3*c^8*d^6 - 10*a^3*c^6*d^8 + 5*a^3*c^4*d^10 - a^3*c^2*d^12)*f*x*cos(f*x + e)^2 + 192*(a^3*c^11*d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^11 - a^3*c*d^13)*f*x*cos(f*x + e) + 48*(a^3*c^10*d^4 - 5*a^3*c^8*d^6 + 10*a^3*c^6*d^8 - 10*a^3*c^4*d^10 + 5*a^3*c^2*d^12 - a^3*d^14)*f*x + 3*(63*a^3*c^4*d^9 - 36*a^3*c^2*d^11 + 8*a^3*d^13 - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a^3 + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c^6*d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a^3*c^4*d^9 - 4*(6*a^2*b + b^3)*c^13 + 20*(2*a^3 + 3*a*b^2)*c^12*d - 9*(8*a^2*b + 3*b^3)*c^11*d^2 - 5*(8*a^3 - 9*a*b^2)*c^10*d^3 - (9*a^2*b + 4*b^3)*c^9*d^4)*cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^10 - 4*(6*a^2*b + b^3)*c^12*d + 20*(2*a^3 + 3*a*b^2)*c^11*d^2 - 9*(8*a^2*b + 3*b^3)*c^10*d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*cos(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^11 - 4*(6*a^2*b + b^3)*c^11*d^2 + 20*(2*a^3 + 3*a*b^2)*c^10*d^3 - 9*(8*a^2*b + 3*b^3)*c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*cos(f*x + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^10 + 8*a^3*c*d^12 - 4*(6*a^2*b + b^3)*c^10*d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^5 - 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^3*c^12*d^2 + 18*a*b^2*c^11*d^3 - 116*a^3*c^3*d^11 + 24*a^3*c*d^13 - (150*a^2*b + 41*b^3)*c^10*d^4 + 77*(2*a^3 + 3*a*b^2)*c^9*d^5 - (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a*b^2)*c^7*d^7 + (165*a^2*b + 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9 + (72*a*b^2*c^14 - 18*a^2*b*c^5*d^9 + 50*a^3*c^4*d^10 - 4*(72*a^2*b + 17*b

$$\begin{aligned}
& ^3)*c^{13}d + 60*(4*a^3 + 3*a*b^2)*c^{12}d^2 + (312*a^2*b + 29*b^3)*c^{11}d^3 \\
& - (520*a^3 + 267*a*b^2)*c^{10}d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 + \\
& 21*a*b^2)*c^8*d^6 + (87*a^2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d^8 \\
& ^8)*\cos(f*x + e)^3 + (12*b^3*c^{14} + 108*a*b^2*c^{13}d + 104*a^3*c^3*d^{11} - (\\
& 648*a^2*b + 203*b^3)*c^{12}d^2 + 15*(40*a^3 + 51*a*b^2)*c^{11}d^3 + (339*a^2*b \\
& b + 47*b^3)*c^{10}d^4 - (1189*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^ \\
& 3)*c^8*d^6 + (997*a^3 + 84*a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8 \\
& *(64*a^3 + 3*a*b^2)*c^5*d^9)*\cos(f*x + e)^2 + (8*b^3*c^{13}d + 72*a*b^2*c^{12} \\
& *d^2 - 407*a^3*c^4*d^{10} + 84*a^3*c^2*d^{12} - 8*(66*a^2*b + 19*b^3)*c^{11}d^3 \\
& + 8*(65*a^3 + 93*a*b^2)*c^{10}d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 + \\
& 759*a*b^2)*c^8*d^6 + (471*a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)* \\
& c^6*d^8 - 3*(9*a^2*b + 4*b^3)*c^5*d^9)*\cos(f*x + e))*\sin(f*x + e))/((c^{19} - \\
& 5*c^{17}d^2 + 10*c^{15}d^4 - 10*c^{13}d^6 + 5*c^{11}d^8 - c^9*d^{10})*f*\cos(f*x \\
& + e)^4 + 4*(c^{18}d - 5*c^{16}d^3 + 10*c^{14}d^5 - 10*c^{12}d^7 + 5*c^{10}d^9 - \\
& c^8*d^{11})*f*\cos(f*x + e)^3 + 6*(c^{17}d^2 - 5*c^{15}d^4 + 10*c^{13}d^6 - 10*c^ \\
& 11*d^8 + 5*c^9*d^{10} - c^7*d^{12})*f*\cos(f*x + e)^2 + 4*(c^{16}d^3 - 5*c^{14}d^5 \\
& + 10*c^{12}d^7 - 10*c^{10}d^9 + 5*c^8*d^{11} - c^6*d^{13})*f*\cos(f*x + e) + (c^{1 \\
& 5}d^4 - 5*c^{13}d^6 + 10*c^{11}d^8 - 10*c^9*d^{10} + 5*c^7*d^{12} - c^5*d^{14})*f), \\
& 1/24*(24*(a^3*c^{14} - 5*a^3*c^{12}d^2 + 10*a^3*c^{10}d^4 - 10*a^3*c^8*d^6 + 5 \\
& *a^3*c^6*d^8 - a^3*c^4*d^{10})*f*x*\cos(f*x + e)^4 + 96*(a^3*c^{13}d - 5*a^3*c^ \\
& 11*d^3 + 10*a^3*c^9*d^5 - 10*a^3*c^7*d^7 + 5*a^3*c^5*d^9 - a^3*c^3*d^{11})*f* \\
& x*\cos(f*x + e)^3 + 144*(a^3*c^{12}d^2 - 5*a^3*c^{10}d^4 + 10*a^3*c^8*d^6 - 10 \\
& *a^3*c^6*d^8 + 5*a^3*c^4*d^{10} - a^3*c^2*d^{12})*f*x*\cos(f*x + e)^2 + 96*(a^3* \\
& c^{11}d^3 - 5*a^3*c^9*d^5 + 10*a^3*c^7*d^7 - 10*a^3*c^5*d^9 + 5*a^3*c^3*d^{11} \\
& - a^3*c*d^{13})*f*x*\cos(f*x + e) + 24*(a^3*c^{10}d^4 - 5*a^3*c^8*d^6 + 10*a^3 \\
& *c^6*d^8 - 10*a^3*c^4*d^{10} + 5*a^3*c^2*d^{12} - a^3*d^{14})*f*x - 3*(63*a^3*c^4 \\
& *d^9 - 36*a^3*c^2*d^{11} + 8*a^3*d^{13} - 4*(6*a^2*b + b^3)*c^9*d^4 + 20*(2*a^3 \\
& + 3*a*b^2)*c^8*d^5 - 9*(8*a^2*b + 3*b^3)*c^7*d^6 - 5*(8*a^3 - 9*a*b^2)*c^6 \\
& *d^7 - (9*a^2*b + 4*b^3)*c^5*d^8 + (63*a^3*c^8*d^5 - 36*a^3*c^6*d^7 + 8*a^3 \\
& *c^4*d^9 - 4*(6*a^2*b + b^3)*c^{13} + 20*(2*a^3 + 3*a*b^2)*c^{12}d - 9*(8*a^2* \\
& b + 3*b^3)*c^{11}d^2 - 5*(8*a^3 - 9*a*b^2)*c^{10}d^3 - (9*a^2*b + 4*b^3)*c^9* \\
& d^4)*\cos(f*x + e)^4 + 4*(63*a^3*c^7*d^6 - 36*a^3*c^5*d^8 + 8*a^3*c^3*d^{10} - \\
& 4*(6*a^2*b + b^3)*c^{12}d + 20*(2*a^3 + 3*a*b^2)*c^{11}d^2 - 9*(8*a^2*b + 3* \\
& b^3)*c^{10}d^3 - 5*(8*a^3 - 9*a*b^2)*c^9*d^4 - (9*a^2*b + 4*b^3)*c^8*d^5)*co \\
& s(f*x + e)^3 + 6*(63*a^3*c^6*d^7 - 36*a^3*c^4*d^9 + 8*a^3*c^2*d^{11} - 4*(6*a \\
& ^2*b + b^3)*c^{11}d^2 + 20*(2*a^3 + 3*a*b^2)*c^{10}d^3 - 9*(8*a^2*b + 3*b^3)* \\
& c^9*d^4 - 5*(8*a^3 - 9*a*b^2)*c^8*d^5 - (9*a^2*b + 4*b^3)*c^7*d^6)*\cos(f*x \\
& + e)^2 + 4*(63*a^3*c^5*d^8 - 36*a^3*c^3*d^{10} + 8*a^3*c*d^{12} - 4*(6*a^2*b + \\
& b^3)*c^{10}d^3 + 20*(2*a^3 + 3*a*b^2)*c^9*d^4 - 9*(8*a^2*b + 3*b^3)*c^8*d^5 \\
& - 5*(8*a^3 - 9*a*b^2)*c^7*d^6 - (9*a^2*b + 4*b^3)*c^6*d^7)*\cos(f*x + e))*sq \\
& rt(-c^2 + d^2)*\arctan(-\sqrt{-c^2 + d^2}*(d*\cos(f*x + e) + c))/((c^2 - d^2)*s \\
& in(f*x + e))) + (2*b^3*c^{12}d^2 + 18*a*b^2*c^{11}d^3 - 116*a^3*c^3*d^{11} + 24 \\
& *a^3*c*d^{13} - (150*a^2*b + 41*b^3)*c^{10}d^4 + 77*(2*a^3 + 3*a*b^2)*c^9*d^5 \\
& - (15*a^2*b + 29*b^3)*c^8*d^6 - (271*a^3 + 201*a*b^2)*c^7*d^7 + (165*a^2*b \\
& + 68*b^3)*c^6*d^8 + (209*a^3 - 48*a*b^2)*c^5*d^9 + (72*a*b^2*c^{14} - 18*a^2* \\
& b*c^5*d^9 + 50*a^3*c^4*d^{10} - 4*(72*a^2*b + 17*b^3)*c^{13}d + 60*(4*a^3 + 3* \\
& a*b^2)*c^{12}d^2 + (312*a^2*b + 29*b^3)*c^{11}d^3 - (520*a^3 + 267*a*b^2)*c^{1 \\
& 0}d^4 - (93*a^2*b - 41*b^3)*c^9*d^5 + (475*a^3 + 21*a*b^2)*c^8*d^6 + (87*a^ \\
& 2*b - 2*b^3)*c^7*d^7 - (245*a^3 + 6*a*b^2)*c^6*d^8)*\cos(f*x + e)^3 + (12*b^ \\
& 3*c^{14} + 108*a*b^2*c^{13}d + 104*a^3*c^3*d^{11} - (648*a^2*b + 203*b^3)*c^{12}d \\
& ^2 + 15*(40*a^3 + 51*a*b^2)*c^{11}d^3 + (339*a^2*b + 47*b^3)*c^{10}d^4 - (118 \\
& 9*a^3 + 933*a*b^2)*c^9*d^5 + (321*a^2*b + 152*b^3)*c^8*d^6 + (997*a^3 + 84* \\
& a*b^2)*c^7*d^7 - 4*(3*a^2*b + 2*b^3)*c^6*d^8 - 8*(64*a^3 + 3*a*b^2)*c^5*d^9 \\
&)*\cos(f*x + e)^2 + (8*b^3*c^{13}d + 72*a*b^2*c^{12}d^2 - 407*a^3*c^4*d^{10} + 8 \\
& 4*a^3*c^2*d^{12} - 8*(66*a^2*b + 19*b^3)*c^{11}d^3 + 8*(65*a^3 + 93*a*b^2)*c^1 \\
& 0*d^4 + (84*a^2*b - 47*b^3)*c^9*d^5 - (964*a^3 + 759*a*b^2)*c^8*d^6 + (471* \\
& a^2*b + 203*b^3)*c^7*d^7 + (767*a^3 - 57*a*b^2)*c^6*d^8 - 3*(9*a^2*b + 4*b^ \\
& 3)*c^5*d^9)*\cos(f*x + e))*\sin(f*x + e))/((c^{19} - 5*c^{17}d^2 + 10*c^{15}d^4 - \\
& 10*c^{13}d^6 + 5*c^{11}d^8 - c^9*d^{10})*f*\cos(f*x + e)^4 + 4*(c^{18}d - 5*c^{16}
\end{aligned}$$

$$*d^3 + 10*c^{14}*d^5 - 10*c^{12}*d^7 + 5*c^{10}*d^9 - c^8*d^{11}) * f * \cos(f*x + e)^3 + 6*(c^{17}*d^2 - 5*c^{15}*d^4 + 10*c^{13}*d^6 - 10*c^{11}*d^8 + 5*c^9*d^{10} - c^7*d^{12}) * f * \cos(f*x + e)^2 + 4*(c^{16}*d^3 - 5*c^{14}*d^5 + 10*c^{12}*d^7 - 10*c^{10}*d^9 + 5*c^8*d^{11} - c^6*d^{13}) * f * \cos(f*x + e) + (c^{15}*d^4 - 5*c^{13}*d^6 + 10*c^{11}*d^8 - 10*c^9*d^{10} + 5*c^7*d^{12} - c^5*d^{14}) * f)]$$

giac [B] time = 15.07, size = 3311, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="giac")

[Out] $\frac{1}{12} * (3 * (24 * a^2 * b * c^9 + 4 * b^3 * c^9 - 40 * a^3 * c^8 * d - 60 * a * b^2 * c^8 * d + 72 * a^2 * b * c^7 * d^2 + 27 * b^3 * c^7 * d^2 + 40 * a^3 * c^6 * d^3 - 45 * a * b^2 * c^6 * d^3 + 9 * a^2 * b * c^5 * d^4 + 4 * b^3 * c^5 * d^4 - 63 * a^3 * c^4 * d^5 + 36 * a^3 * c^2 * d^7 - 8 * a^3 * d^9) * (\pi * \operatorname{floor}(\frac{1}{2} * (f * x + e) / \pi + \frac{1}{2}) * \operatorname{sgn}(-2 * c + 2 * d) + \arctan(-\frac{c * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e) - d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)}{\sqrt{-c^2 + d^2}})) / ((c^{13} - 4 * c^{11} * d^2 + 6 * c^9 * d^4 - 4 * c^7 * d^6 + c^5 * d^8) * \sqrt{-c^2 + d^2}) + 12 * (f * x + e) * a^3 / c^5 - (72 * a * b^2 * c^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 12 * b^3 * c^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 28 * a^2 * b * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 108 * a * b^2 * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 60 * b^3 * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 240 * a^3 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 648 * a^2 * b * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 324 * a * b^2 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 189 * b^3 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 600 * a^3 * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 504 * a^2 * b * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 89 * a * b^2 * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 183 * b^3 * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 240 * a^3 * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 459 * a^2 * b * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 801 * a * b^2 * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 183 * b^3 * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 435 * a^3 * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 513 * a^2 * b * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 189 * a * b^2 * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 189 * b^3 * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 249 * a^3 * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 153 * a^2 * b * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 63 * a * b^2 * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 60 * b^3 * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 291 * a^3 * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 45 * a^2 * b * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 72 * a * b^2 * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 12 * b^3 * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 2 * 73 * a^3 * c^3 * d^8 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 12 * a^3 * c^2 * d^9 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 84 * a^3 * c * d^{10} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 + 24 * a^3 * d^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^7 - 216 * a * b^2 * c^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 12 * b^3 * c^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 864 * a^2 * b * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 108 * a * b^2 * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 212 * b^3 * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 720 * a^3 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 648 * a^2 * b * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 684 * a * b^2 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 197 * b^3 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 600 * a^3 * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 600 * a^2 * b * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 819 * a * b^2 * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 27 * b^3 * c^8 * d^3 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 1360 * a^3 * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 141 * a^2 * b * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 861 * a * b^2 * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 27 * b^3 * c^7 * d^4 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 1051 * a^3 * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 237 * a^2 * b * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 807 * a * b^2 * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 197 * b^3 * c^6 * d^5 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 1029 * a^3 * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 507 * a^2 * b * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 39 * a * b^2 * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 212 * b^3 * c^5 * d^6 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 759 * a^3 * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 27 * a^2 * b * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 120 * a * b^2 * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 12 * b^3 * c^4 * d^7 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 473 * a^3 * c^3 * d^8 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 380 * a^3 * c^2 * d^9 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 - 84 * a^3 * c * d^{10} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 72 * a^3 * d^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^5 + 216 * a * b^2 * c^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 + 12 * b^3 * c^{11} * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 - 864 * a^2 * b * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 + 108 * a * b^2 * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 - 212 * b^3 * c^{10} * d * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 + 720 * a^3 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 - 648 * a^2 * b * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 + 684 * a * b^2 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3 - 197 * b^3 * c^9 * d^2 * \tan(\frac{1}{2} * f * x + \frac{1}{2} * e)^3$

$$\begin{aligned} & x + 1/2e)^3 + 600a^3c^8d^3\tan(1/2fx + 1/2e)^3 + 600a^2b^3c^8d^3\tan(1/2fx + 1/2e)^3 + 819a^2b^2c^8d^3\tan(1/2fx + 1/2e)^3 + 27b^3c^8d^3\tan(1/2fx + 1/2e)^3 - 1360a^3c^7d^4\tan(1/2fx + 1/2e)^3 + 141a^2b^3c^7d^4\tan(1/2fx + 1/2e)^3 - 861a^2b^2c^7d^4\tan(1/2fx + 1/2e)^3 - 27b^3c^7d^4\tan(1/2fx + 1/2e)^3 - 1051a^3c^6d^5\tan(1/2fx + 1/2e)^3 + 237a^2b^3c^6d^5\tan(1/2fx + 1/2e)^3 - 807a^2b^2c^6d^5\tan(1/2fx + 1/2e)^3 + 197b^3c^6d^5\tan(1/2fx + 1/2e)^3 + 1029a^3c^5d^6\tan(1/2fx + 1/2e)^3 + 507a^2b^3c^5d^6\tan(1/2fx + 1/2e)^3 - 39a^2b^2c^5d^6\tan(1/2fx + 1/2e)^3 + 212b^3c^5d^6\tan(1/2fx + 1/2e)^3 + 759a^3c^4d^7\tan(1/2fx + 1/2e)^3 + 27a^2b^3c^4d^7\tan(1/2fx + 1/2e)^3 - 120a^2b^2c^4d^7\tan(1/2fx + 1/2e)^3 - 12b^3c^4d^7\tan(1/2fx + 1/2e)^3 - 473a^3c^3d^8\tan(1/2fx + 1/2e)^3 - 380a^3c^2d^9\tan(1/2fx + 1/2e)^3 + 84a^3c^2d^10\tan(1/2fx + 1/2e)^3 + 72a^3d^11\tan(1/2fx + 1/2e)^3 - 72a^2b^2c^11\tan(1/2fx + 1/2e) - 12b^3c^11\tan(1/2fx + 1/2e) + 288a^2b^3c^10d\tan(1/2fx + 1/2e) - 108a^2b^2c^10d\tan(1/2fx + 1/2e) + 60b^3c^10d\tan(1/2fx + 1/2e) - 240a^3c^9d^2\tan(1/2fx + 1/2e) + 648a^2b^3c^9d^2\tan(1/2fx + 1/2e) - 324a^2b^2c^9d^2\tan(1/2fx + 1/2e) + 189b^3c^9d^2\tan(1/2fx + 1/2e) - 600a^3c^8d^3\tan(1/2fx + 1/2e) + 504a^2b^3c^8d^3\tan(1/2fx + 1/2e) - 891a^2b^2c^8d^3\tan(1/2fx + 1/2e) + 183b^3c^8d^3\tan(1/2fx + 1/2e) - 240a^3c^7d^4\tan(1/2fx + 1/2e) + 459a^2b^3c^7d^4\tan(1/2fx + 1/2e) - 801a^2b^2c^7d^4\tan(1/2fx + 1/2e) + 183b^3c^7d^4\tan(1/2fx + 1/2e) + 435a^3c^6d^5\tan(1/2fx + 1/2e) + 513a^2b^3c^6d^5\tan(1/2fx + 1/2e) - 189a^2b^2c^6d^5\tan(1/2fx + 1/2e) + 189b^3c^6d^5\tan(1/2fx + 1/2e) + 249a^3c^5d^6\tan(1/2fx + 1/2e) + 153a^2b^3c^5d^6\tan(1/2fx + 1/2e) - 63a^2b^2c^5d^6\tan(1/2fx + 1/2e) + 60b^3c^5d^6\tan(1/2fx + 1/2e) - 291a^3c^4d^7\tan(1/2fx + 1/2e) - 45a^2b^3c^4d^7\tan(1/2fx + 1/2e) - 72a^2b^2c^4d^7\tan(1/2fx + 1/2e) - 12b^3c^4d^7\tan(1/2fx + 1/2e) - 273a^3c^3d^8\tan(1/2fx + 1/2e) + 12a^3c^2d^9\tan(1/2fx + 1/2e) + 84a^3c^2d^10\tan(1/2fx + 1/2e) + 24a^3d^11\tan(1/2fx + 1/2e))/((c^12 - 4c^10d^2 + 6c^8d^4 - 4c^6d^6 + c^4d^8)*(c*tan(1/2fx + 1/2e)^2 - d*tan(1/2fx + 1/2e)^2 - c - d)^4))/f \end{aligned}$$

maple [B] time = 0.80, size = 8573, normalized size = 13.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more details)Is 4*c^2-4*d^2 positive or negative?

mupad [B] time = 16.35, size = 21021, normalized size = 33.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b/\cos(e + f*x))^3/(c + d/\cos(e + f*x))^5, x)$

[Out] $(\text{atan}(\frac{((c + d)^9*(c - d)^9)^{1/2} * ((\tan(e/2 + (f*x)/2) * (64*a^6*c^18 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 3600*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d)) / (2*(c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (((32*a^3*c^27 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 + 16*a^3*c^11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14*d^13 + 528*a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a^3*c^18*d^9 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 1852*a^3*c^22*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - 16*b^3*c^14*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + 320*b^3*c^18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 + 320*b^3*c^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + 180*a*b^2*c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^2*c^18*d^9 + 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - 720*a*b^2*c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c^25*d^2 - 36*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 144*a^2*b*c^17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c^20*d^7 + 1200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 96*a^2*b*c^24*d^3 - 96*a^2*b*c^25*d^2 - 240*a*b^2*c^26*d - 96*a^2*b*c^26*d)) / (c^26*d + c^27 - c^12*d^15 - c^13*d^14 + 7*c^14*d^13 + 7*c^15*d^12 - 21*c^16*d^11 - 21*c^17*d^10 + 35*c^18*d^9 + 35*c^19*d^8 - 35*c^20*d^7 - 35*c^21*d^6 + 21*c^22*d^5 + 21*c^23*d^4 - 7*c^24*d^3 - 7*c^25*d^2) - (\tan(e/2 + (f*x)/2) * ((c + d)^9*(c - d)^9)^{1/2} * (4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)) * (128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13*d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 - 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22*d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2)) / (16*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2) * (c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2))) * ((c + d)^9*(c - d)^9)^{1/2} * (4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)) / (8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2))) * (4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d) * i) / (8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2))$

$$\begin{aligned}
& 19*d^4 - 9*c^{21}*d^2)) + (((c + d)^9*(c - d)^9)^{(1/2)}*((\tan(e/2 + (f*x)/2)*(\\
& 64*a^6*c^{18} + 128*a^6*d^{18} + 16*b^6*c^{18} - 128*a^6*c*d^{17} - 128*a^6*c^{17}*d \\
& + 192*a^2*b^4*c^{18} + 576*a^4*b^2*c^{18} - 1024*a^6*c^2*d^{16} + 1024*a^6*c^3*d^{15} \\
& + 3584*a^6*c^4*d^{14} - 3584*a^6*c^5*d^{13} - 6968*a^6*c^6*d^{12} + 7168*a^6*c \\
& ^7*d^{11} + 8385*a^6*c^8*d^{10} - 8960*a^6*c^9*d^9 - 7024*a^6*c^{10}*d^8 + 7168*a \\
& ^6*c^{11}*d^7 + 4848*a^6*c^{12}*d^6 - 3584*a^6*c^{13}*d^5 - 1920*a^6*c^{14}*d^4 + 1 \\
& 024*a^6*c^{15}*d^3 + 1152*a^6*c^{16}*d^2 + 16*b^6*c^{10}*d^8 + 216*b^6*c^{12}*d^6 + \\
& 761*b^6*c^{14}*d^4 + 216*b^6*c^{16}*d^2 - 360*a*b^5*c^{11}*d^7 - 2910*a*b^5*c^{13} \\
& *d^5 - 3600*a*b^5*c^{15}*d^3 - 3200*a^3*b^3*c^{17}*d - 144*a^5*b*c^5*d^{13} - 504 \\
& *a^5*b*c^7*d^{11} + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^{11}*d^7 + 2016*a^5*b*c^{13} \\
& *d^5 - 3840*a^5*b*c^{15}*d^3 + 72*a^2*b^4*c^{10}*d^8 + 3087*a^2*b^4*c^{12}*d^6 + \\
& 9552*a^2*b^4*c^{14}*d^4 + 5472*a^2*b^4*c^{16}*d^2 - 64*a^3*b^3*c^5*d^{13} - 144*a \\
& ^3*b^3*c^7*d^{11} + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^{11}*d^7 - 6224*a^3*b \\
& ^3*c^{13}*d^5 - 12640*a^3*b^3*c^{15}*d^3 + 720*a^4*b^2*c^6*d^{12} - 2280*a^4*b^2 \\
& *c^8*d^{10} + 1431*a^4*b^2*c^{10}*d^8 + 5256*a^4*b^2*c^{12}*d^6 + 4416*a^4*b^2*c^{14} \\
& *d^4 + 8256*a^4*b^2*c^{16}*d^2 - 480*a*b^5*c^{17}*d - 1920*a^5*b*c^{17}*d))/((2* \\
& (c^{22}*d + c^{23} - c^8*d^{15} - c^9*d^{14} + 7*c^{10}*d^{13} + 7*c^{11}*d^{12} - 21*c^{12} \\
& *d^{11} - 21*c^{13}*d^{10} + 35*c^{14}*d^9 + 35*c^{15}*d^8 - 35*c^{16}*d^7 - 35*c^{17}*d^6 \\
& + 21*c^{18}*d^5 + 21*c^{19}*d^4 - 7*c^{20}*d^3 - 7*c^{21}*d^2)) - (((32*a^3*c^{27} + \\
& 16*b^3*c^{27} + 96*a^2*b*c^{27} - 160*a^3*c^{26}*d - 16*b^3*c^{26}*d - 32*a^3*c^{10} \\
& *d^{17} + 16*a^3*c^{11}*d^{16} + 272*a^3*c^{12}*d^{15} - 132*a^3*c^{13}*d^{14} - 1020*a^3 \\
& *c^{14}*d^{13} + 528*a^3*c^{15}*d^{12} + 2160*a^3*c^{16}*d^{11} - 1112*a^3*c^{17}*d^{10} - \\
& 2920*a^3*c^{18}*d^9 + 1280*a^3*c^{19}*d^8 + 2752*a^3*c^{20}*d^7 - 836*a^3*c^{21}*d^6 \\
& - 1852*a^3*c^{22}*d^5 + 352*a^3*c^{23}*d^4 + 800*a^3*c^{24}*d^3 - 128*a^3*c^{25} \\
& *d^2 - 16*b^3*c^{14}*d^{13} + 16*b^3*c^{15}*d^{12} - 44*b^3*c^{16}*d^{11} + 44*b^3*c^{17} \\
& *d^{10} + 320*b^3*c^{18}*d^9 - 320*b^3*c^{19}*d^8 - 520*b^3*c^{20}*d^7 + 520*b^3*c^2 \\
& 1*d^6 + 320*b^3*c^{22}*d^5 - 320*b^3*c^{23}*d^4 - 44*b^3*c^{24}*d^3 + 44*b^3*c^{25} \\
& *d^2 + 180*a*b^2*c^{15}*d^{12} - 180*a*b^2*c^{16}*d^{11} - 480*a*b^2*c^{17}*d^{10} + 48 \\
& 0*a*b^2*c^{18}*d^9 + 120*a*b^2*c^{19}*d^8 - 120*a*b^2*c^{20}*d^7 + 720*a*b^2*c^{21} \\
& *d^6 - 720*a*b^2*c^{22}*d^5 - 780*a*b^2*c^{23}*d^4 + 780*a*b^2*c^{24}*d^3 + 240*a \\
& *b^2*c^{25}*d^2 - 36*a^2*b*c^{14}*d^{13} + 36*a^2*b*c^{15}*d^{12} - 144*a^2*b*c^{16}*d^{11} \\
& + 144*a^2*b*c^{17}*d^{10} + 840*a^2*b*c^{18}*d^9 - 840*a^2*b*c^{19}*d^8 - 1200*a \\
& ^2*b*c^{20}*d^7 + 1200*a^2*b*c^{21}*d^6 + 540*a^2*b*c^{22}*d^5 - 540*a^2*b*c^{23} \\
& *d^4 + 96*a^2*b*c^{24}*d^3 - 96*a^2*b*c^{25}*d^2 - 240*a*b^2*c^{26}*d - 96*a^2*b*c^{26} \\
& *d)/(c^{26}*d + c^{27} - c^{12}*d^{15} - c^{13}*d^{14} + 7*c^{14}*d^{13} + 7*c^{15}*d^{12} - \\
& 21*c^{16}*d^{11} - 21*c^{17}*d^{10} + 35*c^{18}*d^9 + 35*c^{19}*d^8 - 35*c^{20}*d^7 - 35* \\
& c^{21}*d^6 + 21*c^{22}*d^5 + 21*c^{23}*d^4 - 7*c^{24}*d^3 - 7*c^{25}*d^2) + (\tan(e/2 \\
& + (f*x)/2)*((c + d)^9*(c - d)^9)^{(1/2)}*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 \\
& - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3 \\
& *c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c \\
& ^7*d^2 - 60*a*b^2*c^8*d)*(128*c^{27}*d - 128*c^{10}*d^{18} + 128*c^{11}*d^{17} + 1024 \\
& *c^{12}*d^{16} - 1024*c^{13}*d^{15} - 3584*c^{14}*d^{14} + 3584*c^{15}*d^{13} + 7168*c^{16} \\
& *d^{12} - 7168*c^{17}*d^{11} - 8960*c^{18}*d^{10} + 8960*c^{19}*d^9 + 7168*c^{20}*d^8 - 716 \\
& 8*c^{21}*d^7 - 3584*c^{22}*d^6 + 3584*c^{23}*d^5 + 1024*c^{24}*d^4 - 1024*c^{25}*d^3 \\
& - 128*c^{26}*d^2))/(16*(c^{23} - c^5*d^{18} + 9*c^7*d^{16} - 36*c^9*d^{14} + 84*c^{11} \\
& *d^{12} - 126*c^{13}*d^{10} + 126*c^{15}*d^8 - 84*c^{17}*d^6 + 36*c^{19}*d^4 - 9*c^{21}*d^2) \\
& *(c^{22}*d + c^{23} - c^8*d^{15} - c^9*d^{14} + 7*c^{10}*d^{13} + 7*c^{11}*d^{12} - 21*c^{12} \\
& *d^{11} - 21*c^{13}*d^{10} + 35*c^{14}*d^9 + 35*c^{15}*d^8 - 35*c^{16}*d^7 - 35*c^{17} \\
& *d^6 + 21*c^{18}*d^5 + 21*c^{19}*d^4 - 7*c^{20}*d^3 - 7*c^{21}*d^2)))*((c + d)^9*(c \\
& - d)^9)^{(1/2)}*(4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3 \\
& *c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 \\
& - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)) \\
& /(8*(c^{23} - c^5*d^{18} + 9*c^7*d^{16} - 36*c^9*d^{14} + 84*c^{11}*d^{12} - 126*c^{13} \\
& *d^{10} + 126*c^{15}*d^8 - 84*c^{17}*d^6 + 36*c^{19}*d^4 - 9*c^{21}*d^2)))*(4*b^3*c^9 - \\
& 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 \\
& + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2 \\
& *b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d)*1i)/(8*(c^{23} - c^5*d^{18} + \\
& 9*c^7*d^{16} - 36*c^9*d^{14} + 84*c^{11}*d^{12} - 126*c^{13}*d^{10} + 126*c^{15}*d^8 - 84 \\
& *c^{17}*d^6 + 36*c^{19}*d^4 - 9*c^{21}*d^2)))/((64*a^9*d^{17} - 192*a^8*b*c^{17} - 32
\end{aligned}$$

$$\begin{aligned}
& *a^9*c*d^{16} + 320*a^9*c^{16}*d + 16*a^3*b^6*c^{17} + 192*a^5*b^4*c^{17} - 32*a^6* \\
& b^3*c^{17} + 576*a^7*b^2*c^{17} - 544*a^9*c^2*d^{15} + 264*a^9*c^3*d^{14} + 2040*a^ \\
& 9*c^4*d^{13} - 856*a^9*c^5*d^{12} - 4320*a^9*c^6*d^{11} + 1649*a^9*c^7*d^{10} + 584 \\
& 0*a^9*c^8*d^9 - 2416*a^9*c^9*d^8 - 5504*a^9*c^{10}*d^7 + 2936*a^9*c^{11}*d^6 + \\
& 3704*a^9*c^{12}*d^5 - 1600*a^9*c^{13}*d^4 - 1600*a^9*c^{14}*d^3 + 1280*a^9*c^{15}*d \\
& ^2 - 480*a^4*b^5*c^{16}*d - 3168*a^6*b^3*c^{16}*d + 480*a^7*b^2*c^{16}*d - 72*a^8 \\
& *b*c^4*d^{13} - 72*a^8*b*c^5*d^{12} - 216*a^8*b*c^6*d^{11} - 288*a^8*b*c^7*d^{10} + \\
& 1986*a^8*b*c^8*d^9 + 1680*a^8*b*c^9*d^8 - 4224*a^8*b*c^{10}*d^7 - 2400*a^8*b \\
& *c^{11}*d^6 + 936*a^8*b*c^{12}*d^5 + 1080*a^8*b*c^{13}*d^4 - 4032*a^8*b*c^{14}*d^3 \\
& + 192*a^8*b*c^{15}*d^2 + 16*a^3*b^6*c^9*d^8 + 216*a^3*b^6*c^{11}*d^6 + 761*a^3* \\
& b^6*c^{13}*d^4 + 216*a^3*b^6*c^{15}*d^2 - 360*a^4*b^5*c^{10}*d^7 - 2910*a^4*b^5*c \\
& ^{12}*d^5 - 3600*a^4*b^5*c^{14}*d^3 + 72*a^5*b^4*c^9*d^8 + 3087*a^5*b^4*c^{11}*d^ \\
& 6 + 9552*a^5*b^4*c^{13}*d^4 + 5472*a^5*b^4*c^{15}*d^2 - 32*a^6*b^3*c^4*d^{13} - 3 \\
& 2*a^6*b^3*c^5*d^{12} - 56*a^6*b^3*c^6*d^{11} - 88*a^6*b^3*c^7*d^{10} + 736*a^6*b^ \\
& 3*c^8*d^9 + 640*a^6*b^3*c^9*d^8 - 2564*a^6*b^3*c^{10}*d^7 - 1040*a^6*b^3*c^{11} \\
& *d^6 - 6864*a^6*b^3*c^{12}*d^5 + 640*a^6*b^3*c^{13}*d^4 - 12552*a^6*b^3*c^{14}*d^ \\
& 3 - 88*a^6*b^3*c^{15}*d^2 + 360*a^7*b^2*c^5*d^{12} + 360*a^7*b^2*c^6*d^{11} - 132 \\
& 0*a^7*b^2*c^7*d^{10} - 960*a^7*b^2*c^8*d^9 + 1191*a^7*b^2*c^9*d^8 + 240*a^7*b \\
& ^2*c^{10}*d^7 + 3816*a^7*b^2*c^{11}*d^6 + 1440*a^7*b^2*c^{12}*d^5 + 5976*a^7*b^2* \\
& c^{13}*d^4 - 1560*a^7*b^2*c^{14}*d^3 + 7776*a^7*b^2*c^{15}*d^2 - 1728*a^8*b*c^{16} \\
& d)/(c^{26}*d + c^{27} - c^{12}*d^{15} - c^{13}*d^{14} + 7*c^{14}*d^{13} + 7*c^{15}*d^{12} - 21* \\
& c^{16}*d^{11} - 21*c^{17}*d^{10} + 35*c^{18}*d^9 + 35*c^{19}*d^8 - 35*c^{20}*d^7 - 35*c^2 \\
& 1*d^6 + 21*c^{22}*d^5 + 21*c^{23}*d^4 - 7*c^{24}*d^3 - 7*c^{25}*d^2) - (((c + d)^9* \\
& (c - d)^9)^{(1/2)}*((tan(e/2 + (f*x)/2)*(64*a^6*c^{18} + 128*a^6*d^{18} + 16*b^6* \\
& c^{18} - 128*a^6*c*d^{17} - 128*a^6*c^{17}*d + 192*a^2*b^4*c^{18} + 576*a^4*b^2*c^{1 \\
& 8} - 1024*a^6*c^2*d^{16} + 1024*a^6*c^3*d^{15} + 3584*a^6*c^4*d^{14} - 3584*a^6*c^ \\
& 5*d^{13} - 6968*a^6*c^6*d^{12} + 7168*a^6*c^7*d^{11} + 8385*a^6*c^8*d^{10} - 8960*a \\
& ^6*c^9*d^9 - 7024*a^6*c^{10}*d^8 + 7168*a^6*c^{11}*d^7 + 4848*a^6*c^{12}*d^6 - 35 \\
& 84*a^6*c^{13}*d^5 - 1920*a^6*c^{14}*d^4 + 1024*a^6*c^{15}*d^3 + 1152*a^6*c^{16}*d^2 \\
& + 16*b^6*c^{10}*d^8 + 216*b^6*c^{12}*d^6 + 761*b^6*c^{14}*d^4 + 216*b^6*c^{16}*d^2 \\
& - 360*a*b^5*c^{11}*d^7 - 2910*a*b^5*c^{13}*d^5 - 3600*a*b^5*c^{15}*d^3 - 3200*a^ \\
& 3*b^3*c^{17}*d - 144*a^5*b*c^5*d^{13} - 504*a^5*b*c^7*d^{11} + 3666*a^5*b*c^9*d^9 \\
& - 6624*a^5*b*c^{11}*d^7 + 2016*a^5*b*c^{13}*d^5 - 3840*a^5*b*c^{15}*d^3 + 72*a^2 \\
& *b^4*c^{10}*d^8 + 3087*a^2*b^4*c^{12}*d^6 + 9552*a^2*b^4*c^{14}*d^4 + 5472*a^2*b^ \\
& 4*c^{16}*d^2 - 64*a^3*b^3*c^5*d^{13} - 144*a^3*b^3*c^7*d^{11} + 1376*a^3*b^3*c^9* \\
& d^9 - 3604*a^3*b^3*c^{11}*d^7 - 6224*a^3*b^3*c^{13}*d^5 - 12640*a^3*b^3*c^{15}*d^ \\
& 3 + 720*a^4*b^2*c^6*d^{12} - 2280*a^4*b^2*c^8*d^{10} + 1431*a^4*b^2*c^{10}*d^8 + \\
& 5256*a^4*b^2*c^{12}*d^6 + 4416*a^4*b^2*c^{14}*d^4 + 8256*a^4*b^2*c^{16}*d^2 - 480 \\
& *a*b^5*c^{17}*d - 1920*a^5*b*c^{17}*d))/((2*(c^{22}*d + c^{23} - c^8*d^{15} - c^9*d^{14} \\
& + 7*c^{10}*d^{13} + 7*c^{11}*d^{12} - 21*c^{12}*d^{11} - 21*c^{13}*d^{10} + 35*c^{14}*d^9 + \\
& 35*c^{15}*d^8 - 35*c^{16}*d^7 - 35*c^{17}*d^6 + 21*c^{18}*d^5 + 21*c^{19}*d^4 - 7*c^2 \\
& 0*d^3 - 7*c^{21}*d^2)) + (((32*a^3*c^{27} + 16*b^3*c^{27} + 96*a^2*b*c^{27} - 160*a \\
& ^3*c^{26}*d - 16*b^3*c^{26}*d - 32*a^3*c^{10}*d^{17} + 16*a^3*c^{11}*d^{16} + 272*a^3*c \\
& ^{12}*d^{15} - 132*a^3*c^{13}*d^{14} - 1020*a^3*c^{14}*d^{13} + 528*a^3*c^{15}*d^{12} + 216 \\
& 0*a^3*c^{16}*d^{11} - 1112*a^3*c^{17}*d^{10} - 2920*a^3*c^{18}*d^9 + 1280*a^3*c^{19}*d^ \\
& 8 + 2752*a^3*c^{20}*d^7 - 836*a^3*c^{21}*d^6 - 1852*a^3*c^{22}*d^5 + 352*a^3*c^{23} \\
& *d^4 + 800*a^3*c^{24}*d^3 - 128*a^3*c^{25}*d^2 - 16*b^3*c^{14}*d^{13} + 16*b^3*c^{15} \\
& *d^{12} - 44*b^3*c^{16}*d^{11} + 44*b^3*c^{17}*d^{10} + 320*b^3*c^{18}*d^9 - 320*b^3*c^ \\
& 19*d^8 - 520*b^3*c^{20}*d^7 + 520*b^3*c^{21}*d^6 + 320*b^3*c^{22}*d^5 - 320*b^3*c \\
& ^{23}*d^4 - 44*b^3*c^{24}*d^3 + 44*b^3*c^{25}*d^2 + 180*a*b^2*c^{15}*d^{12} - 180*a*b \\
& ^2*c^{16}*d^{11} - 480*a*b^2*c^{17}*d^{10} + 480*a*b^2*c^{18}*d^9 + 120*a*b^2*c^{19}*d^ \\
& 8 - 120*a*b^2*c^{20}*d^7 + 720*a*b^2*c^{21}*d^6 - 720*a*b^2*c^{22}*d^5 - 780*a*b^ \\
& 2*c^{23}*d^4 + 780*a*b^2*c^{24}*d^3 + 240*a*b^2*c^{25}*d^2 - 36*a^2*b*c^{14}*d^{13} + \\
& 36*a^2*b*c^{15}*d^{12} - 144*a^2*b*c^{16}*d^{11} + 144*a^2*b*c^{17}*d^{10} + 840*a^2*b \\
& *c^{18}*d^9 - 840*a^2*b*c^{19}*d^8 - 1200*a^2*b*c^{20}*d^7 + 1200*a^2*b*c^{21}*d^6 \\
& + 540*a^2*b*c^{22}*d^5 - 540*a^2*b*c^{23}*d^4 + 96*a^2*b*c^{24}*d^3 - 96*a^2*b*c^ \\
& 25*d^2 - 240*a*b^2*c^{26}*d - 96*a^2*b*c^{26}*d)/(c^{26}*d + c^{27} - c^{12}*d^{15} - c \\
& ^{13}*d^{14} + 7*c^{14}*d^{13} + 7*c^{15}*d^{12} - 21*c^{16}*d^{11} - 21*c^{17}*d^{10} + 35*c^1 \\
& 8*d^9 + 35*c^{19}*d^8 - 35*c^{20}*d^7 - 35*c^{21}*d^6 + 21*c^{22}*d^5 + 21*c^{23}*d^4
\end{aligned}$$

$$\begin{aligned}
& - 7c^{24}d^3 - 7c^{25}d^2) - (\tan(e/2 + (f*x)/2)*((c + d)^9*(c - d)^9)^{(1/2)} \\
& * (4b^3c^9 - 8a^3d^9 + 24a^2b^3c^9 - 40a^3c^8d + 36a^3c^2d^7 - 63a^3c^4d^5 \\
& + 40a^3c^6d^3 + 4b^3c^5d^4 + 27b^3c^7d^2 - 45a*b^2c^6d^3 + 9a^2b^3c^5d^4 \\
& + 72a^2b^3c^7d^2 - 60a*b^2c^8d)*(128c^{27}d - 128c^{10}d^{18} + 128c^{11}d^{17} \\
& + 1024c^{12}d^{16} - 1024c^{13}d^{15} - 3584c^{14}d^{14} + 3584c^{15}d^{13} + 7168c^{16}d^{12} \\
& - 7168c^{17}d^{11} - 8960c^{18}d^{10} + 8960c^{19}d^9 + 7168c^{20}d^8 - 7168c^{21}d^7 \\
& - 3584c^{22}d^6 + 3584c^{23}d^5 + 1024c^{24}d^4 - 1024c^{25}d^3 - 128c^{26}d^2)) / (16*(c^{23} - c^5d^{18} \\
& + 9c^7d^{16} - 36c^9d^{14} + 84c^{11}d^{12} - 126c^{13}d^{10} + 126c^{15}d^8 - 84c^{17}d^6 \\
& + 36c^{19}d^4 - 9c^{21}d^2)*(c^{22}d + c^{23} - c^8d^{15} - c^9d^{14} + 7c^{10}d^{13} \\
& + 7c^{11}d^{12} - 21c^{12}d^{11} - 21c^{13}d^{10} + 35c^{14}d^9 + 35c^{15}d^8 - 35c^{16}d^7 \\
& - 35c^{17}d^6 + 21c^{18}d^5 + 21c^{19}d^4 - 7c^{20}d^3 - 7c^{21}d^2)) * ((c + d)^9*(c - d)^9)^{(1/2)} \\
& * (4b^3c^9 - 8a^3d^9 + 24a^2b^3c^9 - 40a^3c^8d + 36a^3c^2d^7 - 63a^3c^4d^5 + 40a^3c^6d^3 \\
& + 4b^3c^5d^4 + 27b^3c^7d^2 - 45a*b^2c^6d^3 + 9a^2b^3c^5d^4 + 72a^2b^3c^7d^2 - 60a*b^2c^8d) \\
& / (8*(c^{23} - c^5d^{18} + 9c^7d^{16} - 36c^9d^{14} + 84c^{11}d^{12} - 126c^{13}d^{10} + 126c^{15}d^8 \\
& - 84c^{17}d^6 + 36c^{19}d^4 - 9c^{21}d^2)) * (4b^3c^9 - 8a^3d^9 + 24a^2b^3c^9 - 40a^3c^8d \\
& + 36a^3c^2d^7 - 63a^3c^4d^5 + 40a^3c^6d^3 + 4b^3c^5d^4 + 27b^3c^7d^2 - 45a*b^2c^6d^3 \\
& + 9a^2b^3c^5d^4 + 72a^2b^3c^7d^2 - 60a*b^2c^8d) / (8*(c^{23} - c^5d^{18} + 9c^7d^{16} - 36c^9d^{14} \\
& + 84c^{11}d^{12} - 126c^{13}d^{10} + 126c^{15}d^8 - 84c^{17}d^6 + 36c^{19}d^4 - 9c^{21}d^2)) \\
& + (((c + d)^9*(c - d)^9)^{(1/2)} * ((\tan(e/2 + (f*x)/2)*(64a^6c^{18} + 128a^6c^{18}d^{18} \\
& + 16b^6c^{18} - 128a^6c^8d^{17} - 128a^6c^{17}d + 192a^2b^4c^{18} + 576a^4b^2c^{18} - 1024a^6c^2d^{16} \\
& + 1024a^6c^3d^{15} + 3584a^6c^4d^{14} - 3584a^6c^5d^{13} - 6968a^6c^6d^{12} + 7168a^6c^7d^{11} + 8385a^6c^8d^{10} \\
& - 8960a^6c^9d^9 - 7024a^6c^{10}d^8 + 7168a^6c^{11}d^7 + 4848a^6c^{12}d^6 - 3584a^6c^{13}d^5 \\
& - 1920a^6c^{14}d^4 + 1024a^6c^{15}d^3 + 1152a^6c^{16}d^2 + 16b^6c^{10}d^8 + 216b^6c^{12}d^6 + 761b^6c^{14}d^4 + 216b^6c^{16}d^2 \\
& - 360a*b^5c^{11}d^7 - 2910a*b^5c^{13}d^5 - 3600a*b^5c^{15}d^3 - 3200a^3b^3c^{17}d - 144a^5b^3c^5d^{13} \\
& - 504a^5b^3c^7d^{11} + 3666a^5b^3c^9d^9 - 6624a^5b^3c^{11}d^7 + 2016a^5b^3c^{13}d^5 - 3840a^5b^3c^{15}d^3 \\
& + 72a^2b^4c^{10}d^8 + 3087a^2b^4c^{12}d^6 + 9552a^2b^4c^{14}d^4 + 5472a^2b^4c^{16}d^2 - 64a^3b^3c^5d^{13} \\
& - 144a^3b^3c^7d^{11} + 1376a^3b^3c^9d^9 - 3604a^3b^3c^{11}d^7 - 6224a^3b^3c^{13}d^5 - 12640a^3b^3c^{15}d^3 \\
& + 720a^4b^2c^6d^{12} - 2280a^4b^2c^8d^{10} + 1431a^4b^2c^{10}d^8 + 5256a^4b^2c^{12}d^6 + 4416a^4b^2c^{14}d^4 \\
& + 8256a^4b^2c^{16}d^2 - 480a*b^5c^{17}d - 1920a^5b^3c^{17}d)) / (2*(c^{22}d + c^{23} - c^8d^{15} \\
& - c^9d^{14} + 7c^{10}d^{13} + 7c^{11}d^{12} - 21c^{12}d^{11} - 21c^{13}d^{10} + 35c^{14}d^9 + 35c^{15}d^8 \\
& - 35c^{16}d^7 - 35c^{17}d^6 + 21c^{18}d^5 + 21c^{19}d^4 - 7c^{20}d^3 - 7c^{21}d^2)) - (((32a^3c^{27} + 16b^3c^{27} \\
& + 96a^2b^3c^{27} - 160a^3c^{26}d - 16b^3c^{26}d - 32a^3c^{10}d^{17} + 16a^3c^{11}d^{16} + 272a^3c^{12}d^{15} \\
& - 132a^3c^{13}d^{14} - 1020a^3c^{14}d^{13} + 528a^3c^{15}d^{12} + 2160a^3c^{16}d^{11} - 1112a^3c^{17}d^{10} \\
& - 2920a^3c^{18}d^9 + 1280a^3c^{19}d^8 + 2752a^3c^{20}d^7 - 836a^3c^{21}d^6 - 1852a^3c^{22}d^5 + 352a^3c^{23}d^4 \\
& + 800a^3c^{24}d^3 - 128a^3c^{25}d^2 - 16b^3c^{14}d^{13} + 16b^3c^{15}d^{12} - 44b^3c^{16}d^{11} \\
& + 44b^3c^{17}d^{10} + 320b^3c^{18}d^9 - 320b^3c^{19}d^8 - 520b^3c^{20}d^7 + 520b^3c^{21}d^6 + 320b^3c^{22}d^5 \\
& - 320b^3c^{23}d^4 - 44b^3c^{24}d^3 + 44b^3c^{25}d^2 + 180a*b^2c^{15}d^{12} - 180a*b^2c^{16}d^{11} \\
& - 480a*b^2c^{17}d^{10} + 480a*b^2c^{18}d^9 + 120a*b^2c^{19}d^8 - 120a*b^2c^{20}d^7 + 720a*b^2c^{21}d^6 \\
& - 720a*b^2c^{22}d^5 - 780a*b^2c^{23}d^4 + 780a*b^2c^{24}d^3 + 240a*b^2c^{25}d^2 - 36a^2b^3c^{14}d^{13} \\
& + 36a^2b^3c^{15}d^{12} - 144a^2b^3c^{16}d^{11} + 144a^2b^3c^{17}d^{10} + 840a^2b^3c^{18}d^9 - 840a^2b^3c^{19}d^8 \\
& - 1200a^2b^3c^{20}d^7 + 1200a^2b^3c^{21}d^6 + 540a^2b^3c^{22}d^5 - 540a^2b^3c^{23}d^4 + 96a^2b^3c^{24}d^3 \\
& - 96a^2b^3c^{25}d^2 - 240a*b^2c^{26}d - 96a^2b^3c^{26}d) / (c^{26}d + c^{27} - c^{12}d^{15} - c^{13}d^{14} \\
& + 7c^{14}d^{13} + 7c^{15}d^{12} - 21c^{16}d^{11} - 21c^{17}d^{10} + 35c^{18}d^9 + 35c^{19}d^8 - 35c^{20}d^7 \\
& - 35c^{21}d^6 + 21c^{22}d^5 + 21c^{23}d^4 - 7c^{24}d^3 - 7c^{25}d^2) + (\tan(e/2 + (f*x)/2)*((c + d)^9*
\end{aligned}$$

$$\begin{aligned}
& (c - d)^9)^{(1/2)} * (4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36* \\
& a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7* \\
& d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8* \\
& d) * (128*c^27*d - 128*c^10*d^18 + 128*c^11*d^17 + 1024*c^12*d^16 - 1024*c^13 \\
& *d^15 - 3584*c^14*d^14 + 3584*c^15*d^13 + 7168*c^16*d^12 - 7168*c^17*d^11 - \\
& 8960*c^18*d^10 + 8960*c^19*d^9 + 7168*c^20*d^8 - 7168*c^21*d^7 - 3584*c^22 \\
& *d^6 + 3584*c^23*d^5 + 1024*c^24*d^4 - 1024*c^25*d^3 - 128*c^26*d^2) / (16*(\\
& c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + \\
& 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2) * (c^22*d + c^23 - c^ \\
& 8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21*c^13*d^10 \\
& + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18*d^5 + 21 \\
& *c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) * ((c + d)^9 * (c - d)^9)^{(1/2)} * (4*b^3*c^ \\
& ^9 - 8*a^3*d^9 + 24*a^2*b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4* \\
& d^5 + 40*a^3*c^6*d^3 + 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + \\
& 9*a^2*b*c^5*d^4 + 72*a^2*b*c^7*d^2 - 60*a*b^2*c^8*d) / (8*(c^23 - c^5*d^18 + \\
& 9*c^7*d^16 - 36*c^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 8 \\
& 4*c^17*d^6 + 36*c^19*d^4 - 9*c^21*d^2)) * (4*b^3*c^9 - 8*a^3*d^9 + 24*a^2*b* \\
& c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + 4*b \\
& ^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^2*b \\
& *c^7*d^2 - 60*a*b^2*c^8*d) / (8*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c^9*d^14 \\
& + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^19*d^4 - \\
& 9*c^21*d^2)) * ((c + d)^9 * (c - d)^9)^{(1/2)} * (4*b^3*c^9 - 8*a^3*d^9 + 24*a^2 \\
& *b*c^9 - 40*a^3*c^8*d + 36*a^3*c^2*d^7 - 63*a^3*c^4*d^5 + 40*a^3*c^6*d^3 + \\
& 4*b^3*c^5*d^4 + 27*b^3*c^7*d^2 - 45*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 72*a^ \\
& 2*b*c^7*d^2 - 60*a*b^2*c^8*d) * i) / (4*f*(c^23 - c^5*d^18 + 9*c^7*d^16 - 36*c \\
& ^9*d^14 + 84*c^11*d^12 - 126*c^13*d^10 + 126*c^15*d^8 - 84*c^17*d^6 + 36*c^ \\
& 19*d^4 - 9*c^21*d^2)) - (2*a^3*atan(-(a^3*((tan(e/2 + (f*x)/2)*(64*a^6*c^1 \\
& 8 + 128*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2* \\
& b^4*c^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584* \\
& a^6*c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + \\
& 8385*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^ \\
& 7 + 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^ \\
& 15*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c \\
& ^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 360 \\
& 0*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c^7 \\
& *d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 - 38 \\
& 40*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*a^2* \\
& b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b^3*c^ \\
& 7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*c^13*d \\
& ^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^8*d^10 \\
& + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14*d^4 + 8 \\
& 256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d) / (2*(c^22*d + \\
& c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*d^11 - 21* \\
& c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^6 + 21*c^18 \\
& *d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (a^3*((32*a^3*c^27 + 16*b^ \\
& 3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^3*c^10*d^17 \\
& + 16*a^3*c^11*d^16 + 272*a^3*c^12*d^15 - 132*a^3*c^13*d^14 - 1020*a^3*c^14* \\
& d^13 + 528*a^3*c^15*d^12 + 2160*a^3*c^16*d^11 - 1112*a^3*c^17*d^10 - 2920*a \\
& ^3*c^18*d^9 + 1280*a^3*c^19*d^8 + 2752*a^3*c^20*d^7 - 836*a^3*c^21*d^6 - 18 \\
& 52*a^3*c^22*d^5 + 352*a^3*c^23*d^4 + 800*a^3*c^24*d^3 - 128*a^3*c^25*d^2 - \\
& 16*b^3*c^14*d^13 + 16*b^3*c^15*d^12 - 44*b^3*c^16*d^11 + 44*b^3*c^17*d^10 + \\
& 320*b^3*c^18*d^9 - 320*b^3*c^19*d^8 - 520*b^3*c^20*d^7 + 520*b^3*c^21*d^6 \\
& + 320*b^3*c^22*d^5 - 320*b^3*c^23*d^4 - 44*b^3*c^24*d^3 + 44*b^3*c^25*d^2 + \\
& 180*a*b^2*c^15*d^12 - 180*a*b^2*c^16*d^11 - 480*a*b^2*c^17*d^10 + 480*a*b^ \\
& 2*c^18*d^9 + 120*a*b^2*c^19*d^8 - 120*a*b^2*c^20*d^7 + 720*a*b^2*c^21*d^6 - \\
& 720*a*b^2*c^22*d^5 - 780*a*b^2*c^23*d^4 + 780*a*b^2*c^24*d^3 + 240*a*b^2*c \\
& ^25*d^2 - 36*a^2*b*c^14*d^13 + 36*a^2*b*c^15*d^12 - 144*a^2*b*c^16*d^11 + 1 \\
& 44*a^2*b*c^17*d^10 + 840*a^2*b*c^18*d^9 - 840*a^2*b*c^19*d^8 - 1200*a^2*b*c \\
& ^20*d^7 + 1200*a^2*b*c^21*d^6 + 540*a^2*b*c^22*d^5 - 540*a^2*b*c^23*d^4 + 9
\end{aligned}$$

$$\begin{aligned}
& 6*a^2*b*c^{24*d^3} - 96*a^2*b*c^{25*d^2} - 240*a*b^2*c^{26*d} - 96*a^2*b*c^{26*d}) / \\
& (c^{26*d} + c^{27} - c^{12*d^{15}} - c^{13*d^{14}} + 7*c^{14*d^{13}} + 7*c^{15*d^{12}} - 21*c^{16*d^{11}} - 21*c^{17*d^{10}} + 35*c^{18*d^9} + 35*c^{19*d^8} - 35*c^{20*d^7} - 35*c^{21*d^6} \\
& + 21*c^{22*d^5} + 21*c^{23*d^4} - 7*c^{24*d^3} - 7*c^{25*d^2}) - (a^3*\tan(e/2 + (f*x)/2)*(128*c^{27*d} - 128*c^{10*d^{18}} + 128*c^{11*d^{17}} + 1024*c^{12*d^{16}} - 1024*c^{13*d^{15}} - 3584*c^{14*d^{14}} + 3584*c^{15*d^{13}} + 7168*c^{16*d^{12}} - 7168*c^{17*d^{11}} - 8960*c^{18*d^{10}} + 8960*c^{19*d^9} + 7168*c^{20*d^8} - 7168*c^{21*d^7} - 3584*c^{22*d^6} + 3584*c^{23*d^5} + 1024*c^{24*d^4} - 1024*c^{25*d^3} - 128*c^{26*d^2})* \\
& 1i)/(2*c^5*(c^{22*d} + c^{23} - c^{8*d^{15}} - c^{9*d^{14}} + 7*c^{10*d^{13}} + 7*c^{11*d^{12}} - 21*c^{12*d^{11}} - 21*c^{13*d^{10}} + 35*c^{14*d^9} + 35*c^{15*d^8} - 35*c^{16*d^7} - 35*c^{17*d^6} + 21*c^{18*d^5} + 21*c^{19*d^4} - 7*c^{20*d^3} - 7*c^{21*d^2}))*1i)/c^5)) / c^5 + (a^3*((\tan(e/2 + (f*x)/2)*(64*a^6*c^{18} + 128*a^6*d^{18} + 16*b^6*c^{18} - 128*a^6*c*d^{17} - 128*a^6*c^{17*d} + 192*a^2*b^4*c^{18} + 576*a^4*b^2*c^{18} - 1024*a^6*c^2*d^{16} + 1024*a^6*c^3*d^{15} + 3584*a^6*c^4*d^{14} - 3584*a^6*c^5*d^{13} - 6968*a^6*c^6*d^{12} + 7168*a^6*c^7*d^{11} + 8385*a^6*c^8*d^{10} - 8960*a^6*c^9*d^9 - 7024*a^6*c^{10*d^8} + 7168*a^6*c^{11*d^7} + 4848*a^6*c^{12*d^6} - 3584*a^6*c^{13*d^5} - 1920*a^6*c^{14*d^4} + 1024*a^6*c^{15*d^3} + 1152*a^6*c^{16*d^2} + 16*b^6*c^{10*d^8} + 216*b^6*c^{12*d^6} + 761*b^6*c^{14*d^4} + 216*b^6*c^{16*d^2} - 360*a*b^5*c^{11*d^7} - 2910*a*b^5*c^{13*d^5} - 3600*a*b^5*c^{15*d^3} - 3200*a^3*b^3*c^{17*d} - 144*a^5*b*c^5*d^{13} - 504*a^5*b*c^7*d^{11} + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^{11*d^7} + 2016*a^5*b*c^{13*d^5} - 3840*a^5*b*c^{15*d^3} + 72*a^2*b^4*c^{10*d^8} + 3087*a^2*b^4*c^{12*d^6} + 9552*a^2*b^4*c^{14*d^4} + 5472*a^2*b^4*c^{16*d^2} - 64*a^3*b^3*c^5*d^{13} - 144*a^3*b^3*c^7*d^{11} + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^{11*d^7} - 6224*a^3*b^3*c^{13*d^5} - 12640*a^3*b^3*c^{15*d^3} + 720*a^4*b^2*c^6*d^{12} - 2280*a^4*b^2*c^8*d^{10} + 1431*a^4*b^2*c^{10*d^8} + 5256*a^4*b^2*c^{12*d^6} + 4416*a^4*b^2*c^{14*d^4} + 8256*a^4*b^2*c^{16*d^2} - 480*a*b^5*c^{17*d} - 1920*a^5*b*c^{17*d}))/ (2*(c^{22*d} + c^{23} - c^{8*d^{15}} - c^{9*d^{14}} + 7*c^{10*d^{13}} + 7*c^{11*d^{12}} - 21*c^{12*d^{11}} - 21*c^{13*d^{10}} + 35*c^{14*d^9} + 35*c^{15*d^8} - 35*c^{16*d^7} - 35*c^{17*d^6} + 21*c^{18*d^5} + 21*c^{19*d^4} - 7*c^{20*d^3} - 7*c^{21*d^2}) - (a^3*((32*a^3*c^{27} + 16*b^3*c^{27} + 96*a^2*b*c^{27} - 160*a^3*c^{26*d} - 16*b^3*c^{26*d} - 32*a^3*c^{10*d^{17}} + 16*a^3*c^{11*d^{16}} + 272*a^3*c^{12*d^{15}} - 132*a^3*c^{13*d^{14}} - 1020*a^3*c^{14*d^{13}} + 528*a^3*c^{15*d^{12}} + 2160*a^3*c^{16*d^{11}} - 1112*a^3*c^{17*d^{10}} - 2920*a^3*c^{18*d^9} + 1280*a^3*c^{19*d^8} + 2752*a^3*c^{20*d^7} - 836*a^3*c^{21*d^6} - 1852*a^3*c^{22*d^5} + 352*a^3*c^{23*d^4} + 800*a^3*c^{24*d^3} - 128*a^3*c^{25*d^2} - 16*b^3*c^{14*d^{13}} + 16*b^3*c^{15*d^{12}} - 44*b^3*c^{16*d^{11}} + 44*b^3*c^{17*d^{10}} + 320*b^3*c^{18*d^9} - 320*b^3*c^{19*d^8} - 520*b^3*c^{20*d^7} + 520*b^3*c^{21*d^6} + 320*b^3*c^{22*d^5} - 320*b^3*c^{23*d^4} - 44*b^3*c^{24*d^3} + 44*b^3*c^{25*d^2} + 180*a*b^2*c^{15*d^{12}} - 180*a*b^2*c^{16*d^{11}} - 480*a*b^2*c^{17*d^{10}} + 480*a*b^2*c^{18*d^9} + 120*a*b^2*c^{19*d^8} - 120*a*b^2*c^{20*d^7} + 720*a*b^2*c^{21*d^6} - 720*a*b^2*c^{22*d^5} - 780*a*b^2*c^{23*d^4} + 780*a*b^2*c^{24*d^3} + 240*a*b^2*c^{25*d^2} - 36*a^2*b*c^{14*d^{13}} + 36*a^2*b*c^{15*d^{12}} - 144*a^2*b*c^{16*d^{11}} + 144*a^2*b*c^{17*d^{10}} + 840*a^2*b*c^{18*d^9} - 840*a^2*b*c^{19*d^8} - 1200*a^2*b*c^{20*d^7} + 1200*a^2*b*c^{21*d^6} + 540*a^2*b*c^{22*d^5} - 540*a^2*b*c^{23*d^4} + 96*a^2*b*c^{24*d^3} - 96*a^2*b*c^{25*d^2} - 240*a*b^2*c^{26*d} - 96*a^2*b*c^{26*d}))/ (c^{26*d} + c^{27} - c^{12*d^{15}} - c^{13*d^{14}} + 7*c^{14*d^{13}} + 7*c^{15*d^{12}} - 21*c^{16*d^{11}} - 21*c^{17*d^{10}} + 35*c^{18*d^9} + 35*c^{19*d^8} - 35*c^{20*d^7} - 35*c^{21*d^6} + 21*c^{22*d^5} + 21*c^{23*d^4} - 7*c^{24*d^3} - 7*c^{25*d^2}) + (a^3*\tan(e/2 + (f*x)/2)*(128*c^{27*d} - 128*c^{10*d^{18}} + 128*c^{11*d^{17}} + 1024*c^{12*d^{16}} - 1024*c^{13*d^{15}} - 3584*c^{14*d^{14}} + 3584*c^{15*d^{13}} + 7168*c^{16*d^{12}} - 7168*c^{17*d^{11}} - 8960*c^{18*d^{10}} + 8960*c^{19*d^9} + 7168*c^{20*d^8} - 7168*c^{21*d^7} - 3584*c^{22*d^6} + 3584*c^{23*d^5} + 1024*c^{24*d^4} - 1024*c^{25*d^3} - 128*c^{26*d^2})*1i)/(2*c^5*(c^{22*d} + c^{23} - c^{8*d^{15}} - c^{9*d^{14}} + 7*c^{10*d^{13}} + 7*c^{11*d^{12}} - 21*c^{12*d^{11}} - 21*c^{13*d^{10}} + 35*c^{14*d^9} + 35*c^{15*d^8} - 35*c^{16*d^7} - 35*c^{17*d^6} + 21*c^{18*d^5} + 21*c^{19*d^4} - 7*c^{20*d^3} - 7*c^{21*d^2}))*1i)/c^5)) / c^5) / ((64*a^9*d^{17} - 192*a^8*b*c^{17} - 32*a^9*c*d^{16} + 320*a^9*c^{16*d} + 16*a^3*b^6*c^{17} + 192*a^5*b^4*c^{17} - 32*a^6*b^3*c^{17} + 576*a^7*b^2*c^{17} - 544*a^9*c^2*d^{15} + 264*a^9*c^3*d^{14} + 2040*a^9*c^4*d^{13} - 856*a^9*c^5*d^{12} - 4320*a^9*c^6*d^{11} + 1649*a^9*c^7*d^{10} + 5840*a^9*c^8*d^9 - 2416*a^9*c^9*d^8 - 5504*a^9*c^{10*d^7} + 2936
\end{aligned}$$

$$\begin{aligned}
& *a^9*c^{11}*d^6 + 3704*a^9*c^{12}*d^5 - 1600*a^9*c^{13}*d^4 - 1600*a^9*c^{14}*d^3 + \\
& 1280*a^9*c^{15}*d^2 - 480*a^4*b^5*c^{16}*d - 3168*a^6*b^3*c^{16}*d + 480*a^7*b^2 \\
& *c^{16}*d - 72*a^8*b*c^4*d^{13} - 72*a^8*b*c^5*d^{12} - 216*a^8*b*c^6*d^{11} - 288* \\
& a^8*b*c^7*d^{10} + 1986*a^8*b*c^8*d^9 + 1680*a^8*b*c^9*d^8 - 4224*a^8*b*c^{10}* \\
& d^7 - 2400*a^8*b*c^{11}*d^6 + 936*a^8*b*c^{12}*d^5 + 1080*a^8*b*c^{13}*d^4 - 4032 \\
& *a^8*b*c^{14}*d^3 + 192*a^8*b*c^{15}*d^2 + 16*a^3*b^6*c^9*d^8 + 216*a^3*b^6*c^{11} \\
& *d^6 + 761*a^3*b^6*c^{13}*d^4 + 216*a^3*b^6*c^{15}*d^2 - 360*a^4*b^5*c^{10}*d^7 \\
& - 2910*a^4*b^5*c^{12}*d^5 - 3600*a^4*b^5*c^{14}*d^3 + 72*a^5*b^4*c^9*d^8 + 3087 \\
& *a^5*b^4*c^{11}*d^6 + 9552*a^5*b^4*c^{13}*d^4 + 5472*a^5*b^4*c^{15}*d^2 - 32*a^6* \\
& b^3*c^4*d^{13} - 32*a^6*b^3*c^5*d^{12} - 56*a^6*b^3*c^6*d^{11} - 88*a^6*b^3*c^7*d \\
& ^{10} + 736*a^6*b^3*c^8*d^9 + 640*a^6*b^3*c^9*d^8 - 2564*a^6*b^3*c^{10}*d^7 - 1 \\
& 040*a^6*b^3*c^{11}*d^6 - 6864*a^6*b^3*c^{12}*d^5 + 640*a^6*b^3*c^{13}*d^4 - 12552 \\
& *a^6*b^3*c^{14}*d^3 - 88*a^6*b^3*c^{15}*d^2 + 360*a^7*b^2*c^5*d^{12} + 360*a^7*b^2 \\
& *c^6*d^{11} - 1320*a^7*b^2*c^7*d^{10} - 960*a^7*b^2*c^8*d^9 + 1191*a^7*b^2*c^9 \\
& *d^8 + 240*a^7*b^2*c^{10}*d^7 + 3816*a^7*b^2*c^{11}*d^6 + 1440*a^7*b^2*c^{12}*d^5 \\
& + 5976*a^7*b^2*c^{13}*d^4 - 1560*a^7*b^2*c^{14}*d^3 + 7776*a^7*b^2*c^{15}*d^2 - \\
& 1728*a^8*b*c^{16}*d)/(c^{26}*d + c^{27} - c^{12}*d^{15} - c^{13}*d^{14} + 7*c^{14}*d^{13} + 7 \\
& *c^{15}*d^{12} - 21*c^{16}*d^{11} - 21*c^{17}*d^{10} + 35*c^{18}*d^9 + 35*c^{19}*d^8 - 35*c \\
& ^{20}*d^7 - 35*c^{21}*d^6 + 21*c^{22}*d^5 + 21*c^{23}*d^4 - 7*c^{24}*d^3 - 7*c^{25}*d^2 \\
&) - (a^3*((tan(e/2 + (f*x)/2)*(64*a^6*c^{18} + 128*a^6*d^{18} + 16*b^6*c^{18} - 1 \\
& 28*a^6*c*d^{17} - 128*a^6*c^{17}*d + 192*a^2*b^4*c^{18} + 576*a^4*b^2*c^{18} - 1024 \\
& *a^6*c^2*d^{16} + 1024*a^6*c^3*d^{15} + 3584*a^6*c^4*d^{14} - 3584*a^6*c^5*d^{13} - \\
& 6968*a^6*c^6*d^{12} + 7168*a^6*c^7*d^{11} + 8385*a^6*c^8*d^{10} - 8960*a^6*c^9*d \\
& ^9 - 7024*a^6*c^{10}*d^8 + 7168*a^6*c^{11}*d^7 + 4848*a^6*c^{12}*d^6 - 3584*a^6*c \\
& ^{13}*d^5 - 1920*a^6*c^{14}*d^4 + 1024*a^6*c^{15}*d^3 + 1152*a^6*c^{16}*d^2 + 16*b^6 \\
& *c^{10}*d^8 + 216*b^6*c^{12}*d^6 + 761*b^6*c^{14}*d^4 + 216*b^6*c^{16}*d^2 - 360*a \\
& *b^5*c^{11}*d^7 - 2910*a*b^5*c^{13}*d^5 - 3600*a*b^5*c^{15}*d^3 - 3200*a^3*b^3*c^ \\
& 17*d - 144*a^5*b*c^5*d^{13} - 504*a^5*b*c^7*d^{11} + 3666*a^5*b*c^9*d^9 - 6624* \\
& a^5*b*c^{11}*d^7 + 2016*a^5*b*c^{13}*d^5 - 3840*a^5*b*c^{15}*d^3 + 72*a^2*b^4*c^{11} \\
& 0*d^8 + 3087*a^2*b^4*c^{12}*d^6 + 9552*a^2*b^4*c^{14}*d^4 + 5472*a^2*b^4*c^{16}*d \\
& ^2 - 64*a^3*b^3*c^5*d^{13} - 144*a^3*b^3*c^7*d^{11} + 1376*a^3*b^3*c^9*d^9 - 36 \\
& 04*a^3*b^3*c^{11}*d^7 - 6224*a^3*b^3*c^{13}*d^5 - 12640*a^3*b^3*c^{15}*d^3 + 720* \\
& a^4*b^2*c^6*d^{12} - 2280*a^4*b^2*c^8*d^{10} + 1431*a^4*b^2*c^{10}*d^8 + 5256*a^4 \\
& *b^2*c^{12}*d^6 + 4416*a^4*b^2*c^{14}*d^4 + 8256*a^4*b^2*c^{16}*d^2 - 480*a*b^5*c \\
& ^{17}*d - 1920*a^5*b*c^{17}*d))/(2*(c^{22}*d + c^{23} - c^8*d^{15} - c^9*d^{14} + 7*c^{11} \\
& 0*d^{13} + 7*c^{11}*d^{12} - 21*c^{12}*d^{11} - 21*c^{13}*d^{10} + 35*c^{14}*d^9 + 35*c^{15} \\
& *d^8 - 35*c^{16}*d^7 - 35*c^{17}*d^6 + 21*c^{18}*d^5 + 21*c^{19}*d^4 - 7*c^{20}*d^3 - \\
& 7*c^{21}*d^2)) + (a^3*((32*a^3*c^{27} + 16*b^3*c^{27} + 96*a^2*b*c^{27} - 160*a^3*c \\
& ^{26}*d - 16*b^3*c^{26}*d - 32*a^3*c^{10}*d^{17} + 16*a^3*c^{11}*d^{16} + 272*a^3*c^{12} \\
& *d^{15} - 132*a^3*c^{13}*d^{14} - 1020*a^3*c^{14}*d^{13} + 528*a^3*c^{15}*d^{12} + 2160*a^ \\
& 3*c^{16}*d^{11} - 1112*a^3*c^{17}*d^{10} - 2920*a^3*c^{18}*d^9 + 1280*a^3*c^{19}*d^8 + \\
& 2752*a^3*c^{20}*d^7 - 836*a^3*c^{21}*d^6 - 1852*a^3*c^{22}*d^5 + 352*a^3*c^{23}*d^4 \\
& + 800*a^3*c^{24}*d^3 - 128*a^3*c^{25}*d^2 - 16*b^3*c^{14}*d^{13} + 16*b^3*c^{15}*d^{11} \\
& 2 - 44*b^3*c^{16}*d^{11} + 44*b^3*c^{17}*d^{10} + 320*b^3*c^{18}*d^9 - 320*b^3*c^{19}*d \\
& ^8 - 520*b^3*c^{20}*d^7 + 520*b^3*c^{21}*d^6 + 320*b^3*c^{22}*d^5 - 320*b^3*c^{23} \\
& *d^4 - 44*b^3*c^{24}*d^3 + 44*b^3*c^{25}*d^2 + 180*a*b^2*c^{15}*d^{12} - 180*a*b^2*c \\
& ^{16}*d^{11} - 480*a*b^2*c^{17}*d^{10} + 480*a*b^2*c^{18}*d^9 + 120*a*b^2*c^{19}*d^8 - \\
& 120*a*b^2*c^{20}*d^7 + 720*a*b^2*c^{21}*d^6 - 720*a*b^2*c^{22}*d^5 - 780*a*b^2*c^ \\
& 23*d^4 + 780*a*b^2*c^{24}*d^3 + 240*a*b^2*c^{25}*d^2 - 36*a^2*b*c^{14}*d^{13} + 36* \\
& a^2*b*c^{15}*d^{12} - 144*a^2*b*c^{16}*d^{11} + 144*a^2*b*c^{17}*d^{10} + 840*a^2*b*c^{18} \\
& *d^9 - 840*a^2*b*c^{19}*d^8 - 1200*a^2*b*c^{20}*d^7 + 1200*a^2*b*c^{21}*d^6 + 54 \\
& 0*a^2*b*c^{22}*d^5 - 540*a^2*b*c^{23}*d^4 + 96*a^2*b*c^{24}*d^3 - 96*a^2*b*c^{25}*d \\
& ^2 - 240*a*b^2*c^{26}*d - 96*a^2*b*c^{26}*d)/(c^{26}*d + c^{27} - c^{12}*d^{15} - c^{13} \\
& *d^{14} + 7*c^{14}*d^{13} + 7*c^{15}*d^{12} - 21*c^{16}*d^{11} - 21*c^{17}*d^{10} + 35*c^{18} \\
& *d^9 + 35*c^{19}*d^8 - 35*c^{20}*d^7 - 35*c^{21}*d^6 + 21*c^{22}*d^5 + 21*c^{23}*d^4 - 7 \\
& *c^{24}*d^3 - 7*c^{25}*d^2) - (a^3*tan(e/2 + (f*x)/2)*(128*c^{27}*d - 128*c^{10}*d^ \\
& 18 + 128*c^{11}*d^{17} + 1024*c^{12}*d^{16} - 1024*c^{13}*d^{15} - 3584*c^{14}*d^{14} + 358 \\
& 4*c^{15}*d^{13} + 7168*c^{16}*d^{12} - 7168*c^{17}*d^{11} - 8960*c^{18}*d^{10} + 8960*c^{19} \\
& *d^9 + 7168*c^{20}*d^8 - 7168*c^{21}*d^7 - 3584*c^{22}*d^6 + 3584*c^{23}*d^5 + 1024*
\end{aligned}$$

$$\begin{aligned}
& c^{24}d^4 - 1024c^{25}d^3 - 128c^{26}d^2) * i) / (2c^5 * (c^{22}d + c^{23} - c^8d^{15} - c^9d^{14} + 7c^{10}d^{13} + 7c^{11}d^{12} - 21c^{12}d^{11} - 21c^{13}d^{10} + 35c^{14}d^9 + 35c^{15}d^8 - 35c^{16}d^7 - 35c^{17}d^6 + 21c^{18}d^5 + 21c^{19}d^4 - 7c^{20}d^3 - 7c^{21}d^2)) * i) / c^5 * i) / c^5 + (a^3 * ((\tan(e/2 + (f*x)/2) * (64a^6c^{18} + 128a^6d^{18} + 16b^6c^{18} - 128a^6c^*d^{17} - 128a^6c^{17}d + 192a^2b^4c^{18} + 576a^4b^2c^{18} - 1024a^6c^2d^{16} + 1024a^6c^3d^{15} + 3584a^6c^4d^{14} - 3584a^6c^5d^{13} - 6968a^6c^6d^{12} + 7168a^6c^7d^{11} + 8385a^6c^8d^{10} - 8960a^6c^9d^9 - 7024a^6c^{10}d^8 + 7168a^6c^{11}d^7 + 4848a^6c^{12}d^6 - 3584a^6c^{13}d^5 - 1920a^6c^{14}d^4 + 1024a^6c^{15}d^3 + 1152a^6c^{16}d^2 + 16b^6c^{10}d^8 + 216b^6c^{12}d^6 + 761b^6c^{14}d^4 + 216b^6c^{16}d^2 - 360a*b^5c^{11}d^7 - 2910a*b^5c^{13}d^5 - 3600a*b^5c^{15}d^3 - 3200a^3b^3c^{17}d - 144a^5b*c^5d^{13} - 504a^5b*c^7d^{11} + 3666a^5b*c^9d^9 - 6624a^5b*c^{11}d^7 + 2016a^5b*c^{13}d^5 - 3840a^5b*c^{15}d^3 + 72a^2b^4c^{10}d^8 + 3087a^2b^4c^{12}d^6 + 9552a^2b^4c^{14}d^4 + 5472a^2b^4c^{16}d^2 - 64a^3b^3c^5d^{13} - 144a^3b^3c^7d^{11} + 1376a^3b^3c^9d^9 - 3604a^3b^3c^{11}d^7 - 6224a^3b^3c^{13}d^5 - 12640a^3b^3c^{15}d^3 + 720a^4b^2c^6d^{12} - 2280a^4b^2c^8d^{10} + 1431a^4b^2c^{10}d^8 + 5256a^4b^2c^{12}d^6 + 4416a^4b^2c^{14}d^4 + 8256a^4b^2c^{16}d^2 - 480a*b^5c^{17}d - 1920a^5b*c^{17}d)) / (2 * (c^{22}d + c^{23} - c^8d^{15} - c^9d^{14} + 7c^{10}d^{13} + 7c^{11}d^{12} - 21c^{12}d^{11} - 21c^{13}d^{10} + 35c^{14}d^9 + 35c^{15}d^8 - 35c^{16}d^7 - 35c^{17}d^6 + 21c^{18}d^5 + 21c^{19}d^4 - 7c^{20}d^3 - 7c^{21}d^2)) - (a^3 * ((32a^3c^{27} + 16b^3c^{27} + 96a^2b*c^{27} - 160a^3c^{26}d - 16b^3c^{26}d - 32a^3c^{10}d^{17} + 16a^3c^{11}d^{16} + 272a^3c^{12}d^{15} - 132a^3c^{13}d^{14} - 1020a^3c^{14}d^{13} + 528a^3c^{15}d^{12} + 2160a^3c^{16}d^{11} - 1112a^3c^{17}d^{10} - 2920a^3c^{18}d^9 + 1280a^3c^{19}d^8 + 2752a^3c^{20}d^7 - 836a^3c^{21}d^6 - 1852a^3c^{22}d^5 + 352a^3c^{23}d^4 + 800a^3c^{24}d^3 - 128a^3c^{25}d^2 - 16b^3c^{14}d^{13} + 16b^3c^{15}d^{12} - 44b^3c^{16}d^{11} + 44b^3c^{17}d^{10} + 320b^3c^{18}d^9 - 320b^3c^{19}d^8 - 520b^3c^{20}d^7 + 520b^3c^{21}d^6 + 320b^3c^{22}d^5 - 320b^3c^{23}d^4 - 44b^3c^{24}d^3 + 44b^3c^{25}d^2 + 180a*b^2c^{15}d^{12} - 180a*b^2c^{16}d^{11} - 480a*b^2c^{17}d^{10} + 480a*b^2c^{18}d^9 + 120a*b^2c^{19}d^8 - 120a*b^2c^{20}d^7 + 720a*b^2c^{21}d^6 - 720a*b^2c^{22}d^5 - 780a*b^2c^{23}d^4 + 780a*b^2c^{24}d^3 + 240a*b^2c^{25}d^2 - 36a^2b*c^{14}d^{13} + 36a^2b*c^{15}d^{12} - 144a^2b*c^{16}d^{11} + 144a^2b*c^{17}d^{10} + 840a^2b*c^{18}d^9 - 840a^2b*c^{19}d^8 - 1200a^2b*c^{20}d^7 + 1200a^2b*c^{21}d^6 + 540a^2b*c^{22}d^5 - 540a^2b*c^{23}d^4 + 96a^2b*c^{24}d^3 - 96a^2b*c^{25}d^2 - 240a*b^2c^{26}d - 96a^2b*c^{26}d)) / (c^{26}d + c^{27} - c^{12}d^{15} - c^{13}d^{14} + 7c^{14}d^{13} + 7c^{15}d^{12} - 21c^{16}d^{11} - 21c^{17}d^{10} + 35c^{18}d^9 + 35c^{19}d^8 - 35c^{20}d^7 - 35c^{21}d^6 + 21c^{22}d^5 + 21c^{23}d^4 - 7c^{24}d^3 - 7c^{25}d^2) + (a^3 * \tan(e/2 + (f*x)/2) * (128c^{27}d - 128c^{10}d^{18} + 128c^{11}d^{17} + 1024c^{12}d^{16} - 1024c^{13}d^{15} - 3584c^{14}d^{14} + 3584c^{15}d^{13} + 7168c^{16}d^{12} - 7168c^{17}d^{11} - 8960c^{18}d^{10} + 8960c^{19}d^9 + 7168c^{20}d^8 - 7168c^{21}d^7 - 3584c^{22}d^6 + 3584c^{23}d^5 + 1024c^{24}d^4 - 1024c^{25}d^3 - 128c^{26}d^2) * i) / (2c^5 * (c^{22}d + c^{23} - c^8d^{15} - c^9d^{14} + 7c^{10}d^{13} + 7c^{11}d^{12} - 21c^{12}d^{11} - 21c^{13}d^{10} + 35c^{14}d^9 + 35c^{15}d^8 - 35c^{16}d^7 - 35c^{17}d^6 + 21c^{18}d^5 + 21c^{19}d^4 - 7c^{20}d^3 - 7c^{21}d^2)) * i) / c^5 * i) / c^5)) / (c^5 * f) - ((\tan(e/2 + (f*x)/2) ^ 7 * (8a^3d^8 + 4b^3c^8 - 24a*b^2c^8 - 4a^3c*d^7 + 32b^3c^7d - 32a^3c^2d^6 + 15a^3c^3d^5 + 40a^3c^4d^4 - 40a^3c^5d^3 - 80a^3c^6d^2 + 4b^3c^4d^4 + 32b^3c^5d^3 + 21b^3c^6d^2 - 24a*b^2c^4d^4 - 51a*b^2c^5d^3 - 144a*b^2c^6d^2 + 15a^2b*c^4d^4 + 96a^2b*c^5d^3 + 72a^2b*c^6d^2 - 36a*b^2c^7d + 96a^2b*c^7d)) / (4 * (c^4d - c^5) * (c + d)^4) - (\tan(e/2 + (f*x)/2) * (4b^3c^8 - 8a^3d^8 + 24a*b^2c^8 - 4a^3c*d^7 - 32b^3c^7d + 32a^3c^2d^6 + 15a^3c^3d^5 - 40a^3c^4d^4 - 40a^3c^5d^3 + 80a^3c^6d^2 + 4b^3c^4d^4 - 32b^3c^5d^3 + 21b^3c^6d^2 + 24a*b^2c^4d^4 - 51a*b^2c^5d^3 + 144a*b^2c^6d^2 + 15a^2b*c^4d^4 - 96a^2b*c^5d^3 + 72a^2b*c^6d^2 - 36a*b^2c^7d - 96a^2b*c^7d)) / (4 * (c + d) * (c^8 - 4c^7d + c^4d^4 - 4c^5d^3 + 6c^6d^2)) + (\tan(e/2 + (f*x)/
\end{aligned}$$

$$2)^5 \cdot (72a^3d^8 + 12b^3c^8 - 216ab^2c^8 - 12a^3cd^7 + 224b^3c^7d - 320a^3c^2d^6 + 69a^3c^3d^5 + 520a^3c^4d^4 - 120a^3c^5d^3 - 720a^3c^6d^2 + 12b^3c^4d^4 + 224b^3c^5d^3 + 39b^3c^6d^2 - 120ab^2c^4d^4 - 81ab^2c^5d^3 - 1008ab^2c^6d^2 - 27a^2b^2c^4d^4 + 480a^2b^2c^5d^3 + 216a^2b^2c^6d^2 - 108ab^2c^7d + 864a^2b^2c^7d) / (12(c+d)^3(c^6 - 2c^5d + c^4d^2)) + (\tan(e/2 + (fx)/2)^3 \cdot (72a^3d^8 - 12b^3c^8 - 216ab^2c^8 + 12a^3cd^7 + 224b^3c^7d - 320a^3c^2d^6 - 69a^3c^3d^5 + 520a^3c^4d^4 + 120a^3c^5d^3 - 720a^3c^6d^2 - 12b^3c^4d^4 + 224b^3c^5d^3 - 39b^3c^6d^2 - 120ab^2c^4d^4 + 81ab^2c^5d^3 - 1008ab^2c^6d^2 + 27a^2b^2c^4d^4 + 480a^2b^2c^5d^3 - 216a^2b^2c^6d^2 + 108ab^2c^7d + 864a^2b^2c^7d) / (12(c+d)^2(3c^6d - c^7 + c^4d^3 - 3c^5d^2))) / (f \cdot (\tan(e/2 + (fx)/2)^4(6c^4 + 6d^4 - 12c^2d^2) + \tan(e/2 + (fx)/2)^2(8cd^3 - 8c^3d - 4c^4 + 4d^4) - \tan(e/2 + (fx)/2)^6(8cd^3 - 8c^3d + 4c^4 - 4d^4) + \tan(e/2 + (fx)/2)^8(c^4 - 4c^3d - 4cd^3 + d^4 + 6c^2d^2) + 4cd^3 + 4c^3d + c^4 + d^4 + 6c^2d^2))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^3}{(c + d \sec(e + fx))^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)

[Out] Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**5, x)

3.198 $\int \sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx)) dx$

Optimal. Leaf size=320

$$\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)}{bf} - \frac{2c\sqrt{a+b}\cot(e+fx)}{bf}$$

[Out] $-2*(a-b)*d*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e)))/(a-b)^{1/2}/b/f+2*(b*(c-d)+a*d)*\cot(f*x+e)*\text{EllipticF}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2},((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e)))/(a-b)^{1/2}/b/f-2*c*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2},(a+b)/a,((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e)))/(a-b)^{1/2}/f$

Rubi [A] time = 0.28, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3916, 3784, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)}{bf} - \frac{2c\sqrt{a+b}\cot(e+fx)}{bf}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*d*\text{Cot}[e+f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(b*f)+(2*\text{Sqrt}[a+b]*(b*(c-d)+a*d)*\text{Cot}[e+f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(b*f)-(2*\text{Sqrt}[a+b]*c*\text{Cot}[e+f*x]*\text{EllipticPi}[(a+b)/a,\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]],(a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/f$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3916

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Int[(Csc[e + f*x]*(b*c + a*d + b*d*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[cs
sc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[cs
sc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx)) dx = (ac) \int \frac{1}{\sqrt{a + b \sec(e + fx)}} dx + \int \frac{\sec(e + fx)(bc + ad + bd \sec(e + fx))}{\sqrt{a + b \sec(e + fx)}} dx$$

$$= -\frac{2\sqrt{a + b} c \cot(e + fx) \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(e+fx))}{a-b}}}{f}$$

$$= -\frac{2(a-b)\sqrt{a+b} d \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(e+fx))}{a-b}}}{bf}$$

Mathematica [C] time = 17.94, size = 913, normalized size = 2.85

$$\frac{2d \cos(e + fx) \sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx)) \sin(e + fx)}{f(d + c \cos(e + fx))} + \frac{2\sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx)) \left(a \sqrt{\frac{b(1-\sec(e+fx))}{a-b}} \right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]

```
[Out] (2*d*cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])*Sin[e + f*x
])/((f*(d + c*cos[e + f*x])) + (2*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x
]))*(a*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] + b*Sqrt[(-a + b)/(a + b)]
*d*Tan[(e + f*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^3 + a*S
qrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*d*Tan
[(e + f*x)/2]^5 + (2*I)*a*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-
a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)
/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
+ (2*I)*a*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f
*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b
)] - I*(a - b)*d*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*
Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a
- b)*(c - d)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]],
```


$n(f*x+e), ((a-b)/(a+b))^{(1/2)} * a*d*\sin(f*x+e) + (\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)} * ((b+a*\cos(f*x+e))/(1+\cos(f*x+e)) / (a+b))^{(1/2)} * \text{EllipticE}((-1+\cos(f*x+e)) / \sin(f*x+e), ((a-b)/(a+b))^{(1/2)}) * b*d*\sin(f*x+e) - 2 * (\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)} * ((b+a*\cos(f*x+e))/(1+\cos(f*x+e)) / (a+b))^{(1/2)} * \text{EllipticPi}((-1+\cos(f*x+e)) / \sin(f*x+e), -1, ((a-b)/(a+b))^{(1/2)}) * a*c*\sin(f*x+e) - \cos(f*x+e)^2 * a*d + \cos(f*x+e) * a*d - \cos(f*x+e) * b*d + b*d) / \sin(f*x+e)^5 / (b+a*\cos(f*x+e))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(fx + e) + a} (d \sec(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{b}{\cos(e + fx)}} \left(c + \frac{d}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)

[Out] int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))*(a+b*sec(f*x+e))^(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x)), x)

$$3.199 \quad \int \frac{\sqrt{a+b \sec(e+fx)}}{c+d \sec(e+fx)} dx$$

Optimal. Leaf size=220

$$\frac{2(bc - ad) \tan(e + fx) \sqrt{\frac{a+b \sec(e+fx)}{a+b}} \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) - 2\sqrt{a+b} \cot(e + fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\tan^2(e + fx)}}{cf(c+d) \sqrt{-\tan^2(e + fx)} \sqrt{a+b \sec(e + fx)}}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}, (a+b)/a, ((a+b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*(b*(1-\sec(f*x+e)))/(a+b)^{(1/2)}*(-b*(1+\sec(f*x+e)))/(a-b)^{(1/2)}/c/f+2*(-a*d+b*c)*\text{EllipticPi}(1/2*(1-\sec(f*x+e))^{(1/2)}*2^{(1/2)}, 2*d/(c+d), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\sec(f*x+e))/(a+b))^{(1/2)}*\tan(f*x+e)/c/(c+d)/f/(a+b*\sec(f*x+e))^{(1/2)}/(-\tan(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.24, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3926, 3784, 3973}

$$\frac{2(bc - ad) \tan(e + fx) \sqrt{\frac{a+b \sec(e+fx)}{a+b}} \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) - 2\sqrt{a+b} \cot(e + fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\tan^2(e + fx)}}{cf(c+d) \sqrt{-\tan^2(e + fx)} \sqrt{a+b \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]

[Out] $(-2*\text{Sqrt}[a + b]*\text{Cot}[e + f*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b))*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(c*f) + (2*(b*c - a*d)*\text{EllipticPi}[(2*d)/(c + d), \text{ArcSin}[\text{Sqrt}[1 - \text{Sec}[e + f*x]]/\text{Sqrt}[2]], (2*b)/(a + b)]*\text{Sqrt}[(a + b*\text{Sec}[e + f*x])/(a + b)]*\text{Tan}[e + f*x])/(c*(c + d)*f*\text{Sqrt}[a + b*\text{Sec}[e + f*x]]*\text{Sqrt}[-\text{Tan}[e + f*x]^2])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3926

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := Dist[a/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[(b*c - a*d)/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3973

Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))), x_Symbol] := Simp[(-2*Cot[e + f*x]*Sqrt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]


```
[Out] -2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c^2+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c^2-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*a*d^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a*c^2+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a*d^2)*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f*x+e)^2/c/(c-d)/(c+d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(fx + e) + a}}{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{\cos(e+fx)}}}{c + \frac{d}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec(e + fx)}}{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

3.200 $\int (a + b \sec(e + fx))^{3/2} (c + d \sec(e + fx)) dx$

Optimal. Leaf size=380

$$\frac{2\sqrt{a+b} (3a^2d + ab(6c - 4d) - b^2(3c - d)) \cot(e + fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b}}\right)\right)}{3bf}$$

[Out] $-2/3*(a-b)*(4*a*d+3*b*c)*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/b/f+2/3*(a*b*(6*c-4*d)-b^2*(3*c-d)+3*a^2*d)*\cot(f*x+e)*\text{EllipticF}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/b/f-2*a*c*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/f+2/3*b*d*(a+b*\sec(f*x+e))^{1/2}*\tan(f*x+e)/f$

Rubi [A] time = 0.43, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3918, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b} (3a^2d + ab(6c - 4d) - b^2(3c - d)) \cot(e + fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b}}\right)\right)}{3bf}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]), x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(3*b*c+4*a*d)*\text{Cot}[e+f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(3*b*f) + (2*\text{Sqrt}[a+b]*(a*b*(6*c-4*d)-b^2*(3*c-d)+3*a^2*d)*\text{Cot}[e+f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(3*b*f) - (2*a*\text{Sqrt}[a+b]*c*\text{Cot}[e+f*x]*\text{EllipticPi}[(a+b)/a, \text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/f + (2*b*d*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]*\text{Tan}[e+f*x])/3*f)$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3918

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m +

$(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*\text{Csc}[e + f*x] + b*(b*c*m + a*d*(2*m - 1))*\text{Csc}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{GtQ}\{m, 1\} \&\& \text{NeQ}\{a^2 - b^2, 0\} \&\& \text{IntegerQ}\{2*m\}$

Rule 3921

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_.))*(d_.) + (c_.)]/\text{Sqrt}[\text{csc}[e_.] + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[1/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] + \text{Dist}[d, \text{Int}[\text{Csc}[e + f*x]/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{NeQ}\{a^2 - b^2, 0\}$

Rule 4004

$\text{Int}[(\text{csc}[e_.] + (f_.)*(x_.))*(\text{csc}[e_.] + (f_.)*(x_.))*(B_.) + (A_.)]/\text{Sqrt}[\text{csc}[e_.] + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Simp}[(-2*(A*b - a*B)*\text{Rt}[a + (b*B)/A, 2]*\text{Sqrt}[(b*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Sqrt}[-(b*(1 + \text{Csc}[e + f*x]))/(a - b)])*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Csc}[e + f*x]]/\text{Rt}[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*\text{Cot}[e + f*x]), x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x\} \&\& \text{NeQ}\{a^2 - b^2, 0\} \&\& \text{EqQ}\{A^2 - B^2, 0\}$

Rule 4058

$\text{Int}[(A_.) + \text{csc}[e_.] + (f_.)*(x_.)]*(B_.) + \text{csc}[e_.] + (f_.)*(x_.)]^2*(C_.)]/\text{Sqrt}[\text{csc}[e_.] + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Int}[(A + (B - C)*\text{Csc}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x] + \text{Dist}[C, \text{Int}[(\text{Csc}[e + f*x]*(1 + \text{Csc}[e + f*x]))/\text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x\} \&\& \text{NeQ}\{a^2 - b^2, 0\}$

Rubi steps

$$\begin{aligned} \int (a + b \sec(e + fx))^{3/2} (c + d \sec(e + fx)) dx &= \frac{2bd\sqrt{a + b \sec(e + fx)} \tan(e + fx)}{3f} + \frac{2}{3} \int \frac{\frac{3a^2c}{2} + \frac{1}{2}(6abc + 3)}{\sqrt{a + b \sec(e + fx)}} dx \\ &= \frac{2bd\sqrt{a + b \sec(e + fx)} \tan(e + fx)}{3f} + \frac{2}{3} \int \frac{\frac{3a^2c}{2} + \left(-\frac{1}{2}b(3bc + 4ad)\right)}{\sqrt{a + b \sec(e + fx)}} dx \\ &= -\frac{2(a - b)\sqrt{a + b} (3bc + 4ad) \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right)\right)}{3bf} \\ &= -\frac{2(a - b)\sqrt{a + b} (3bc + 4ad) \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right)\right)}{3bf} \end{aligned}$$

Mathematica [B] time = 24.76, size = 6063, normalized size = 15.96

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]

[Out] Result too large to show

fricas [F] time = 1.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bd \sec(fx + e)^2 + ac + (bc + ad) \sec(fx + e)\right) \sqrt{b \sec(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a)^{\frac{3}{2}} (d \sec(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)

maple [B] time = 1.88, size = 2337, normalized size = 6.15

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x)

[Out]
$$-2/3/f*(-1+\cos(f*x+e))^2*(4*\cos(f*x+e)^3*a^2*d-4*\cos(f*x+e)^2*a^2*d+3*\cos(f*x+e)^3*a*b*c-b^2*d+\cos(f*x+e)^3*a*b*d-3*\cos(f*x+e)^2*a*b*c+4*\cos(f*x+e)^2*a*b*d-5*\cos(f*x+e)*a*b*d+3*\cos(f*x+e)^2*b^2*c-3*\cos(f*x+e)*b^2*c+\cos(f*x+e)^2*b^2*d-3*\cos(f*x+e)^2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*b^2*c+6*\cos(f*x+e)^2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticPi((-1+\cos(f*x+e))/\sin(f*x+e),-1,((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*c-3*\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*c+3*\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*d+3*\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*b^2*c+\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*b^2*d-4*\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*d-3*\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*b^2*c+6*\cos(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticPi((-1+\cos(f*x+e))/\sin(f*x+e),-1,((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*c+3*\cos(f*x+e)^2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*d+3*\cos(f*x+e)^2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*b^2*d-4*\cos(f*x+e)^2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*\sin(f*x+e)*a^2*d+4*\cos(f*x+e)^2*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*E$$

```

lIipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d-3
*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*
x+e)))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*sin(f*x+e)*a*b*c-4*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*c
os(f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+6*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*EllipticF((-1+cos(
f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c+4*cos(f*x+e)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*
EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d-
3*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x
+e)))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))
*sin(f*x+e)*a*b*c-4*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(
f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+6*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c*((b+a*cos(f*x+e))/
cos(f*x+e))^(1/2)*(1+cos(f*x+e))^2/(b+a*cos(f*x+e))/cos(f*x+e)/sin(f*x+e)^5

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a)^{\frac{3}{2}} (d \sec(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{\cos(e + fx)} \right)^{\frac{3}{2}} \left(c + \frac{d}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(e + fx))^{\frac{3}{2}} (c + d \sec(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)*(c + d*sec(e + f*x)), x)
```

$$3.201 \quad \int \frac{(a+b \sec(e+fx))^{3/2}}{c+d \sec(e+fx)} dx$$

Optimal. Leaf size=326

$$\frac{2(bc-ad)^2 \tan(e+fx) \sqrt{\frac{a+b \sec(e+fx)}{a+b}} \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) 2a\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{cdf(c+d) \sqrt{-\tan^2(e+fx)} \sqrt{a+b \sec(e+fx)}}$$

[Out] $2*b*\cot(f*x+e)*\text{EllipticF}((a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}, ((a+b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*(b*(1-\sec(f*x+e))/(a+b))^{(1/2)}*(-b*(1+\sec(f*x+e))/(a-b))^{(1/2)}/d/f-2*a*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}, (a+b)/a, ((a+b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*(b*(1-\sec(f*x+e))/(a+b))^{(1/2)}*(-b*(1+\sec(f*x+e))/(a-b))^{(1/2)}/c/f-2*(-a*d+b*c)^2*\text{EllipticPi}(1/2*(1-\sec(f*x+e))^{(1/2)}*2^{(1/2)}, 2*d/(c+d), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\sec(f*x+e))/(a+b))^{(1/2)}*\tan(f*x+e)/c/d/(c+d)/f/(a+b*\sec(f*x+e))^{(1/2)}/(-\tan(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3928, 3921, 3784, 3832, 3973}

$$\frac{2(bc-ad)^2 \tan(e+fx) \sqrt{\frac{a+b \sec(e+fx)}{a+b}} \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) 2a\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{cdf(c+d) \sqrt{-\tan^2(e+fx)} \sqrt{a+b \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]), x]

[Out] $(2*b*\text{Sqrt}[a+b]*\text{Cot}[e+f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(d*f) - (2*a*\text{Sqrt}[a+b]*\text{Cot}[e+f*x]*\text{EllipticPi}[(a+b)/a, \text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(c*f) - (2*(b*c-a*d)^2*\text{EllipticPi}[(2*d)/(c+d), \text{ArcSin}[\text{Sqrt}[1-\text{Sec}[e+f*x]]/\text{Sqrt}[2]], (2*b)/(a+b)]*\text{Sqrt}[(a+b*\text{Sec}[e+f*x])/(a+b)]*\text{Tan}[e+f*x]/(c*d*(c+d)*f*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]*\text{Sqrt}[-\text{Tan}[e+f*x]^2])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d}, x]

$d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3928

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^{(3/2)})/(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] \rightarrow \text{Dist}[1/(c*d), \text{Int}[(a^2*d + b^2*c*\text{Csc}[e + f*x])/ \text{Sqrt}[a + b*\text{Csc}[e + f*x]], x], x] - \text{Dist}[(b*c - a*d)^2/(c*d), \text{Int}[\text{Csc}[e + f*x]/(\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*(c + d*\text{Csc}[e + f*x])), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 3973

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]/(\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] \rightarrow \text{Simp}[(-2*\text{Cot}[e + f*x]*\text{Sqrt}[(a + b*\text{Csc}[e + f*x])/(a + b)]*\text{EllipticPi}[(2*d)/(c + d), \text{ArcSin}[\text{Sqrt}[1 - \text{Csc}[e + f*x]]/\text{Sqrt}[2]], (2*b)/(a + b)]/(f*(c + d)*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[-\text{Cot}[e + f*x]^2]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec(e + fx))^{3/2}}{c + d \sec(e + fx)} dx &= \frac{\int \frac{a^2 d + b^2 c \sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx}{cd} - \frac{(bc - ad)^2 \int \frac{\sec(e + fx)}{\sqrt{a + b \sec(e + fx)}(c + d \sec(e + fx))} dx}{cd} \\ &= -\frac{2(bc - ad)^2 \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1 - \sec(e + fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) \sqrt{\frac{a + b \sec(e + fx)}{a+b}} \tan(e + fx) + a^2 \int \frac{1}{\sqrt{a + b \sec(e + fx)}} dx}{cd(c + d)f\sqrt{a + b \sec(e + fx)}\sqrt{-\tan^2(e + fx)}} \\ &= \frac{2b\sqrt{a + b} \cot(e + fx) F\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1 - \sec(e + fx))}{a+b}} \sqrt{\frac{b(1 + \sec(e + fx))}{a-b}}}{df} \end{aligned}$$

Mathematica [A] time = 6.31, size = 230, normalized size = 0.71

$$\frac{4 \cos^2\left(\frac{1}{2}(e + fx)\right) \sqrt{\frac{\cos(e + fx)}{\cos(e + fx) + 1}} \sqrt{\frac{a \cos(e + fx) + b}{(a + b)(\cos(e + fx) + 1)}} \sqrt{a + b \sec(e + fx)} \left(c(a - b)^2(c + d) F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1 - \sec(e + fx))}{a+b}} \sqrt{\frac{b(1 + \sec(e + fx))}{a-b}}\right)}{cf(c - d)(c + d)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]

[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)^2*c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(a^2*(c^2 - d^2)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + (b*c - a*d)^2*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*Sqrt[a + b*Sec[e + f*x]]/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)

maple [A] time = 1.70, size = 581, normalized size = 1.78

$$2\sqrt{\frac{b+a\cos(fx+e)}{\cos(fx+e)}} \sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}} \sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))(a+b)}} (1 + \cos(fx + e))^2 \left(\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}, \sqrt{\frac{a-b}{a+b}}\right) a^2 c^2 + \text{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)

[Out] $-2/f*((b+a*\cos(f*x+e))/\cos(f*x+e))^{(1/2)}*(\cos(f*x+e)/(1+\cos(f*x+e)))^{(1/2)}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))^{(1/2)}*(1+\cos(f*x+e))^2*(\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*a^2*c^2+\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*a^2*c*d-2*\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*a*b*c^2-2*\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*a*b*c*d+\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*b^2*c^2+\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{(1/2)})*b^2*c*d-2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^{(1/2)})*a^2*d^2+4*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^{(1/2)})*a*b*c*d-2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^{(1/2)})*b^2*c^2-2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),-1,((a-b)/(a+b))^{(1/2)})*a^2*c^2+2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),-1,((a-b)/(a+b))^{(1/2)})*a^2*d^2)*(-1+\cos(f*x+e))/(b+a*\cos(f*x+e))/\sin(f*x+e)^2/c/(c-d)/(c+d)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{3/2}}{c + \frac{d}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)

[Out] `int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{\frac{3}{2}}}{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)), x)`

[Out] `Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x)), x)`

3.202 $\int (a + b \sec(e + fx))^{5/2} (c + d \sec(e + fx)) dx$

Optimal. Leaf size=442

$$\frac{2(a-b)\sqrt{a+b} (23a^2d + 35abc + 9b^2d) \cot(e + fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{\frac{b(\sec(e+fx)+1)}{a-b}} E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b}}\right)\right) \frac{a+b}{a-b}}{15bf}$$

[Out] $-2/15*(a-b)*(23*a^2*d+35*a*b*c+9*b^2*d)*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/b/f+2/15*(a^2*b*(45*c-23*d)-a*b^2*(35*c-17*d)+b^3*(5*c-9*d)+15*a^3*d)*\cot(f*x+e)*\text{EllipticF}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/b/f-2*a^2*c*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/f+2/5*b*d*(a+b*\sec(f*x+e))^{3/2}*\tan(f*x+e)/f+2/15*b*(8*a*d+5*b*c)*(a+b*\sec(f*x+e))^{1/2})*\tan(f*x+e)/f$

Rubi [A] time = 0.63, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3918, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b} (a^2b(45c - 23d) + 15a^3d - ab^2(35c - 17d) + b^3(5c - 9d)) \cot(e + fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{\frac{b(\sec(e+fx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b}}\right)\right) \frac{a+b}{a-b}}{15bf}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(35*a*b*c+23*a^2*d+9*b^2*d)*\text{Cot}[e+f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(15*b*f) + (2*\text{Sqrt}[a+b]*(a^2*b*(45*c-23*d)-a*b^2*(35*c-17*d)+b^3*(5*c-9*d)+15*a^3*d)*\text{Cot}[e+f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/(15*b*f) - (2*a^2*\text{Sqrt}[a+b]*c*\text{Cot}[e+f*x]*\text{EllipticPi}[(a+b)/a, \text{ArcSin}[\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/\text{Sqrt}[a+b]], (a+b)/(a-b)]*\text{Sqrt}[(b*(1-\text{Sec}[e+f*x]))/(a+b)]*\text{Sqrt}[-((b*(1+\text{Sec}[e+f*x]))/(a-b))]/f + (2*b*(5*b*c+8*a*d)*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]*\text{Tan}[e+f*x])/f + (2*b*d*(a+b*\text{Sec}[e+f*x])^{3/2}*\text{Tan}[e+f*x])/(5*f)$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3918

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4056

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int (a + b \sec(e + fx))^{5/2} (c + d \sec(e + fx)) dx &= \frac{2bd(a + b \sec(e + fx))^{3/2} \tan(e + fx)}{5f} + \frac{2}{5} \int \sqrt{a + b \sec(e + fx)} dx \\ &= \frac{2b(5bc + 8ad)\sqrt{a + b \sec(e + fx)} \tan(e + fx)}{15f} + \frac{2bd(a + b \sec(e + fx))^{3/2}}{5f} \\ &= \frac{2b(5bc + 8ad)\sqrt{a + b \sec(e + fx)} \tan(e + fx)}{15f} + \frac{2bd(a + b \sec(e + fx))^{3/2}}{5f} \\ &= -\frac{2(a - b)\sqrt{a + b} (35abc + 23a^2d + 9b^2d) \cot(e + fx) E(\sin^{-1}(\frac{a + b \sec(e + fx)}{\sqrt{a + b}}))}{15bf} \\ &= -\frac{2(a - b)\sqrt{a + b} (35abc + 23a^2d + 9b^2d) \cot(e + fx) E(\sin^{-1}(\frac{a + b \sec(e + fx)}{\sqrt{a + b}}))}{15bf} \end{aligned}$$

Mathematica [B] time = 26.00, size = 7138, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]), x]

[Out] Result too large to show

fricas [F] time = 1.47, size = 0, normalized size = 0.00

integral $\left((b^2 d \sec(fx + e))^3 + a^2 c + (b^2 c + 2 abd) \sec(fx + e)^2 + (2 abc + a^2 d) \sec(fx + e) \right) \sqrt{b \sec(fx + e) + a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)), x, algorithm="fricas")

[Out] integral((b^2*d*sec(f*x + e)^3 + a^2*c + (b^2*c + 2*a*b*d)*sec(f*x + e)^2 + (2*a*b*c + a^2*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a)^{\frac{5}{2}} (d \sec(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)

maple [B] time = 2.20, size = 3285, normalized size = 7.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)), x)

[Out] $-2/15/f*(1+\cos(f*x+e))^2*((b+a*\cos(f*x+e))/\cos(f*x+e))^{1/2}*(-1+\cos(f*x+e))^2*(15*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a^3*d+5*\cos(f*x+e)^3*b^3*c+23*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a^2*b*d+35*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a*b^2*c+17*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a*b^2*d-35*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a^2*b*c-23*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a^2*b*d-35*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticE((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a*b^2*c-9*\cos(f*x+e)^3*\sin(f*x+e)*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}*EllipticF((-1+\cos(f*x+e))/\sin(f*x+e), ((a-b)/(a+b))^{1/2})*a^2$

```

*b*c+23*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos
(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),
(a-b)/(a+b))^(1/2))*a^2*b*d+35*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+c
os(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*c+17*cos(f*x+e)^2*sin(f*x+
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b)
)^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*d-3
5*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e
))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/
(a+b))^(1/2))*a^2*b*c-23*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*d-35*cos(f*x+e)^2*sin(f*x+e)*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2
)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2*c-9*cos(f
*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+c
os(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*a*b^2*d+45*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b*c-5*cos(f*x+e)*b^3*c+23*cos(f*x+e)^4*a^
3*d-23*cos(f*x+e)^3*a^3*d+9*cos(f*x+e)^3*b^3*d-6*cos(f*x+e)^2*b^3*d-3*b^3*d
-35*cos(f*x+e)^3*a^2*b*c+23*cos(f*x+e)^3*a^2*b*d+35*cos(f*x+e)^3*a*b^2*c+5*
cos(f*x+e)^3*a*b^2*d-34*cos(f*x+e)^2*a^2*b*d-40*cos(f*x+e)^2*a*b^2*c-14*cos
(f*x+e)*a*b^2*d+35*cos(f*x+e)^4*a^2*b*c+11*cos(f*x+e)^4*a^2*b*d+5*cos(f*x+e
)^4*a*b^2*c+9*cos(f*x+e)^4*a*b^2*d-15*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*Elliptic
F((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*c-23*cos(f*x+e)^2*sin
(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))
/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*
d+9*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x
+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*b^3*d+5*cos(f*x+e)^3*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*c+9*cos(f*x+e)^3*sin(f*x+e)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*El
lipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*d-23*cos(f*x+e)
^3*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*
x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*a^3*d-9*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*c
os(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*b^3*d+30*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+
cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3*c+30*cos(f*x+e)^3*sin(f*
x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+
b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^3
*c-15*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f
*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a
-b)/(a+b))^(1/2))*a^3*c+15*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^3*d+5*cos(f*x+e)^2*sin(f*x+e)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)
*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3*c-9*cos(f*x+
e)^3*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(
f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/
2))*b^3*d/(b+a*cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^5

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a)^2 (d \sec(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{\cos(e + fx)} \right)^{5/2} \left(c + \frac{d}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)

[Out] int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(e + fx))^{5/2} (c + d \sec(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)

[Out] Integral((a + b*sec(e + f*x))**(5/2)*(c + d*sec(e + f*x)), x)

$$3.203 \quad \int \frac{c+d \sec(e+fx)}{\sqrt{a+b \sec(e+fx)}} dx$$

Optimal. Leaf size=208

$$\frac{2d\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2c\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{bf}$$

[Out] 2*d*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a/f

Rubi [A] time = 0.12, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3921, 3784, 3832}

$$\frac{2d\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} F\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) + 2c\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{bf}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]

[Out] (2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))]/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))]/(a*f)

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{c + d \sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx = c \int \frac{1}{\sqrt{a + b \sec(e + fx)}} dx + d \int \frac{\sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx$$

$$= \frac{2\sqrt{a + b} d \cot(e + fx) F\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(e + fx))}{a + b}} \sqrt{\frac{b(1 + \sec(e + fx))}{a - b}}}{bf}$$

Mathematica [A] time = 2.69, size = 145, normalized size = 0.70

$$\frac{4 \cos^2\left(\frac{1}{2}(e + fx)\right) \sqrt{\frac{\cos(e + fx)}{\cos(e + fx) + 1}} \sec(e + fx) \sqrt{\frac{a \cos(e + fx) + b}{(a + b)(\cos(e + fx) + 1)}} \left((d - c) F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e + fx)\right)\right) \middle| \frac{a - b}{a + b}\right) + 2c \Pi\left(-1, \frac{a - b}{a + b}\right) \right)}{f \sqrt{a + b \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]

[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d \sec(fx + e) + c}{\sqrt{b \sec(fx + e) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

maple [A] time = 1.72, size = 215, normalized size = 1.03

$$\frac{2 \sqrt{\frac{b + a \cos(fx + e)}{\cos(fx + e)}} \sqrt{\frac{\cos(fx + e)}{1 + \cos(fx + e)}} \sqrt{\frac{b + a \cos(fx + e)}{(1 + \cos(fx + e))(a + b)}} (1 + \cos(fx + e))^2 (-1 + \cos(fx + e)) \left(\text{EllipticF}\left(\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) \right)}{f (b + a \cos(fx + e)) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)

[Out] -2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x+e))

e))*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*c)/(b+a*cos(f*x+e))/sin(f*x+e)^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + \frac{d}{\cos(e+fx)}}{\sqrt{a + \frac{b}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2),x)

[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral((c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)

$$3.204 \quad \int \frac{1}{\sqrt{a+b \sec(e+fx)} (c+d \sec(e+fx))} dx$$

Optimal. Leaf size=216

$$\frac{2d \tan(e+fx) \sqrt{\frac{a+b \sec(e+fx)}{a+b}} \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) + 2\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx))}{a-b}}}{cf(c+d) \sqrt{-\tan^2(e+fx)} \sqrt{a+b \sec(e+fx)}} - \frac{2\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx))}{a-b}}}{acf}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}, (a+b)/a, ((a+b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*(b*(1-\sec(f*x+e))/(a+b))^{(1/2)}*(-b*(1+\sec(f*x+e)))/(a-b)^{(1/2)}/a/c/f-2*d*\text{EllipticPi}(1/2*(1-\sec(f*x+e))^{(1/2)}*2^{(1/2)}, 2*d/(c+d), 2^{(1/2)}*(b/(a+b))^{(1/2)}*((a+b*\sec(f*x+e))/(a+b))^{(1/2)}*\tan(f*x+e)/c/(c+d)/f/(a+b*\sec(f*x+e))^{(1/2)}/(-\tan(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.24, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3930, 3784, 3973}

$$\frac{2d \tan(e+fx) \sqrt{\frac{a+b \sec(e+fx)}{a+b}} \Pi\left(\frac{2d}{c+d}; \sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right) \middle| \frac{2b}{a+b}\right) + 2\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx))}{a-b}}}{cf(c+d) \sqrt{-\tan^2(e+fx)} \sqrt{a+b \sec(e+fx)}} - \frac{2\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx))}{a-b}}}{acf}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]

[Out] $(-2*\text{Sqrt}[a + b]*\text{Cot}[e + f*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(a*c*f) - (2*d*\text{EllipticPi}[(2*d)/(c + d), \text{ArcSin}[\text{Sqrt}[1 - \text{Sec}[e + f*x]]/\text{Sqrt}[2]], (2*b)/(a + b)]*\text{Sqrt}[(a + b*\text{Sec}[e + f*x))/(a + b)]*\text{Tan}[e + f*x]/(c*(c + d)*f*\text{Sqrt}[a + b*\text{Sec}[e + f*x]]*\text{Sqrt}[-\text{Tan}[e + f*x]^2])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3930

Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))), x_Symbol] :> Dist[1/c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/c, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 3973

Int[csc[(e_.) + (f_.)*(x_.)]/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sqrt[(a + b*Csc[e + f*x))/(a + b]]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx))} dx = \frac{\int \frac{1}{\sqrt{a + b \sec(e + fx)}} dx}{c} - \frac{d \int \frac{\sec(e + fx)}{\sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx))} dx}{c}$$

$$= -\frac{2\sqrt{a + b} \cot(e + fx) \Pi\left(\frac{a + b}{a}; \sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \cos(e + fx))}{a + b}}}{acf}$$

Mathematica [A] time = 8.62, size = 251, normalized size = 1.16

$$\frac{2 \sec^{\frac{3}{2}}(e + fx) \sqrt{\sec(e + fx) + 1} \sqrt{\cos(e + fx) \sec^2\left(\frac{1}{2}(e + fx)\right)} \sqrt{\frac{a \cos(e + fx) + b}{(a + b)(\cos(e + fx) + 1)}} (c \cos(e + fx) + d) \left(c(c + d) \sqrt{\sec^2\left(\frac{1}{2}(e + fx)\right)} + cf(c - d)(c + d) \sqrt{\sec^2\left(\frac{1}{2}(e + fx)\right)}\right)}{cf(c - d)(c + d) \sqrt{\sec^2\left(\frac{1}{2}(e + fx)\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]

[Out] (-2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(d + c*Cos[e + f*x])*(c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(c^2 - d^2)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + d^2*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]])/(c*(c - d)*(c + d)*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(fx + e) + a} (d \sec(fx + e) + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)

maple [A] time = 1.80, size = 318, normalized size = 1.47

$$2 \sqrt{\frac{b + a \cos(fx + e)}{\cos(fx + e)}} \sqrt{\frac{\cos(fx + e)}{1 + \cos(fx + e)}} \sqrt{\frac{b + a \cos(fx + e)}{(1 + \cos(fx + e))(a + b)}} (1 + \cos(fx + e))^2 (-1 + \cos(fx + e)) \left(c^2 \text{EllipticF}\left(\frac{-1 + \cos(fx + e)}{\sin(fx + e)}\right) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)

[Out] $-2/f*((b+a*\cos(f*x+e))/\cos(f*x+e))^{1/2}*(\cos(f*x+e)/(1+\cos(f*x+e)))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e))/(a+b))^{1/2}*(1+\cos(f*x+e))^2*(-1+\cos(f*x+e))*(c^2*\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2}))+d*\text{EllipticF}((-1+\cos(f*x+e))/\sin(f*x+e),((a-b)/(a+b))^{1/2})*c-2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^{1/2})*d^2-2*c^2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),-1,((a-b)/(a+b))^{1/2}))+2*\text{EllipticPi}((-1+\cos(f*x+e))/\sin(f*x+e),-1,((a-b)/(a+b))^{1/2})*d^2)/(b+a*\cos(f*x+e))/\sin(f*x+e)^2/c/(c-d)/(c+d)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(fx + e) + a} (d \sec(fx + e) + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{\cos(e+fx)}} \left(c + \frac{d}{\cos(e+fx)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)

[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)

[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)

3.205 $\int \frac{c+d \sec(e+fx)}{(a+b \sec(e+fx))^{3/2}} dx$

Optimal. Leaf size=376

$$\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}} - \frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\right)}{a^2f}$$

[Out] $2*(-a*d+b*c)*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/a/b/f/(a+b)^{1/2}-2*(-a*d+b*c)*\cot(f*x+e)*\text{EllipticF}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/a/b/f/(a+b)^{1/2}-2*c*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/a^2/f+2*b*(-a*d+b*c)*\tan(f*x+e)/a/(a^2-b^2)/f/(a+b*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.43, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3923, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}} - \frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\right)}{a^2f}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2), x]

[Out] $(2*(b*c - a*d)*\text{Cot}[e + f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(a*b*\text{Sqrt}[a + b]*f) - (2*(b*c - a*d)*\text{Cot}[e + f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(a*b*\text{Sqrt}[a + b]*f) - (2*\text{Sqrt}[a + b]*c*\text{Cot}[e + f*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(a^2*f) + (2*b*(b*c - a*d)*\text{Tan}[e + f*x])/(a*(a^2 - b^2)*f*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D

ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4004

Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B)]/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]

Rule 4058

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{c + d \sec(e + fx)}{(a + b \sec(e + fx))^{3/2}} dx = \frac{2b(bc - ad) \tan(e + fx)}{a(a^2 - b^2) f \sqrt{a + b \sec(e + fx)}} - \frac{2 \int \frac{-\frac{1}{2}(a^2 - b^2)c + \frac{1}{2}a(bc - ad) \sec(e + fx) + \frac{1}{2}b(bc - ad) \sec^2(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx}{a(a^2 - b^2)}$$

$$= \frac{2b(bc - ad) \tan(e + fx)}{a(a^2 - b^2) f \sqrt{a + b \sec(e + fx)}} - \frac{2 \int \frac{-\frac{1}{2}(a^2 - b^2)c + (\frac{1}{2}a(bc - ad) - \frac{1}{2}b(bc - ad)) \sec(e + fx)}{\sqrt{a + b \sec(e + fx)}} dx}{a(a^2 - b^2)}$$

$$= \frac{2(bc - ad) \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(e + fx))}{a + b}} \sqrt{\frac{b(1 + \sec(e + fx))}{a - b}}}{ab\sqrt{a + b} f}$$

$$= \frac{2(bc - ad) \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{a + b}}\right) \middle| \frac{a + b}{a - b}\right) \sqrt{\frac{b(1 - \sec(e + fx))}{a + b}} \sqrt{\frac{b(1 + \sec(e + fx))}{a - b}}}{ab\sqrt{a + b} f}$$

Mathematica [C] time = 14.82, size = 1491, normalized size = 3.97

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]

[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]*(c + d*Sec[e + f*x])*((2*(-(b*c) + a*d)*Sin[e + f*x])/(a*(a^2 - b^2)) - (2*(-(b^2*c*Sin[e + f*x]) + a*b*d*Sin[e +


```

f*x]))/(a*(a^2 - b^2)*(b + a*cos[e + f*x])))/(f*(d + c*cos[e + f*x])*(a +
b*sec[e + f*x])^(3/2)) + (2*(b + a*cos[e + f*x])^(3/2)*sqrt[sec[e + f*x]]*
(c + d*sec[e + f*x])*sqrt[(a + b - a*tan[(e + f*x)/2]^2 + b*tan[(e + f*x)/2
]^2)/(1 + tan[(e + f*x)/2]^2)]*(a*b*sqrt[(-a + b)/(a + b)]*c*tan[(e + f*x)/
2] + b^2*sqrt[(-a + b)/(a + b)]*c*tan[(e + f*x)/2] - a^2*sqrt[(-a + b)/(a +
b)]*d*tan[(e + f*x)/2] - a*b*sqrt[(-a + b)/(a + b)]*d*tan[(e + f*x)/2] - 2
*a*b*sqrt[(-a + b)/(a + b)]*c*tan[(e + f*x)/2]^3 + 2*a^2*sqrt[(-a + b)/(a +
b)]*d*tan[(e + f*x)/2]^3 + a*b*sqrt[(-a + b)/(a + b)]*c*tan[(e + f*x)/2]^5
- b^2*sqrt[(-a + b)/(a + b)]*c*tan[(e + f*x)/2]^5 - a^2*sqrt[(-a + b)/(a +
b)]*d*tan[(e + f*x)/2]^5 + a*b*sqrt[(-a + b)/(a + b)]*d*tan[(e + f*x)/2]^5
- (2*I)*a^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*tan[(e + f*x)/2]], (a + b)/(a - b)]*sqrt[1 - tan[(e + f*x)/2]^2]*sqrt[(a
+ b - a*tan[(e + f*x)/2]^2 + b*tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*c
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*tan[(e + f
*x)/2]], (a + b)/(a - b)]*sqrt[1 - tan[(e + f*x)/2]^2]*sqrt[(a + b - a*tan[
(e + f*x)/2]^2 + b*tan[(e + f*x)/2]^2)/(a + b)] - (2*I)*a^2*c*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*tan[(e + f*x)/2]], (a +
b)/(a - b)]*tan[(e + f*x)/2]^2*sqrt[1 - tan[(e + f*x)/2]^2]*sqrt[(a + b - a
*tan[(e + f*x)/2]^2 + b*tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*c*Elliptic
Pi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*tan[(e + f*x)/2]],
(a + b)/(a - b)]*tan[(e + f*x)/2]^2*sqrt[1 - tan[(e + f*x)/2]^2]*sqrt[(a +
b - a*tan[(e + f*x)/2]^2 + b*tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-(b*
c) + a*d)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*tan[(e + f*x)/2]], (a
+ b)/(a - b)]*sqrt[1 - tan[(e + f*x)/2]^2]*(1 + tan[(e + f*x)/2]^2)*sqrt[(a
+ b - a*tan[(e + f*x)/2]^2 + b*tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(2
*b*c + a*(c - d))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*tan[(e + f*x)/
2]], (a + b)/(a - b)]*sqrt[1 - tan[(e + f*x)/2]^2]*(1 + tan[(e + f*x)/2]^2)
*sqrt[(a + b - a*tan[(e + f*x)/2]^2 + b*tan[(e + f*x)/2]^2)/(a + b)))/(a*S
qrt[(-a + b)/(a + b)]*(a^2 - b^2)*f*(d + c*cos[e + f*x])*(a + b*sec[e + f*x
])^(3/2)*(-1 + tan[(e + f*x)/2]^2)*sqrt[(1 + tan[(e + f*x)/2]^2)/(1 - tan[(
e + f*x)/2]^2)]*(a*(-1 + tan[(e + f*x)/2]^2) - b*(1 + tan[(e + f*x)/2]^2)))

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{(b \sec(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

maple [B] time = 1.80, size = 2010, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*4^(1/2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)^2*a^2*d-sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*c+sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*d+cos(f*x+e)*a*b*c-cos(f*x+e)*a^2*d-cos(f*x+e)^2*a*b*c-cos(f*x+e)^2*a*b*d+cos(f*x+e)*a*b*d+cos(f*x+e)^2*b^2*c-cos(f*x+e)*b^2*c-2*cos(f*x+e)*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*b^2*c-sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a*b*c+sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a*b*d+sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a*b*c-sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a*b*d-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*d+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c+2*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*a^2*c-sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*d+sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*b^2*c+2*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*a^2*c-2*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*b^2*c-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d+cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*c-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*a*b*d/(b+a*cos(f*x+e))/sin(f*x+e)/a/(a+b)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{(b \sec(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + \frac{d}{\cos(e+fx)}}{\left(a + \frac{b}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2), x)

[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec(e + fx)}{\left(a + b \sec(e + fx)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(3/2), x)

[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(3/2), x)

$$3.206 \quad \int \frac{c+d \sec(e+fx)}{(a+b \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=495

$$\frac{2c\sqrt{a+b} \cot(e+fx) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}} \sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}} \Pi\left(\frac{a+b}{a}; \sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right)}{a^3 f} + \frac{2b(bc-ad) \tan(e+fx)}{3af(a^2-b^2)(a+b \sec(e+fx))^{3/2}}$$

[Out] $2/3*(-4*a^3*d+7*a^2*b*c-3*b^3*c)*\cot(f*x+e)*\text{EllipticE}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/a^2/(a-b)/b/(a+b)^{3/2}/f-2/3*(-3*a^3*d+6*a^2*b*c+a^2*b*d-a*b^2*c-3*b^3*c)*\cot(f*x+e)*\text{EllipticF}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, ((a+b)/(a-b))^{1/2})*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/a^2/(a-b)/b/(a+b)^{3/2}/f-2*c*\cot(f*x+e)*\text{EllipticPi}((a+b*\sec(f*x+e))^{1/2}/(a+b)^{1/2}, (a+b)/a, ((a+b)/(a-b))^{1/2})*(a+b)^{1/2}*(b*(1-\sec(f*x+e))/(a+b))^{1/2}*(-b*(1+\sec(f*x+e))/(a-b))^{1/2}/a^3/f+2/3*b*(-a*d+b*c)*\tan(f*x+e)/a/(a^2-b^2)/f/(a+b*\sec(f*x+e))^{3/2}+2/3*b*(-4*a^3*d+7*a^2*b*c-3*b^3*c)*\tan(f*x+e)/a^2/(a^2-b^2)^2/f/(a+b*\sec(f*x+e))^{1/2}$

Rubi [A] time = 0.78, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3923, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b(7a^2bc - 4a^3d - 3b^3c) \tan(e+fx)}{3a^2f(a^2-b^2)^2 \sqrt{a+b \sec(e+fx)}} + \frac{2b(bc-ad) \tan(e+fx)}{3af(a^2-b^2)(a+b \sec(e+fx))^{3/2}} - \frac{2(6a^2bc + a^2bd - 3a^3d - ab^2c - 3b^3c)}{3af(a^2-b^2)(a+b \sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2), x]

[Out] $(2*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*\text{Cot}[e + f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(3*a^2*(a - b)*b*(a + b)^{3/2}*f) - (2*(6*a^2*b*c - a*b^2*c - 3*b^3*c - 3*a^3*d + a^2*b*d)*\text{Cot}[e + f*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(3*a^2*(a - b)*b*(a + b)^{3/2}*f) - (2*\text{Sqrt}[a + b]*c*\text{Cot}[e + f*x]*\text{EllipticPi}[(a + b)/a, \text{ArcSin}[\text{Sqrt}[a + b*\text{Sec}[e + f*x]]/\text{Sqrt}[a + b]], (a + b)/(a - b)]*\text{Sqrt}[(b*(1 - \text{Sec}[e + f*x]))/(a + b)]*\text{Sqrt}[-((b*(1 + \text{Sec}[e + f*x]))/(a - b))]/(a^3*f) + (2*b*(b*c - a*d)*\text{Tan}[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*\text{Sec}[e + f*x])^{3/2}) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*\text{Tan}[e + f*x])/(3*a^2*(a^2 - b^2)^2*f*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])$

Rule 3784

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3832

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)]/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Simp[(-2*(A*b - a*B)*Rt[a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]]/Rt[a + (b*B)/A, 2]], (a*A + b*B)/(a*A - b*B))/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + d \sec(e + fx)}{(a + b \sec(e + fx))^{5/2}} dx &= \frac{2b(bc - ad) \tan(e + fx)}{3a(a^2 - b^2) f(a + b \sec(e + fx))^{3/2}} - \frac{2 \int \frac{-\frac{3}{2}(a^2 - b^2)c + \frac{3}{2}a(bc - ad) \sec(e + fx) - \frac{1}{2}b(bc - ad) \sec^2(e + fx)}{(a + b \sec(e + fx))^{3/2}} dx}{3a(a^2 - b^2)} \\
&= \frac{2b(bc - ad) \tan(e + fx)}{3a(a^2 - b^2) f(a + b \sec(e + fx))^{3/2}} + \frac{2b(7a^2bc - 3b^3c - 4a^3d) \tan(e + fx)}{3a^2(a^2 - b^2)^2 f \sqrt{a + b \sec(e + fx)}} + \frac{4 \int}{\dots} \\
&= \frac{2b(bc - ad) \tan(e + fx)}{3a(a^2 - b^2) f(a + b \sec(e + fx))^{3/2}} + \frac{2b(7a^2bc - 3b^3c - 4a^3d) \tan(e + fx)}{3a^2(a^2 - b^2)^2 f \sqrt{a + b \sec(e + fx)}} + \frac{4 \int}{\dots} \\
&= \frac{2(7a^2bc - 3b^3c - 4a^3d) \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{3a^2(a-b)b(a+b)^{3/2}f} \\
&= \frac{2(7a^2bc - 3b^3c - 4a^3d) \cot(e + fx) E\left(\sin^{-1}\left(\frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{a+b}}\right) \middle| \frac{a+b}{a-b}\right) \sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{3a^2(a-b)b(a+b)^{3/2}f}
\end{aligned}$$

Mathematica [C] time = 17.43, size = 2083, normalized size = 4.21

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2), x]

[Out] ((b + a*Cos[e + f*x])^3*Sec[e + f*x]^2*(c + d*Sec[e + f*x])*((2*(-7*a^2*b*c + 3*b^3*c + 4*a^3*d)*Sin[e + f*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*c*Sin[e + f*x] - a*b^2*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[e + f*x])^2) - (2*(-8*a^2*b^2*c*Sin[e + f*x] + 4*b^4*c*Sin[e + f*x] + 5*a^3*b*d*Sin[e + f*x] - a*b^3*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[e + f*x])))/(f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)) + (2*(b + a*Cos[e + f*x])^(5/2)*Sec[e + f*x]^(3/2)*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2])*((7*a^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 3*b^4*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2] - 4*a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2] - 14*a^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^3 + 6*a*b^3*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^3 + 8*a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^3 + 7*a^3*b*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 + 3*b^4*Sqrt[(-a + b)/(a + b)]*c*Tan[(e + f*x)/2]^5 - 4*a^4*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 + 4*a^3*b*Sqrt[(-a + b)/(a + b)]*d*Tan[(e + f*x)/2]^5 - (6*I)*a^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (12*I)*a^2*b^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (6*I)*b^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (6*I)*a^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]

]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (12 *I)*a^2*b^2*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (6*I)*b^4*c*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-7*a^2*b*c + 3*b^3*c + 4*a^3*d)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a - b)*(-4*a*b^2*c - 6*b^3*c + 3*a^3*(c - d) + a^2*b*(9*c + d))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)])))/(3*a^2*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)*(-1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e + f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{b \sec(fx + e) + a} (d \sec(fx + e) + c)}{b^3 \sec(fx + e)^3 + 3ab^2 \sec(fx + e)^2 + 3a^2b \sec(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{(b \sec(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)

maple [B] time = 1.81, size = 5710, normalized size = 11.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d \sec(fx + e) + c}{(b \sec(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + \frac{d}{\cos(e+fx)}}{\left(a + \frac{b}{\cos(e+fx)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2),x)

[Out] int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + d \sec(e + fx)}{(a + b \sec(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(5/2),x)

[Out] Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(5/2), x)

3.207 $\int \sqrt{a + b \sec(e + fx)} \sqrt{c + d \sec(e + fx)} dx$

Optimal. Leaf size=389

$$2 \cot(e + fx)(a + b \sec(e + fx)) \sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} \Pi\left(\frac{b(c+d)}{(a+b)d}; \sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}} \sqrt{c+d \sec(e+fx)}}{\sqrt{a+b \sec(e+fx)}}\right)\right) \\ f \sqrt{\frac{a+b}{c+d}}$$

[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2), b*(c+d)/(a+b)/d, ((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e))*((-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/((a+b)/(c+d))^(1/2)-2*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sec(f*x+e))^(1/2), a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*x+e))*(c+d)^(1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)

Rubi [A] time = 0.45, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3932, 3936, 3982}

$$2 \cot(e + fx)(a + b \sec(e + fx)) \sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} \Pi\left(\frac{b(c+d)}{(a+b)d}; \sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}} \sqrt{c+d \sec(e+fx)}}{\sqrt{a+b \sec(e+fx)}}\right)\right) \\ f \sqrt{\frac{a+b}{c+d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]], x]

[Out] (-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x])]/(Sqrt[a + b]*f) + (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x])]/(Sqrt[(a + b)/(c + d)]*f)

Rule 3932

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)], x_Symbol] := Dist[c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]], x], x] + Dist[d, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/Sqrt[c + d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3936

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)], x_Symbol] := Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(c*f*Rt[(a + b)/(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3982

```
Int[(csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)], x_Symbol] := Simp[(-2*(a + b*Csc[e + f*x])*Sqrt[-((b*c - a*d)*(1 - Csc[e + f*x]))]/((c + d)*(a + b*Csc[e + f*x])))*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))]/((c - d)*(a + b*Csc[e + f*x]))]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a + b \sec(e + fx)} \sqrt{c + d \sec(e + fx)} dx = c \int \frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{c + d \sec(e + fx)}} dx + d \int \frac{\sec(e + fx) \sqrt{a + b \sec(e + fx)}}{\sqrt{c + d \sec(e + fx)}} dx$$

$$= -\frac{2\sqrt{c+d} \cot(e + fx) \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right)}{\sqrt{a}}$$

Mathematica [C] time = 32.95, size = 39925, normalized size = 102.63

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)

maple [A] time = 2.29, size = 543, normalized size = 1.40

$$2 \left(2 \operatorname{EllipticPi} \left(\frac{(-1 + \cos(fx+e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, \frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) bd + 2 \operatorname{EllipticPi} \left(\frac{(-1 + \cos(fx+e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, -\frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) ac - \operatorname{EllipticF} \left(\frac{(-1 + \cos(fx+e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, \frac{a+b}{a-b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x)`

[Out] $2/f*(2*\text{EllipticPi}((-1+\cos(f*x+e))*((a-b)/(a+b))^{1/2}/\sin(f*x+e), (a+b)/(a-b)), ((c-d)/(c+d))^{1/2}/((a-b)/(a+b))^{1/2})*b*d+2*\text{EllipticPi}((-1+\cos(f*x+e))*((a-b)/(a+b))^{1/2}/\sin(f*x+e), -(a+b)/(a-b), ((c-d)/(c+d))^{1/2}/((a-b)/(a+b))^{1/2})*a*c-\text{EllipticF}((-1+\cos(f*x+e))*((a-b)/(a+b))^{1/2}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{1/2})*a*c+\text{EllipticF}((-1+\cos(f*x+e))*((a-b)/(a+b))^{1/2}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{1/2})*a*d+\text{EllipticF}((-1+\cos(f*x+e))*((a-b)/(a+b))^{1/2}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{1/2})*b*c-\text{EllipticF}((-1+\cos(f*x+e))*((a-b)/(a+b))^{1/2}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{1/2})*b*d*\cos(f*x+e)*((b+a*\cos(f*x+e))/\cos(f*x+e))^{1/2}*((d+c*\cos(f*x+e))/\cos(f*x+e))^{1/2}*\sin(f*x+e)^2*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d))^{1/2}*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b))^{1/2}/(-1+\cos(f*x+e))/(d+c*\cos(f*x+e))/(b+a*\cos(f*x+e))/((a-b)/(a+b))^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{b}{\cos(e + fx)}} \sqrt{c + \frac{d}{\cos(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)`

[Out] `int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec(e + fx)} \sqrt{c + d \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)`

[Out] `Integral(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)), x)`

$$3.208 \quad \int \frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx$$

Optimal. Leaf size=198

$$\frac{2\sqrt{c+d} \cot(e+fx)(a+b \sec(e+fx)) \sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right)}{cf\sqrt{a+b}}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sec(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sec(f*x+e))^{(1/2)}, a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)}*(a+b*\sec(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sec(f*x+e))/(c-d)/(a+b*\sec(f*x+e)))^{(1/2)}/c/f/(a+b)^{(1/2)})$

Rubi [A] time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {3936}

$$\frac{2\sqrt{c+d} \cot(e+fx)(a+b \sec(e+fx)) \sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right)}{cf\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]], x]

[Out] $(-2*\text{Sqrt}[c + d]*\text{Cot}[e + f*x]*\text{EllipticPi}[(a*(c + d))/((a + b)*c), \text{ArcSin}[(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])/(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*\text{Sqrt}[-(((b*c - a*d)*(1 - \text{Sec}[e + f*x])))/((c + d)*(a + b*\text{Sec}[e + f*x]))]*\text{Sqrt}[(b*c - a*d)*(1 + \text{Sec}[e + f*x])]/((c - d)*(a + b*\text{Sec}[e + f*x]))]*(a + b*\text{Sec}[e + f*x]))/(\text{Sqrt}[a + b]*c*f)$

Rule 3936

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))]/((c - d)*(a + b*Csc[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x])))/((c + d)*(a + b*Csc[e + f*x]))]*EllipticPi[(a*(c + d))/(c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(c*f*Rt[(a + b)/(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx = -\frac{2\sqrt{c+d} \cot(e+fx) \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}}}{\sqrt{a+b} cf}$$

Mathematica [A] time = 5.87, size = 336, normalized size = 1.70

$$4 \sin^2\left(\frac{1}{2}(e+fx)\right) \csc(e+fx) \sqrt{a+b \sec(e+fx)} \sqrt{\frac{(c+d) \cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}} \sqrt{\frac{(a+b) \csc^2\left(\frac{1}{2}(e+fx)\right) (c \cos(e+fx)+d)}{ad-bc}} \left(c(a+b) F\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}} \right) \right) \right) / (cf(a+b) \sqrt{c+d \sec(e+fx)})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]

[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*((a + b)*c*EllipticF[ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))] - a*(c + d)*EllipticPi[(b*c - a*d)/(a*c + b*c), ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d)))*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x)/2]^2/((a + b)*c*f*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(fx + e) + a}}{\sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)

maple [A] time = 2.15, size = 352, normalized size = 1.78

$$\frac{2 \left(2 \operatorname{EllipticPi} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, -\frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) a - \operatorname{EllipticF} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}} \right) a + \operatorname{EllipticF} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}} \right) a \right)}{f(-1 + \cos(fx + e)) \left(\frac{a-b}{a+b} \right)^{1/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)

[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), -(a+b)/(a-b), ((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a+EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b)*cos(f*x+e)*((d+c*cos(f*x+e))/(1+cos(f*x+e)))/(c+d))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*sin(f*x+e)^2*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e)))/(b+a*cos(f*x+e))/((a-b)/(a+b))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(fx + e) + a}}{\sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{\cos(e+fx)}}}{\sqrt{c + \frac{d}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)

[Out] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{c + d \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x))/sqrt(c + d*sec(e + f*x)), x)

$$3.209 \quad \int \frac{\sqrt{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=598

$$\frac{2\sqrt{c+d} \cot(e+fx)(a+b \sec(e+fx)) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right)}{c^2 f \sqrt{a+b}}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sec(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sec(f*x+e))^{(1/2)}, a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)}*(a+b*\sec(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sec(f*x+e))/(c-d)/(a+b*\sec(f*x+e)))^{(1/2)}/c^2/f/(a+b)^{(1/2)}-2*d*\cot(f*x+e)*\text{EllipticE}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}, ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(1+\sec(f*x+e))*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\sec(f*x+e))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}/c/(c-d)/f/(c+d)^{(1/2)}/(-(-a*d+b*c)*(1+\sec(f*x+e))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}-2*(a-b)*d*\cot(f*x+e)*\text{EllipticF}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}, ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(c+d*\sec(f*x+e))*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\sec(f*x+e))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sec(f*x+e))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}/c/(c-d)/(-a*d+b*c)/f/(c+d)^{(1/2)}$

Rubi [A] time = 0.90, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3939, 3936, 3986, 3984, 3994}

$$\frac{2\sqrt{c+d} \cot(e+fx)(a+b \sec(e+fx)) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right)}{c^2 f \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2), x]

[Out] $(-2*\text{Sqrt}[c + d]*\text{Cot}[e + f*x]*\text{EllipticPi}[(a*(c + d))/((a + b)*c), \text{ArcSin}[(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])/(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*\text{Sqrt}[-(((b*c - a*d)*(1 - \text{Sec}[e + f*x]))/((c + d)*(a + b*\text{Sec}[e + f*x])))]*\text{Sqrt}[(b*c - a*d)*(1 + \text{Sec}[e + f*x])]/((c - d)*(a + b*\text{Sec}[e + f*x]))]*(a + b*\text{Sec}[e + f*x])/(\text{Sqrt}[a + b]*c^2*f) - (2*\text{Sqrt}[a + b]*d*\text{Cot}[e + f*x]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*(1 + \text{Sec}[e + f*x])* \text{Sqrt}[(b*c - a*d)*(1 - \text{Sec}[e + f*x])]/((a + b)*(c + d*\text{Sec}[e + f*x]))]/(c*(c - d)*\text{Sqrt}[c + d]*f*\text{Sqrt}[-(((b*c - a*d)*(1 + \text{Sec}[e + f*x]))/((a - b)*(c + d*\text{Sec}[e + f*x])))] - (2*(a - b)*\text{Sqrt}[a + b]*d*\text{Cot}[e + f*x]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*\text{Sqrt}[(b*c - a*d)*(1 - \text{Sec}[e + f*x])]/((a + b)*(c + d*\text{Sec}[e + f*x]))]*\text{Sqrt}[-(((b*c - a*d)*(1 + \text{Sec}[e + f*x]))/((a - b)*(c + d*\text{Sec}[e + f*x])))]*(c + d*\text{Sec}[e + f*x])/((c*(c - d)*\text{Sqrt}[c + d]*(b*c - a*d)*f)$

Rule 3936

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[(b*c - a*d)*(1 + Csc[e + f*x])]/((c - d)*(a + b*Csc[e + f*x]))*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a

+ b*Csc[e + f*x]], ((a - b)*(c + d))/((a + b)*(c - d)))/(c*f*Rt[(a + b)/(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3939

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(3/2), x_Symbol] := Dist[1/c, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]], x], x] - Dist[d/c, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]

Rule 3984

Int[csc[(e_.) + (f_.)*(x_.)]/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)]), x_Symbol] := Simp[(-2*(c + d*Csc[e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x]))])*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3986

Int[(csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)])/(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(3/2), x_Symbol] := Dist[(a - b)/(c - d), Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dist[(b*c - a*d)/(c - d), Int[(Csc[e + f*x]*(1 + Csc[e + f*x])]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3994

Int[(sec[(e_.) + (f_.)*(x_.)]*((A_.) + (B_.)*sec[(e_.) + (f_.)*(x_.)]))/(Sqrt[(a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sec[(e_.) + (f_.)*(x_.)])^(3/2)), x_Symbol] := Simp[(2*A*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x]))]*EllipticE[ArcSin[(Rt[(c + d)/(a + b), 2]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]]], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Tan[e + f*x]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B]

Rubi steps

$$\int \frac{\sqrt{a + b \sec(e + fx)}}{(c + d \sec(e + fx))^{3/2}} dx = \frac{\int \frac{\sqrt{a + b \sec(e + fx)}}{\sqrt{c + d \sec(e + fx)}} dx}{c} - \frac{d \int \frac{\sec(e + fx) \sqrt{a + b \sec(e + fx)}}{(c + d \sec(e + fx))^{3/2}} dx}{c}$$

$$= - \frac{2\sqrt{c + d} \cot(e + fx) \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \sqrt{-\frac{(bc-ad)}{(c+d)(a+b)}}}{\sqrt{a + b} c^2 f}$$

$$= - \frac{2\sqrt{c + d} \cot(e + fx) \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \sqrt{-\frac{(bc-ad)}{(c+d)(a+b)}}}{\sqrt{a + b} c^2 f}$$

Mathematica [B] time = 9.28, size = 1708, normalized size = 2.86

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]

[Out] ((d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*((4*b*c*(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(a*c + b*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*a*d*((Sqrt[-a + b]/(a + b))*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[-a + b]/(a + b))*Sin[(e + f*x)/2]]/Sqrt[(b + a*Cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d)))/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x])]*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])))/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]])))/((c - d)*(c + d

$$\begin{aligned} &)^{(1/2)} * b * c^2 - \sin(f * x + e) * \cos(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b) \\ &)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticE}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * a * d^2 + \\ &\sin(f * x + e) * \cos(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticE}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * b * d^2 - 2 * \sin(f * x + e) * \cos(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticPi}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), -(a + b) / (a - b), ((c - d) / (c + d))^{(1/2)} / ((a - b) / (a + b))^{(1/2)}) * a * c^2 + 2 * \sin(f * x + e) * \cos(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticPi}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), -(a + b) / (a - b), ((c - d) / (c + d))^{(1/2)} / ((a - b) / (a + b))^{(1/2)}) * a * d^2 + \text{EllipticF}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * a * c * d * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \sin(f * x + e) - \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticF}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * b * c * d - \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticE}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * a * c * d + \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticE}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * b * c * d + \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticE}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * b * d^2 - 2 * \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticPi}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), -(a + b) / (a - b), ((c - d) / (c + d))^{(1/2)} / ((a - b) / (a + b))^{(1/2)}) * a * c^2 - \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticF}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * b * c^2 - \sin(f * x + e) * ((b + a * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (a + b)^{(1/2)} * ((d + c * \cos(f * x + e)) / (1 + \cos(f * x + e))) / (c + d)^{(1/2)} * \text{EllipticE}((-1 + \cos(f * x + e)) * ((a - b) / (a + b))^{(1/2)} / \sin(f * x + e), ((a + b) * (c - d) / (a - b) / (c + d))^{(1/2)}) * a * d^2 * \cos(f * x + e) * ((b + a * \cos(f * x + e)) / \cos(f * x + e))^{(1/2)} * ((d + c * \cos(f * x + e)) / \cos(f * x + e))^{(1/2)} / \sin(f * x + e) / (d + c * \cos(f * x + e)) / (b + a * \cos(f * x + e)) / c / (c + d) / (c - d) / ((a - b) / (a + b))^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(fx + e) + a}}{(d \sec(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{\cos(e+fx)}}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)

[Out] `int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec(e + fx)}}{(c + d \sec(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)`

[Out] `Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(3/2), x)`

3.210 $\int \frac{\sqrt{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{5/2}} dx$

Optimal. Leaf size=899

$$\frac{2\sqrt{a+b \sec(e+fx)} \sin(e+fx)d^2}{3c(c^2-d^2)f(d+c \cos(e+fx))\sqrt{c+d \sec(e+fx)}} + \frac{2(a-b)\sqrt{a+b} (6bc^3-7adc^2-2bd^2c+3ad^3) \sqrt{-\frac{(bc-ad)}{(a+b)}}}{3c(c^2-d^2)f(d+c \cos(e+fx))\sqrt{c+d \sec(e+fx)}}$$

[Out] $2/3*d^2*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c/(c^2-d^2)/f/(d+c*\cos(f*x+e))/(c+d*\sec(f*x+e))^{(1/2)}+2/3*(a-b)*d*(-7*a*c^2*d+3*a*d^3+6*b*c^3-2*b*c*d^2)*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticE((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)*(-(-a*d+b*c)*(1-\cos(f*x+e)))/(a+b)/(d+c*\cos(f*x+e))^{(1/2)*(-(-a*d+b*c)*(1+\cos(f*x+e)))/(a-b)/(d+c*\cos(f*x+e))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^2/(c-d)^2/(c+d)^{(3/2)}/(-a*d+b*c)^2/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}+2/3*(b*c^2*(3*c^2+3*c*d-2*d^2)-a*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticF((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)*(-(-a*d+b*c)*(1-\cos(f*x+e)))/(a+b)/(d+c*\cos(f*x+e))^{(1/2)*(-(-a*d+b*c)*(1+\cos(f*x+e)))/(a-b)/(d+c*\cos(f*x+e))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^3/(c-d)^2/(c+d)^{(3/2)}/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticPi((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)*(-(-a*d+b*c)*(1-\cos(f*x+e)))/(a+b)/(d+c*\cos(f*x+e))^{(1/2)*(-(-a*d+b*c)*(1+\cos(f*x+e)))/(a-b)/(d+c*\cos(f*x+e))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^3/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 2.26, antiderivative size = 899, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3942, 3048, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b \sec(e+fx)} \sin(e+fx)d^2}{3c(c^2-d^2)f(d+c \cos(e+fx))\sqrt{c+d \sec(e+fx)}} + \frac{2(a-b)\sqrt{a+b} (6bc^3-7adc^2-2bd^2c+3ad^3) \sqrt{-\frac{(bc-ad)}{(a+b)}}}{3c(c^2-d^2)f(d+c \cos(e+fx))\sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]`

[Out] $(2*(a-b)*\text{Sqrt}[a+b]*d*(6*b*c^3-7*a*c^2*d-2*b*c*d^2+3*a*d^3)*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(3*c^2*(c-d)^2*(c+d)^{(3/2)}*(b*c-a*d)^2*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])+(2*\text{Sqrt}[a+b]*(b*c^2*(3*c^2+3*c*d-2*d^2)-a*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(3*c^3*(c-d)^2*(c+d)^{(3/2)}*(b*c-a*d)*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])-(2*\text{Sqrt}[a+b]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticPi}(((a+b)*c)/(a*(c+d)),\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d$

+ c*cos[e + f*x]]), ((a + b)*(c - d))/((a - b)*(c + d))*sqrt[a + b*sec[e + f*x]]/(c^3*sqrt[c + d]*f*sqrt[b + a*cos[e + f*x]]*sqrt[c + d*sec[e + f*x]]) + (2*d^2*sqrt[a + b*sec[e + f*x]]*sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*cos[e + f*x])*sqrt[c + d*sec[e + f*x]])

Rule 2811

Int[sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Simp[(2*(a + b*sin[e + f*x])*sqrt[(b*c - a*d)*(1 + sin[e + f*x])]/((c - d)*(a + b*sin[e + f*x])))*sqrt[-((b*c - a*d)*(1 - sin[e + f*x]))/(c + d)*(a + b*sin[e + f*x])])*ellipticPi[(b*(c + d))/(d*(a + b)), ArcSin[Rt[(a + b)/(c + d), 2]*sqrt[c + d*sin[e + f*x]]/sqrt[a + b*sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(d*f*Rt[(a + b)/(c + d), 2]*cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]

Rule 2818

Int[1/(sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*sin[e + f*x])*sqrt[(b*c - a*d)*(1 - sin[e + f*x])]/((a + b)*(c + d*sin[e + f*x])))*sqrt[-((b*c - a*d)*(1 + sin[e + f*x])]/((a - b)*(c + d*sin[e + f*x]))])*ellipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(sqrt[a + b*sin[e + f*x]]/sqrt[c + d*sin[e + f*x]])], (a + b)*(c - d)/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]

Rule 2996

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Simp[(-2*A*(c - d)*(a + b*sin[e + f*x])*sqrt[(b*c - a*d)*(1 + sin[e + f*x])]/((c - d)*(a + b*sin[e + f*x])))*sqrt[-((b*c - a*d)*(1 - sin[e + f*x])]/((c + d)*(a + b*sin[e + f*x]))])*ellipticE[ArcSin[Rt[(a + b)/(c + d), 2]*sqrt[c + d*sin[e + f*x]]/sqrt[a + b*sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]

Rule 2998

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(sqrt[a + b*sin[e + f*x]]*sqrt[c + d*sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + sin[e + f*x])/(a + b*sin[e + f*x])^(3/2)*sqrt[c + d*sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3048

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(c^2*C + A*d^2)*cos[e + f*x]*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^(m - 1)*(c + d*sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*sin[e + f*x] - b*(A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

Rule 3053

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3942

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^n, x_Symbol] := Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

$$\int \frac{\sqrt{a + b \sec(e + fx)}}{(c + d \sec(e + fx))^{5/2}} dx = \frac{(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}) \int \frac{\cos^2(e + fx) \sqrt{b + a \cos(e + fx)}}{(d + c \cos(e + fx))^{5/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}}$$

$$= \frac{2d^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx)) \sqrt{c + d \sec(e + fx)}} + \frac{(2\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)})}{c^3 \sqrt{b + a \cos(e + fx)}}$$

$$= \frac{2d^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx)) \sqrt{c + d \sec(e + fx)}} + \frac{(a \sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)})}{c^3 \sqrt{b + a \cos(e + fx)}}$$

$$= \frac{2\sqrt{a + b} \sqrt{-\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{-\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}} (d + c \cos(e + fx))^{3/2} \csc(e + fx)}{c^3 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}}$$

$$= \frac{2(a - b) \sqrt{a + b} d (6bc^3 - 7ac^2d - 2bcd^2 + 3ad^3) \sqrt{-\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{-\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}}}{3c^2(c - d)^2(c + d)^{3/2}(bc - ad)}$$

Mathematica [B] time = 6.90, size = 1990, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2), x]

[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) - (2*(6*b*c^3*d*Sin[e + f*x] - 7*a*c^2*d^2*Sin[e + f*x] - 2*b*c*d^3*Sin[e + f*x] + 3*a*d^4*Sin[e + f*x]))/(3*c*(b*c - a*d)*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))/(f*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((4*(b*c - a*d)*(3*b^2*c^4 - 3*a*b*c^3*d - a^2*c^2*d^2 + b^2*c

```

c^2*d^2 - a*b*c*d^3 + a^2*d^4)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*S
qrt[((c + d)*(b + a*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-
a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*E
llipticF[ArcSin[Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/
((a + b)*(c + d)*Sqrt[b + a*cos[e + f*x]]*Sqrt[d + c*cos[e + f*x]]) + 4*(b*
c - a*d)*(3*a*b*c^4 - 3*a^2*c^3*d + 6*b^2*c^3*d - 7*a*b*c^2*d^2 - a^2*c*d^3
- 2*b^2*c*d^3 + 4*a*b*d^4)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sq
rt[((c + d)*(b + a*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a
- b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*E
llipticF[ArcSin[Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/
((a + b)*(c + d)*Sqrt[b + a*cos[e + f*x]]*Sqrt[d + c*cos[e + f*x]]) - (Sqrt[
((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*cos[e + f*x])*Cs
c[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e +
f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), A
rcSin[Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c
*Sqrt[b + a*cos[e + f*x]]*Sqrt[d + c*cos[e + f*x]]) + 2*(6*a*b*c^3*d - 7*a
^2*c^2*d^2 - 2*a*b*c*d^3 + 3*a^2*d^4)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[
(e + f*x)/2]*Sqrt[d + c*cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a +
b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/
((-a + b)*(c + d)))/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*cos[e +
f*x]])*Sqrt[b + a*cos[e + f*x]]*Sqrt[(b + a*cos[e + f*x])/(a + b)]*Sqrt[((a
+ b)*(d + c*cos[e + f*x]))/((c + d)*(b + a*cos[e + f*x]))]) - (2*(b*c - a*
d)*(((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c
+ d)*(b + a*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*
(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF
[ArcSin[Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)
]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)
*(c + d)*Sqrt[b + a*cos[e + f*x]]*Sqrt[d + c*cos[e + f*x]]) - ((b*c + a*d)*
Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*cos[e + f*x
])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*
c), ArcSin[Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a
*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a +
b)*c*Sqrt[b + a*cos[e + f*x]]*Sqrt[d + c*cos[e + f*x]])))/(a*c) + (Sqrt[d
+ c*cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*cos[e + f*x]])))/(3*c*(c - d
)^2*(c + d)^2*(b*c - a*d)*f*Sqrt[b + a*cos[e + f*x]]*(c + d*Sec[e + f*x])^(
5/2))

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(fx + e) + a}}{(d \sec(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)

maple [B] time = 3.08, size = 15724, normalized size = 17.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(fx + e) + a}}{(d \sec(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(5/2),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec(e + fx)}}{(c + d \sec(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(5/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(5/2), x)

$$3.211 \quad \int \frac{(a+b \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=744

$$\frac{2\sqrt{a+b}(bc-a(2c-d)) \csc(e+fx) \sqrt{a+b \sec(e+fx)} (c \cos(e+fx)+d)^{3/2} \sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c \cos(e+fx)+d)}} \sqrt{\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c \cos(e+fx)+d)}}}{c^2 f(c-d) \sqrt{c+d} \sqrt{a \cos(e+fx)+b} \sqrt{c+d \sec(e+fx)}}$$

[Out] $-2*(a-b)*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticE((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c/(c-d)/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2*(b*c-a*(2*c-d))*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticF((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^2/(c-d)/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2*a*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticPi((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^2/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 1.11, antiderivative size = 744, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3942, 2798, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b}(bc-a(2c-d)) \csc(e+fx) \sqrt{a+b \sec(e+fx)} (c \cos(e+fx)+d)^{3/2} \sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c \cos(e+fx)+d)}} \sqrt{\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c \cos(e+fx)+d)}}}{c^2 f(c-d) \sqrt{c+d} \sqrt{a \cos(e+fx)+b} \sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*\text{Sqrt}[-((b*c-a*d)*(1-\text{Cos}[e+f*x]))]/((a+b)*(d+c*\text{Cos}[e+f*x])))*\text{Sqrt}[-((b*c-a*d)*(1+\text{Cos}[e+f*x]))]/((a-b)*(d+c*\text{Cos}[e+f*x]))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*csc[e+f*x]*EllipticE[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(c*(c-d)*\text{Sqrt}[c+d]*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])-(2*\text{Sqrt}[a+b]*(b*c-a*(2*c-d))*\text{Sqrt}[-((b*c-a*d)*(1-\text{Cos}[e+f*x]))]/((a+b)*(d+c*\text{Cos}[e+f*x])))*\text{Sqrt}[-((b*c-a*d)*(1+\text{Cos}[e+f*x]))]/((a-b)*(d+c*\text{Cos}[e+f*x]))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*csc[e+f*x]*EllipticF[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(c^2*(c-d)*\text{Sqrt}[c+d]*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])-(2*a*\text{Sqrt}[a+b]*\text{Sqrt}[-((b*c-a*d)*(1-\text{Cos}[e+f*x]))]/((a+b)*(d+c*\text{Cos}[e+f*x])))*\text{Sqrt}[-((b*c-a*d)*(1+\text{Cos}[e+f*x]))]/((a-b)*(d+c*\text{Cos}[e+f*x]))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*csc[e+f*x]*EllipticPi[(((a+b)*c)/(a*(c+d)),\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(c^2*\text{Sqrt}[c+d]*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])]$

Rule 2798

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2)/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2811

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]]]/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(d*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]

Rule 2818

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x]))])*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]

Rule 2996

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))])*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]]]/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]

Rule 2998

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3942

Int[(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(csc[(e_) + (f_)*(x_)]*(d_) + (c_))^(n_), x_Symbol] := Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^{3/2}} dx &= \frac{(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}) \int \frac{(b + a \cos(e + fx))^{3/2}}{(d + c \cos(e + fx))^{3/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} \\
&= \frac{(a^2 \sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}) \int \frac{\sqrt{d + c \cos(e + fx)}}{\sqrt{b + a \cos(e + fx)}} dx}{c^2 \sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} + \frac{((bc - ad) \sqrt{d + c \cos(e + fx)}) \int \frac{1}{\sqrt{b + a \cos(e + fx)}} dx}{c^2 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\
&= -\frac{2a \sqrt{a + b} \sqrt{-\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{-\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}} (d + c \cos(e + fx))^{3/2} \operatorname{csc}(e + fx)}{c^2 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\
&= -\frac{2(a - b) \sqrt{a + b} \sqrt{-\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{-\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}} (d + c \cos(e + fx))^{3/2} \operatorname{csc}(e + fx)}{c(c - d) \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}}
\end{aligned}$$

Mathematica [B] time = 9.72, size = 1750, normalized size = 2.35

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]

[Out] (2*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(3/2)*(-b*c*Sin[e + f*x]) + a*d*Sin[e + f*x])/((-c^2 + d^2)*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(3/2)) + ((d + c*Cos[e + f*x])^(3/2)*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(a*b*c - b^2*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(a^2*c - b^2*c)*(b*c - a*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 2*(-a*b*c) + a^2*d)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]]], (2*(b*c - a*d))/((-a + b)*(c + d)))/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x]])*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/((c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Si

$$\begin{aligned}
& s(f*x+e)/(c+d)^{(1/2)}*EllipticPi((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f \\
& *x+e), -(a+b)/(a-b), ((c-d)/(c+d))^{(1/2)}/((a-b)/(a+b))^{(1/2)})*a^2*c^2+2*\sin(f \\
& *x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f* \\
& x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticPi((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), -(a+b)/(a-b), ((c-d)/(c+d))^{(1/2)}/((a-b)/(a+b))^{(1/2)})*a^2* \\
& d^2+EllipticF((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a^2*c*d*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((\\
& d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*\sin(f*x+e)-2*EllipticF((-1+\cos(f* \\
& x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b \\
& *c^2*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos \\
& (f*x+e)))/(c+d)^{(1/2)}*\sin(f*x+e)+a*b*d^2*((a-b)/(a+b))^{(1/2)}-b^2*c*d*((a-b) \\
& /(a+b))^{(1/2)}-2*\sin(f*x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b \\
&))^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticF((-1+\cos(f* \\
& x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b*c \\
& *d+2*\sin(f*x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((\\
& d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticE((-1+\cos(f*x+e))*((a-b) \\
&)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b*c*d-((a-b)/(\\
& a+b))^{(1/2)}*\cos(f*x+e)^2*a*b*c^2-((a-b)/(a+b))^{(1/2)}*\cos(f*x+e)*a^2*c*d+((a \\
& -b)/(a+b))^{(1/2)}*\cos(f*x+e)*a*b*c^2-((a-b)/(a+b))^{(1/2)}*\cos(f*x+e)*a*b*d^2+ \\
& ((a-b)/(a+b))^{(1/2)}*\cos(f*x+e)*b^2*c*d+((a-b)/(a+b))^{(1/2)}*\cos(f*x+e)^2*a^2 \\
& *c*d-\sin(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x \\
& +e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticE((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a^2*d^2+2*EllipticE((-1+\cos \\
& (f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a \\
& *b*c*d*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e))/(1+co \\
& s(f*x+e)))/(c+d)^{(1/2)}*\sin(f*x+e)-2*\sin(f*x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e)) \\
&)/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}* \\
& EllipticF((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b) \\
&)/(c+d))^{(1/2)})*a*b*c^2+\sin(f*x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e \\
&))/(a+b))^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticF((-1 \\
& +\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}) \\
&)*b^2*c*d-\sin(f*x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)} \\
& *((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticE((-1+\cos(f*x+e))* \\
& ((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a^2*c*d+\sin \\
& (f*x+e)*\cos(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(\\
& f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticE((-1+\cos(f*x+e))*((a-b)/(a+b)) \\
&)^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b*c^2+\sin(f*x+e)*\cos(f \\
& *x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e))/(1+co \\
& s(f*x+e)))/(c+d)^{(1/2)}*EllipticE((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f* \\
& x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b*d^2-\sin(f*x+e)*\cos(f*x+e)*((b+a*c \\
& os(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+ \\
& d))^{(1/2)}*EllipticE((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)* \\
& (c-d)/(a-b)/(c+d))^{(1/2)})*b^2*c*d-2*\sin(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f*x+ \\
& e)))/(a+b))^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*EllipticF((- \\
& 1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}) \\
&)*a*b*c*d-a*b*c*d*((a-b)/(a+b))^{(1/2)}+\sin(f*x+e)*((b+a*\cos(f*x+e))/(1+\cos(f* \\
& x+e)))/(a+b))^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*Elliptic \\
& F((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}) \\
&)*a^2*c^2+EllipticF((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a \\
& +b)*(c-d)/(a-b)/(c+d))^{(1/2)})*b^2*c*d*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b \\
&))^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*\sin(f*x+e)-EllipticE \\
& ((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}) \\
&)*a^2*c*d*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e \\
&))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*\sin(f*x+e)+EllipticE((-1+\cos(f*x+e))*((a-b)/ \\
& (a+b))^{(1/2)}/\sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b*c^2*((b+a*\cos(\\
& f*x+e))/(1+\cos(f*x+e)))/(a+b)^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d) \\
&)^{(1/2)}*\sin(f*x+e)+EllipticE((-1+\cos(f*x+e))*((a-b)/(a+b))^{(1/2)}/\sin(f*x+e), \\
& ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*a*b*d^2*((b+a*\cos(f*x+e))/(1+\cos(f*x+e)))/(\\
& a+b))^{(1/2)}*((d+c*\cos(f*x+e))/(1+\cos(f*x+e)))/(c+d)^{(1/2)}*\sin(f*x+e)-Ellipt
\end{aligned}$$

icE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c*d*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*sin(f*x+e)+sin(f*x+e)*cos(f*x+e)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a^2*c^2+sin(f*x+e)*cos(f*x+e)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c^2-sin(f*x+e)*cos(f*x+e)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a^2*d^2-sin(f*x+e)*cos(f*x+e)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e))/(c+d))^(1/2)*EllipticE((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2*c^2*cos(f*x+e)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))/c/(c-d)/((a-b)/(a+b))^(1/2)/(c+d)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{(d \sec(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{3/2}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2),x)

[Out] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{\frac{3}{2}}}{(c + d \sec(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(3/2),x)

[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(3/2), x)

$$3.212 \quad \int \frac{(a+b \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=919

$$\frac{2(a-b)\sqrt{a+b} (3bc^3 - 7adc^2 + bd^2c + 3ad^3) \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} \csc(e+fx) E\left(\sin^{-1}\left(\frac{2(a-b)\sqrt{a+b} (3bc^3 - 7adc^2 + bd^2c + 3ad^3) \sqrt{b+a \cos(e+fx)}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a \cos(e+fx)}}\right)\right)}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a \cos(e+fx)}} \sqrt{c+}$$

[Out] $-2/3*d*(-a*d+b*c)*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c/(c^2-d^2)/f/(d+c*\cos(f*x+e))/(c+d*\sec(f*x+e))^{(1/2)}-2/3*(a-b)*(-7*a*c^2*d+3*a*d^3+3*b*c^3+b*c*d^2)*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticE((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^2/(c-d)^2/(c+d)^{(3/2)}/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2/3*(b^2*c^3*(3*c+d)-2*a*b*c^2*(3*c^2+2*c*d-d^2)+a^2*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticF((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^3/(c-d)^2/(c+d)^{(3/2)}/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2*a*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticPi((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^3/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 2.13, antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3942, 2989, 3053, 2811, 2998, 2818, 2996}

$$\frac{2(a-b)\sqrt{a+b} (3bc^3 - 7adc^2 + bd^2c + 3ad^3) \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} \csc(e+fx) E\left(\sin^{-1}\left(\frac{2(a-b)\sqrt{a+b} (3bc^3 - 7adc^2 + bd^2c + 3ad^3) \sqrt{b+a \cos(e+fx)}}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a \cos(e+fx)}}\right)\right)}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a \cos(e+fx)}} \sqrt{c+}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2), x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(3*b*c^3-7*a*c^2*d+b*c*d^2+3*a*d^3)*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*csc[e+f*x]*EllipticE[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(3*c^2*(c-d)^2*(c+d)^{(3/2)}*(b*c-a*d)*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]-(2*\text{Sqrt}[a+b]*(b^2*c^3*(3*c+d)-2*a*b*c^2*(3*c^2+2*c*d-d^2)+a^2*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*csc[e+f*x]*EllipticF[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(3*c^3*(c-d)^2*(c+d)^{(3/2)}*(b*c-a*d)*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]-(2*a*\text{Sqrt}[a+b]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*csc[e+f*x]*EllipticPi[((a+b)*c)/(a*(c+d)),\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]$

$$\frac{1}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}} \left(\frac{(a+b)(c-d)}{(a-b)(c+d)} \sqrt{a+b\sec[e+fx]} \right) / (c^3\sqrt{c+d}f\sqrt{b+a\cos[e+fx]}\sqrt{c+d\sec[e+fx]}) - (2d(b*c - a*d)\sqrt{a+b\sec[e+fx]}\sin[e+fx]) / (3c(c^2 - d^2)f(d+c\cos[e+fx])\sqrt{c+d\sec[e+fx]})$$
Rule 2811

$$\text{Int}[\sqrt{(a_.) + (b_.)\sin[(e_.) + (f_.)x]} / \sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}], x_Symbol] \rightarrow \text{Simp}[(2(a + b\sin[e + fx])\sqrt{((b*c - a*d)*(1 + \sin[e + fx]))}) / ((c - d)(a + b\sin[e + fx]))] \sqrt{-((b*c - a*d)*(1 - \sin[e + fx]))} / ((c + d)(a + b\sin[e + fx]))] \text{EllipticPi}[(b*(c + d)) / (d*(a + b)), \text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]\sqrt{c + d\sin[e + fx]}] / \sqrt{a + b\sin[e + fx]}], ((a - b)(c + d)) / ((a + b)(c - d))] / (d*f*\text{Rt}[(a + b)/(c + d), 2]\cos[e + fx]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(a + b)/(c + d)]$$
Rule 2818

$$\text{Int}[1 / (\sqrt{(a_.) + (b_.)\sin[(e_.) + (f_.)x]} \sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}), x_Symbol] \rightarrow \text{Simp}[(2(c + d\sin[e + fx])\sqrt{((b*c - a*d)*(1 - \sin[e + fx]))}) / ((a + b)(c + d\sin[e + fx]))] \sqrt{-((b*c - a*d)*(1 + \sin[e + fx]))} / ((a - b)(c + d\sin[e + fx]))] \text{EllipticF}[\text{ArcSin}[\text{Rt}[(c + d)/(a + b), 2](\sqrt{a + b\sin[e + fx]} / \sqrt{c + d\sin[e + fx]})], (a + b)(c - d) / ((a - b)(c + d))] / (f*(b*c - a*d)\text{Rt}[(c + d)/(a + b), 2]\cos[e + fx]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/(a + b)]$$
Rule 2989

$$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]^{(m_.)} ((A_.) + (B_.)\sin[(e_.) + (f_.)x])^{(n_.)}], x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)(B*c - A*d)\cos[e + fx](a + b\sin[e + fx])^{(m - 1)}(c + d\sin[e + fx])^{(n + 1)} / (d*f*(n + 1)(c^2 - d^2)), x] + \text{Dist}[1 / (d*(n + 1)(c^2 - d^2)), \text{Int}[(a + b\sin[e + fx])^{(m - 2)}(c + d\sin[e + fx])^{(n + 1)}] \text{Simp}[b*(b*c - a*d)(B*c - A*d)(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) - a*(b*c - a*d)(B*c - A*d)(n + 2)] \sin[e + fx] + b*(d*(A*b*c + a*B*c - a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))] \sin[e + fx]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LtQ}[n, -1]$$
Rule 2996

$$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x] / (((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{(3/2)} \sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}), x_Symbol] \rightarrow \text{Simp}[(-2*A*(c - d)(a + b\sin[e + fx])\sqrt{((b*c - a*d)*(1 + \sin[e + fx]))}) / ((c - d)(a + b\sin[e + fx]))] \sqrt{-((b*c - a*d)*(1 - \sin[e + fx]))} / ((c + d)(a + b\sin[e + fx]))] \text{EllipticE}[\text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]\sqrt{c + d\sin[e + fx]}] / \sqrt{a + b\sin[e + fx]}], ((a - b)(c + d)) / ((a + b)(c - d))] / (f*(b*c - a*d)^2*\text{Rt}[(a + b)/(c + d), 2]\cos[e + fx]), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(a + b)/(c + d)]$$
Rule 2998

$$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x] / (((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{(3/2)} \sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}), x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1 / (\sqrt{a + b\sin[e + fx]}\sqrt{c + d\sin[e + fx]}), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \sin[e + fx]) / ((a + b\sin[e + fx])^{(3/2)}\sqrt{c + d\sin[e + fx]})], x], x] /; \text{FreeQ}\{a, b, c, d, e,$$

f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]

Rule 3053

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3942

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^{5/2}} dx &= \frac{\left(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}\right) \int \frac{\cos(e + fx)(b + a \cos(e + fx))^{3/2}}{(d + c \cos(e + fx))^{5/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} \\ &= -\frac{2d(bc - ad)\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))\sqrt{c + d \sec(e + fx)}} + \frac{(2\sqrt{d + c \cos(e + fx)} \sqrt{a - b \sec(e + fx)})}{c^3\sqrt{b + a \cos(e + fx)}} \\ &= -\frac{2d(bc - ad)\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{3c(c^2 - d^2)f(d + c \cos(e + fx))\sqrt{c + d \sec(e + fx)}} + \frac{(a^2\sqrt{d + c \cos(e + fx)} \sqrt{a - b \sec(e + fx)})}{c^3\sqrt{b + a \cos(e + fx)}} \\ &= -\frac{2a\sqrt{a + b} \sqrt{\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}} (d + c \cos(e + fx))^{3/2} \csc(e + fx)}{c^3\sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\ &= -\frac{2(a - b)\sqrt{a + b} (3bc^3 - 7ac^2d + bcd^2 + 3ad^3) \sqrt{\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}}}{3c^2(c - d)^2(c + d)^{3/2}(bc - ad)} \end{aligned}$$

Mathematica [B] time = 6.74, size = 1960, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2), x]

[Out] ((d + c*Cos[e + f*x])^3*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((2*(-(b*c*d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e + f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(3/2)*((2*(-(b*c*d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e + f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2))

$$\frac{5}{2} * \text{Sec}[e + f*x] * (a + b * \text{Sec}[e + f*x])^{3/2} * ((4 * (b*c - a*d) * (3 * a * b * c^3 + a^2 * c^2 * d - 4 * b^2 * c^2 * d + a * b * c * d^2 - a^2 * d^3) * \text{Sqrt}[\frac{(c + d) * \text{Cot}[(e + f*x)/2]}{c - d}] * \text{Sqrt}[\frac{(c + d) * (b + a * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Csc}[e + f*x] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] / \text{Sqrt}[2]], (2 * (b*c - a*d)) / ((a + b) * (c - d))] * \text{Sin}[\frac{(e + f*x)/2]^4}{(a + b) * (c + d) * \text{Sqrt}[b + a * \text{Cos}[e + f*x]] * \text{Sqrt}[d + c * \text{Cos}[e + f*x]]]) + 4 * (b*c - a*d) * (3 * a^2 * c^3 - 3 * b^2 * c^3 + 4 * a * b * c^2 * d + a^2 * c * d^2 - b^2 * c * d^2 - 4 * a * b * d^3) * ((\text{Sqrt}[\frac{(c + d) * \text{Cot}[(e + f*x)/2]}{c - d}] * \text{Sqrt}[\frac{(c + d) * (b + a * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Csc}[e + f*x] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] / \text{Sqrt}[2]], (2 * (b*c - a*d)) / ((a + b) * (c - d))] * \text{Sin}[(e + f*x)/2]^4) / ((a + b) * (c + d) * \text{Sqrt}[b + a * \text{Cos}[e + f*x]] * \text{Sqrt}[d + c * \text{Cos}[e + f*x]]) - (\text{Sqrt}[\frac{(c + d) * \text{Cot}[(e + f*x)/2]}{c - d}] * \text{Sqrt}[\frac{(c + d) * (b + a * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Csc}[e + f*x] * \text{EllipticPi}[(b*c - a*d) / ((a + b) * c), \text{ArcSin}[\text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] / \text{Sqrt}[2]], (2 * (b*c - a*d)) / ((a + b) * (c - d))] * \text{Sin}[(e + f*x)/2]^4) / ((a + b) * c * \text{Sqrt}[b + a * \text{Cos}[e + f*x]] * \text{Sqrt}[d + c * \text{Cos}[e + f*x]])) + 2 * (-3 * a * b * c^3 + 7 * a^2 * c^2 * d - a * b * c * d^2 - 3 * a^2 * d^3) * ((\text{Sqrt}[-a + b] / (a + b)) * (a + b) * \text{Cos}[(e + f*x)/2] * \text{Sqrt}[d + c * \text{Cos}[e + f*x]] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[-a + b] / (a + b)] * \text{Sin}[(e + f*x)/2]) / \text{Sqrt}[(b + a * \text{Cos}[e + f*x]) / (a + b)]], (2 * (b*c - a*d)) / ((-a + b) * (c + d))) / (a * c * \text{Sqrt}[\frac{(a + b) * \text{Cos}[(e + f*x)/2]^2}{(b + a * \text{Cos}[e + f*x])}] * \text{Sqrt}[b + a * \text{Cos}[e + f*x]] * \text{Sqrt}[\frac{(b + a * \text{Cos}[e + f*x])}{(a + b)}] * \text{Sqrt}[\frac{(a + b) * (d + c * \text{Cos}[e + f*x])}{(c + d) * (b + a * \text{Cos}[e + f*x])}]) - (2 * (b*c - a*d) * ((b*c + (a + b) * d) * \text{Sqrt}[\frac{(c + d) * \text{Cot}[(e + f*x)/2]}{c - d}] * \text{Sqrt}[\frac{(c + d) * (b + a * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Csc}[e + f*x] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] / \text{Sqrt}[2]], (2 * (b*c - a*d)) / ((a + b) * (c - d))] * \text{Sin}[(e + f*x)/2]^4) / ((a + b) * (c + d) * \text{Sqrt}[b + a * \text{Cos}[e + f*x]] * \text{Sqrt}[d + c * \text{Cos}[e + f*x]]) - ((b*c + a*d) * \text{Sqrt}[\frac{(c + d) * \text{Cot}[(e + f*x)/2]}{c - d}] * \text{Sqrt}[\frac{(c + d) * (b + a * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] * \text{Csc}[e + f*x] * \text{EllipticPi}[(b*c - a*d) / ((a + b) * c), \text{ArcSin}[\text{Sqrt}[\frac{(-a - b) * (d + c * \text{Cos}[e + f*x]) * \text{Csc}[(e + f*x)/2]^2}{(b*c - a*d)}] / \text{Sqrt}[2]], (2 * (b*c - a*d)) / ((a + b) * (c - d))] * \text{Sin}[(e + f*x)/2]^4) / ((a + b) * c * \text{Sqrt}[b + a * \text{Cos}[e + f*x]] * \text{Sqrt}[d + c * \text{Cos}[e + f*x]])) / (a * c) + (\text{Sqrt}[d + c * \text{Cos}[e + f*x]] * \text{Sin}[e + f*x]) / (c * \text{Sqrt}[b + a * \text{Cos}[e + f*x]])) / (3 * c * (c - d)^2 * (c + d)^2 * f * (b + a * \text{Cos}[e + f*x])^{3/2} * (c + d * \text{Sec}[e + f*x])^{5/2})$$

fricas [F] time = 9.29, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(b \sec(fx + e) + a)^{\frac{3}{2}} \sqrt{d \sec(fx + e) + c}}{d^3 \sec^3(fx + e) + 3cd^2 \sec^2(fx + e) + 3c^2d \sec(fx + e) + c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^3*sec(f*x + e)^3 + 3*c*d^2*sec(f*x + e)^2 + 3*c^2*d*sec(f*x + e) + c^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{(d \sec(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

maple [B] time = 2.23, size = 13060, normalized size = 14.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{(d \sec(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{\frac{3}{2}}}{(c + d \sec(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(5/2), x)
```

$$3.213 \quad \int \frac{(a+b \sec(e+fx))^{3/2}}{(c+d \sec(e+fx))^{7/2}} dx$$

Optimal. Leaf size=1122

$$\frac{2(b+a \cos(e+fx))\sqrt{a+b \sec(e+fx)} \sin(e+fx)d^2}{5c(c^2-d^2)f(d+c \cos(e+fx))^2\sqrt{c+d \sec(e+fx)}} - \frac{2(10bc^3-13adc^2-2bd^2c+5ad^3)\sqrt{a+b \sec(e+fx)}}{15c^2(c^2-d^2)^2f(d+c \cos(e+fx))\sqrt{c+d \sec(e+fx)}}$$

[Out] $2/5*d^2*(b+a*\cos(f*x+e))*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c/(c^2-d^2)/f/(d+c*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2/15*d*(-13*a*c^2*d+5*a*d^3+10*b*c^3-2*b*c*d^2)*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c^2/(c^2-d^2)^2/f/(d+c*\cos(f*x+e))/(c+d*\sec(f*x+e))^{(1/2)}+2/15*(a-b)*(2*a*b*c*d*(35*c^4-8*c^2*d^2+5*d^4)-a^2*d^2*(58*c^4-41*c^2*d^2+15*d^4)-b^2*(15*c^6+19*c^4*d^2-2*c^2*d^4))*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticE((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^3/(c-d)^3/(c+d)^{(5/2)}/(-a*d+b*c)^2/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2/15*(b^2*c^3*(15*c^3+10*c^2*d+9*c*d^2-2*d^3)-2*a*b*c^2*(15*c^4+20*c^3*d-4*c^2*d^2-4*c*d^3+5*d^4)+a^2*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5))*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticF((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^4/(c-d)^3/(c+d)^{(5/2)}/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2*a*(d+c*\cos(f*x+e))^{(3/2)}*csc(f*x+e)*EllipticPi((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^4/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 3.16, antiderivative size = 1122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3942, 3048, 3047, 3053, 2811, 2998, 2818, 2996}

$$\frac{2(b+a \cos(e+fx))\sqrt{a+b \sec(e+fx)} \sin(e+fx)d^2}{5c(c^2-d^2)f(d+c \cos(e+fx))^2\sqrt{c+d \sec(e+fx)}} - \frac{2(10bc^3-13adc^2-2bd^2c+5ad^3)\sqrt{a+b \sec(e+fx)}}{15c^2(c^2-d^2)^2f(d+c \cos(e+fx))\sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2), x]

[Out] $(2*(a-b)*\text{Sqrt}[a+b]*(2*a*b*c*d*(35*c^4-8*c^2*d^2+5*d^4)-a^2*d^2*(58*c^4-41*c^2*d^2+15*d^4)-b^2*(15*c^6+19*c^4*d^2-2*c^2*d^4))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(15*c^3*(c-d)^3*(c+d)^{(5/2)}*(b*c-a*d)^2*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])-(2*\text{Sqrt}[a+b]*(b^2*c^3*(15*c^3+10*c^2*d+9*c*d^2-2*d^3)-2*a*b*c^2*(15*c^4+20*c^3*d-4*c^2*d^2-4*c*d^3+5*d^4)+a^2*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]$


```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*Sin[e + f*x] +
b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3048

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :=
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3053

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3942

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_.))^(n_), x_Symbol] := Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs
c[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b +
a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2
] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^{7/2}} dx &= \frac{\left(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}\right) \int \frac{\cos^2(e+fx)(b+a \cos(e+fx))^{3/2}}{(d+c \cos(e+fx))^{7/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} \\
&= \frac{2d^2(b + a \cos(e + fx))\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} + \frac{(2\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)})}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} \\
&= \frac{2d^2(b + a \cos(e + fx))\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} - \frac{2d(10bc^3 - 13ac^2d - 2bcd)}{15c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} \\
&= \frac{2d^2(b + a \cos(e + fx))\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} - \frac{2d(10bc^3 - 13ac^2d - 2bcd)}{15c^2(c^2 - d^2)^2 f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} \\
&= -\frac{2a\sqrt{a + b} \sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c \cos(e+fx))}} (d + c \cos(e + fx))^{3/2} \csc(e + fx)}{c^4 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\
&= \frac{2(a - b)\sqrt{a + b} (2abcd(35c^4 - 8c^2d^2 + 5d^4) - a^2d^2(58c^4 - 41c^2d^2 + 15d^4) - b^2(15c^4 - 10c^2d^2 + 5d^4))}{c^4 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}}
\end{aligned}$$

Mathematica [B] time = 7.41, size = 2385, normalized size = 2.13

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2), x]

[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((-2*(-(b*c*d^2*Sin[e + f*x]) + a*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) - (4*(5*b*c^3*d*Sin[e + f*x] - 8*a*c^2*d^2*Sin[e + f*x] - b*c*d^3*Sin[e + f*x] + 4*a*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^2) + (2*(15*b^2*c^6*Sin[e + f*x] - 70*a*b*c^5*d*Sin[e + f*x] + 58*a^2*c^4*d^2*Sin[e + f*x] + 19*b^2*c^4*d^2*Sin[e + f*x] + 16*a*b*c^3*d^3*Sin[e + f*x] - 41*a^2*c^2*d^4*Sin[e + f*x] - 2*b^2*c^2*d^4*Sin[e + f*x] - 10*a*b*c*d^5*Sin[e + f*x] + 15*a^2*d^6*Sin[e + f*x]))/(15*c^2*(b*c - a*d)*(c^2 - d^2)^3*(d + c*Cos[e + f*x])))/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(-15*a*b^2*c^6 + 5*a^2*b*c^5*d + 25*b^3*c^5*d + 13*a^3*c^4*d^2 - 38*a*b^2*c^4*d^2 + 25*a^2*b*c^3*d^3 + 7*b^3*c^3*d^3 - 18*a^3*c^2*d^4 - 11*a*b^2*c^2*d^4 + 2*a^2*b*c*d^5 + 5*a^3*d^6)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d])*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d)))*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-15*a^2*b*c^6 + 15*b^3*c^6 + 15*a^3*c^5*d - 55*a*b^2*c^5*d + 33*a^2*b*c^4*d^2 + 19*b^3*c^4*d^2 + 13*a^3*c^3*d^3 + 35*a*b^2*c^3*d^3 - 70*a^2*b*c^2*d^4 - 2*b^3*c^2*d^4 + 4*a^3*c*d^5 - 12*a*b^2*c*d^5 + 20*a^2*b*d^6)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d])*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d]

)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 2*(15*a*b^2*c^6 - 70*a^2*b*c^5*d + 58*a^3*c^4*d^2 + 19*a*b^2*c^4*d^2 + 16*a^2*b*c^3*d^3 - 41*a^3*c^2*d^4 - 2*a*b^2*c^2*d^4 - 10*a^2*b*c*d^5 + 15*a^3*d^6)*((Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])/Sqrt[(b + a*Cos[e + f*x])/(a + b)]]], (2*(b*c - a*d))/((-a + b)*(c + d)))/(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*Cos[e + f*x]))*Sqrt[b + a*Cos[e + f*x]]*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*Cos[e + f*x]))/(c + d)*(b + a*Cos[e + f*x]))]) - (2*(b*c - a*d)*((b*c + (a + b)*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x]))*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])/(a*c) + (Sqrt[d + c*Cos[e + f*x]]*Sin[e + f*x])/(c*Sqrt[b + a*Cos[e + f*x]])/((15*c^2*(c - d)^3*(c + d)^3*(-(b*c) + a*d)*f*(b + a*Cos[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^(7/2))

fricas [F] time = 8.70, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(b \sec(fx + e) + a)^{\frac{3}{2}} \sqrt{d \sec(fx + e) + c}}{d^4 \sec(fx + e)^4 + 4cd^3 \sec(fx + e)^3 + 6c^2d^2 \sec(fx + e)^2 + 4c^3d \sec(fx + e) + c^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="fricas")

[Out] integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^4*sec(f*x + e)^4 + 4*c*d^3*sec(f*x + e)^3 + 6*c^2*d^2*sec(f*x + e)^2 + 4*c^3*d*sec(f*x + e) + c^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{(d \sec(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)

maple [B] time = 3.40, size = 39420, normalized size = 35.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{3}{2}}}{(d \sec(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)`

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(7/2),x)`

[Out] `\text{Hanged}`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(7/2),x)`

[Out] Timed out

$$3.214 \quad \int \frac{(a+b \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^{5/2}} dx$$

Optimal. Leaf size=891

$$\frac{2\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} (d+c \cos(e+fx))^{3/2} \csc(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b}}{\sqrt{a+b}\sqrt{c+d}}\right)\right)}{c^3 \sqrt{c+d} f \sqrt{b+a \cos(e+fx)} \sqrt{c+d \sec(e+fx)}}$$

[Out] $2/3*(-a*d+b*c)^2*\sin(f*x+e)*(a+b*\sec(f*x+e))^{1/2}/c/(c^2-d^2)/f/(d+c*\cos(f*x+e))/(c+d*\sec(f*x+e))^{1/2}-2/3*(a-b)*(7*a*c^2-3*a*d^2-4*b*c*d)*(d+c*\cos(f*x+e))^{3/2}*csc(f*x+e)*EllipticE((c+d)^{1/2}*(b+a*\cos(f*x+e))^{1/2}/(a+b)^{1/2}/(d+c*\cos(f*x+e))^{1/2}),((a+b)*(c-d)/(a-b)/(c+d))^{1/2}*(a+b)^{1/2}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{1/2}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{1/2}*(a+b*\sec(f*x+e))^{1/2}/c^2/(c-d)^2/(c+d)^{3/2}/f/(b+a*\cos(f*x+e))^{1/2}/(c+d*\sec(f*x+e))^{1/2}+2/3*(b^2*c^2*(c+3*d)-a*b*c*(7*c^2+4*c*d-3*d^2)+a^2*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(d+c*\cos(f*x+e))^{3/2}*csc(f*x+e)*EllipticF((c+d)^{1/2}*(b+a*\cos(f*x+e))^{1/2}/(a+b)^{1/2}/(d+c*\cos(f*x+e))^{1/2}),((a+b)*(c-d)/(a-b)/(c+d))^{1/2}*(a+b)^{1/2}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{1/2}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{1/2}*(a+b*\sec(f*x+e))^{1/2}/c^3/(c-d)^2/(c+d)^{3/2}/f/(b+a*\cos(f*x+e))^{1/2}/(c+d*\sec(f*x+e))^{1/2}-2*a^2*(d+c*\cos(f*x+e))^{3/2}*csc(f*x+e)*EllipticPi((c+d)^{1/2}*(b+a*\cos(f*x+e))^{1/2}/(a+b)^{1/2}/(d+c*\cos(f*x+e))^{1/2}), (a+b)*c/a/(c+d), ((a+b)*(c-d)/(a-b)/(c+d))^{1/2}*(a+b)^{1/2}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{1/2}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{1/2}*(a+b*\sec(f*x+e))^{1/2}/c^3/f/(c+d)^{1/2}/(b+a*\cos(f*x+e))^{1/2}/(c+d*\sec(f*x+e))^{1/2}$

Rubi [A] time = 2.01, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3942, 2792, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} (d+c \cos(e+fx))^{3/2} \csc(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b}}{\sqrt{a+b}\sqrt{c+d}}\right)\right)}{c^3 \sqrt{c+d} f \sqrt{b+a \cos(e+fx)} \sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2), x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*(7*a*c^2-4*b*c*d-3*a*d^2)*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{3/2})*\text{Csc}[e+f*x]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])], ((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(3*c^2*(c-d)^2*(c+d)^{3/2})*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]+(2*\text{Sqrt}[a+b]*(b^2*c^2*(c+3*d)-a*b*c*(7*c^2+4*c*d-3*d^2)+a^2*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{3/2})*\text{Csc}[e+f*x]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])], ((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(3*c^3*(c-d)^2*(c+d)^{3/2})*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]-(2*a^2*\text{Sqrt}[a+b]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{3/2})*\text{Csc}[e+f*x]*\text{EllipticPi}[(a+b)*c/(a*(c+d)), \text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])], ((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(c^3*\text{Sqrt}[a+b]*\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]))$

$\text{rt}[c + d]*f*\text{Sqrt}[b + a*\text{Cos}[e + f*x]]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]] + (2*(b*c - a*d)^2*\text{Sqrt}[a + b*\text{Sec}[e + f*x]]*\text{Sin}[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*\text{Cos}[e + f*x])*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])$

Rule 2792

$\text{Int}[(a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(m_)}*((c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(n_)}, x_Symbol] :> -\text{Simp}[(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 2)}*(c + d*\text{Sin}[e + f*x])^{(n + 1)})/(d*f*(n + 1)*(c^2 - d^2)), x] + \text{Dist}[1/(d*(n + 1)*(c^2 - d^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 3)}*(c + d*\text{Sin}[e + f*x])^{(n + 1)}*\text{Simp}[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*\text{Sin}[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\text{Sin}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegersQ}[2*m, 2*n])$

Rule 2811

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]]/\text{Sqrt}[(c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]], x_Symbol] :> \text{Simp}[(2*(a + b*\text{Sin}[e + f*x])* \text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((c - d)*(a + b*\text{Sin}[e + f*x]))*\text{Sqrt}[-((b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((c + d)*(a + b*\text{Sin}[e + f*x])))]*\text{EllipticPi}[(b*(c + d))/(d*(a + b)), \text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]], ((a - b)*(c + d))/((a + b)*(c - d))]/(d*f*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(a + b)/(c + d)]$

Rule 2818

$\text{Int}[1/(\text{Sqrt}[(a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]], x_Symbol] :> \text{Simp}[(2*(c + d*\text{Sin}[e + f*x])* \text{Sqrt}[(b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))*\text{Sqrt}[-((b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((a - b)*(c + d*\text{Sin}[e + f*x])))]*\text{EllipticF}[\text{ArcSin}[\text{Rt}[(c + d)/(a + b), 2]*(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*\text{Rt}[(c + d)/(a + b), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/(a + b)]$

Rule 2996

$\text{Int}[(A_.) + (B_.)*\text{sin}[e_.) + (f_.)*(x_)]/((a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(3/2)}*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]], x_Symbol] :> \text{Simp}[(-2*A*(c - d)*(a + b*\text{Sin}[e + f*x])* \text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((c - d)*(a + b*\text{Sin}[e + f*x]))*\text{Sqrt}[-((b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((c + d)*(a + b*\text{Sin}[e + f*x])))]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(a + b)/(c + d)]$

Rule 2998

$\text{Int}[(A_.) + (B_.)*\text{sin}[e_.) + (f_.)*(x_)]/((a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(3/2)}*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]], x_Symbol] :> \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f*x])/(a + b*\text{Sin}[e + f*x])^{(3/2)}*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

&& NeQ[A, B]

Rule 3053

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3942

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^{5/2}} dx &= \frac{(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}) \int \frac{(b + a \cos(e + fx))^{5/2}}{(d + c \cos(e + fx))^{5/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} \\ &= \frac{2(bc - ad)^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx)) \sqrt{c + d \sec(e + fx)}} + \frac{(2\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}) \int \frac{(b + a \cos(e + fx))^{5/2}}{(d + c \cos(e + fx))^{5/2}} dx}{c^3 \sqrt{b + a \cos(e + fx)}} \\ &= \frac{2(bc - ad)^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{3c(c^2 - d^2) f(d + c \cos(e + fx)) \sqrt{c + d \sec(e + fx)}} + \frac{(a^3 \sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}) \int \frac{(b + a \cos(e + fx))^{5/2}}{(d + c \cos(e + fx))^{5/2}} dx}{c^3 \sqrt{b + a \cos(e + fx)}} \\ &= -\frac{2a^2 \sqrt{a + b} \sqrt{\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}} (d + c \cos(e + fx))^{3/2} \operatorname{csch}^{-1}\left(\frac{d + c \cos(e + fx)}{a + b \sec(e + fx)}\right)}{c^3 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\ &= -\frac{2(a - b) \sqrt{a + b} (7ac^2 - 4bcd - 3ad^2) \sqrt{\frac{(bc - ad)(1 - \cos(e + fx))}{(a + b)(d + c \cos(e + fx))}} \sqrt{\frac{(bc - ad)(1 + \cos(e + fx))}{(a - b)(d + c \cos(e + fx))}}}{3c^2(c - d)^2(c + d)^{3/2} f} \end{aligned}$$

Mathematica [B] time = 6.73, size = 2026, normalized size = 2.27

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2), x]

[Out] ((d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*Sin[e + f*x] - 2*a*b*c*d*Sin[e + f*x] + a^2*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) + (2*(7*a*b*c^3*Sin[e + f*x] - 7*a^2*c^2*d*Sin[e + f*x] - 4*b^2*c^2*d*Sin[e + f*x] + a*b*c*d^2*Sin[e + f*x] + 3*a^2*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*(a + b*Sec[e + f*x])

$$\begin{aligned} & \sqrt[5]{2} \cdot ((4 \cdot (b \cdot c - a \cdot d) \cdot (2 \cdot a^2 \cdot b \cdot c^3 + b^3 \cdot c^3 + a^3 \cdot c^2 \cdot d - 8 \cdot a \cdot b^2 \cdot c^2 \cdot d + \\ & 2 \cdot a^2 \cdot b \cdot c \cdot d^2 + 3 \cdot b^3 \cdot c \cdot d^2 - a^3 \cdot d^3) \cdot \sqrt{((c + d) \cdot \cot((e + f \cdot x)/2))^2} / (c - d) \cdot \sqrt{((c + d) \cdot (b + a \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \\ & \cdot \sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \csc[e + f \cdot x] \cdot \text{EllipticF}[\text{ArcSin}[\sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d)] / \sqrt{2}], (2 \cdot (b \cdot c - a \cdot d)) / ((a + b) \cdot (c - d))] \cdot \sin[(e + f \cdot x) / 2]^4 / ((a + b) \cdot (c + d) \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)} \cdot \sqrt{d + c \cdot \cos(e + f \cdot x)}) \\ & + 4 \cdot (b \cdot c - a \cdot d) \cdot (3 \cdot a^3 \cdot c^3 - 7 \cdot a \cdot b^2 \cdot c^3 + 4 \cdot b^3 \cdot c^2 \cdot d + a^3 \cdot c \cdot d^2 + 3 \cdot a \cdot b^2 \cdot c \cdot d^2 - 4 \cdot a^2 \cdot b \cdot d^3) \cdot ((\sqrt{((c + d) \cdot \cot((e + f \cdot x)/2))^2} / (c - d) \cdot \sqrt{((c + d) \cdot (b + a \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \csc[e + f \cdot x] \cdot \text{EllipticF}[\text{ArcSin}[\sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d)] / \sqrt{2}], (2 \cdot (b \cdot c - a \cdot d)) / ((a + b) \cdot (c - d))] \cdot \sin[(e + f \cdot x) / 2]^4 / ((a + b) \cdot (c + d) \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)} \cdot \sqrt{d + c \cdot \cos(e + f \cdot x)}) - (\sqrt{((c + d) \cdot \cot((e + f \cdot x)/2))^2} / (c - d) \cdot \sqrt{((c + d) \cdot (b + a \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \csc[e + f \cdot x] \cdot \text{EllipticPi}[(b \cdot c - a \cdot d) / ((a + b) \cdot c), \text{ArcSin}[\sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d)] / \sqrt{2}], (2 \cdot (b \cdot c - a \cdot d)) / ((a + b) \cdot (c - d))] \cdot \sin[(e + f \cdot x) / 2]^4 / ((a + b) \cdot c \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)} \cdot \sqrt{d + c \cdot \cos(e + f \cdot x)})) + 2 \cdot (-7 \cdot a^2 \cdot b \cdot c^3 + 7 \cdot a^3 \cdot c^2 \cdot d + 4 \cdot a \cdot b^2 \cdot c^2 \cdot d - a^2 \cdot b \cdot c \cdot d^2 - 3 \cdot a^3 \cdot d^3) \cdot ((\sqrt{(-a + b) / (a + b)}) \cdot (a + b) \cdot \cos[(e + f \cdot x) / 2] \cdot \sqrt{d + c \cdot \cos(e + f \cdot x)} \cdot \text{EllipticE}[\text{ArcSin}[(\sqrt{(-a + b) / (a + b)}) \cdot \sin[(e + f \cdot x) / 2]] / \sqrt{(b + a \cdot \cos(e + f \cdot x)) / (a + b)}]), (2 \cdot (b \cdot c - a \cdot d)) / ((-a + b) \cdot (c + d))] / (a \cdot c \cdot \sqrt{((a + b) \cdot \cos[(e + f \cdot x) / 2])^2} / (b + a \cdot \cos(e + f \cdot x))) \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)} \cdot \sqrt{(b + a \cdot \cos(e + f \cdot x)) / (a + b)} \cdot \sqrt{((a + b) \cdot (d + c \cdot \cos(e + f \cdot x))) / ((c + d) \cdot (b + a \cdot \cos(e + f \cdot x)))}) - (2 \cdot (b \cdot c - a \cdot d) \cdot ((b \cdot c + (a + b) \cdot d) \cdot \sqrt{((c + d) \cdot \cot((e + f \cdot x)/2))^2} / (c - d)) \cdot \sqrt{((c + d) \cdot (b + a \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \csc[e + f \cdot x] \cdot \text{EllipticF}[\text{ArcSin}[\sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d)] / \sqrt{2}], (2 \cdot (b \cdot c - a \cdot d)) / ((a + b) \cdot (c - d))] \cdot \sin[(e + f \cdot x) / 2]^4 / ((a + b) \cdot (c + d) \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)} \cdot \sqrt{d + c \cdot \cos(e + f \cdot x)}) - ((b \cdot c + a \cdot d) \cdot \sqrt{((c + d) \cdot \cot((e + f \cdot x)/2))^2} / (c - d) \cdot \sqrt{((c + d) \cdot (b + a \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d) \cdot \csc[e + f \cdot x] \cdot \text{EllipticPi}[(b \cdot c - a \cdot d) / ((a + b) \cdot c), \text{ArcSin}[\sqrt{((-a - b) \cdot (d + c \cdot \cos(e + f \cdot x)) \cdot \csc((e + f \cdot x)/2))^2} / (b \cdot c - a \cdot d)] / \sqrt{2}], (2 \cdot (b \cdot c - a \cdot d)) / ((a + b) \cdot (c - d))] \cdot \sin[(e + f \cdot x) / 2]^4 / ((a + b) \cdot c \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)} \cdot \sqrt{d + c \cdot \cos(e + f \cdot x)})) / (a \cdot c) + (\sqrt{d + c \cdot \cos(e + f \cdot x)} \cdot \sin[e + f \cdot x]) / (c \cdot \sqrt{b + a \cdot \cos(e + f \cdot x)})) / (3 \cdot c \cdot (c - d)^2 \cdot (c + d)^2 \cdot f \cdot (b + a \cdot \cos(e + f \cdot x))^{\sqrt{5}/2} \cdot (c + d \cdot \sec(e + f \cdot x))^{\sqrt{5}/2}) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{5}{2}}}{(d \sec(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)

maple [B] time = 2.30, size = 15922, normalized size = 17.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{5}{2}}}{(d \sec(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(5/2),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(5/2),x)

[Out] Timed out

$$3.215 \quad \int \frac{(a+b \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^{7/2}} dx$$

Optimal. Leaf size=1150

$$\frac{2\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} (d+c \cos(e+fx))^{3/2} \csc(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{b+a}}{\sqrt{a+b} \sqrt{d+c}}\right)\right)}{c^4 \sqrt{c+d} f \sqrt{b+a \cos(e+fx)} \sqrt{c+d \sec(e+fx)}}$$

[Out] $-2/5*d*(-a*d+b*c)*(b+a*\cos(f*x+e))*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c/(c^2-d^2)/f/(d+c*\cos(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))^{(1/2)+2/15*(-a*d+b*c)*(-13*a*c^2*d+5*a*d^3+5*b*c^3+3*b*c*d^2)*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c^2/(c^2-d^2)^{2/2}/f/(d+c*\cos(f*x+e))/(c+d*\sec(f*x+e))^{(1/2)+2/15*(a-b)*(b^2*c^2*d*(29*c^2+3*d^2)-a*b*c*(35*c^4+34*c^2*d^2-5*d^4)+a^2*(58*c^4*d-41*c^2*d^3+15*d^5))}*(d+c*\cos(f*x+e))^{(3/2)*\csc(f*x+e)*\text{EllipticE}((c+d)^{(1/2)*(b+a*\cos(f*x+e))^{(1/2)/(a+b)^{(1/2)/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(a+b)^{(1/2)*(-(-a*d+b*c)*(1-\cos(f*x+e)))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)*(-(-a*d+b*c)*(1+\cos(f*x+e)))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)*(a+b*\sec(f*x+e))^{(1/2)}/c^3/(c-d)^3/(c+d)^{(5/2)/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))^{(1/2)+2/15*(b^3*c^4*(5*c^2+24*c*d+3*d^2)-a*b^2*c^3*(35*c^3+42*c^2*d+21*c*d^2-2*d^3)+a^2*b*c^2*(45*c^4+48*c^3*d+c^2*d^2-8*c*d^3+10*d^4)-a^3*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5)}}*(d+c*\cos(f*x+e))^{(3/2)*\csc(f*x+e)*\text{EllipticF}((c+d)^{(1/2)*(b+a*\cos(f*x+e))^{(1/2)/(a+b)^{(1/2)/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(a+b)^{(1/2)*(-(-a*d+b*c)*(1-\cos(f*x+e)))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)*(-(-a*d+b*c)*(1+\cos(f*x+e)))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)*(a+b*\sec(f*x+e))^{(1/2)}/c^4/(c-d)^3/(c+d)^{(5/2)/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))^{(1/2)-2*a^2*(d+c*\cos(f*x+e))^{(3/2)*\csc(f*x+e)*\text{EllipticPi}((c+d)^{(1/2)*(b+a*\cos(f*x+e))^{(1/2)/(a+b)^{(1/2)/(d+c*\cos(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(a+b)^{(1/2)*(-(-a*d+b*c)*(1-\cos(f*x+e)))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)*(-(-a*d+b*c)*(1+\cos(f*x+e)))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)*(a+b*\sec(f*x+e))^{(1/2)}/c^4/f/(c+d)^{(1/2)/(b+a*\cos(f*x+e))^{(1/2)/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 3.44, antiderivative size = 1150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3942, 2989, 3047, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b} \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} (d+c \cos(e+fx))^{3/2} \csc(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{b+a}}{\sqrt{a+b} \sqrt{d+c}}\right)\right)}{c^4 \sqrt{c+d} f \sqrt{b+a \cos(e+fx)} \sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2), x]

[Out] $(2*(a-b)*\text{Sqrt}[a+b]*(b^2*c^2*d*(29*c^2+3*d^2)-a*b*c*(35*c^4+34*c^2*d^2-5*d^4)+a^2*(58*c^4*d-41*c^2*d^3+15*d^5))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(15*c^3*(c-d)^3*(c+d)^{(5/2)*(b*c-a*d)*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])+(2*\text{Sqrt}[a+b]*(b^3*c^4*(5*c^2+24*c*d+3*d^2)-a*b^2*c^3*(35*c^3+42*c^2*d+21*c*d^2-2*d^3)+a^2*b*c^2*(45*c^4+48*c^3*d+c^2*d^2-8*c*d^3+10*d^4)-a^3*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*d^5))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])$

$$\begin{aligned} & \sqrt{\frac{c+d}{a+b}} \operatorname{Csc}[e+fx] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] \\ & \sqrt{\frac{a+b\sec[e+fx]}{15c^4(c-d)^3(c+d)^{5/2}(b*c-a*d)*f}} \\ & \sqrt{\frac{b+a\cos[e+fx]}{c+d\sec[e+fx]}} - \frac{2a^2\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}{\sqrt{-(b*c-a*d)(1-\cos[e+fx])}} \\ & \sqrt{\frac{-(b*c-a*d)(1+\cos[e+fx])}{(a-b)(d+c\cos[e+fx])}} * (d+c\cos[e+fx])^{3/2} \operatorname{Csc}[e+fx] \operatorname{EllipticPi}\left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d}\sqrt{b+a\cos[e+fx]}}{\sqrt{a+b}\sqrt{d+c\cos[e+fx]}}\right]\right], \\ & \frac{(a+b)(c-d)}{(a-b)(c+d)} \sqrt{\frac{a+b\sec[e+fx]}{c^4\sqrt{c+d}*f\sqrt{b+a\cos[e+fx]}\sqrt{c+d\sec[e+fx]}}} - \frac{2d*(b*c-a*d)*(b+a\cos[e+fx])\sqrt{a+b\sec[e+fx]}\sin[e+fx]}{5c*(c^2-d^2)*f*(d+c\cos[e+fx])^2\sqrt{c+d\sec[e+fx]}} \\ & + \frac{2*(b*c-a*d)*(5*b*c^3-13*a*c^2*d+3*b*c*d^2+5*a*d^3)\sqrt{a+b\sec[e+fx]}\operatorname{Sin}[e+fx]}{(15*c^2*(c^2-d^2)^2*f*(d+c\cos[e+fx])\sqrt{c+d\sec[e+fx]}} \end{aligned}$$

Rule 2811

$$\begin{aligned} & \operatorname{Int}\left[\frac{\sqrt{(a_.) + (b_.)\sin[(e_.) + (f_.)x]}}{\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}}\right], x_Symbol] \rightarrow \operatorname{Simp}\left[\frac{2*(a + b\sin[e + fx])\sqrt{((b*c - a*d)*(1 + \sin[e + fx]))}}{(c - d)*(a + b\sin[e + fx])}\sqrt{-(b*c - a*d)*(1 - \sin[e + fx])}}{(c + d)*(a + b\sin[e + fx])}\right] \\ & \operatorname{EllipticPi}\left[\frac{b*(c + d)}{d*(a + b)}, \operatorname{ArcSin}\left[\frac{\operatorname{Rt}[(a + b)/(c + d), 2]\sqrt{c + d\sin[e + fx]}}{\sqrt{a + b\sin[e + fx]}}\right], \frac{(a - b)(c + d)}{(a + b)(c - d)}\right] \\ & \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \& \& \operatorname{NeQ}[b*c - a*d, 0] \& \& \operatorname{NeQ}[a^2 - b^2, 0] \& \& \operatorname{NeQ}[c^2 - d^2, 0] \& \& \operatorname{PosQ}[(a + b)/(c + d)] \end{aligned}$$

Rule 2818

$$\begin{aligned} & \operatorname{Int}\left[\frac{1}{\sqrt{(a_.) + (b_.)\sin[(e_.) + (f_.)x]}\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}}\right], x_Symbol] \rightarrow \operatorname{Simp}\left[\frac{2*(c + d\sin[e + fx])\sqrt{((b*c - a*d)*(1 - \sin[e + fx]))}}{(a + b)*(c + d\sin[e + fx])}\sqrt{-(b*c - a*d)*(1 + \sin[e + fx])}}{(a - b)*(c + d\sin[e + fx])}\right] \\ & \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Rt}[(c + d)/(a + b), 2]\sqrt{a + b\sin[e + fx]}}{\sqrt{c + d\sin[e + fx]}}\right], \frac{(a + b)(c - d)}{(a - b)(c + d)}\right] \\ & \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \& \& \operatorname{NeQ}[b*c - a*d, 0] \& \& \operatorname{NeQ}[a^2 - b^2, 0] \& \& \operatorname{NeQ}[c^2 - d^2, 0] \& \& \operatorname{PosQ}[(c + d)/(a + b)] \end{aligned}$$

Rule 2989

$$\begin{aligned} & \operatorname{Int}\left[\frac{((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{(m_.)}((A_.) + (B_.)\sin[(e_.) + (f_.)x])^{(n_.)}}{\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}}\right], x_Symbol] \rightarrow -\operatorname{Simp}\left[\frac{(b*c - a*d)*(B*c - A*d)\cos[e + fx]*(a + b\sin[e + fx])^{(m-1)}(c + d\sin[e + fx])^{(n+1)}}{d*f*(n+1)*(c^2 - d^2)}, x\right] \\ & + \operatorname{Dist}\left[\frac{1}{d*(n+1)*(c^2 - d^2)}, \operatorname{Int}\left[\frac{(a + b\sin[e + fx])^{(m-2)}(c + d\sin[e + fx])^{(n+1)}}{b*(b*c - a*d)*(B*c - A*d)*(m-1) + a*d*(a*A*c + b*B*c - (A*b + a*B)*d)*(n+1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n+1) - a*(b*c - a*d)*(B*c - A*d)*(n+2))*\sin[e + fx] + b*(d*(A*b*c + a*B*c - a*A*d)*(m+n+1) - b*B*(c^2*m + d^2*(n+1)))*\sin[e + fx]^2}, x\right], x\right] \\ & \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, x\} \& \& \operatorname{NeQ}[b*c - a*d, 0] \& \& \operatorname{NeQ}[a^2 - b^2, 0] \& \& \operatorname{NeQ}[c^2 - d^2, 0] \& \& \operatorname{GtQ}[m, 1] \& \& \operatorname{LtQ}[n, -1] \end{aligned}$$

Rule 2996

$$\begin{aligned} & \operatorname{Int}\left[\frac{(A_.) + (B_.)\sin[(e_.) + (f_.)x]}{((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{3/2}\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}}\right], x_Symbol] \rightarrow \operatorname{Simp}\left[\frac{-2*A*(c - d)*(a + b\sin[e + fx])\sqrt{((b*c - a*d)*(1 + \sin[e + fx]))}}{(c - d)*(a + b\sin[e + fx])}\sqrt{-(b*c - a*d)*(1 - \sin[e + fx])}}{(c + d)*(a + b\sin[e + fx])}\right] \\ & \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Rt}[(a + b)/(c + d), 2]\sqrt{c + d\sin[e + fx]}}{\sqrt{a + b\sin[e + fx]}}\right], \frac{(a - b)(c + d)}{(a + b)}\right] \end{aligned}$$

$\frac{*(c - d))}{(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x} /; FreeQ[\{a, b, c, d, e, f, A, B\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 - b^2, 0] \&\& NeQ[c^2 - d^2, 0] \&\& EqQ[A, B] \&\& PosQ[(a + b)/(c + d)]$

Rule 2998

$Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]) / (((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^{3/2} * Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x]) / ((a + b*Sin[e + f*x])^{3/2} * Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 - b^2, 0] \&\& NeQ[c^2 - d^2, 0] \&\& NeQ[A, B]$

Rule 3047

$Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m * ((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n * ((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x] * (a + b*Sin[e + f*x])^m * (c + d*Sin[e + f*x])^{n+1}) / (d*f*(n+1)*(c^2 - d^2)), x] + Dist[1/(d*(n+1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^{m-1} * (c + d*Sin[e + f*x])^{n+1} * Simp[A*d*(b*d*m + a*c*(n+1)) + (c*C - B*d)*(b*c*m + a*d*(n+1)) - (d*(A*(a*d*(n+2) - b*c*(n+1)) + B*(b*d*(n+1) - a*c*(n+2))) - C*(b*c*d*(n+1) - a*(c^2 + d^2*(n+1)))] * Sin[e + f*x] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1))] * Sin[e + f*x]^2, x], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, C\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 - b^2, 0] \&\& NeQ[c^2 - d^2, 0] \&\& GtQ[m, 0] \&\& LtQ[n, -1]$

Rule 3053

$Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2) / (((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^{3/2} * Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]] / Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x]) / ((a + b*Sin[e + f*x])^{3/2} * Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, C\}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 - b^2, 0] \&\& NeQ[c^2 - d^2, 0]$

Rule 3942

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{m_} * (csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^{n_}, x_Symbol] :> Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Cs c[e + f*x]]) / (Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[(b + a*Sin[e + f*x])^m * (d + c*Sin[e + f*x])^n / Sin[e + f*x]^{m+n}], x], x] /; FreeQ[\{a, b, c, d, e, f, m, n\}, x] \&\& NeQ[b*c - a*d, 0] \&\& IntegerQ[m + 1/2] \&\& IntegerQ[n + 1/2] \&\& LeQ[-2, m + n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^{7/2}} dx &= \frac{\left(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}\right) \int \frac{\cos(e+fx)(b+a \cos(e+fx))^{5/2}}{(d+c \cos(e+fx))^{7/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} \\
&= -\frac{2d(bc - ad)(b + a \cos(e + fx))\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} + \frac{(2\sqrt{d + c \cos(e + fx)})}{15c} \\
&= -\frac{2d(bc - ad)(b + a \cos(e + fx))\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} + \frac{2(bc - ad)(5\sqrt{d + c \cos(e + fx)})}{15c} \\
&= -\frac{2d(bc - ad)(b + a \cos(e + fx))\sqrt{a + b \sec(e + fx)} \sin(e + fx)}{5c(c^2 - d^2) f(d + c \cos(e + fx))^2 \sqrt{c + d \sec(e + fx)}} + \frac{2(bc - ad)(5\sqrt{d + c \cos(e + fx)})}{15c} \\
&= -\frac{2a^2 \sqrt{a + b} \sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c \cos(e+fx))}} (d + c \cos(e + fx))^{3/2} \operatorname{cs}(\operatorname{arcsin}(\frac{d + c \cos(e + fx)}{\sqrt{b + a \cos(e + fx)}}))}{c^4 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\
&= \frac{2(a - b)\sqrt{a + b} (b^2 c^2 d (29c^2 + 3d^2) - abc (35c^4 + 34c^2 d^2 - 5d^4) + a^2 (58c^4 d - 4d^5))}{15c^4 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}}
\end{aligned}$$

Mathematica [B] time = 7.32, size = 2344, normalized size = 2.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]

[Out] ((d + c*Cos[e + f*x])^4*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((-2*(b^2*c^2*d*Sin[e + f*x] - 2*a*b*c*d^2*Sin[e + f*x] + a^2*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) + (2*(5*b^2*c^4*Sin[e + f*x] - 21*a*b*c^3*d*Sin[e + f*x] + 16*a^2*c^2*d^2*Sin[e + f*x] + 3*b^2*c^2*d^2*Sin[e + f*x] + 5*a*b*c*d^3*Sin[e + f*x] - 8*a^2*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^2) + (2*(35*a*b*c^5*Sin[e + f*x] - 58*a^2*c^4*d*Sin[e + f*x] - 29*b^2*c^4*d*Sin[e + f*x] + 34*a*b*c^3*d^2*Sin[e + f*x] + 41*a^2*c^2*d^3*Sin[e + f*x] - 3*b^2*c^2*d^3*Sin[e + f*x] - 5*a*b*c*d^4*Sin[e + f*x] - 15*a^2*d^5*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^3))/((f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(10*a^2*b*c^5 + 5*b^3*c^5 + 13*a^3*c^4*d - 48*a*b^2*c^4*d + 15*a^2*b*c^3*d^2 + 27*b^3*c^3*d^2 - 18*a^3*c^2*d^3 - 16*a*b^2*c^2*d^3 + 7*a^2*b*c*d^4 + 5*a^3*d^5)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d])*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d)))*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(15*a^3*c^5 - 35*a*b^2*c^5 + 23*a^2*b*c^4*d + 29*b^3*c^4*d + 13*a^3*c^3*d^2 - 5*a*b^2*c^3*d^2 - 75*a^2*b*c^2*d^3 + 3*b^3*c^2*d^3 + 4*a^3*c*d^4 + 8*a*b^2*c*d^4 + 20*a^2*b*d^5)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d])*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d])*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]]/Sqrt[2]], (2

```

*(b*c - a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[
b + a*cos[e + f*x]]*Sqrt[d + c*cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x
)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a - b)*
(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a
*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*cos[e + f
*x]]*Sqrt[d + c*cos[e + f*x]]) + 2*(-35*a^2*b*c^5 + 58*a^3*c^4*d + 29*a*b^
2*c^4*d - 34*a^2*b*c^3*d^2 - 41*a^3*c^2*d^3 + 3*a*b^2*c^2*d^3 + 5*a^2*b*c*d
^4 + 15*a^3*d^5)*(Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d +
c*cos[e + f*x]]*EllipticE[ArcSin[(Sqrt[(-a + b)/(a + b)]*Sin[(e + f*x)/2])
/Sqrt[(b + a*cos[e + f*x])/(a + b)]], (2*(b*c - a*d))/((-a + b)*(c + d)))/
(a*c*Sqrt[((a + b)*Cos[(e + f*x)/2]^2)/(b + a*cos[e + f*x]])*Sqrt[b + a*cos
[e + f*x]]*Sqrt[(b + a*cos[e + f*x])/(a + b)]*Sqrt[((a + b)*(d + c*cos[e +
f*x]))/((c + d)*(b + a*cos[e + f*x]))]) - (2*(b*c - a*d)*(((b*c + (a + b)*d
)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*cos[e + f
*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b
)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c -
a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*cos
[e + f*x]]*Sqrt[d + c*cos[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(e
+ f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*cos[e + f*x])*Csc[(e + f*x)/2]^2
)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[((-a
- b)*(d + c*cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*
c - a*d))/((a + b)*(c - d))*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b + a*cos[
e + f*x]]*Sqrt[d + c*cos[e + f*x]])))/(a*c) + (Sqrt[d + c*cos[e + f*x]]*Sin
[e + f*x])/(c*Sqrt[b + a*cos[e + f*x]])))/(15*c^2*(c - d)^3*(c + d)^3*f*(b
+ a*cos[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^(7/2))

```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{5}{2}}}{(d \sec(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

maple [B] time = 2.98, size = 32283, normalized size = 28.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x)
```

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{5}{2}}}{(d \sec(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(7/2),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(7/2),x)

[Out] Timed out

$$3.216 \quad \int \frac{(a+b \sec(e+fx))^{5/2}}{(c+d \sec(e+fx))^{9/2}} dx$$

Optimal. Leaf size=1428

$$\frac{2\sqrt{a+b} \sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} (d+c \cos(e+fx))^{3/2} \csc(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{b+a}}{\sqrt{a+b} \sqrt{d+c}}\right)\right)}{c^5 \sqrt{c+d} f \sqrt{b+a \cos(e+fx)} \sqrt{c+d \sec(e+fx)}}$$

[Out] $2/7*d^2*(b+a*\cos(f*x+e))^2*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c/(c^2-d^2)/f/(d+c*\cos(f*x+e))^3/(c+d*\sec(f*x+e))^{(1/2)}-2/35*d*(-19*a*c^2*d+7*a*d^3+14*b*c^3-2*b*c*d^2)*(b+a*\cos(f*x+e))*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c^2/(c^2-d^2)^2/f/(d+c*\cos(f*x+e))^2/(c+d*\sec(f*x+e))^{(1/2)}-2/105*(2*a*b*c*d*(91*c^4-2*c^2*d^2+7*d^4)-a^2*d^2*(162*c^4-101*c^2*d^2+35*d^4)-b^2*(35*c^6+67*c^4*d^2-6*c^2*d^4))*\sin(f*x+e)*(a+b*\sec(f*x+e))^{(1/2)}/c^3/(c^2-d^2)^3/f/(d+c*\cos(f*x+e))/(c+d*\sec(f*x+e))^{(1/2)}+2/105*(a-b)*(2*b^3*c^3*d*(133*c^4+62*c^2*d^2-3*d^4)+2*a^2*b*c*d*(406*c^6+73*c^4*d^2+132*c^2*d^4-35*d^6)-a*b^2*c^2*(245*c^6+852*c^4*d^2+41*c^2*d^4+14*d^6)-a^3*(582*c^6*d^2-485*c^4*d^4+392*c^2*d^6-105*d^8))*(d+c*\cos(f*x+e))^{(3/2)}*\csc(f*x+e)*\text{EllipticE}((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^4/(c-d)^4/(c+d)^{(7/2)}/(-a*d+b*c)^2/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}+2/105*(b^3*c^4*(35*c^4+231*c^3*d+67*c^2*d^2+57*c*d^3-6*d^4)-a*b^2*c^3*(245*c^5+413*c^4*d+439*c^3*d^2+53*c^2*d^3-12*c*d^4+14*d^5)+a^2*b*c^2*(315*c^6+497*c^5*d+219*c^4*d^2-73*c^3*d^3+208*c^2*d^4+56*c*d^5-70*d^6)-a^3*d*(525*c^7+57*c^6*d-699*c^5*d^2+214*c^4*d^3+672*c^3*d^4-280*c^2*d^5-210*c*d^6+105*d^7))*(d+c*\cos(f*x+e))^{(3/2)}*\csc(f*x+e)*\text{EllipticF}((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^5/(c-d)^4/(c+d)^{(7/2)}/(-a*d+b*c)/f/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}-2*a^2*(d+c*\cos(f*x+e))^{(3/2)}*\csc(f*x+e)*\text{EllipticPi}((c+d)^{(1/2)}*(b+a*\cos(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(d+c*\cos(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(a+b)^{(1/2)}*(-(-a*d+b*c)*(1-\cos(f*x+e))/(a+b)/(d+c*\cos(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\cos(f*x+e))/(a-b)/(d+c*\cos(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/c^5/f/(c+d)^{(1/2)}/(b+a*\cos(f*x+e))^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}$

Rubi [A] time = 5.44, antiderivative size = 1428, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3942, 3048, 3047, 3053, 2811, 2998, 2818, 2996}

$$\frac{2\sqrt{a+b} \sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c \cos(e+fx))}} (d+c \cos(e+fx))^{3/2} \csc(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{b+a}}{\sqrt{a+b} \sqrt{d+c}}\right)\right)}{c^5 \sqrt{c+d} f \sqrt{b+a \cos(e+fx)} \sqrt{c+d \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2), x]

[Out] $(2*(a-b)*\text{Sqrt}[a+b]*(2*b^3*c^3*d*(133*c^4+62*c^2*d^2-3*d^4)+2*a^2*b*c*d*(406*c^6+73*c^4*d^2+132*c^2*d^4-35*d^6)-a*b^2*c^2*(245*c^6+852*c^4*d^2+41*c^2*d^4+14*d^6)-a^3*(582*c^6*d^2-485*c^4*d^4+392*c^2*d^6-105*d^8))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/((a+b)*(d+c*\text{Cos}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/((a-b)*(d+c*\text{Cos}[e+f*x])))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]]/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])]$

, ((a + b)*(c - d))/((a - b)*(c + d))*Sqrt[a + b*Sec[e + f*x]]/(105*c^4*(c - d)^4*(c + d)^(7/2)*(b*c - a*d)^2*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 + 57*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*d^3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 73*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d - 699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^7))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))*Sqrt[a + b*Sec[e + f*x]]/(105*c^5*(c - d)^4*(c + d)^(7/2)*(b*c - a*d)*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) - (2*a^2*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])^(3/2)*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))*Sqrt[a + b*Sec[e + f*x]]/(c^5*Sqrt[c + d]*f*Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]) + (2*d^2*(b + a*Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3*Sqrt[c + d*Sec[e + f*x]]) - (2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(35*c^2*(c^2 - d^2)^2*f*(d + c*Cos[e + f*x])^2*Sqrt[c + d*Sec[e + f*x]]) - (2*(2*a*b*c*d*(91*c^4 - 2*c^2*d^2 + 7*d^4) - a^2*d^2*(162*c^4 - 101*c^2*d^2 + 35*d^4) - b^2*(35*c^6 + 67*c^4*d^2 - 6*c^2*d^4))*Sqrt[a + b*Sec[e + f*x]]*Sin[e + f*x])/(105*c^3*(c^2 - d^2)^3*f*(d + c*Cos[e + f*x])*Sqrt[c + d*Sec[e + f*x]])

Rule 2811

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(d*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]

Rule 2818

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]])/Sqrt[c + d*Sin[e + f*x]]], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]

Rule 2996

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]

Rule 2998

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3047

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))]*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

Rule 3048

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x] - b*(A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))]*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

Rule 3053

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3942

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_), x_Symbol] := Dist[(Sqrt[d + c*Sin[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int[(b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^{9/2}} dx &= \frac{\left(\sqrt{d + c \cos(e + fx)} \sqrt{a + b \sec(e + fx)}\right) \int \frac{\cos^2(e+fx)(b+a \cos(e+fx))^{5/2}}{(d+c \cos(e+fx))^{9/2}} dx}{\sqrt{b + a \cos(e + fx)} \sqrt{c + d \sec(e + fx)}} \\
&= \frac{2d^2(b + a \cos(e + fx))^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{7c(c^2 - d^2) f(d + c \cos(e + fx))^3 \sqrt{c + d \sec(e + fx)}} + \frac{(2\sqrt{d + c \cos(e + fx)})}{35c^2 (d + c \cos(e + fx))^{3/2}} \\
&= \frac{2d^2(b + a \cos(e + fx))^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{7c(c^2 - d^2) f(d + c \cos(e + fx))^3 \sqrt{c + d \sec(e + fx)}} - \frac{2d(14bc^3 - 19ac^2d - 13c^4d)}{35c^2 (d + c \cos(e + fx))^{3/2}} \\
&= \frac{2d^2(b + a \cos(e + fx))^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{7c(c^2 - d^2) f(d + c \cos(e + fx))^3 \sqrt{c + d \sec(e + fx)}} - \frac{2d(14bc^3 - 19ac^2d - 13c^4d)}{35c^2 (d + c \cos(e + fx))^{3/2}} \\
&= \frac{2d^2(b + a \cos(e + fx))^2 \sqrt{a + b \sec(e + fx)} \sin(e + fx)}{7c(c^2 - d^2) f(d + c \cos(e + fx))^3 \sqrt{c + d \sec(e + fx)}} - \frac{2d(14bc^3 - 19ac^2d - 13c^4d)}{35c^2 (d + c \cos(e + fx))^{3/2}} \\
&= - \frac{2a^2 \sqrt{a + b} \sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c \cos(e+fx))}} \sqrt{\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c \cos(e+fx))}} (d + c \cos(e + fx))^{3/2} \operatorname{cs}(\operatorname{arcsin}(\frac{d + c \cos(e + fx)}{\sqrt{b + a \cos(e + fx)}}))}{c^5 \sqrt{c + d} f \sqrt{b + a \cos(e + fx)}} \\
&= \frac{2(a - b) \sqrt{a + b} (2b^3 c^3 d (133c^4 + 62c^2 d^2 - 3d^4) + 2a^2 bcd (406c^6 + 73c^4 d^2 + 13c^4 d^2))}{35c^2 (d + c \cos(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [B] time = 8.29, size = 2979, normalized size = 2.09

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]

[Out] ((d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*d^2*Sin[e + f*x] - 2*a*b*c*d^3*Sin[e + f*x] + a^2*d^4*Sin[e + f*x]))/(7*c^3*(c^2 - d^2)*(d + c*Cos[e + f*x])^4) + (2*(-14*b^2*c^4*d*Sin[e + f*x] + 43*a*b*c^3*d^2*Sin[e + f*x] - 29*a^2*c^2*d^3*Sin[e + f*x] + 2*b^2*c^2*d^3*Sin[e + f*x] - 19*a*b*c*d^4*Sin[e + f*x] + 17*a^2*d^5*Sin[e + f*x]))/(35*c^3*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^3) + (2*(35*b^2*c^6*Sin[e + f*x] - 224*a*b*c^5*d*Sin[e + f*x] + 234*a^2*c^4*d^2*Sin[e + f*x] + 67*b^2*c^4*d^2*Sin[e + f*x] + 52*a*b*c^3*d^3*Sin[e + f*x] - 209*a^2*c^2*d^4*Sin[e + f*x] - 6*b^2*c^2*d^4*Sin[e + f*x] - 20*a*b*c*d^5*Sin[e + f*x] + 71*a^2*d^6*Sin[e + f*x]))/(105*c^3*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^2) + (2*(245*a*b^2*c^8*Sin[e + f*x] - 812*a^2*b*c^7*d*Sin[e + f*x] - 266*b^3*c^7*d*Sin[e + f*x] + 582*a^3*c^6*d^2*Sin[e + f*x] + 852*a*b^2*c^6*d^2*Sin[e + f*x] - 146*a^2*b*c^5*d^3*Sin[e + f*x] - 124*b^3*c^5*d^3*Sin[e + f*x] - 485*a^3*c^4*d^4*Sin[e + f*x] + 41*a*b^2*c^4*d^4*Sin[e + f*x] - 264*a^2*b*c^3*d^5*Sin[e + f*x] + 6*b^3*c^3*d^5*Sin[e + f*x] + 392*a^3*c^2*d^6*Sin[e + f*x] + 14*a*b^2*c^2*d^6*Sin[e + f*x] + 70*a^2*b*c*d^7*Sin[e + f*x] - 105*a^3*d^8*Sin[e + f*x]))/(105*c^3*(b*c - a*d)*(c^2 - d^2)^4*(d + c*Cos[e + f*x])))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(9/2)) + ((d + c*Cos[e + f*x])^(9/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(-70*a^2*b^2*c^8 - 35*b^4*c^8 - 77*a^3*b*c^7*d + 427*a*b^3*c^7*d + 162*a^4*c^6*d^2 - 522*a^2*b^2*c^6*d^2

$$\begin{aligned}
& 2 - 298*b^4*c^6*d^2 + 348*a^3*b*c^5*d^3 + 666*a*b^3*c^5*d^3 - 263*a^4*c^4*d^4 \\
& - 586*a^2*b^2*c^4*d^4 - 51*b^4*c^4*d^4 + 127*a^3*b*c^3*d^5 + 59*a*b^3*c^3*d^5 + 136*a^4*c^2*d^6 \\
& + 26*a^2*b^2*c^2*d^6 - 14*a^3*b*c*d^7 - 35*a^4*d^8) * \text{Sqrt}[\frac{(c+d)\text{Cot}[(e+fx)/2]^2}{(c-d)}] * \text{Sqrt}[\frac{(c+d)(b+a\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] * \text{Sqrt}[\frac{((-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2)}{(b*c-a*d)}] * \text{Csc}[e+fx] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\frac{(-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] / \text{Sqrt}[2]]], (2*(b*c-a*d))/((a+b)*(c-d))] * \text{Sin}[(e+fx)/2]^4 / ((a+b)*(c+d)*\text{Sqrt}[b+a\text{Cos}[e+fx]]) * \text{Sqrt}[d+c\text{Cos}[e+fx]] + 4*(b*c-a*d)*(-105*a^3*b*c^8 + 245*a*b^3*c^8 + 105*a^4*c^7*d - 567*a^2*b^2*c^7*d - 266*b^4*c^7*d + 190*a^3*b*c^6*d^2 + 586*a*b^3*c^6*d^2 + 162*a^4*c^5*d^3 + 706*a^2*b^2*c^5*d^3 - 124*b^4*c^5*d^3 - 1261*a^3*b*c^4*d^4 - 83*a*b^3*c^4*d^4 + 145*a^4*c^3*d^5 - 223*a^2*b^2*c^3*d^5 + 6*b^4*c^3*d^5 + 548*a^3*b*c^2*d^6 + 20*a*b^3*c^2*d^6 - 28*a^4*c*d^7 + 84*a^2*b^2*c*d^7 - 140*a^3*b*d^8) * ((\text{Sqrt}[\frac{(c+d)\text{Cot}[(e+fx)/2]^2}{(c-d)}] * \text{Sqrt}[\frac{(c+d)(b+a\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] * \text{Sqrt}[\frac{((-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2)}{(b*c-a*d)}] * \text{Csc}[e+fx] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\frac{(-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] / \text{Sqrt}[2]]], (2*(b*c-a*d))/((a+b)*(c-d))] * \text{Sin}[(e+fx)/2]^4 / ((a+b)*(c+d)*\text{Sqrt}[b+a\text{Cos}[e+fx]]) * \text{Sqrt}[d+c\text{Cos}[e+fx]]) - (\text{Sqrt}[\frac{(c+d)\text{Cot}[(e+fx)/2]^2}{(c-d)}] * \text{Sqrt}[\frac{(c+d)(b+a\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] * \text{Sqrt}[\frac{((-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2)}{(b*c-a*d)}] * \text{Csc}[e+fx] * \text{EllipticPi}[(b*c-a*d)/((a+b)*c), \text{ArcSin}[\text{Sqrt}[\frac{(-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] / \text{Sqrt}[2]]], (2*(b*c-a*d))/((a+b)*(c-d))] * \text{Sin}[(e+fx)/2]^4 / ((a+b)*c*\text{Sqrt}[b+a\text{Cos}[e+fx]]) * \text{Sqrt}[d+c\text{Cos}[e+fx]])) + 2*(245*a^2*b^2*c^8 - 812*a^3*b*c^7*d - 266*a*b^3*c^7*d + 582*a^4*c^6*d^2 + 852*a^2*b^2*c^6*d^2 - 146*a^3*b*c^5*d^3 - 124*a*b^3*c^5*d^3 - 485*a^4*c^4*d^4 + 41*a^2*b^2*c^4*d^4 - 264*a^3*b*c^3*d^5 + 6*a*b^3*c^3*d^5 + 392*a^4*c^2*d^6 + 14*a^2*b^2*c^2*d^6 + 70*a^3*b*c*d^7 - 105*a^4*d^8) * ((\text{Sqrt}[(-a+b)/(a+b)]*(a+b)*\text{Cos}[(e+fx)/2]*\text{Sqrt}[d+c\text{Cos}[e+fx]] * \text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[(-a+b)/(a+b)]*\text{Sin}[(e+fx)/2])/ \text{Sqrt}[(b+a\text{Cos}[e+fx])/(a+b)]]], (2*(b*c-a*d))/((-a+b)*(c+d)))/((a*c*\text{Sqrt}[\frac{(a+b)\text{Cos}[(e+fx)/2]^2}{(b+a\text{Cos}[e+fx])}] * \text{Sqrt}[b+a\text{Cos}[e+fx]] * \text{Sqrt}[(b+a\text{Cos}[e+fx])/(a+b)] * \text{Sqrt}[\frac{(a+b)(d+c\text{Cos}[e+fx])}{(c+d)(b+a\text{Cos}[e+fx])}])) - (2*(b*c-a*d)*((b*c+(a+b)*d)*\text{Sqrt}[\frac{(c+d)\text{Cot}[(e+fx)/2]^2}{(c-d)}] * \text{Sqrt}[\frac{(c+d)(b+a\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] * \text{Sqrt}[\frac{((-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2)}{(b*c-a*d)}] * \text{Csc}[e+fx] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\frac{(-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] / \text{Sqrt}[2]]], (2*(b*c-a*d))/((a+b)*(c-d))] * \text{Sin}[(e+fx)/2]^4 / ((a+b)*(c+d)*\text{Sqrt}[b+a\text{Cos}[e+fx]]) * \text{Sqrt}[d+c\text{Cos}[e+fx]]) - ((b*c+a*d)*\text{Sqrt}[\frac{(c+d)\text{Cot}[(e+fx)/2]^2}{(c-d)}] * \text{Sqrt}[\frac{(c+d)(b+a\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] * \text{Sqrt}[\frac{((-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2)}{(b*c-a*d)}] * \text{Csc}[e+fx] * \text{EllipticPi}[(b*c-a*d)/((a+b)*c), \text{ArcSin}[\text{Sqrt}[\frac{(-a-b)(d+c\text{Cos}[e+fx])\text{Csc}[(e+fx)/2]^2}{(b*c-a*d)}] / \text{Sqrt}[2]]], (2*(b*c-a*d))/((a+b)*(c-d))] * \text{Sin}[(e+fx)/2]^4 / ((a+b)*c*\text{Sqrt}[b+a\text{Cos}[e+fx]]) * \text{Sqrt}[d+c\text{Cos}[e+fx]]) / (a*c) + (\text{Sqrt}[d+c\text{Cos}[e+fx]] * \text{Sin}[e+fx]) / (c*\text{Sqrt}[b+a\text{Cos}[e+fx]])) / (105*c^3*(c-d)^4*(c+d)^4*(-(b*c)+a*d)*f*(b+a\text{Cos}[e+fx])^(5/2)*(c+d*\text{Sec}[e+fx])^(9/2))
\end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{5}{2}}}{(d \sec(fx + e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)

maple [B] time = 4.94, size = 75468, normalized size = 52.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{5}{2}}}{(d \sec(fx + e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(9/2),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(9/2),x)

[Out] Timed out

$$3.217 \quad \int \frac{(c+d \sec(e+fx))^{3/2}}{\sqrt{a+b \sec(e+fx)}} dx$$

Optimal. Leaf size=652

$$\frac{2(bc-ad) \cot(e+fx) \sqrt{a+b \sec(e+fx)} \sqrt{c+d \sec(e+fx)} \sqrt{\frac{(bc-ad)(\sec(e+fx)-1)}{(c+d)(a+b \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b \sec(e+fx))}} F\left(\sin^{-1}\left(\sqrt{\frac{(a+b)(c+d \sec(e+fx))}{(c+d)(a+b \sec(e+fx))}}\right)\right)}{abf \sqrt{\frac{(a+b)(c+d \sec(e+fx))}{(c+d)(a+b \sec(e+fx))}}}$$

[Out] $-2*c*(c+d)*\cot(f*x+e)*\text{EllipticPi}(((a+b)*(c+d*\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}, a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)}*(a+b*\sec(f*x+e))^{(3/2)*((-a*d+b*c)*(1+\sec(f*x+e))/(c-d)/(a+b*\sec(f*x+e)))^{(1/2)}*((a+b)*(-a*d+b*c)*(-1+\sec(f*x+e))*(c+d*\sec(f*x+e))/(c+d)^2/(a+b*\sec(f*x+e))^2)^{(1/2)}/a/(a+b)/f/(c+d*\sec(f*x+e))^{(1/2)}+2*d*(c+d)*\cot(f*x+e)*\text{EllipticPi}(((a+b)*(c+d*\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}, b*(c+d)/(a+b)/d, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)}*(a+b*\sec(f*x+e))^{(3/2)*((-a*d+b*c)*(1+\sec(f*x+e))/(c-d)/(a+b*\sec(f*x+e)))^{(1/2)}*(-(a+b)*(a*d-b*c)*(-1+\sec(f*x+e))*(c+d*\sec(f*x+e))/(c+d)^2/(a+b*\sec(f*x+e))^2)^{(1/2)}/b/(a+b)/f/(c+d*\sec(f*x+e))^{(1/2)}+2*(-a*d+b*c)*\cot(f*x+e)*\text{EllipticF}(((a+b)*(c+d*\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)}*((-a*d+b*c)*(-1+\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sec(f*x+e))/(c-d)/(a+b*\sec(f*x+e)))^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}*(c+d*\sec(f*x+e))^{(1/2)}/a/b/f/((a+b)*(c+d*\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)})$

Rubi [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+d \sec(e+fx))^{3/2}}{\sqrt{a+b \sec(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]], x]

[Out] Defer[Int][(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]], x]

Rubi steps

$$\int \frac{(c+d \sec(e+fx))^{3/2}}{\sqrt{a+b \sec(e+fx)}} dx = \int \frac{(c+d \sec(e+fx))^{3/2}}{\sqrt{a+b \sec(e+fx)}} dx$$

Mathematica [C] time = 32.78, size = 49385, normalized size = 75.74

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]], x]

[Out] Result too large to show

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec(fx + e) + c)^{\frac{3}{2}}}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)

maple [A] time = 2.10, size = 491, normalized size = 0.75

$$2 \left(2 \operatorname{EllipticPi} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, \frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) d^2 + 2 \operatorname{EllipticPi} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, -\frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) c^2 - \operatorname{EllipticF} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, \frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x)

[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), (a+b)/(a-b)), ((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*d^2+2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), -(a+b)/(a-b), ((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c^2-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c^2+2*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c*d-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d^2)*cos(f*x+e)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^2*((d+c*cos(f*x+e))/(1+cos(f*x+e)))/(c+d))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))/((a-b)/(a+b))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec(fx + e) + c)^{\frac{3}{2}}}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c + \frac{d}{\cos(e+fx)} \right)^{3/2}}{\sqrt{a + \frac{b}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2), x)`

[Out] `int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^{\frac{3}{2}}}{\sqrt{a + b \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2), x)`

[Out] `Integral((c + d*sec(e + f*x))**(3/2)/sqrt(a + b*sec(e + f*x)), x)`

$$3.218 \quad \int \frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{a+b \sec(e+fx)}} dx$$

Optimal. Leaf size=198

$$\frac{2\sqrt{a+b} \cot(e+fx)(c+d \sec(e+fx)) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d \sec(e+fx))}} \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}\right)\right)}{af\sqrt{c+d}}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}, (a+b)*c/a/(c+d), ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(c+d*\sec(f*x+e))* (a+b)^{(1/2)}*((-a*d+b*c)*(1-\sec(f*x+e)))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sec(f*x+e)))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}/a/f/(c+d)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {3936}

$$\frac{2\sqrt{a+b} \cot(e+fx)(c+d \sec(e+fx)) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d \sec(e+fx))}} \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}\right)\right)}{af\sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]], x]

[Out] $(-2*\text{Sqrt}[a + b]*\text{Cot}[e + f*x]*\text{EllipticPi}(((a + b)*c)/(a*(c + d)), \text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sec}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sec}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))*\text{Sqrt}[\frac{(b*c - a*d)*(1 - \text{Sec}[e + f*x])}{(a + b)*(c + d*\text{Sec}[e + f*x])}])*\text{Sqrt}[\frac{-((b*c - a*d)*(1 + \text{Sec}[e + f*x])}{(a - b)*(c + d*\text{Sec}[e + f*x])}])]*(c + d*\text{Sec}[e + f*x])]/(a*\text{Sqrt}[c + d]*f)$

Rule 3936

Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x]))])*Sqrt[-((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x]))])*EllipticPi[(a*(c + d))/(c*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(c*f*Rt[(a + b)/(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{\sqrt{c+d \sec(e+fx)}}{\sqrt{a+b \sec(e+fx)}} dx = -\frac{2\sqrt{a+b} \cot(e+fx) \Pi\left(\frac{(a+b)c}{a(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}\right)\right) \frac{(a+b)(c-d)}{(a-b)(c+d)} \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d \sec(e+fx))}}}{af\sqrt{c+d}}$$

Mathematica [A] time = 5.54, size = 325, normalized size = 1.64

$$4 \sin^2\left(\frac{1}{2}(e+fx)\right) \csc(e+fx) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(e+fx)\right)}{a-b}} \sqrt{c+d \sec(e+fx)} \sqrt{\frac{(c+d) \csc^2\left(\frac{1}{2}(e+fx)\right) (a \cos(e+fx)+b)}{bc-ad}} \left(a(c+d)F\right)$$

$af(c+d)\sqrt{a+b \sec(e+fx)}$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]

[Out] (4*Sqrt[((a + b)*Cot[(e + f*x)/2]^2)/(a - b)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*(a*(c + d)*EllipticF[ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] - (a + b)*c*EllipticPi[(-(b*c) + a*d)/(a*(c + d)), ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d)))]*Sqrt[c + d*Sec[e + f*x]]*Sin[(e + f*x)/2]^2/(a*(c + d)*f*Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Sqrt[a + b*Sec[e + f*x]])

fricas [F] time = 4.52, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{d \sec(fx + e) + c}}{\sqrt{b \sec(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec(fx + e) + c}}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

maple [A] time = 2.13, size = 352, normalized size = 1.78

$$2 \left(2 \text{EllipticPi} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, -\frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) c - \text{EllipticF} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}} \right) c + \text{EllipticF} \left(\frac{(-1 + \cos(fx + e)) \sqrt{\frac{a-b}{a+b}}}{\sin(fx + e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}} \right) \right) f(-1 + \cos(fx + e))(d +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x)

[Out] 2/f*(2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), -(a+b)/(a-b), ((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c - EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c + EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d*cos(f*x+e)*((d+c*cos(f*x+e))/(1+cos(f*x+e)))/(c+d)^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e)))/(a+b)^(1/2)*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))/((a-b)/(a+b))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec(fx + e) + c}}{\sqrt{b \sec(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + \frac{d}{\cos(e+fx)}}}{\sqrt{a + \frac{b}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2),x)

[Out] int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d \sec(e + fx)}}{\sqrt{a + b \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)

$$3.219 \quad \int \frac{1}{\sqrt{a+b \sec(e+fx)} \sqrt{c+d \sec(e+fx)}} dx$$

Optimal. Leaf size=398

$$\frac{2b\sqrt{a+b} \cot(e+fx)(c+d \sec(e+fx)) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d \sec(e+fx))}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}\right)\right)}{af\sqrt{c+d}(bc-ad)}$$

[Out] $-2*\cot(f*x+e)*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sec(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sec(f*x+e))^{(1/2)}, a*(c+d)/(a+b)/c, ((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)}*(a+b*\sec(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sec(f*x+e))/(c+d)/(a+b*\sec(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sec(f*x+e))/(c-d)/(a+b*\sec(f*x+e)))^{(1/2)}/a/c/f/(a+b)^{(1/2)}-2*b*\cot(f*x+e)*\text{EllipticF}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)}, ((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)}*(c+d*\sec(f*x+e))*(a+b)^{(1/2)}*((-a*d+b*c)*(1-\sec(f*x+e))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sec(f*x+e))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}/a/(-a*d+b*c)/f/(c+d)^{(1/2)}$

Rubi [A] time = 0.44, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3938, 3936, 3984}

$$\frac{2b\sqrt{a+b} \cot(e+fx)(c+d \sec(e+fx)) \sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d \sec(e+fx))}} \sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d \sec(e+fx))}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}\right)\right)}{af\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] $(-2*\text{Sqrt}[c+d]*\text{Cot}[e+f*x]*\text{EllipticPi}[(a*(c+d))/((a+b)*c), \text{ArcSin}[(\text{Sqrt}[a+b]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])/(\text{Sqrt}[c+d]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]])], ((a-b)*(c+d))/((a+b)*(c-d))*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Sec}[e+f*x])))/((c+d)*(a+b*\text{Sec}[e+f*x])))]*\text{Sqrt}[(b*c-a*d)*(1+\text{Sec}[e+f*x])]/((c-d)*(a+b*\text{Sec}[e+f*x]))]*(a+b*\text{Sec}[e+f*x])/((a*\text{Sqrt}[a+b]*c*f)-(2*b*\text{Sqrt}[a+b]*\text{Cot}[e+f*x]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])], ((a+b)*(c-d))/((a-b)*(c+d)))*\text{Sqrt}[(b*c-a*d)*(1-\text{Sec}[e+f*x])]/((a+b)*(c+d*\text{Sec}[e+f*x])))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Sec}[e+f*x])))/((a-b)*(c+d*\text{Sec}[e+f*x])))]*(c+d*\text{Sec}[e+f*x])/((a*\text{Sqrt}[c+d]*(b*c-a*d)*f)$

Rule 3936

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)], x_Symbol] :> Simp[(2*(a + b*Csc[e + f*x])*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f*x])))]*EllipticPi[(a*(c + d))/(c*(a + b)), ArcSin[Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Csc[e + f*x]]]/Sqrt[a + b*Csc[e + f*x]], ((a - b)*(c + d))/((a + b)*(c - d))]/(c*f*Rt[(a + b)/(c + d), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3938

Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]], x], x] - Dist[b/a, Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]

&& NeQ[b*c - a*d, 0]

Rule 3984

Int[csc[(e_.) + (f_.)*(x_.)]/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)]), x_Symbol] := Simp[(-2*(c + d*Csc[e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x]))])*Sqrt[-((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))])*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \sec(e + fx)} \sqrt{c + d \sec(e + fx)}} dx = \frac{\int \frac{\sqrt{a+b \sec(e+fx)}}{\sqrt{c+d \sec(e+fx)}} dx}{a} - \frac{b \int \frac{\sec(e+fx)}{\sqrt{a+b \sec(e+fx)} \sqrt{c+d \sec(e+fx)}} dx}{a}$$

$$= -\frac{2\sqrt{c+d} \cot(e+fx) \Pi\left(\frac{a(c+d)}{(a+b)c}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sec(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sec(e+fx)}}\right)\right)}{a}$$

Mathematica [C] time = 2.37, size = 249, normalized size = 0.63

$$\frac{4i \cos^2\left(\frac{1}{2}(e + fx)\right) \sec(e + fx) \sqrt{\frac{a \cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}} \sqrt{\frac{c \cos(e+fx)+d}{(c+d)(\cos(e+fx)+1)}} \left(F\left(i \sinh^{-1}\left(\sqrt{\frac{b-a}{a+b}} \tan\left(\frac{1}{2}(e + fx)\right)\right)\right)\right)_{(a+b)}^{(a-b)}}{f \sqrt{\frac{b-a}{a+b}} \sqrt{a + b \sec(e + fx)} \sqrt{c + d \sec(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] ((4*I)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))])*(EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))] - 2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))])*Sec[e + f*x]/(Sqrt[(-a + b)/(a + b)]*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])

fricas [F] time = 5.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c}}{bd \sec(fx + e)^2 + ac + (bc + ad) \sec(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

maple [A] time = 2.15, size = 292, normalized size = 0.73

$$\frac{2 \left(\text{EllipticF} \left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, \sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}} \right) - 2 \text{EllipticPi} \left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)}, -\frac{a+b}{a-b}, \sqrt{\frac{c-d}{c+d}} \right) \right) \cos(fx+e) (\sin^2(fx+e))}{f(-1+\cos(fx+e))(d+c\cos(fx+e))(b+a\cos(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x)

[Out] -2/f*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e)))/(a+b))^(1/2)*((d+c*cos(f*x+e))/(1+cos(f*x+e)))/(c+d))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))/(a-b)/(a+b))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(fx + e) + a} \sqrt{d \sec(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{\cos(e+fx)}} \sqrt{c + \frac{d}{\cos(e+fx)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)

[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec(e + fx)} \sqrt{c + d \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x)

[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)

$$3.220 \quad \int \frac{1}{\sqrt{a+b \sec(e+fx)} (c+d \sec(e+fx))^{3/2}} dx$$

Optimal. Leaf size=622

$$\frac{2d\sqrt{a+b}(2c-d)\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b}}{\sqrt{a+b}\sqrt{c+d}}\right)\right)}{c^2f(c-d)\sqrt{c+d}(bc-ad)}$$

[Out] $-2*(a-b)*d^2*\cot(f*x+e)*\text{EllipticE}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(c+d*\sec(f*x+e))*(a+b)^{(1/2)}*((-a*d+b*c)*(1-\sec(f*x+e))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sec(f*x+e))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}/c/(c-d)/(-a*d+b*c)^{2/f}/(c+d)^{(1/2)}-2*(2*c-d)*d*\cot(f*x+e)*\text{EllipticF}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(c+d*\sec(f*x+e))*(a+b)^{(1/2)}*((-a*d+b*c)*(1-\sec(f*x+e))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sec(f*x+e))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}/c^2/(c-d)/(-a*d+b*c)/f/(c+d)^{(1/2)}-2*\cot(f*x+e)*\text{EllipticPi}((c+d)^{(1/2)}*(a+b*\sec(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sec(f*x+e))^{(1/2)},(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*(c+d*\sec(f*x+e))*(a+b)^{(1/2)}*((-a*d+b*c)*(1-\sec(f*x+e))/(a+b)/(c+d*\sec(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sec(f*x+e))/(a-b)/(c+d*\sec(f*x+e)))^{(1/2)}/a/c^2/f/(c+d)^{(1/2)}$

Rubi [A] time = 1.32, antiderivative size = 763, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3942, 3054, 2811, 2998, 2818, 2996}

$$\frac{2d\sqrt{a+b}(2c-d)\csc(e+fx)\sqrt{a+b\sec(e+fx)}(c\cos(e+fx)+d)^{3/2}\sqrt{\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}}{c^2f(c-d)\sqrt{c+d}(bc-ad)\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]

[Out] $(-2*(a-b)*\text{Sqrt}[a+b]*d^2*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/(a+b)*(d+c*\text{Cos}[e+f*x]))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/(a-b)*(d+c*\text{Cos}[e+f*x]))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(c*(c-d)*\text{Sqrt}[c+d]*(b*c-a*d)^2*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]-(2*\text{Sqrt}[a+b]*(2*c-d)*d*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/(a+b)*(d+c*\text{Cos}[e+f*x]))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/(a-b)*(d+c*\text{Cos}[e+f*x]))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(c^2*(c-d)*\text{Sqrt}[c+d]*(b*c-a*d)*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]]-(2*\text{Sqrt}[a+b]*\text{Sqrt}[-(((b*c-a*d)*(1-\text{Cos}[e+f*x]))/(a+b)*(d+c*\text{Cos}[e+f*x]))]*\text{Sqrt}[-(((b*c-a*d)*(1+\text{Cos}[e+f*x]))/(a-b)*(d+c*\text{Cos}[e+f*x]))]*(d+c*\text{Cos}[e+f*x])^{(3/2)}*\text{Csc}[e+f*x]*\text{EllipticPi}[(a+b)*c/(a*(c+d),\text{ArcSin}[(\text{Sqrt}[c+d]*\text{Sqrt}[b+a*\text{Cos}[e+f*x]])/(\text{Sqrt}[a+b]*\text{Sqrt}[d+c*\text{Cos}[e+f*x]])],((a+b)*(c-d))/((a-b)*(c+d))]*\text{Sqrt}[a+b*\text{Sec}[e+f*x]]/(a*c^2*\text{Sqrt}[c+d]*f*\text{Sqrt}[b+a*\text{Cos}[e+f*x]]*\text{Sqrt}[c+d*\text{Sec}[e+f*x]])$

Rule 2811

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*

$$\frac{1 - \sin[e + f*x]}{(c + d)*(a + b*\sin[e + f*x])} * \text{EllipticPi}[\frac{b*(c + d)}{d*(a + b)}, \text{ArcSin}[\frac{\text{Rt}[(a + b)/(c + d), 2]*\text{Sqrt}[c + d*\sin[e + f*x]]}{\text{Sqrt}[a + b*\sin[e + f*x]]}], \frac{(a - b)*(c + d)}{(a + b)*(c - d)}] / (d*f*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]), x] /;$$
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]

Rule 2818

$$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[(2*(c + d*\sin[e + f*x])* \text{Sqrt}[(b*c - a*d)*(1 - \sin[e + f*x])]/((a + b)*(c + d*\sin[e + f*x])))* \text{Sqrt}[-((b*c - a*d)*(1 + \sin[e + f*x])]/((a - b)*(c + d*\sin[e + f*x])))]* \text{EllipticF}[\text{ArcSin}[\text{Rt}[(c + d)/(a + b), 2]*(\text{Sqrt}[a + b*\sin[e + f*x]]/\text{Sqrt}[c + d*\sin[e + f*x]])], \frac{(a + b)*(c - d)}{(a - b)*(c + d)}] / (f*(b*c - a*d)*\text{Rt}[(c + d)/(a + b), 2]*\text{Cos}[e + f*x]), x] /;$$
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]

Rule 2996

$$\text{Int}[(A_) + (B_)*\sin[(e_) + (f_)*(x_)]]/(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{3/2}*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[(-2*A*(c - d)*(a + b*\sin[e + f*x])* \text{Sqrt}[(b*c - a*d)*(1 + \sin[e + f*x])]/((c - d)*(a + b*\sin[e + f*x])))* \text{Sqrt}[-((b*c - a*d)*(1 - \sin[e + f*x])]/((c + d)*(a + b*\sin[e + f*x])))]* \text{EllipticE}[\text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]*\text{Sqrt}[c + d*\sin[e + f*x]]/\text{Sqrt}[a + b*\sin[e + f*x]]], \frac{(a - b)*(c + d)}{(a + b)*(c - d)}] / (f*(b*c - a*d)^2*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]), x] /;$$
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]

Rule 2998

$$\text{Int}[(A_) + (B_)*\sin[(e_) + (f_)*(x_)]]/(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{3/2}*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\sin[e + f*x]]*\text{Sqrt}[c + d*\sin[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \sin[e + f*x])]/((a + b*\sin[e + f*x])^{3/2}*\text{Sqrt}[c + d*\sin[e + f*x]]), x], x] /;$$
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3054

$$\text{Int}[(A_) + (C_)*\sin[(e_) + (f_)*(x_)]^2/(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{3/2}*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[C/b^2, \text{Int}[\text{Sqrt}[a + b*\sin[e + f*x]]/\text{Sqrt}[c + d*\sin[e + f*x]], x], x] + \text{Dist}[1/b^2, \text{Int}[(A*b^2 - a^2*C - 2*a*b*C*\sin[e + f*x])]/((a + b*\sin[e + f*x])^{3/2}*\text{Sqrt}[c + d*\sin[e + f*x]]), x], x] /;$$
FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3942

$$\text{Int}[(\text{csc}[(e_) + (f_)*(x_)]*(b_) + (a_))^{(m_)}*(\text{csc}[(e_) + (f_)*(x_)]*(d_) + (c_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(\text{Sqrt}[d + c*\sin[e + f*x]]*\text{Sqrt}[a + b*\text{Csc}[e + f*x]])/(\text{Sqrt}[b + a*\sin[e + f*x]]*\text{Sqrt}[c + d*\text{Csc}[e + f*x]]), \text{Int}[(b + a*\sin[e + f*x])^m*(d + c*\sin[e + f*x])^n/\text{Sin}[e + f*x]^{(m + n)}, x], x] /;$$
FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]

Rubi steps

[In] integrate(1/(c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{\cos(e+fx)}} \left(c + \frac{d}{\cos(e+fx)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)),x)

[Out] int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sec(e + fx)} (c + d \sec(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))**(3/2)), x)

$$3.221 \quad \int \frac{\sqrt[3]{a+b \sec(e+fx)}}{\sqrt[3]{c+d \sec(e+fx)}} dx$$

Optimal. Leaf size=89

$$\frac{\sqrt[3]{a+b \sec(e+fx)} \sqrt[3]{c \cos(e+fx)+d} \operatorname{Int}\left(\frac{\sqrt[3]{a \cos(e+fx)+b}}{\sqrt[3]{c \cos(e+fx)+d}}, x\right)}{\sqrt[3]{a \cos(e+fx)+b} \sqrt[3]{c+d \sec(e+fx)}}$$

[Out] $(d+c*\cos(f*x+e))^{(1/3)}*(a+b*\sec(f*x+e))^{(1/3)}*\operatorname{Unintegrable}((b+a*\cos(f*x+e))^{(1/3)}/(d+c*\cos(f*x+e))^{(1/3)}, x)/(b+a*\cos(f*x+e))^{(1/3)}/(c+d*\sec(f*x+e))^{(1/3)}$

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{\sqrt[3]{c+d \sec(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Int}[(a+b*\operatorname{Sec}[e+f*x])^{(1/3)}/(c+d*\operatorname{Sec}[e+f*x])^{(1/3)}, x]$

[Out] $((d+c*\operatorname{Cos}[e+f*x])^{(1/3)}*(a+b*\operatorname{Sec}[e+f*x])^{(1/3)}*\operatorname{Defer}[\operatorname{Int}[(b+a*\operatorname{Cos}[e+f*x])^{(1/3)}/(d+c*\operatorname{Cos}[e+f*x])^{(1/3)}, x])/((b+a*\operatorname{Cos}[e+f*x])^{(1/3)}*(c+d*\operatorname{Sec}[e+f*x])^{(1/3)})$

Rubi steps

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{\sqrt[3]{c+d \sec(e+fx)}} dx = \frac{(\sqrt[3]{d+c \cos(e+fx)} \sqrt[3]{a+b \sec(e+fx)}) \int \frac{\sqrt[3]{b+a \cos(e+fx)}}{\sqrt[3]{d+c \cos(e+fx)}} dx}{\sqrt[3]{b+a \cos(e+fx)} \sqrt[3]{c+d \sec(e+fx)}}$$

Mathematica [A] time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{\sqrt[3]{c+d \sec(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Integrate}[(a+b*\operatorname{Sec}[e+f*x])^{(1/3)}/(c+d*\operatorname{Sec}[e+f*x])^{(1/3)}, x]$

[Out] $\operatorname{Integrate}[(a+b*\operatorname{Sec}[e+f*x])^{(1/3)}/(c+d*\operatorname{Sec}[e+f*x])^{(1/3)}, x]$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((a+b*\sec(f*x+e))^{(1/3)}/(c+d*\sec(f*x+e))^{(1/3)}, x, \text{algorithm}=\text{"fricas"})$

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx+e)+a)^{\frac{1}{3}}}{(d \sec(fx+e)+c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{1}{3}}}{(c + d \sec(fx + e))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)

[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{1}{3}}}{(d \sec(fx + e) + c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{1/3}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3),x)

[Out] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + b \sec(e + fx)}}{\sqrt[3]{c + d \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(1/3),x)

[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(1/3), x)

$$3.222 \quad \int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}} dx$$

Optimal. Leaf size=32

$$\text{Int} \left(\frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}}, x \right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]

[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]

Rubi steps

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}} dx = \int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}} dx$$

Mathematica [A] time = 52.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{4/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx+e) + a)^{\frac{1}{3}}}{(d \sec(fx+e) + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)

maple [A] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{1}{3}}}{(c + d \sec(fx + e))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)

[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{1}{3}}}{(d \sec(fx + e) + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{1/3}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3),x)

[Out] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + b \sec(e + fx)}}{(c + d \sec(e + fx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(4/3),x)

[Out] Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(4/3), x)

$$3.223 \quad \int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}} dx$$

Optimal. Leaf size=32

$$\text{Int} \left(\frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}}, x \right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]

[Out] Defer[Int][(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]

Rubi steps

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}} dx = \int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}} dx$$

Mathematica [A] time = 89.28, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+b \sec(e+fx)}}{(c+d \sec(e+fx))^{7/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx+e) + a)^{\frac{1}{3}}}{(d \sec(fx+e) + c)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)

maple [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{1}{3}}}{(c + d \sec(fx + e))^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)

[Out] int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{1}{3}}}{(d \sec(fx + e) + c)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(7/3),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(7/3),x)

[Out] Timed out

$$3.224 \quad \int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{2/3}} dx$$

Optimal. Leaf size=89

$$\frac{(a+b \sec(e+fx))^{2/3}(c \cos(e+fx)+d)^{2/3} \operatorname{Int}\left(\frac{(a \cos(e+fx)+b)^{2/3}}{(c \cos(e+fx)+d)^{2/3}}, x\right)}{(a \cos(e+fx)+b)^{2/3}(c+d \sec(e+fx))^{2/3}}$$

[Out] (d+c*cos(f*x+e))^(2/3)*(a+b*sec(f*x+e))^(2/3)*Unintegrable((b+a*cos(f*x+e))^(2/3)/(d+c*cos(f*x+e))^(2/3),x)/(b+a*cos(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3)

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]

[Out] ((d + c*Cos[e + f*x])^(2/3)*(a + b*Sec[e + f*x])^(2/3)*Defer[Int][(b + a*Cos[e + f*x])^(2/3)/(d + c*Cos[e + f*x])^(2/3), x])/((b + a*Cos[e + f*x])^(2/3)*(c + d*Sec[e + f*x])^(2/3))

Rubi steps

$$\int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{2/3}} dx = \frac{\left((d+c \cos(e+fx))^{2/3}(a+b \sec(e+fx))^{2/3}\right) \int \frac{(b+a \cos(e+fx))^{2/3}}{(d+c \cos(e+fx))^{2/3}} dx}{(b+a \cos(e+fx))^{2/3}(c+d \sec(e+fx))^{2/3}}$$

Mathematica [A] time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx+e)+a)^{\frac{2}{3}}}{(d \sec(fx+e)+c)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

maple [A] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{2}{3}}}{(c + d \sec(fx + e))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{2}{3}}}{(d \sec(fx + e) + c)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{2/3}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3),x)
```

```
[Out] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{\frac{2}{3}}}{(c + d \sec(e + fx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(2/3), x)
```

$$3.225 \quad \int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{5/3}} dx$$

Optimal. Leaf size=32

$$\text{Int} \left(\frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{5/3}}, x \right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{5/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]

[Out] Defer[Int] [(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]

Rubi steps

$$\int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{5/3}} dx = \int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{5/3}} dx$$

Mathematica [A] time = 53.31, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{5/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{2}{3}}}{(d \sec(fx + e) + c)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)

maple [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{2}{3}}}{(c + d \sec(fx + e))^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3), x)

[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3), x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{2}{3}}}{(d \sec(fx + e) + c)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3), x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{\frac{2}{3}}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3), x)

[Out] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{\frac{2}{3}}}{(c + d \sec(e + fx))^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(5/3), x)

[Out] Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(5/3), x)

$$3.226 \quad \int \frac{(a+b \sec(e+fx))^{2/3}}{(c+d \sec(e+fx))^{8/3}} dx$$

Optimal. Leaf size=32

$$\text{Int} \left(\frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{8/3}}, x \right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{8/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]

[Out] Defer[Int] [(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]

Rubi steps

$$\int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{8/3}} dx = \int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{8/3}} dx$$

Mathematica [A] time = 92.81, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{2/3}}{(c + d \sec(e + fx))^{8/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{2}{3}}}{(d \sec(fx + e) + c)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)

maple [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{2}{3}}}{(c + d \sec(fx + e))^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)

[Out] int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{2}{3}}}{(d \sec(fx + e) + c)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(8/3),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(8/3),x)

[Out] Timed out

$$3.227 \quad \int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{4/3}} dx$$

Optimal. Leaf size=89

$$\frac{(a+b \sec(e+fx))^{4/3}(c \cos(e+fx)+d)^{4/3} \operatorname{Int}\left(\frac{(a \cos(e+fx)+b)^{4/3}}{(c \cos(e+fx)+d)^{4/3}}, x\right)}{(a \cos(e+fx)+b)^{4/3}(c+d \sec(e+fx))^{4/3}}$$

[Out] $(d+c*\cos(f*x+e))^{(4/3)}*(a+b*\sec(f*x+e))^{(4/3)}*\operatorname{Unintegrable}((b+a*\cos(f*x+e))^{(4/3)}/(d+c*\cos(f*x+e))^{(4/3)}, x)/(b+a*\cos(f*x+e))^{(4/3)}/(c+d*\sec(f*x+e))^{(4/3)}$

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{4/3}} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Int}[(a+b*\operatorname{Sec}[e+f*x])^{(4/3)}/(c+d*\operatorname{Sec}[e+f*x])^{(4/3)}, x]$

[Out] $((d+c*\operatorname{Cos}[e+f*x])^{(4/3)}*(a+b*\operatorname{Sec}[e+f*x])^{(4/3)}*\operatorname{Defer}[\operatorname{Int}[(b+a*\operatorname{Cos}[e+f*x])^{(4/3)}/(d+c*\operatorname{Cos}[e+f*x])^{(4/3)}, x]])/((b+a*\operatorname{Cos}[e+f*x])^{(4/3)}*(c+d*\operatorname{Sec}[e+f*x])^{(4/3)})$

Rubi steps

$$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{4/3}} dx = \frac{((d+c \cos(e+fx))^{4/3}(a+b \sec(e+fx))^{4/3}) \int \frac{(b+a \cos(e+fx))^{4/3}}{(d+c \cos(e+fx))^{4/3}} dx}{(b+a \cos(e+fx))^{4/3}(c+d \sec(e+fx))^{4/3}}$$

Mathematica [A] time = 62.59, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{4/3}} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Integrate}[(a+b*\operatorname{Sec}[e+f*x])^{(4/3)}/(c+d*\operatorname{Sec}[e+f*x])^{(4/3)}, x]$

[Out] $\operatorname{Integrate}[(a+b*\operatorname{Sec}[e+f*x])^{(4/3)}/(c+d*\operatorname{Sec}[e+f*x])^{(4/3)}, x]$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((a+b*\sec(f*x+e))^{(4/3)}/(c+d*\sec(f*x+e))^{(4/3)}, x, \operatorname{algorithm}="fricas")$

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx+e)+a)^{\frac{4}{3}}}{(d \sec(fx+e)+c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

maple [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{4}{3}}}{(c + d \sec(fx + e))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

```
[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{4}{3}}}{(d \sec(fx + e) + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\cos(e+fx)}\right)^{\frac{4}{3}}}{\left(c + \frac{d}{\cos(e+fx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3),x)
```

```
[Out] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```


$$3.228 \quad \int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}} dx$$

Optimal. Leaf size=32

$$\text{Int} \left(\frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}}, x \right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]

[Out] Defer[Int] [(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]

Rubi steps

$$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}} dx = \int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}} dx$$

Mathematica [A] time = 102.05, size = 0, normalized size = 0.00

$$\int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{7/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx+e) + a)^{\frac{4}{3}}}{(d \sec(fx+e) + c)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)

maple [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{4}{3}}}{(c + d \sec(fx + e))^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)

[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{4}{3}}}{(d \sec(fx + e) + c)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(7/3),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(7/3),x)

[Out] Timed out

$$3.229 \quad \int \frac{(a+b \sec(e+fx))^{4/3}}{(c+d \sec(e+fx))^{10/3}} dx$$

Optimal. Leaf size=32

$$\text{Int} \left(\frac{(a + b \sec(e + fx))^{4/3}}{(c + d \sec(e + fx))^{10/3}}, x \right)$$

[Out] Unintegrable((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + b \sec(e + fx))^{4/3}}{(c + d \sec(e + fx))^{10/3}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]

[Out] Defer[Int][(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]

Rubi steps

$$\int \frac{(a + b \sec(e + fx))^{4/3}}{(c + d \sec(e + fx))^{10/3}} dx = \int \frac{(a + b \sec(e + fx))^{4/3}}{(c + d \sec(e + fx))^{10/3}} dx$$

Mathematica [A] time = 148.17, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(e + fx))^{4/3}}{(c + d \sec(e + fx))^{10/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]

[Out] Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{4}{3}}}{(d \sec(fx + e) + c)^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3), x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)

maple [A] time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(fx + e))^{\frac{4}{3}}}{(c + d \sec(fx + e))^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)

[Out] int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(fx + e) + a)^{\frac{4}{3}}}{(d \sec(fx + e) + c)^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(10/3),x)

[Out] \text{Hanged}

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(10/3),x)

[Out] Timed out

3.230 $\int \left(c(d \sec(e + fx))^p \right)^n (a + a \sec(e + fx))^m dx$

Optimal. Leaf size=106

$$\frac{\tan(e + fx)(\sec(e + fx) + 1)^{-m-\frac{1}{2}}(a \sec(e + fx) + a)^m F_1\left(np; \frac{1}{2}, \frac{1}{2} - m; np + 1; \sec(e + fx), -\sec(e + fx)\right)}{fnp\sqrt{1 - \sec(e + fx)}} (c)$$

[Out] -AppellF1(n*p, 1/2-m, 1/2, n*p+1, -sec(f*x+e), sec(f*x+e))*(c*(d*sec(f*x+e))^p)^n*(1+sec(f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/p/(1-sec(f*x+e))^(1/2)

Rubi [A] time = 0.19, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3948, 3828, 3827, 133}

$$\frac{\tan(e + fx)(\sec(e + fx) + 1)^{-m-\frac{1}{2}}(a \sec(e + fx) + a)^m F_1\left(np; \frac{1}{2}, \frac{1}{2} - m; np + 1; \sec(e + fx), -\sec(e + fx)\right)}{fnp\sqrt{1 - \sec(e + fx)}} (c)$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]

[Out] -((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d*Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))

Rule 133

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*x)/c), -((f*x)/e)])/((b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 3827

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]

Rule 3828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]

Rule 3948

Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]

Rubi steps

$c[(e + f*x)/2]^2 * \tan[(e + f*x)/2] / 3 + 2 * \tan[(e + f*x)/2]^2 * ((-1 + n*p) * ((-3 * (2 - n*p) * \text{AppellF1}[5/2, m + n*p, 3 - n*p, 7/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * \text{Sec}[(e + f*x)/2]^2 * \tan[(e + f*x)/2]) / 5 + (3 * (m + n*p) * \text{AppellF1}[5/2, 1 + m + n*p, 2 - n*p, 7/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * \text{Sec}[(e + f*x)/2]^2 * \tan[(e + f*x)/2]) / 5 + (m + n*p) * ((-3 * (1 - n*p) * \text{AppellF1}[5/2, 1 + m + n*p, 2 - n*p, 7/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * \text{Sec}[(e + f*x)/2]^2 * \tan[(e + f*x)/2]) / 5 + (3 * (1 + m + n*p) * \text{AppellF1}[5/2, 2 + m + n*p, 1 - n*p, 7/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * \text{Sec}[(e + f*x)/2]^2 * \tan[(e + f*x)/2]) / 5))) / (3 * \text{AppellF1}[1/2, m + n*p, 1 - n*p, 3/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] + 2 * ((-1 + n*p) * \text{AppellF1}[3/2, m + n*p, 2 - n*p, 5/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] + (m + n*p) * \text{AppellF1}[3/2, 1 + m + n*p, 1 - n*p, 5/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * \tan[(e + f*x)/2]^2 + (3 * 2^{(1 + m)} * (m + n*p) * \text{AppellF1}[1/2, m + n*p, 1 - n*p, 3/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * (\text{Sec}[(e + f*x)/2]^2)^{-1 + n*p} * (\cos[(e + f*x)/2]^2 * \text{Sec}[e + f*x])^{-1 + m + n*p} * \tan[(e + f*x)/2] * (-\cos[(e + f*x)/2] * \text{Sec}[e + f*x] * \sin[(e + f*x)/2]) + \cos[(e + f*x)/2]^2 * \text{Sec}[e + f*x] * \tan[e + f*x])) / (3 * \text{AppellF1}[1/2, m + n*p, 1 - n*p, 3/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] + 2 * ((-1 + n*p) * \text{AppellF1}[3/2, m + n*p, 2 - n*p, 5/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] + (m + n*p) * \text{AppellF1}[3/2, 1 + m + n*p, 1 - n*p, 5/2, \tan[(e + f*x)/2]^2, -\tan[(e + f*x)/2]^2] * \tan[(e + f*x)/2]^2)))$

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(d \sec(fx + e)\right)^p c\right)^n (a \sec(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
 [Out] integral(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((d \sec(fx + e))^p c \right)^n (a \sec(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
 [Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)

maple [F] time = 2.51, size = 0, normalized size = 0.00

$$\int \left(c (d \sec(fx + e))^p \right)^n (a + a \sec(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
 [Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((d \sec(fx + e))^p c \right)^n (a \sec(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c \left(\frac{d}{\cos(e + fx)} \right)^p \right)^n \left(a + \frac{a}{\cos(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m,x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec(e + fx) + 1 \right) \right)^m \left(c \left(d \sec(e + fx) \right)^p \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)

[Out] Integral((a*(sec(e + f*x) + 1))^m*(c*(d*sec(e + f*x))^p)^n, x)

3.231 $\int \left(c(d \sec(e + fx))^p \right)^n (a + a \sec(e + fx))^3 dx$

Optimal. Leaf size=275

$$\frac{a^3(4np + 1) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}} + \frac{a^3(4np + 1) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}}$$

[Out] a³*(4*n*p+7)*hypergeom([1/2, -1/2*n*p], [-1/2*n*p+1], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2)-a³*(4*n*p+1)*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n²*p²+1)/(sin(f*x+e)^2)^(1/2)+a³*(2*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)/(n*p+2)+(c*(d*sec(f*x+e))^p)^n*(a³+a³*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)

Rubi [A] time = 0.44, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3948, 3814, 3997, 3787, 3772, 2643}

$$\frac{a^3(4np + 1) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}} + \frac{a^3(4np + 1) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]

[Out] (a³*(7 + 4*n*p)*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]/(f*n*p*(2 + n*p)*Sqrt[Sin[e + f*x]^2]) - (a³*(1 + 4*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]/(f*(1 - n²*p²)*Sqrt[Sin[e + f*x]^2]) + (a³*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x]/(f*(1 + n*p)*(2 + n*p))) + ((c*(d*Sec[e + f*x])^p)^n*(a³ + a³*Sec[e + f*x])*Tan[e + f*x]/(f*(2 + n*p)))

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3814

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)^(m_), x_Symbol] := -Simp[(b²*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2))*(d*Csc[e + f*x])^n/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*

$\text{Csc}[e + f*x]^{(m - 2)} * (d * \text{Csc}[e + f*x])^n * (b * (m + 2*n - 1) + a * (3*m + 2*n - 4) * \text{Csc}[e + f*x]), x, x] /;$ FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]

Rule 3948

$\text{Int}[(c * (d * \text{Sec}[e + f*x])^p)^n * (a + b * \text{Sec}[e + f*x])^m, x_Symbol] :>$ Dist[(c^IntPart[n] * (c * (d * Sec[e + f*x])^p)^FracPart[n]) / (d * Sec[e + f*x])^(p * FracPart[n]), Int[(a + b * Sec[e + f*x])^m * (d * Sec[e + f*x])^(n * p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]

Rule 3997

$\text{Int}[(\text{Csc}[e + f*x] * (d * \text{Csc}[e + f*x])^n)^m * (\text{Csc}[e + f*x] * (b * \text{Cot}[e + f*x] * (d * \text{Csc}[e + f*x])^n / (f * (n + 1)) + \text{Dist}[1 / (n + 1), \text{Int}[(d * \text{Csc}[e + f*x])^n * \text{Simp}[A * a * (n + 1) + B * b * n + (A * b + B * a) * (n + 1) * \text{Csc}[e + f*x], x], x])], x] /;$ FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A * b - a * B, 0] && !LeQ[n, -1]

Rubi steps

$$\begin{aligned} \int (c(d \sec(e + fx))^p)^n (a + a \sec(e + fx))^3 dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + a \sec(e + fx))^3 dx \\ &= \frac{(c(d \sec(e + fx))^p)^n (a^3 + a^3 \sec(e + fx)) \tan(e + fx)}{f(2 + np)} + \frac{(a(d \sec(e + fx))^p)^n (a^3 + a^3 \sec(e + fx)) \tan(e + fx)}{f(1 + np)(2 + np)} \\ &= \frac{a^3(5 + 2np)(c(d \sec(e + fx))^p)^n \tan(e + fx)}{f(1 + np)(2 + np)} + \frac{(c(d \sec(e + fx))^p)^n (a^3 + a^3 \sec(e + fx)) \tan(e + fx)}{f(1 + np)(2 + np)} \\ &= \frac{a^3(5 + 2np)(c(d \sec(e + fx))^p)^n \tan(e + fx)}{f(1 + np)(2 + np)} + \frac{(c(d \sec(e + fx))^p)^n (a^3 + a^3 \sec(e + fx)) \tan(e + fx)}{f(1 + np)(2 + np)} \\ &= \frac{a^3(7 + 4np) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n}{fnp(2 + np)\sqrt{\sin^2(e + fx)}} \end{aligned}$$

Mathematica [C] time = 2.31, size = 343, normalized size = 1.25

$$-ia^3 2^{np-3} \sec^6\left(\frac{1}{2}(e + fx)\right) (\sec(e + fx) + 1)^3 \left(\frac{e^{i(e + fx)}}{1 + e^{2i(e + fx)}}\right)^{np} \left(\frac{12e^{2i(e + fx)} {}_2F_1\left(1, -\frac{np}{2}; \frac{np}{2} + 2; -e^{2i(e + fx)}\right)}{f(np + 2)(1 + e^{2i(e + fx)})}\right) + \frac{8e^{3i(e + fx)}}{fnp(2 + np)\sqrt{\sin^2(e + fx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]

[Out] (-I)*2^(-3 + n*p)*a^3*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(n*p)*((1 + 2*I*E^((2*I)*(e + f*x)))*Hypergeometric2F1[1, -1/2*(n*p), 2 + (n*p)/2, -E^((2*I)*(e + f*x))]) / ((1 + E^((2*I)*(e + f*x)))*f*(2 + n*p)) + (8*I*E^((3*I)*(e + f*x)))*Hypergeometric2F1[1, (-1 - n*p)/2, (5 + n*p)/2, -E^((2*I)*(e + f*x))]

$$\frac{1}{((1 + E^{((2I)(e + f*x)))})^2 * f * (3 + n*p)) + (6 * E^{(I*(e + f*x))} * \text{Hypergeometric2F1}[1, (1 - n*p)/2, (3 + n*p)/2, -E^{((2I)(e + f*x))}] / (f + f*n*p)) + ((1 + E^{((2I)(e + f*x))}) * \text{Hypergeometric2F1}[1, 1 - (n*p)/2, 1 + (n*p)/2, -E^{((2I)(e + f*x))}] / (f*n*p)) * \text{Sec}[(e + f*x)/2]^6 * \text{Sec}[e + f*x]^{-3 - n*p} * (c * (d * \text{Sec}[e + f*x])^p)^n * (1 + \text{Sec}[e + f*x])^3}$$

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3 \sec(fx + e)^3 + 3a^3 \sec(fx + e)^2 + 3a^3 \sec(fx + e) + a^3\right) \left((d \sec(fx + e))^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")

[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3)*((d*sec(f*x + e))^p*c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^3 \left((d \sec(fx + e))^p c\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \left(c(d \sec(fx + e))^p\right)^n (a + a \sec(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c \left(\frac{d}{\cos(e + fx)}\right)^p\right)^n \left(a + \frac{a}{\cos(e + fx)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3,x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \left(c(d \sec(e + fx))^p\right)^n dx + \int 3 \left(c(d \sec(e + fx))^p\right)^n \sec(e + fx) dx + \int 3 \left(c(d \sec(e + fx))^p\right)^n \sec^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e + f*x), x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**2, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**3, x))
```

3.232 $\int \left(c(d \sec(e + fx))^p \right)^n (a + a \sec(e + fx))^2 dx$

Optimal. Leaf size=205

$$\frac{a^2(2np + 1) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}} + \frac{2a^2 \sin(e + fx) \cos(e + fx) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}}$$

[Out] 2*a^2*hypergeom([1/2, -1/2*n*p], [-1/2*n*p+1], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a^2*(2*n*p+1)*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^2*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)

Rubi [A] time = 0.25, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3948, 3788, 3772, 2643, 4046}

$$\frac{a^2(2np + 1) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}} + \frac{2a^2 \sin(e + fx) \cos(e + fx) \left(c(d \sec(e + fx))^p\right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]

[Out] (2*a^2*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 + 2*n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x]/(f*(1 + n*p)))

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3788

Int[(csc[(e_.) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_.) + (f_)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3948

Int[((c_)*((d_)*sec[(e_.) + (f_)*(x_)])^(p_))^(n_)*((a_.) + (b_)*sec[(e_.) + (f_)*(x_)])^(m_), x_Symbol] := Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (c(d \sec(e + fx))^p)^n (a + a \sec(e + fx))^2 dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + a \sec(e + fx))^2 dx \\ &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a^2 + 2a \sec(e + fx) + \sec^2(e + fx)) dx \\ &= \frac{a^2 (c(d \sec(e + fx))^p)^n \tan(e + fx)}{f(1 + np)} + \frac{\left(2a^2 \left(\frac{\cos(e + fx)}{d} \right)^{np} (c(d \sec(e + fx))^p)^n \right) \int \sec(e + fx) dx}{fnp \sqrt{\sin^2(e + fx)}} \\ &= \frac{2a^2 {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n \sin(e + fx)}{fnp \sqrt{\sin^2(e + fx)}} \\ &= \frac{2a^2 {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n \sin(e + fx)}{fnp \sqrt{\sin^2(e + fx)}} \end{aligned}$$

Mathematica [C] time = 2.82, size = 299, normalized size = 1.46

$$\frac{ia^2 2^{np-2} e^{-i(e+fx)} \sec^4\left(\frac{1}{2}(e+fx)\right) (\sec(e+fx) + 1)^2 \left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{np+1} \left(4np(np+1)e^{2i(e+fx)} {}_2F_1\left(1, -\frac{np}{2}; \frac{np}{2} + 2; -e^{2i(e+fx)}\right)\right)}{}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]

[Out] ((-1)*2^(-2 + n*p)*a^2*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1 + n*p) *(4*E^((2*I)*(e + f*x))*n*p*(1 + n*p)*Hypergeometric2F1[1, -1/2*(n*p), 2 + (n*p)/2, -E^((2*I)*(e + f*x))]) + (1 + E^((2*I)*(e + f*x)))*(2 + n*p)*(4*E^(I*(e + f*x))*n*p*Hypergeometric2F1[1, (1 - n*p)/2, (3 + n*p)/2, -E^((2*I)*(e + f*x))]) + (1 + E^((2*I)*(e + f*x)))*(1 + n*p)*Hypergeometric2F1[1, 1 - (n*p)/2, 1 + (n*p)/2, -E^((2*I)*(e + f*x))]))*Sec[(e + f*x)/2]^4*Sec[e + f*x]^(-2 - n*p)*(c*(d*Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^2)/(E^(I*(e + f*x)))*f*n*p*(1 + n*p)*(2 + n*p))

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2\right)\left((d \sec(fx + e))^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*((d*sec(f*x + e))^p*c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^2 \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)

maple [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \left(c (d \sec(fx + e))^p \right)^n (a + a \sec(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a)^2 \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c \left(\frac{d}{\cos(e + fx)} \right)^p \right)^n \left(a + \frac{a}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2,x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \left(c (d \sec(e + fx))^p \right)^n dx + \int 2 \left(c (d \sec(e + fx))^p \right)^n \sec(e + fx) dx + \int \left(c (d \sec(e + fx))^p \right)^n \sec^2(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e)))**p)**n*(a+a*sec(f*x+e))**2,x)

[Out] a**2*(Integral((c*(d*sec(e + f*x)))**p)**n, x) + Integral(2*(c*(d*sec(e + f*x)))**p)**n*sec(e + f*x), x) + Integral((c*(d*sec(e + f*x)))**p)**n*sec(e + f*x)**2, x)

3.233 $\int \left(c(d \sec(e + fx))^p \right)^n (a + a \sec(e + fx)) dx$

Optimal. Leaf size=156

$$\frac{a \sin(e + fx) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p \right)^n}{fnp \sqrt{\sin^2(e + fx)}} - \frac{a \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(2 - np); \cos^2(e + fx)\right)}{f(1 - np)}$$

[Out] a*hypergeom([1/2, -1/2*n*p], [-1/2*n*p+1], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n*p+1)/(sin(f*x+e)^2)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3948, 3787, 3772, 2643}

$$\frac{a \sin(e + fx) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p \right)^n}{fnp \sqrt{\sin^2(e + fx)}} - \frac{a \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(2 - np); \cos^2(e + fx)\right)}{f(1 - np)}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]), x]

[Out] (a*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3948

Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_.))^(n_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FractPart[n])/(d*Sec[e + f*x]^(p*FractPart[n])), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int (c(d \sec(e + fx))^p)^n (a + a \sec(e + fx)) dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + a \sec(e + fx)) dx \\
&= \left(a (d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + a \sec(e + fx)) dx \\
&= \left(a \left(\frac{\cos(e + fx)}{d} \right)^{np} (c(d \sec(e + fx))^p)^n \right) \int \left(\frac{\cos(e + fx)}{d} \right)^{-np} (a + a \sec(e + fx)) dx \\
&= \frac{a {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n \sin(e + fx)}{fnp \sqrt{\sin^2(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 124, normalized size = 0.79

$$\frac{a \sqrt{-\tan^2(e + fx)} \csc(e + fx) (c(d \sec(e + fx))^p)^n \left(np {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(np + 1); \frac{1}{2}(np + 3); \sec^2(e + fx)\right) + (np + 1) \csc(e + fx) \right)}{fnp(np + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]

[Out] (a*Csc[e + f*x]*((1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*sqrt[-Tan[e + f*x]^2])/(f*n*p*(1 + n*p))

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a \sec(fx + e) + a\right) \left(\left(d \sec(fx + e)\right)^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a) \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)

maple [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \left(c (d \sec(fx + e))^p \right)^n (a + a \sec(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(fx + e) + a) \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c \left(\frac{d}{\cos(e + fx)} \right)^p \right)^n \left(a + \frac{a}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)),x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \left(c \left(d \sec(e + fx) \right)^p \right)^n dx + \int \left(c \left(d \sec(e + fx) \right)^p \right)^n \sec(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e)),x)

[Out] a*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x), x))

$$3.234 \quad \int \frac{(c(d \sec(e+fx))^p)^n}{a+a \sec(e+fx)} dx$$

Optimal. Leaf size=208

$$\frac{\sin(e+fx) \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1-np); \frac{1}{2}(3-np); \cos^2(e+fx)\right) (c(d \sec(e+fx))^p)^n (1-np) \sin(e+fx)}{af \sqrt{\sin^2(e+fx)}} + \frac{\sin(e+fx) \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1-np); \frac{1}{2}(3-np); \cos^2(e+fx)\right) (c(d \sec(e+fx))^p)^n (1-np) \sin(e+fx)}{af \sqrt{\sin^2(e+fx)}}$$

[Out] (c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)+(-n*p+1)*cos(f*x+e)^2*hypergeom([1/2, -1/2*n*p+1], [-1/2*n*p+2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/a/f/(-n*p+2)/(sin(f*x+e)^2)^(1/2)

Rubi [A] time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3948, 3820, 3787, 3772, 2643}

$$\frac{\sin(e+fx) \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1-np); \frac{1}{2}(3-np); \cos^2(e+fx)\right) (c(d \sec(e+fx))^p)^n (1-np) \sin(e+fx)}{af \sqrt{\sin^2(e+fx)}} + \frac{\sin(e+fx) \cos(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1-np); \frac{1}{2}(3-np); \cos^2(e+fx)\right) (c(d \sec(e+fx))^p)^n (1-np) \sin(e+fx)}{af \sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]), x]

[Out] ((c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 - n*p)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(a*f*(2 - n*p)*Sqrt[Sin[e + f*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3820

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^

$2 - b^2, 0]$

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_.))^(n_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n]]/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{(c(d \sec(e + fx))^p)^n}{a + a \sec(e + fx)} dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int \frac{(d \sec(e + fx))^{np}}{a + a \sec(e + fx)} dx \\ &= \frac{(c(d \sec(e + fx))^p)^n \sin(e + fx)}{f(a + a \sec(e + fx))} - \frac{(d(1 - np)(d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n)}{a^2} \\ &= \frac{(c(d \sec(e + fx))^p)^n \sin(e + fx)}{f(a + a \sec(e + fx))} + \frac{((1 - np)(d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n)}{a} \\ &= \frac{(c(d \sec(e + fx))^p)^n \sin(e + fx)}{f(a + a \sec(e + fx))} + \frac{\left((1 - np) \left(\frac{\cos(e + fx)}{d} \right)^{np} (c(d \sec(e + fx))^p)^n \right) \int \left(\frac{\cos(e + fx)}{d} \right)^{np}}{a} \\ &= \frac{(c(d \sec(e + fx))^p)^n \sin(e + fx)}{f(a + a \sec(e + fx))} - \frac{\cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx)\right)}{af \sqrt{\sin^2(e + fx)}} \end{aligned}$$

Mathematica [F] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{(c(d \sec(e + fx))^p)^n}{a + a \sec(e + fx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]), x]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((d \sec(fx + e))^p c \right)^n}{a \sec(fx + e) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d \sec(fx + e))^p c \right)^n}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\left(c \left(d \sec(fx + e)\right)^p\right)^n}{a + a \sec(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)

[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d \sec(fx + e)\right)^p c\right)^n}{a \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c \left(\frac{d}{\cos(e+fx)}\right)^p\right)^n}{a + \frac{a}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)),x)

[Out] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\left(c \left(d \sec(e+fx)\right)^p\right)^n}{\sec(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e)),x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x) + 1), x)/a

$$3.235 \quad \int \frac{(c(d \sec(e+fx))^p)^n}{(a+a \sec(e+fx))^2} dx$$

Optimal. Leaf size=248

$$\frac{2(2-np) \sin(e+fx) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2-np); \cos^2(e+fx)\right) (c(d \sec(e+fx))^p)^n (3-2np) \sin(e+fx) \cos(e+fx)}{3a^2 f \sqrt{\sin^2(e+fx)}}$$

[Out] $2/3*(-n*p+2)*\text{hypergeom}([1/2, -1/2*n*p], [-1/2*n*p+1], \cos(f*x+e)^2)*(c*(d*\sec(f*x+e))^p)^n*\sin(f*x+e)/a^2/f/(\sin(f*x+e)^2)^{(1/2)}-1/3*(-2*n*p+3)*\cos(f*x+e)*\text{hypergeom}([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], \cos(f*x+e)^2)*(c*(d*\sec(f*x+e))^p)^n*\sin(f*x+e)/a^2/f/(\sin(f*x+e)^2)^{(1/2)}-2/3*(-n*p+2)*(c*(d*\sec(f*x+e))^p)^n*\tan(f*x+e)/a^2/f/(1+\sec(f*x+e))-1/3*(c*(d*\sec(f*x+e))^p)^n*\tan(f*x+e)/f/(a+a*\sec(f*x+e))^2$

Rubi [A] time = 0.45, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3948, 3817, 4020, 3787, 3772, 2643}

$$\frac{2(2-np) \sin(e+fx) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2-np); \cos^2(e+fx)\right) (c(d \sec(e+fx))^p)^n (3-2np) \sin(e+fx) \cos(e+fx)}{3a^2 f \sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]

[Out] $(2*(2-np)*\text{Hypergeometric2F1}[1/2, -(n*p)/2, (2-np)/2, \text{Cos}[e+f*x]^2]*(c*(d*\text{Sec}[e+f*x])^p)^n*\text{Sin}[e+f*x])/(3*a^2*f*\text{Sqrt}[\text{Sin}[e+f*x]^2]) - ((3-2*n*p)*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1/2, (1-np)/2, (3-np)/2, \text{Cos}[e+f*x]^2]*(c*(d*\text{Sec}[e+f*x])^p)^n*\text{Sin}[e+f*x])/(3*a^2*f*\text{Sqrt}[\text{Sin}[e+f*x]^2]) - (2*(2-np)*(c*(d*\text{Sec}[e+f*x])^p)^n*\text{Tan}[e+f*x])/(3*a^2*f*(1+\text{Sec}[e+f*x])) - ((c*(d*\text{Sec}[e+f*x])^p)^n*\text{Tan}[e+f*x])/(3*f*(a+a*\text{Sec}[e+f*x])^2)$

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n-1)*((Sin[c + d*x]/b)^(n-1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n+1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3817

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e

$+ f*x])^n)/(f*(2*m + 1)), x] + \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegersQ}[2*m, 2*n] \parallel \text{IntegerQ}[m])$

Rule 3948

$\text{Int}[(c_*)*((d_*)*\text{sec}[(e_*) + (f_*)*(x_)])^{(p_*)})^{(n_*)}*((a_*) + (b_*)*\text{sec}[(e_*) + (f_*)*(x_)])^{(m_*)}, x_Symbol] :> \text{Dist}[(c^{\text{IntPart}[n]}*(c*(d*\text{Sec}[e + f*x])^p)^{\text{FracPart}[n]})/(d*\text{Sec}[e + f*x])^{(p*\text{FracPart}[n])}, \text{Int}[(a + b*\text{Sec}[e + f*x])^m*(d*\text{Sec}[e + f*x])^{(n*p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \&\& !\text{IntegerQ}[n]$

Rule 4020

$\text{Int}[(\text{csc}[(e_*) + (f_*)*(x_*)]*(d_*)^{(n_*)}*(\text{csc}[(e_*) + (f_*)*(x_*)]*(b_*) + (a_*)^{(m_*)}*(\text{csc}[(e_*) + (f_*)*(x_*)]*(B_*) + (A_*)))^{(m_*)}, x_Symbol] :> -\text{Simp}[(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*(d*\text{Csc}[e + f*x])^n)/(b*f*(2*m + 1)), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{m+1}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(c(d \sec(e + fx))^p)^n}{(a + a \sec(e + fx))^2} dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int \frac{(d \sec(e + fx))^{np}}{(a + a \sec(e + fx))^2} dx \\ &= -\frac{(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3f(a + a \sec(e + fx))^2} - \frac{\left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int \frac{(d \sec(e + fx))^{np}}{(a + a \sec(e + fx))^2} dx}{3a^2} \\ &= -\frac{2(2 - np)(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3a^2 f(1 + \sec(e + fx))} - \frac{(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3f(a + a \sec(e + fx))^2} \\ &= -\frac{2(2 - np)(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3a^2 f(1 + \sec(e + fx))} - \frac{(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3f(a + a \sec(e + fx))^2} + \\ &= -\frac{2(2 - np)(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3a^2 f(1 + \sec(e + fx))} - \frac{(c(d \sec(e + fx))^p)^n \tan(e + fx)}{3f(a + a \sec(e + fx))^2} + \\ &= \frac{2(2 - np) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n \sin(e + fx)}{3a^2 f \sqrt{\sin^2(e + fx)}} \end{aligned}$$

Mathematica [F] time = 10.83, size = 0, normalized size = 0.00

$$\int \frac{(c(d \sec(e + fx))^p)^n}{(a + a \sec(e + fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2, x]

[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2, x]

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((d \sec(fx + e))^p c \right)^n}{a^2 \sec(fx + e)^2 + 2a^2 \sec(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d \sec(fx + e))^p c \right)^n}{(a \sec(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\left(c (d \sec(fx + e))^p \right)^n}{(a + a \sec(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)

[Out] int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c \left(\frac{d}{\cos(e+fx)} \right)^p \right)^n}{\left(a + \frac{a}{\cos(e+fx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2,x)

[Out] int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(d \sec(e+fx))^p\right)^n}{\frac{\sec^2(e+fx)+2 \sec(e+fx)+1}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e))**2,x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2

$$3.236 \quad \int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx))^m dx$$

Optimal. Leaf size=56

$$(d \sec(e + fx))^{-np} \left(c(d \sec(e + fx))^p \right)^n \text{Int} \left((a + b \sec(e + fx))^m (d \sec(e + fx))^{np}, x \right)$$

[Out] (c*(d*sec(f*x+e))^p)^n*Unintegrable((d*sec(f*x+e))^(n*p)*(a+b*sec(f*x+e))^m,x)/((d*sec(f*x+e))^(n*p))

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]

[Out] ((c*(d*Sec[e + f*x])^p)^n*Defer[Int][(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e + f*x])^m, x])/((d*Sec[e + f*x])^(n*p))

Rubi steps

$$\int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx))^m dx = \left((d \sec(e + fx))^{-np} \left(c(d \sec(e + fx))^p \right)^n \right) \int (d \sec(e + fx))^{np} (a + b \sec(e + fx))^m dx$$

Mathematica [A] time = 2.69, size = 0, normalized size = 0.00

$$\int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]

[Out] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m, x]

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral} \left(\left((d \sec(fx + e))^p c \right)^n (b \sec(fx + e) + a)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((d \sec(fx + e))^p c \right)^n (b \sec(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)

maple [A] time = 1.45, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec(fx + e) \right)^p \right)^n \left(a + b \sec(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(d \sec(fx + e) \right)^p c \right)^n \left(b \sec(fx + e) + a \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c \left(\frac{d}{\cos(e + fx)} \right)^p \right)^n \left(a + \frac{b}{\cos(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m,x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec(e + fx) \right)^p \right)^n \left(a + b \sec(e + fx) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**m,x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**m, x)

3.237 $\int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx))^3 dx$

Optimal. Leaf size=296

$$\frac{a \left(a^2(np + 1) + 3b^2np \right) \sin(e + fx) \cos(e + fx) {}_2F_1 \left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx) \right) \left(c(d \sec(e + fx))^p \right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}}$$

[Out] b*(b^2*(n*p+1)+3*a^2*(n*p+2))*hypergeom([1/2, -1/2*n*p], [-1/2*n*p+1], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2)-a*(3*b^2*n*p+a^2*(n*p+1))*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a*b^2*(2*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)/(n*p+2)+b^2*(c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)

Rubi [A] time = 0.51, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3948, 3842, 4047, 3772, 2643, 4046}

$$\frac{a \left(a^2(np + 1) + 3b^2np \right) \sin(e + fx) \cos(e + fx) {}_2F_1 \left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx) \right) \left(c(d \sec(e + fx))^p \right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]

[Out] (b*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*(2 + n*p)*Sqrt[Sin[e + f*x]^2]) - (a*(3*b^2*n*p + a^2*(1 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 + 2*n*p)*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p)*(2 + n*p)) + (b^2*(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n*p))

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3842

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2))*(d*Csc[e + f*x])^n/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && !IntegerQ[m])

Rule 3948

```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx))^3 dx = \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + b \sec(e + fx))^3 dx$$

$$= \frac{b^2 (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx)) \tan(e + fx)}{f(2 + np)} + \frac{(d \sec(e + fx))^{np} (a + b \sec(e + fx))^2}{f(2 + np)}$$

$$= \frac{b^2 (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx)) \tan(e + fx)}{f(2 + np)} + \frac{(d \sec(e + fx))^{np} (a + b \sec(e + fx))^2}{f(2 + np)}$$

$$= \frac{ab^2(5 + 2np) (c(d \sec(e + fx))^p)^n \tan(e + fx)}{f(1 + np)(2 + np)} + \frac{b^2 (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx)) \tan(e + fx)}{f(2 + np)}$$

$$= \frac{b (b^2(1 + np) + 3a^2(2 + np)) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right)}{fnp(2 + np)\sqrt{\sin^2(e + fx)}}$$

$$= \frac{b (b^2(1 + np) + 3a^2(2 + np)) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right)}{fnp(2 + np)\sqrt{\sin^2(e + fx)}}$$

Mathematica [A] time = 1.31, size = 278, normalized size = 0.94

$$\frac{(-\tan^2(e + fx))^{3/2} \csc^3(e + fx) (c(d \sec(e + fx))^p)^n \left(a^3 (n^3 p^3 + 6n^2 p^2 + 11np + 6) \cos^3(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) + b^2 (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx)) \tan(e + fx) \right)}{fnp(2 + np)\sqrt{\sin^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n*p + 6*n^2*p^2 + n^3*p^3)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(3*a*b*(3 + 4*n*p + n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2]))/f^n)
```

$n*p)/2, \text{Sec}[e + f*x]^2] + (2 + n*p)*(3*a^2*(3 + n*p)*\text{Cos}[e + f*x]^2*\text{Hypergeometric2F1}[1/2, (1 + n*p)/2, (3 + n*p)/2, \text{Sec}[e + f*x]^2] + b^2*(1 + n*p)*\text{Hypergeometric2F1}[1/2, (3 + n*p)/2, (5 + n*p)/2, \text{Sec}[e + f*x]^2]))*(c*(d*\text{Sec}[e + f*x])^p)^n*(-\text{Tan}[e + f*x]^2)^{(3/2)})/(f*n*p*(1 + n*p)*(2 + n*p)*(3 + n*p))$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \sec(fx + e)^3 + 3ab^2 \sec(fx + e)^2 + 3a^2b \sec(fx + e) + a^3\right)\left((d \sec(fx + e))^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")

[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3)*((d*sec(f*x + e))^p*c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a)^3 \left((d \sec(fx + e))^p c\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec(fx + e)\right)^p\right)^n \left(a + b \sec(fx + e)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c \left(\frac{d}{\cos(e + fx)}\right)^p\right)^n \left(a + \frac{b}{\cos(e + fx)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3,x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec(e + fx)\right)^p\right)^n \left(a + b \sec(e + fx)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**3,x)
```

```
[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**3, x)
```

3.238 $\int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx))^2 dx$

Optimal. Leaf size=211

$$\frac{\left(a^2(np + 1) + b^2np \right) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx) \right) \left(c(d \sec(e + fx))^p \right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}} + 2a$$

[Out] 2*a*b*hypergeom([1/2, -1/2*n*p], [-1/2*n*p+1], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-(b^2*n*p+a^2*(n*p+1))*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)

Rubi [A] time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3948, 3788, 3772, 2643, 4046}

$$\frac{\left(a^2(np + 1) + b^2np \right) \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(e + fx) \right) \left(c(d \sec(e + fx))^p \right)^n}{f(1 - n^2p^2) \sqrt{\sin^2(e + fx)}} + 2a$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]

[Out] (2*a*b*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - ((b^2*n*p + a^2*(1 + n*p))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n^2*p^2)*Sqrt[Sin[e + f*x]^2]) + (b^2*(c*(d*Sec[e + f*x])^p)^n*Tan[e + f*x])/(f*(1 + n*p))

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3788

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.)^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3948

Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n])/(d*Sec[e + f*x]^(p*FracPart[n])), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]

] && !IntegerQ[n]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx))^2 dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + b \sec(e + fx))^2 dx \\ &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a^2 + 2ab \sec(e + fx) + b^2 \sec^2(e + fx)) dx \\ &= \frac{b^2 (c(d \sec(e + fx))^p)^n \tan(e + fx)}{f(1 + np)} + \frac{2ab \left(\frac{\cos(e + fx)}{d} \right)^{np} (c(d \sec(e + fx))^p)^n}{fnp \sqrt{\sin^2(e + fx)}} \\ &= \frac{2ab {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n}{fnp \sqrt{\sin^2(e + fx)}} \\ &= \frac{2ab {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n}{fnp \sqrt{\sin^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.54, size = 200, normalized size = 0.95

$$\frac{\sqrt{-\tan^2(e + fx)} \csc(e + fx) \sec(e + fx) (c(d \sec(e + fx))^p)^n \left(a^2 (n^2 p^2 + 3np + 2) \cos^2(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{np}{2}; \frac{np}{2}\right) \right)}{fnp \sqrt{\sin^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]

[Out] (Csc[e + f*x]*(a^2*(2 + 3*n*p + n^2*p^2)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(b*(1 + n*p)*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2] + 2*a*(2 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]))*Sec[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2]/(f*n*p*(1 + n*p))*(2 + n*p))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \sec(fx + e)^2 + 2ab \sec(fx + e) + a^2\right)\left((d \sec(fx + e))^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*((d*sec(f*x + e))^p*c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a)^2 \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec(fx + e) \right)^p \right)^n \left(a + b \sec(fx + e) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec(fx + e) + a \right)^2 \left(\left(d \sec(fx + e) \right)^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c \left(\frac{d}{\cos(e + fx)} \right)^p \right)^n \left(a + \frac{b}{\cos(e + fx)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2,x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \left(d \sec(e + fx) \right)^p \right)^n \left(a + b \sec(e + fx) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**2,x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**2, x)

3.239 $\int \left(c(d \sec(e + fx))^p \right)^n (a + b \sec(e + fx)) dx$

Optimal. Leaf size=156

$$\frac{b \sin(e + fx) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p \right)^n - a \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(2 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p \right)^n}{fnp \sqrt{\sin^2(e + fx)}} \quad f(1 - \cos^2(e + fx))$$

[Out] b*hypergeom([1/2, -1/2*n*p], [-1/2*n*p+1], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2], [-1/2*n*p+3/2], cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n*p+1)/(sin(f*x+e)^2)^(1/2)

Rubi [A] time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3948, 3787, 3772, 2643}

$$\frac{b \sin(e + fx) {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p \right)^n - a \sin(e + fx) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(2 - np); \cos^2(e + fx)\right) \left(c(d \sec(e + fx))^p \right)^n}{fnp \sqrt{\sin^2(e + fx)}} \quad f(1 - \cos^2(e + fx))$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]), x]

[Out] (b*Hypergeometric2F1[1/2, -(n*p)/2, (2 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x]/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3772

Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3948

Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int (c(d \sec(e + fx))^p)^n (a + b \sec(e + fx)) dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} (a + b \sec(e + fx)) dx \\
&= \left(a (d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int (d \sec(e + fx))^{np} dx \\
&= \left(a \left(\frac{\cos(e + fx)}{d} \right)^{np} (c(d \sec(e + fx))^p)^n \right) \int \left(\frac{\cos(e + fx)}{d} \right)^{-np} dx \\
&= \frac{b {}_2F_1\left(\frac{1}{2}, -\frac{np}{2}; \frac{1}{2}(2 - np); \cos^2(e + fx)\right) (c(d \sec(e + fx))^p)^n \sin(e + fx)}{fnp \sqrt{\sin^2(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 125, normalized size = 0.80

$$\frac{\sqrt{-\tan^2(e + fx)} \csc(e + fx) (c(d \sec(e + fx))^p)^n \left(a(np + 1) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{np}{2}; \frac{np}{2} + 1; \sec^2(e + fx)\right) + bnp {}_2F_1\left(\frac{1}{2}, \frac{np}{2}; \frac{np}{2} + 1; \sec^2(e + fx)\right) \right)}{fnp(np + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]

[Out] (Csc[e + f*x]*(a*(1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n*p*(1 + n*p))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \sec(fx + e) + a\right) \left(\left(d \sec(fx + e)\right)^p c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="fricas")

[Out] integral((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a) \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int \left(c (d \sec(fx + e))^p \right)^n (a + b \sec(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)

[Out] int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(fx + e) + a) \left((d \sec(fx + e))^p c \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c \left(\frac{d}{\cos(e + fx)} \right)^p \right)^n \left(a + \frac{b}{\cos(e + fx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)),x)

[Out] int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c (d \sec(e + fx))^p \right)^n (a + b \sec(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e)),x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x)), x)

$$3.240 \quad \int \frac{(c(d \sec(e+fx))^p)^n}{a+b \sec(e+fx)} dx$$

Optimal. Leaf size=206

$$\frac{a \sin(e+fx) \cos(e+fx) \cos^2(e+fx)^{\frac{1}{2}(np-1)} (c(d \sec(e+fx))^p)^n F_1\left(\frac{1}{2}; \frac{1}{2}(np-1), 1; \frac{3}{2}; \sin^2(e+fx), \frac{a^2 \sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)}$$

[Out] -b*AppellF1(1/2,1/2*n*p,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(cos(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/f+a*AppellF1(1/2,1/2*n*p-1/2,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/f

Rubi [A] time = 0.40, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3948, 3869, 2823, 3189, 429}

$$\frac{a \sin(e+fx) \cos(e+fx) \cos^2(e+fx)^{\frac{1}{2}(np-1)} (c(d \sec(e+fx))^p)^n F_1\left(\frac{1}{2}; \frac{1}{2}(np-1), 1; \frac{3}{2}; \sin^2(e+fx), \frac{a^2 \sin^2(e+fx)}{a^2-b^2}\right)}{f(a^2-b^2)}$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]

[Out] -((b*AppellF1[1/2, (n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^((n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)*f) + (a*AppellF1[1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n*p)/2)*(c*(d*Sec[e + f*x])^p)^n*Sin[e + f*x])/((a^2 - b^2)*f)

Rule 429

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2823

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]

Rule 3189

Int[((d_)*sin[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])]/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rule 3869

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +

$a*\text{Sin}[e + f*x]^m/\text{Sin}[e + f*x]^{(m + n)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 3948

$\text{Int}[\left((c_{.}) * ((d_{.}) * \text{sec}[e_{.}] + (f_{.}) * (x_{.}))\right)^{(p_{.})}]^{(n_{.})} * ((a_{.}) + (b_{.}) * \text{sec}[e_{.}] + (f_{.}) * (x_{.}))^{(m_{.})}, x_Symbol] \ :> \ \text{Dist}[(c^{\text{IntPart}[n]} * (c * (d * \text{Sec}[e + f * x])^p)^{\text{FracPart}[n]}) / (d * \text{Sec}[e + f * x])^{(p * \text{FracPart}[n])}], \text{Int}[(a + b * \text{Sec}[e + f * x])^m * (d * \text{Sec}[e + f * x])^{(n * p)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \ \&\& \ !\text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{(c(d \sec(e + fx))^p)^n}{a + b \sec(e + fx)} dx &= \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int \frac{(d \sec(e + fx))^{np}}{a + b \sec(e + fx)} dx \\ &= \left(\cos^{np}(e + fx) (c(d \sec(e + fx))^p)^n \right) \int \frac{\cos^{1-np}(e + fx)}{b + a \cos(e + fx)} dx \\ &= - \left(\left(a \cos^{np}(e + fx) (c(d \sec(e + fx))^p)^n \right) \int \frac{\cos^{2-np}(e + fx)}{b^2 - a^2 \cos^2(e + fx)} dx \right) + \left(b \cos^{np}(e + fx) \right) \\ &= \frac{\left(b \cos^2(e + fx)^{\frac{np}{2}} (c(d \sec(e + fx))^p)^n \right) \text{Subst} \left(\int \frac{(1-x^2)^{-\frac{np}{2}}}{-a^2 + b^2 + a^2 x^2} dx, x, \sin(e + fx) \right)}{f} \\ &= - \frac{b F_1 \left(\frac{1}{2}; \frac{np}{2}, 1; \frac{3}{2}; \sin^2(e + fx), \frac{a^2 \sin^2(e + fx)}{a^2 - b^2} \right) \cos^2(e + fx)^{\frac{np}{2}} (c(d \sec(e + fx))^p)^n \sin(e + fx)}{(a^2 - b^2) f} \end{aligned}$$

Mathematica [B] time = 25.63, size = 5411, normalized size = 26.27

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]

[Out] Result too large to show

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((d \sec(fx + e))^p c \right)^n}{b \sec(fx + e) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d \sec(fx + e))^p c \right)^n}{b \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\left(c(d \sec(fx + e))^p\right)^n}{a + b \sec(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)

[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d \sec(fx + e))^p c\right)^n}{b \sec(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(\frac{d}{\cos(e+fx)}\right)^p\right)^n}{a + \frac{b}{\cos(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)),x)

[Out] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(d \sec(e + fx))^p\right)^n}{a + b \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e)),x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x)), x)

$$3.241 \quad \int \frac{(c(d \sec(e+fx))^p)^n}{(a+b \sec(e+fx))^2} dx$$

Optimal. Leaf size=322

$$\frac{2ab \sin(e+fx) \cos^2(e+fx)^{\frac{np}{2}} (c(d \sec(e+fx))^p)^n F_1\left(\frac{1}{2}; \frac{1}{2}(np-2), 2; \frac{3}{2}; \sin^2(e+fx), \frac{a^2 \sin^2(e+fx)}{a^2-b^2}\right) a^2 \sin(e+fx)}{f(a^2-b^2)^2} + \dots$$

[Out] $-2*a*b*AppellF1(1/2, 1/2*n*p-1, 2, 3/2, \sin(f*x+e)^2, a^2*\sin(f*x+e)^2/(a^2-b^2)) * (\cos(f*x+e)^2)^{(1/2*n*p)} * (c*(d*\sec(f*x+e))^p)^n * \sin(f*x+e) / (a^2-b^2)^2 / f + a^2*AppellF1(1/2, 1/2*n*p-3/2, 2, 3/2, \sin(f*x+e)^2, a^2*\sin(f*x+e)^2/(a^2-b^2)) * \cos(f*x+e) * (\cos(f*x+e)^2)^{(1/2*n*p-1/2)} * (c*(d*\sec(f*x+e))^p)^n * \sin(f*x+e) / (a^2-b^2)^2 / f + b^2*AppellF1(1/2, 1/2*n*p-1/2, 2, 3/2, \sin(f*x+e)^2, a^2*\sin(f*x+e)^2/(a^2-b^2)) * \cos(f*x+e) * (\cos(f*x+e)^2)^{(1/2*n*p-1/2)} * (c*(d*\sec(f*x+e))^p)^n * \sin(f*x+e) / (a^2-b^2)^2 / f$

Rubi [A] time = 0.56, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3948, 3869, 2824, 3189, 429}

$$\frac{2ab \sin(e+fx) \cos^2(e+fx)^{\frac{np}{2}} (c(d \sec(e+fx))^p)^n F_1\left(\frac{1}{2}; \frac{1}{2}(np-2), 2; \frac{3}{2}; \sin^2(e+fx), \frac{a^2 \sin^2(e+fx)}{a^2-b^2}\right) a^2 \sin(e+fx)}{f(a^2-b^2)^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]

[Out] $(-2*a*b*AppellF1[1/2, (-2 + n*p)/2, 2, 3/2, \sin[e + f*x]^2, (a^2*\sin[e + f*x]^2)/(a^2 - b^2)] * (\cos[e + f*x]^2)^{(n*p)/2} * (c*(d*\sec[e + f*x])^p)^n * \sin[e + f*x] / ((a^2 - b^2)^2 * f) + (a^2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, \sin[e + f*x]^2, (a^2*\sin[e + f*x]^2)/(a^2 - b^2)] * \cos[e + f*x] * (\cos[e + f*x]^2)^{((-1 + n*p)/2)} * (c*(d*\sec[e + f*x])^p)^n * \sin[e + f*x] / ((a^2 - b^2)^2 * f) + (b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, \sin[e + f*x]^2, (a^2*\sin[e + f*x]^2)/(a^2 - b^2)] * \cos[e + f*x] * (\cos[e + f*x]^2)^{((-1 + n*p)/2)} * (c*(d*\sec[e + f*x])^p)^n * \sin[e + f*x] / ((a^2 - b^2)^2 * f)$

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2824

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]

Rule 3189

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sine[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d,

e, f, m, p}, x] && !IntegerQ[m]

Rule 3869

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b + a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]

Rule 3948

Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_.)])^(p_.))^n*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Dist[(c^IntPart[n]*(c*(d*Sec[e + f*x])^p)^FracPart[n])/(d*Sec[e + f*x])^(p*FracPart[n]), Int[(a + b*Sec[e + f*x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[n]

Rubi steps

$$\int \frac{(c(d \sec(e + fx))^p)^n}{(a + b \sec(e + fx))^2} dx = \left((d \sec(e + fx))^{-np} (c(d \sec(e + fx))^p)^n \right) \int \frac{(d \sec(e + fx))^{np}}{(a + b \sec(e + fx))^2} dx$$

$$= \left(\cos^{np}(e + fx) (c(d \sec(e + fx))^p)^n \right) \int \frac{\cos^{2-np}(e + fx)}{(b + a \cos(e + fx))^2} dx$$

$$= \left(\cos^{np}(e + fx) (c(d \sec(e + fx))^p)^n \right) \int \left(\frac{b^2 \cos^{2-np}(e + fx)}{(b^2 - a^2 \cos^2(e + fx))^2} - \frac{2ab \cos^{3-np}(e + fx)}{(b^2 - a^2 \cos^2(e + fx))} \right) dx$$

$$= \left(a^2 \cos^{np}(e + fx) (c(d \sec(e + fx))^p)^n \right) \int \frac{\cos^{4-np}(e + fx)}{(-b^2 + a^2 \cos^2(e + fx))^2} dx - \left(2ab \cos^{np}(e + fx) (c(d \sec(e + fx))^p)^n \right) \int \frac{\cos^{3-np}(e + fx)}{b^2 - a^2 \cos^2(e + fx)} dx$$

$$= - \frac{\left(2ab \cos^2(e + fx) \frac{np}{2} (c(d \sec(e + fx))^p)^n \right) \text{Subst} \left(\int \frac{(1-x^2)^{\frac{1}{2}(2-np)}}{(-a^2+b^2+a^2x^2)^2} dx, x, \sin(e + fx) \right)}{f}$$

$$= - \frac{2abF_1 \left(\frac{1}{2}; \frac{1}{2}(-2 + np), 2; \frac{3}{2}; \sin^2(e + fx), \frac{a^2 \sin^2(e+fx)}{a^2-b^2} \right) \cos^2(e + fx) \frac{np}{2} (c(d \sec(e + fx))^p)^n}{(a^2 - b^2)^2 f}$$

Mathematica [B] time = 45.37, size = 14108, normalized size = 43.81

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]

[Out] Result too large to show

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((d \sec(fx + e))^p c \right)^n}{b^2 \sec(fx + e)^2 + 2ab \sec(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")

[Out] integral(((d*sec(f*x + e))^p*c)^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d \sec(fx + e))^p c \right)^n}{(b \sec(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")

[Out] integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c (d \sec(fx + e))^p \right)^n}{(a + b \sec(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)

[Out] int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c \left(\frac{d}{\cos(e+fx)} \right)^p \right)^n}{\left(a + \frac{b}{\cos(e+fx)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2,x)

[Out] int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c (d \sec(e + fx))^p \right)^n}{(a + b \sec(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e))**2,x)

[Out] Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x))**2, x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;
```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```



```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,``^``)
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+``) or
    type(expn,``*``)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```

```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```